Technical Note

Multi-spectral continental shelf scale imaging reveals fish population and their 3D morphology

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1. Materials and Methods

1.1. Extendend Kalman Filter for Accounting Acoustic Attenuation Caused by Dense Fish Shoals

An Extended Kalman Filter (EKF) is designed to account for acoustic attenuation caused by dense fish shoals. In cases where a fish shoal is sufficiently dense and the acoustic transmission frequency is near the resonance peak frequency of the fish swim bladder scattering, acoustic wave attenuates as it further propagates through this dense fish shoal. Fish total population can be under-estimated because of the attenuation. It is therefore desirable to compensate for the accumulated attenuation. The attenuation is caused by a number of factors including the fish depth distribution, the amount of air in their swim bladder, density and acoustic transmission frequency, State of a fish shoal is expressed as an unknown state vector that consists of 7 parameters about a shoal as

$$\mathbf{x} = \left[z_1, H_1, z_2, H_2, r, z_{\mathrm{nb}}, n_{\mathrm{AdB}}\right]^T, \tag{1}$$

where, z_1 and z_2 are respectively the mean shoal depths of a fish shoal that has two layers vertically. H_1 and H_2 are respectively the vertical thickness of each fish layer and r represents the fractional population of the first fish layer with mean depth z_1 and vertical thickness H_1. z_{nb} and $n_{\mathrm{A,dB}}$ are respectively the neutral buoyancy depth and areal number density of this fish shoal. For the region within the shoal that is closest to the acoustic source, the measurements are not attenuated. Within this shoal region, raw scattering strength measurements can be used to infer the and initialize the shoal state vector. At a consecutive range step that is further into the shoal with respect to the acoustic source, the fish shoal state continuously transitions with attenuated measurements. Shoal state at each range step can be estimated using an EKF where the shoal state transition and measurement models are

$$\begin{aligned} \mathbf{x}_k &= \mathbf{x}_{k-1} + \mathbf{w}_k \\ \mathbf{z}_k &= \mathbf{h}\left(\mathbf{x}_k\right) + \mathbf{v}_k. \end{aligned} \tag{2}$$

$\mathbf{x}_k$ and $\mathbf{z}_k$ are respectively the state and measurement at the k^{th} discrete step. $\mathbf{w}_k$ and $\mathbf{v}_k$ are respectively the process noise and measurement noise that are assumed to follow zero-mean multivariate normal distributions with respective covariance matrices $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$. $\mathbf{h}\left(\mathbf{x}_k\right)$ is a nonlinear measurement model as a function of the state vector $\mathbf{x}_k$ given as

$$\mathbf{h}\left(\mathbf{x}_k\right) = \left[h\left(\mathbf{x}_k; f_1\right) \quad h\left(\mathbf{x}_k; f_2\right) \quad \dots \quad h\left(\mathbf{x}_k; f_{N_f}\right)\right]^T, \tag{3}$$

where $h\left(\mathbf{x}_k; f_i\right)$ is the measurement model at the i^{th} transmission frequency among N_f discrete frequency bandwidths. $h\left(\mathbf{x}_k; f_i\right)$ is expressed as

$$h\left(\mathbf{x}_k; f_i\right) = \overline{\mathrm{TS}}\left(z_1, H_1, z_2, H_2, r, z_{\mathrm{nb}}\right) + n_{\mathrm{AdB}}, \tag{4}$$

where $\overline{\mathrm{TS}}\left(z_1, H_1, z_2, H_2, r, z_{\mathrm{nb}}\right)$ is the mean Target Strength (TS) of a swim bladder bearing fish using the Love model with the current state parameters $\mathbf{x}_k$ of the fish shoal. Note that the measurement model $h\left(\mathbf{x}_k; f_i\right)$ does not include attenuation, and accordingly the measurement $\mathbf{z}_k$ denotes the attenuation corrected measurement. This attenuation correction can be done based on the history of the estimated state vectors up to $k-1$ range step. Attenuation factor at each range step calculated using the state estimate at the corresponding range step is accumulated over the range up to the $k-1$ range. Raw measurement at k range step is then scaled up by this accumulated attenuation.

Prediction step of the current EKF is given as

$$\begin{aligned}
\hat{\mathbf{x}}_{k|k-1} &= \hat{\mathbf{x}}_{k-1|k-1} \\
\boldsymbol{P}_{k|k-1} &= \boldsymbol{P}_{k-1|k-1} + \boldsymbol{Q}_k,
\end{aligned} \tag{5}$$

where $\hat{\mathbf{x}}_{k|k-1}$ and $\boldsymbol{P}_{k|k-1}$ are the state estimate and error covariance matrix at range step k given the measurements up to range step $k-1$.

The state estimate and error covariance matrix are updated by

$$\begin{aligned}
\tilde{\mathbf{y}}_k &= \mathbf{z}_k - \hat{\mathbf{x}}_{k|k-1} \\
\boldsymbol{S}_k &= \boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T + \boldsymbol{R}_k \\
\boldsymbol{K}_k &= \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T \boldsymbol{S}_k^{-1} \\
\hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \boldsymbol{K}_k \tilde{\mathbf{y}}_k \\
\boldsymbol{P}_{k|k} &= \left(\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k\right) \boldsymbol{P}_{k|k-1},
\end{aligned} \tag{6}$$

where $\tilde{\mathbf{y}}_k$ is the measurement residual, $\boldsymbol{S}_k$ is the covariance of the residual and $\boldsymbol{K}_k$ is the Kalman gain. $\hat{\mathbf{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$ are the updated state estimate and estimate covariance. $\boldsymbol{H}_k$ is a Jacobian matrix of the measurement model evalated at the predicted state $\hat{\mathbf{x}}_{k|k-1}$ given as

$$\boldsymbol{H}_k = \begin{bmatrix}
\frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial z_1} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial H_1} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial z_2} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial H_2} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial r} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_1)}{\partial z_{\mathrm{nb}}} & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial z_1} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial H_1} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial z_2} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial H_2} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial r} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1}; f_{N_f})}{\partial z_{\mathrm{nb}}} & 1
\end{bmatrix}. \tag{7}$$

The above scheme is for a single measurement at each range step, however, this scheme can be easily extended for multiple measurements by concatenating multiple measurements and measurement models.

2. Results

2.1. Fish Shoal State Estimation Accounting for Attenuation using the Extended Kalman Filter

Here, the EKF is used to estimate fish shoal state when attenuation caused by the fish shoal is present in the measurements. First, a synthetic scattering strength is tested to validate the current EKF model. Scattering strength from a fish shoal of 5 km horizontal thickness with two vertical layers is modeled as shown below.

Neutral buoyancy of the fish shoal is assumed to be 10 m below the sea surface and each vertical layer includes equal fish population. Areal fish number density along the range within the fish shoal is

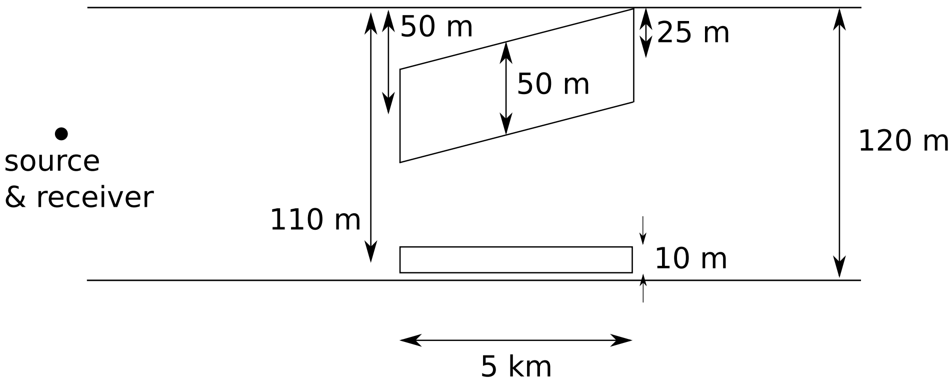

Figure 1. Range-depth configuration of the fish shoal

assumed as shown in Figure **??**. The maximum areal number density is approximately 3 fish/m^2 at 2.5 km range.

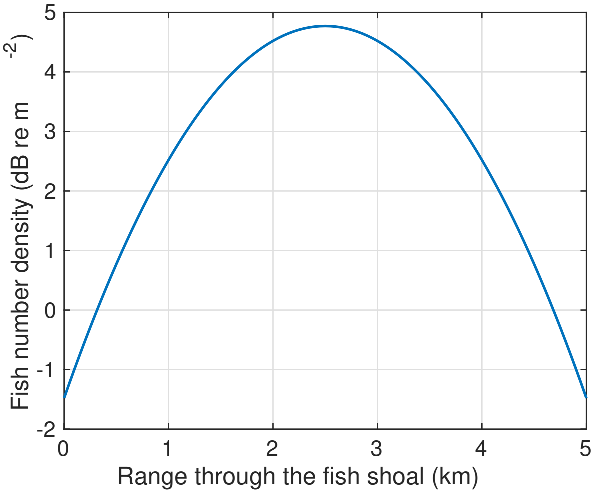

Figure 2. Synthetic areal number density through the fish shoal

Synthetic Scattering Strength (SS) from this fish shoal is shown in Figures **??** and **??**, where Figure **??** includes a zero-mean Gaussian white noise with 3 dB standard deviation and Figure **??** shows the SS without any noise.
SS at 850 and 1465 Hz are strongest at close ranges but also shows the most attenuation as shown in the Target Strength curve in Figure **??**.

EKF is applied to this synthetic SS data where 10 measurements at each range step are used to predict and update the fish shoal state. The measurement noise covariance matrix Q_k is a diagonal matrix since each measurement is assumed uncorrelated and noise at each frequency is assumed to be uncorrelated. Small random perturbation is included in Q_k to resolve a singularity issue caused when multiple measurements following an identical distribution is used. The process noise covariance matrix $\boldsymbol{R}_k$ is also a diagonal matrix since each state parameter is assumed uncorrelated to each other.

First, the EKF model shows the state estimate using data without any noise. The state estimate is shown in Figures **??** and **??**. In general the estimation is better at closer range steps and deviates from the ground truth as the range steps increases. Figure **??** shows the EKF estimation of the fish shoal vertical distribution. Mean occupancy depth estimation of the upper and lower fish layers shown in Figure **??** (a) and (c) respectively show errors from the ground truth mean occupancy depths to within 4 % and 20 % with respect to their ground truth vertical thicknesses. Shoal vertical thickness estimation of the upper and lower fish layers shown in Figure **??** (b) and (d) respectively show errors from the

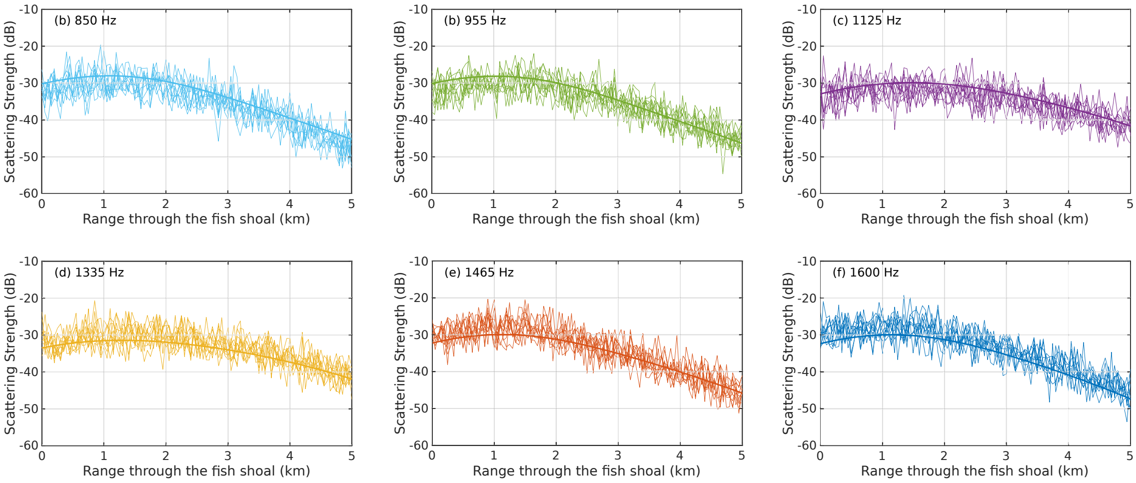

Figure 3. Synthetic Scattering Strength (SS) through the fish shoal with zero-mean Gaussian white noise with 3 dB standard deviation. 10 SS curves are realized at each frequency.

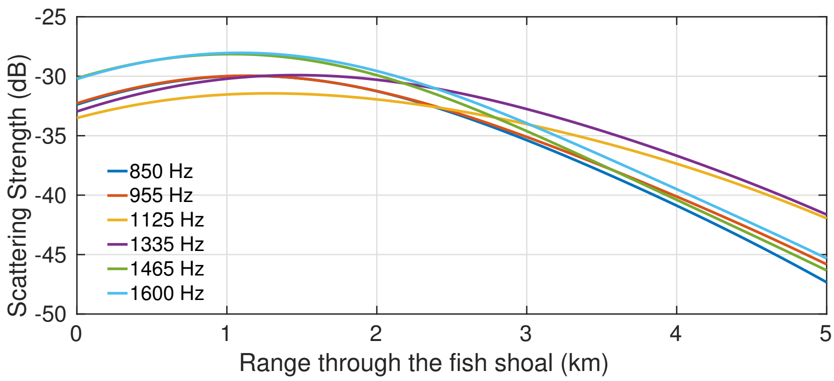

Figure 4. Synthetic Scattering Strength (SS) through the fish shoal without noise

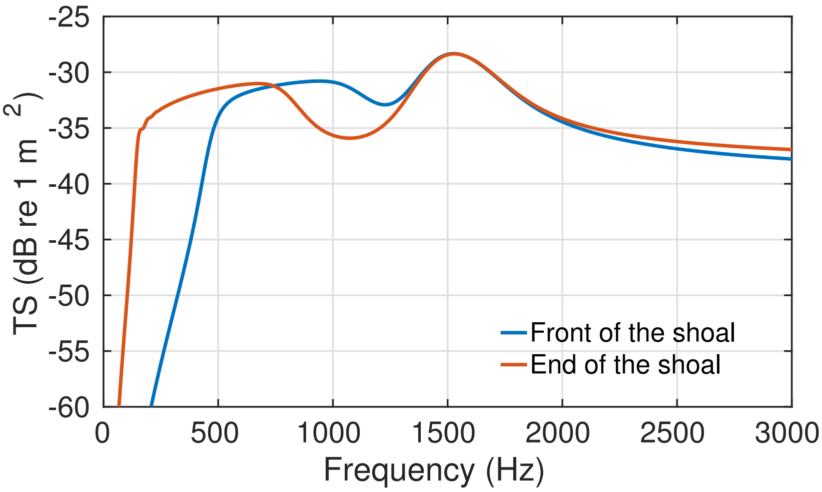

Figure 5. Synthetic Target Strength (TS) at the front and end of the shoal

ground truth vertical thicknesses to within 8 % and 30 % with respect to their ground truth vertical thicknesses. EKF estimation of the fractional population of the upper fish layer, neutral buouancy depth and areal number density are accurate to be within 1% error with respect to their ground truth values.

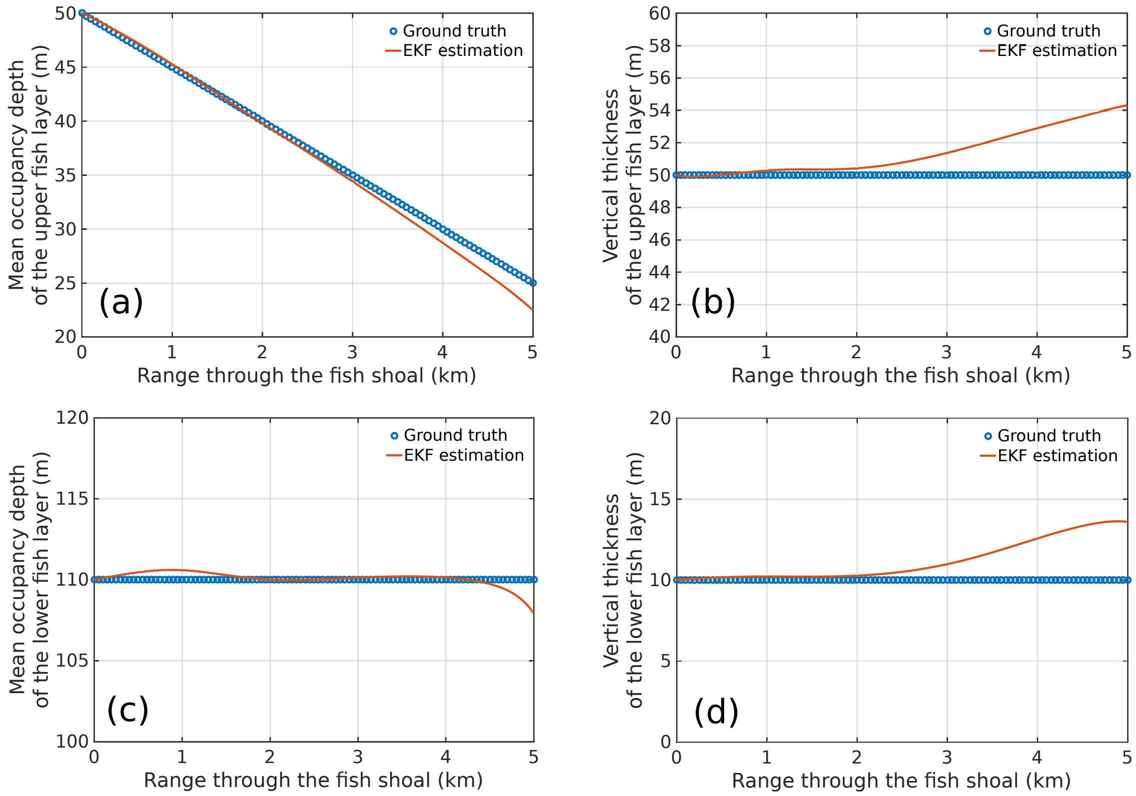

Figure 6. Extended Kalman Filter estimation of the fish shoal depth distribution without any noise in the measurements.

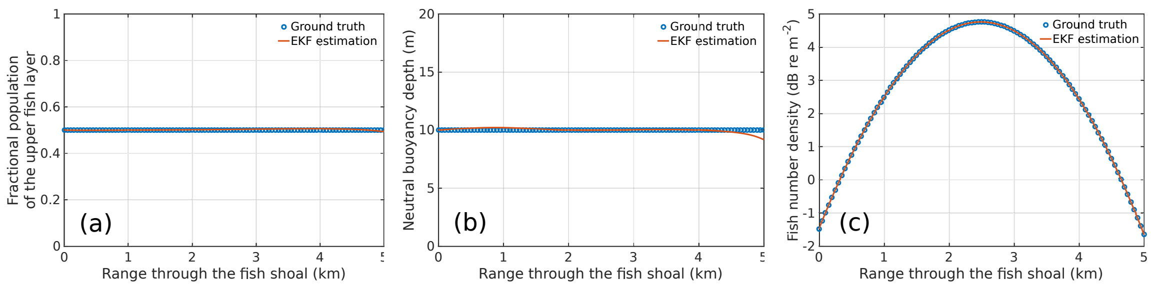

Figure 7. Extended Kalman Filter estimation of (a) the fractional population of the upper fish layer, (b) neutral buoyancy depth and (c) areal number density without any noise in the measurements.

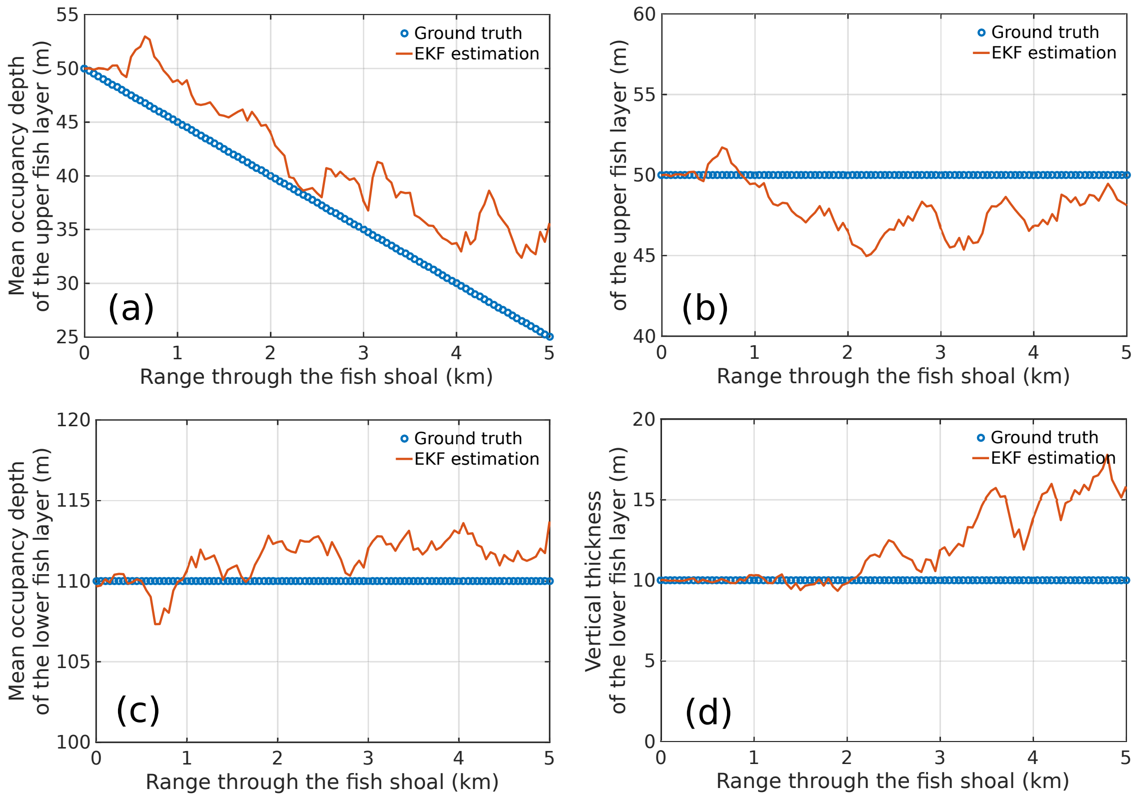

Figure 8. Extended Kalman Filter estimation of the fish shoal depth distribution in the presence of zero-mean Gaussian white measurement noise with 3 dB standard deviation.

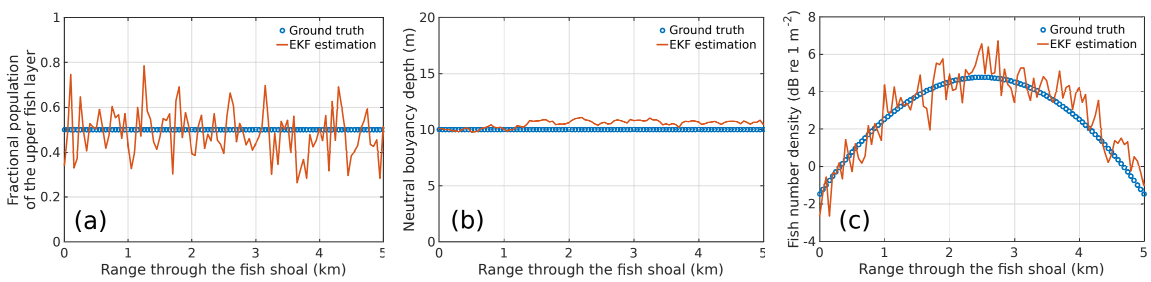

Figure 9. Extended Kalman Filter estimation of (a) the fractional population of the upper fish layer, (b) neutral buoyancy depth and (c) areal number density in the presence of zero-mean Gaussian white measurement noise with 3 dB standard deviation.

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