Technical Note

Multi-spectral continental shelf scale imaging reveals fish population and their 3D morphology

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Abstract: Number density of a fish group and its' 3-D distribution are estimated from 2-D horizontal maps by applying an extended Kalman filter to 2-D multi-frequency acoustic images. The inherent limitation in resolving the vertical fish distribution from 2-D images is overcome by utilizing a unique frequency dependence in acoustic scattering from fish. Air-filled swim bladder inside a fish is a strong acoustic scatterer, where its' frequency response resembles that of a damped-harmonic oscillator. This frequency response is sensitive and nonlinearly related to fish occupancy depth, which enables estimation of their vertical distribution using multi-frequency images.

1. Materials and Methods

1.1. Extendend Kalman Filter for Number Density and Depth Distribution Estimation of Dense Fish Groups

An Extended Kalman Filter (EKF) is designed to estimate the number density and depth distribution of a dense fish group. When a fish group is sufficiently dense and the acoustic transmission frequency is near the resonance frequency of fish swim bladder scattering, acoustic wave significantly attenuates as it further propagates through the dense fish group. In such cases, it is desirable to compensate for the accumulated attenuation since the total population can be under-estimated. This attenuation is determined by a number of factors including fish depth distribution, amount of air inside their swim bladder, number density, and acoustic transmission frequency. The presented EKF accounts for acoustic attenuation caused by transmission through a dense fish group and simultaneously estimates their number density and physical parameters that describe the vertical distribution.

State of a fish group is expressed as an unknown state vector that consists of 7 parameters as

$$\mathbf{x} = \left[z_1, H_1, z_2, H_2, r, z_{\text{nb}}, n_{\text{AdB}}\right]^T, \tag{1}$$

where, z_1 and z_2 are respectively the mean occupancy depths of a fish group that has two vertical layers. H_1 and H_2 are respectively the vertical thickness of each fish layer and r represents the fractional population of the first fish layer with mean depth z_1 and vertical thickness H_1. z_{nb} is the neutral buoyancy depth, a parameter that determines the amount of air in a fish swim bladder, and n_{AdB} is the areal number density in decibels. At the leading edge of a fish group, which is closest to the acoustic source and receiver, the measurements are not attenuated. Within this region, raw Scattering Strength (SS) measurements can be used to infer and initialize the state vector. At consecutive range steps further into the fish group with respect to the acoustic source and receiver, the state vector transitions in accordance with attenuated measurements. The state vector at each range step can be estimated using an EKF, where the state transition and measurement models are given as

$$\begin{aligned} \mathbf{x}_k &= \mathbf{x}_{k-1} + \mathbf{w}_k \\ \mathbf{z}_k &= \mathbf{h}\left(\mathbf{x}_k\right) + \mathbf{v}_k. \end{aligned} \tag{2}$$

$\mathbf{x}_k$ and $\mathbf{z}_k$ are respectively the state and measurement at the k^{th} discrete step. $\mathbf{w}_k$ and $\mathbf{v}_k$ are respectively the process noise and measurement noise that are assumed to follow zero-mean multivariate normal distributions with respective covariance matrices $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$. $\mathbf{h}\left(\mathbf{x}_k\right)$ is a nonlinear measurement model as a function of the state vector $\mathbf{x}_k$ given as

$$\mathbf{h}\left(\mathbf{x}_k\right) = \left[h\left(\mathbf{x}_k; f_1\right) \quad h\left(\mathbf{x}_k; f_2\right) \quad \ldots \quad h\left(\mathbf{x}_k; f_{N_f}\right)\right]^T, \tag{3}$$

where $h\left(\mathbf{x}_k; f_i\right)$ is the measurement model at the i^{th} acoustic transmission frequency, f_i, among N_f discrete frequencies. $h\left(\mathbf{x}_k; f_i\right)$ is expressed as

$$h\left(\mathbf{x}_k; f_i\right) = \overline{\text{TS}}\left(z_1^{(k)}, H_1^{(k)}, z_2^{(k)}, H_2^{(k)}, r^{(k)}, z_{\text{nb}}^{(k)}; f_i\right) + n_{\text{AdB}}^{(k)}, \tag{4}$$

where $\overline{\text{TS}}\left(z_1^{(k)}, H_1^{(k)}, z_2^{(k)}, H_2^{(k)}, r^{(k)}, z_{\text{nb}}^{(k)}; f_i\right)$ is a modeled mean Target Strength (TS) of a swim bladder bearing fish [1] at current fish state $\mathbf{x}_k$ and at frequency f_i. Since the measurement model, $h\left(\mathbf{x}_k; f_i\right)$, does not account for attenuation caused by the fish group, $\mathbf{z}_k$ is the attenuation-corrected measurement. Attenuation in measurements is corrected by accumulating the range-dependent attenuation factor determined by the corresponding state vector at each range step.

Prediction step of the current EKF is given as

$$\begin{aligned} \hat{\mathbf{x}}_{k|k-1} &= \hat{\mathbf{x}}_{k-1|k-1} \\ \boldsymbol{P}_{k|k-1} &= \boldsymbol{P}_{k-1|k-1} + \boldsymbol{Q}_k, \end{aligned} \tag{5}$$

where $\hat{\mathbf{x}}_{k|k-1}$ and $\boldsymbol{P}_{k|k-1}$ are the state estimate and error covariance matrix at range step k given the measurements up to range step $k-1$.

The state estimate and error covariance matrix are updated by

$$\begin{aligned} \tilde{\mathbf{y}}_k &= \mathbf{z}_k - \hat{\mathbf{x}}_{k|k-1} \\ \boldsymbol{S}_k &= \boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T + \boldsymbol{R}_k \\ \boldsymbol{K}_k &= \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T \boldsymbol{S}_k^{-1} \\ \hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \boldsymbol{K}_k \tilde{\mathbf{y}}_k \\ \boldsymbol{P}_{k|k} &= \left(\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k\right) \boldsymbol{P}_{k|k-1}, \end{aligned} \tag{6}$$

where $\tilde{\mathbf{y}}_k$ is the measurement residual, $\boldsymbol{S}_k$ is the covariance of the residual and $\boldsymbol{K}_k$ is the Kalman gain. $\hat{\mathbf{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$ are the updated state estimate and estimate covariance. $\boldsymbol{H}_k$ is a Jacobian matrix of the measurement model evalated at the predicted state $\hat{\mathbf{x}}_{k|k-1}$ given as

$$\boldsymbol{H}_k = \begin{bmatrix} \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial z_1} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial H_1} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial z_2} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial H_2} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial r} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial z_{\text{nb}}} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial z_1} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial H_1} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial z_2} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial H_2} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial r} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial z_{\text{nb}}} & 1 \end{bmatrix}. \tag{7}$$

This EKF for a single measurement at each range step, however, can be easily extended for multiple measurements by concatenating multiple measurements and measurement models.

2. Results

2.1. State Estimation using the Extended Kalman Filter

Here, the EKF is used to estimate the state of a fish group when attenuation is present in measurements. Synthetic SS measurements from a fish group of 5 km horizontal thickness with two vertical layers (Fig. 1) are used to validate the current EKF.

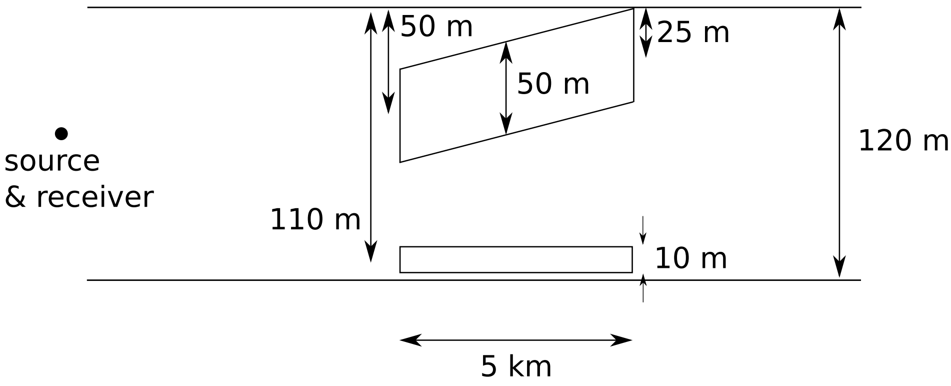

Figure 1. Range-depth configuration of the fish group

Neutral buoyancy of the fish group is assumed to be 10 m below the sea surface and each vertical layer includes equal fish population. Areal number density along the range within the fish group is shown in Fig. 2. The maximum areal number density is approximately 4.5 fish/m^2 at 2.5 km range.

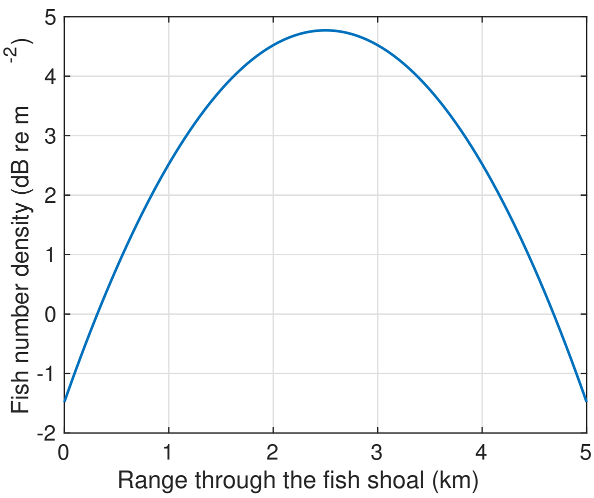

Figure 2. Synthetic areal number density through the fish group

Synthetic SS from this fish group is shown in Figs. 3 and 4, where Fig. 3 includes a zero-mean Gaussian white noise with 3 dB standard deviation and Fig. 4 shows the SS without any noise. SS at 1465 Hz and 1600 Hz are strong at close ranges but also shows the most attenuation as shown in Fig. 4 because the resonance scattering frequency is near 1500 Hz as shown in Fig. 5.

EKF is applied to this synthetic SS data where 10 measurements at each range step are used to predict and update the state vector. The measurement noise covariance matrix $\boldsymbol{Q}_k$ is a diagonal matrix since each measurement is assumed uncorrelated and noise at each frequency is assumed uncorrelated. Small random perturbation is included in $\boldsymbol{Q}_k$ to resolve a singularity issue caused when multiple measurements following an identical distribution is used. The process noise covariance matrix $\boldsymbol{R}_k$ is also a diagonal matrix since each state parameter is assumed uncorrelated with each other.

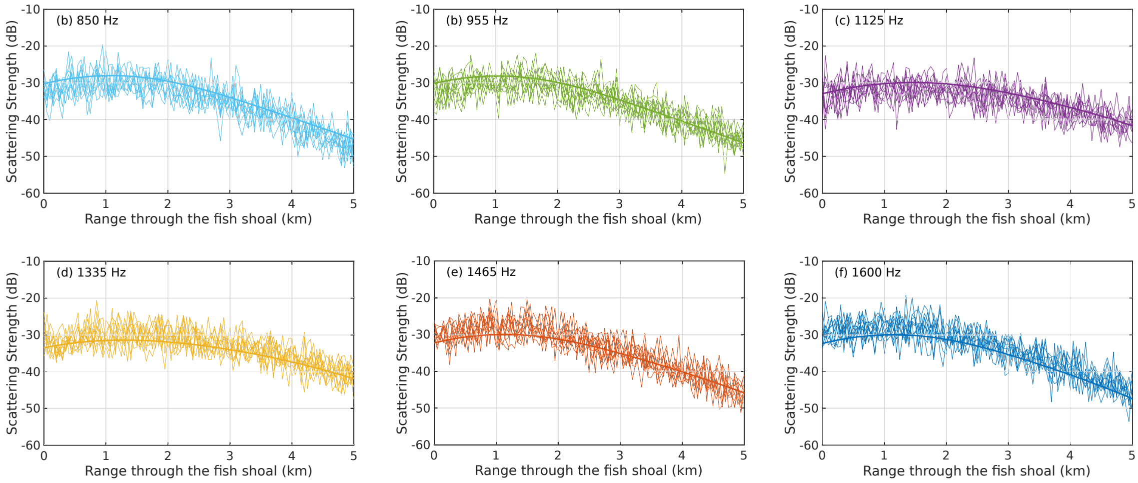

Figure 3. Synthetic Scattering Strength (SS) through the fish group with zero-mean Gaussian white noise with 3 dB standard deviation. 10 SS curves are randomly realized at each frequency.

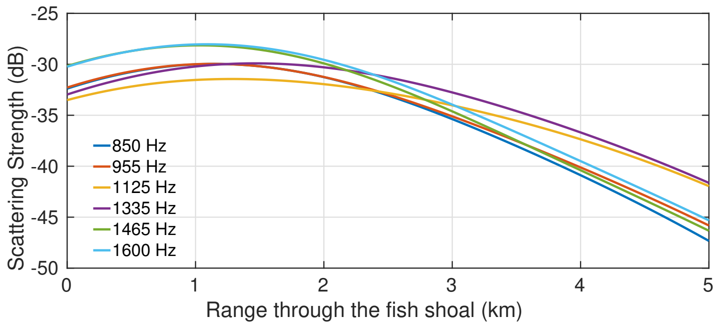

Figure 4. Synthetic Scattering Strength (SS) through the fish group without noise

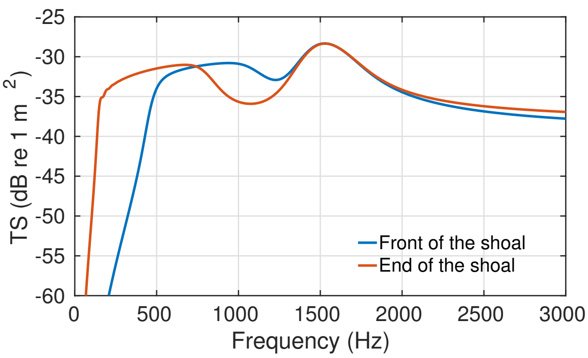

Figure 5. Synthetic Target Strength (TS) at the front and end of the group

For a synthetic data set without any noise, Figs. 6 and 7 show the state estimate using the developed EKF. In general, the estimation is better at closer range steps and deviates from the ground truth as the range steps increases. EKF estimation of the fish vertical distribution is shown in Fig. 6. Mean occupancy depth estimation errors of the upper and lower fish layers (Fig. 6 (a) and (c)) are within 3 % with respect to their ground truth vertical thicknesses. Vertical thickness estimation error of the upper and lower fish layers (Fig. 6 (b) and (d)) are within 20 % with respect to their ground truth vertical thicknesses. EKF estimation of the fractional population of the upper fish layer, neutral buouancy depth and areal number density are accurate to be less than 1% error with respect to their ground truth values.

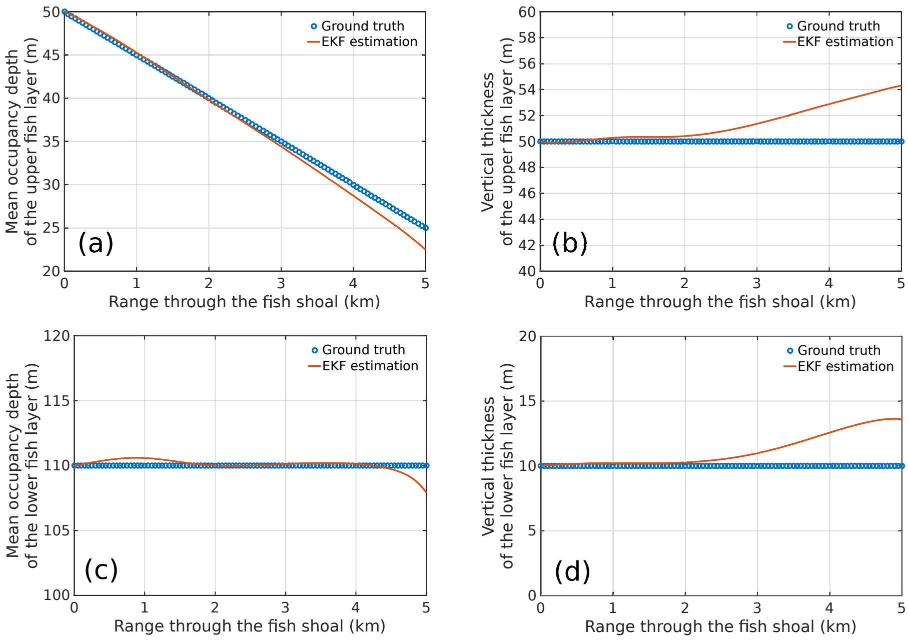

Figure 6. Extended Kalman Filter estimation of the fish depth distribution without any noise in the measurements.

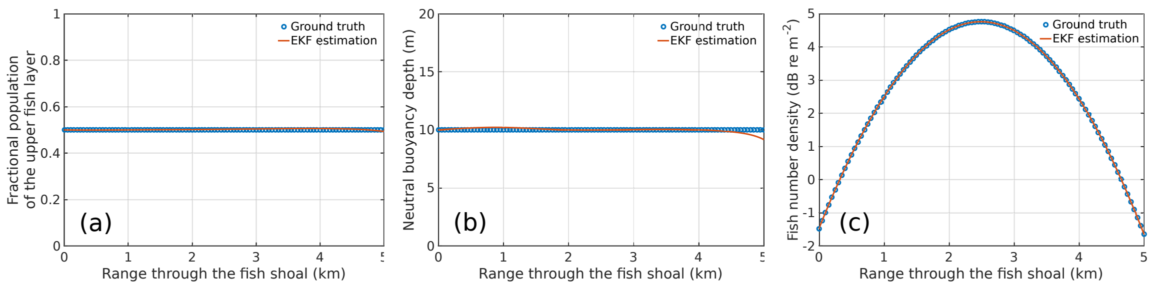

Figure 7. Extended Kalman Filter estimation of (a) the fractional population of the upper fish layer, (b) neutral buoyancy depth and (c) areal number density without any noise in the measurements.

Similarly, EKF estimation results for synthetic measurements with noise are shown in Figs. 8 and 9. Mean occupancy depth estimation errors of the upper and lower fish layers (Fig. 8 (a) and (c)) are within 15 % with respect to their ground truth vertical thicknesses. Vertical thickness estimation errors of the upper and lower fish layers (Fig. 6 (b) and (d)) are within 25 % with respect to their ground truth vertical thicknesses. Estimation of the fractional population of the upper fish layer, neutral buouancy depth and areal number density are accurate to be less than 10% error with respect to their ground truth values.

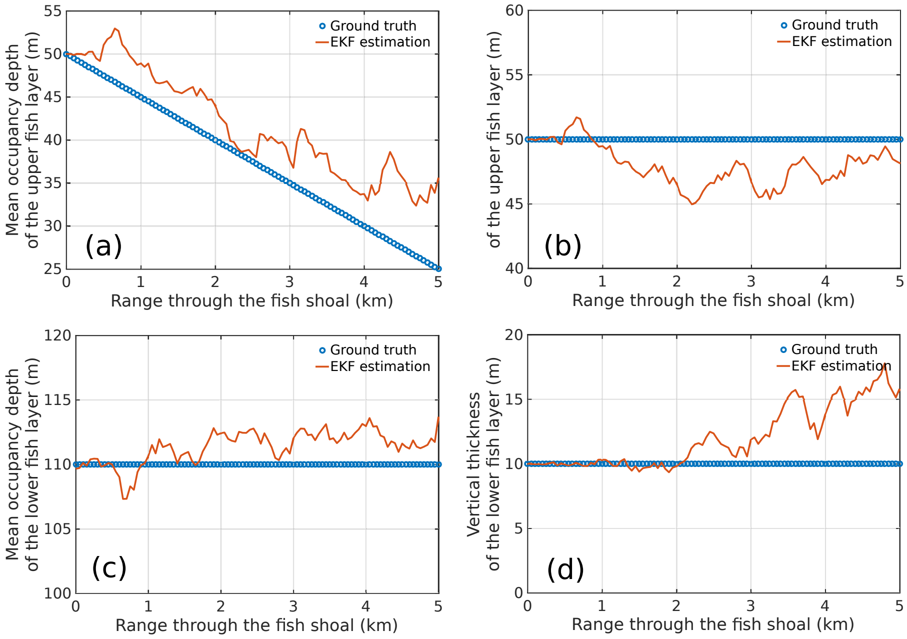

Figure 8. Extended Kalman Filter estimation of the fish depth distribution in the presence of zero-mean Gaussian white measurement noise with 3 dB standard deviation.

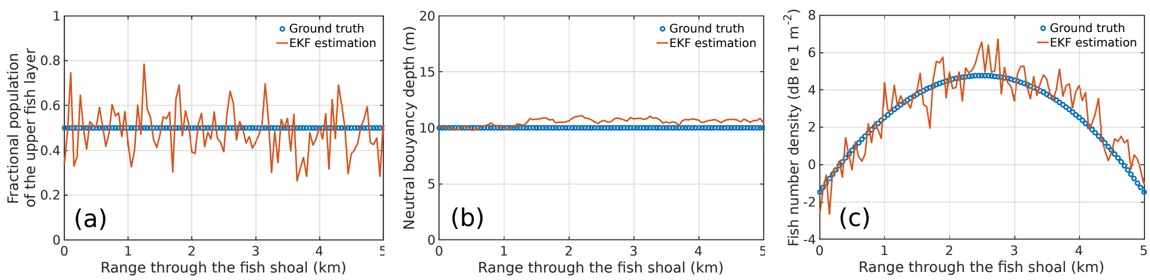

Figure 9. Extended Kalman Filter estimation of (a) the fractional population of the upper fish layer, (b) neutral buoyancy depth and (c) areal number density in the presence of zero-mean Gaussian white measurement noise with 3 dB standard deviation.

References

1. Love, R.H. Resonant acoustic scattering by swimbladder-bearing fish. *The Journal of the Acoustical Society of America* **1978**, *64*, 571–580.