Gaussian Copulas for Large Spatial Fields

Modeling Data-Level Spatial Dependence in Multivariate Generalized Extreme Value Distributions

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Introduction

- ► UKCP Local Projections on a 5km grid over the UK (1980-2080)
- Challenge: Modeling maximum daily precipitation in yearly blocks
 - ▶ 43,920 spatial locations on a 180 x 244 grid
- Two aspects of spatial dependence:
 - 1. GEV parameters (ICAR models)
 - 2. Data-level dependence (Copulas)



Calculating Multivariate Normal Densities

Log Density Formula

$$\log f(\mathbf{x}) \propto \frac{1}{2} \left(\log |\mathbf{Q}| - \mathbf{x}^T \mathbf{Q} \mathbf{x} \right)$$

Key Components

- 1. Log Determinant: $\log |\mathbf{Q}|$
 - Constant for a given precision matrix
- 2. Quadratic Form: $\mathbf{x}^T \mathbf{Q} \mathbf{x}$
 - Needs calculation for each density evaluation

Computational Challenges

- ► Log determinant calculation
 - Time complexity: $O(n^3)$ for naive methods
 - Memory complexity: $O(n^2)$
- Quadratic form calculation
 - ightharpoonup Time complexity: $O(n^2)$
 - Critical for performance in large spatial fields

Spatial Model Considerations

- Some models (e.g., ICAR) avoid log determinant calculation
- Efficient computation crucial for large-scale applications