

Technical Note

## Multi-spectral continental shelf scale imaging reveals fish population and their 3D morphology

## Byunggu Cho

Department of Mechanical Engineering, MIT; bgcho@mit.edu

- Abstract: Number density of a fish group and its' 3-D distribution are estimated from 2-D horizontal
- maps by applying an extended Kalman filter to 2-D multi-frequency acoustic images. The inherent
- limitation in resolving the vertical fish distribution from 2-D images is overcome by utilizing a unique
- frequency dependence in acoustic scattering from fish. Air-filled swim bladder inside a fish is a strong
- acoustic scatterer, where its' frequency response resembles that of a damped-harmonic oscillator.
- 6 This frequency response is sensitive and nonlinearly related to fish occupancy depth, which enables
- estimation of their vertical distribution using multi-frequency images.

## 8 1. Materials and Methods

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1.1. Extendend Kalman Filter for Number Density and Depth Distribution Estimation of Dense Fish Groups

An Extended Kalman Filter (EKF) is designed to estimate the number density and depth distribution of a dense fish group. When a fish group is sufficiently dense and the acoustic transmission frequency is near the resonance frequency of fish swim bladder scattering, acoustic wave significantly attenuates as it further propagates through the dense fish shoal. This attenuation is caused by a number of factors including fish depth distribution, amount of air inside their swim bladder, number density, and acoustic transmission frequency. In such cases, it is desirable to compensate for the accumulated attenuation since the total population can be under-estimated. The presented EKF accounts for acoustic attenuation and simultaneously estimates number density and physical parameters that describe the vertical distribution of a fish group.

State of a fish shoal is expressed as an unknown state vector that consists of 7 parameters about a group as

$$\mathbf{x} = [z_1, H_1, z_2, H_2, r, z_{\text{nb}}, n_{\text{AdB}}]^T,$$
 (1)

where,  $z_1$  and  $z_2$  are respectively the mean shoal depths of a fish shoal that has two layers vertically.  $H_1$  and  $H_2$  are respectively the vertical thickness of each fish layer and r represents the fractional population of the first fish layer with mean depth  $z_1$  and vertical thickness  $H_1$ .  $z_{nb}$  and  $n_{A,dB}$  are respectively the neutral buoyancy depth and areal number density of this fish shoal. For the region within the shoal that is closest to the acoustic source, the measurements are not attenuated. Within this shoal region, raw scattering strength measurements can be used to infer the and initialize the shoal state vector. At a consecutive range step that is further into the shoal with respect to the acoustic source, the fish shoal state continuously transitions with attenuated measurements. Shoal state at each range step can be estimated using an EKF where the shoal state transition and measurement models are

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \mathbf{w}_k$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k.$$
(2)

 $\mathbf{x}_k$  and  $\mathbf{z}_k$  are respectively the state and measurement at the  $k^{\text{th}}$  discrete step.  $\mathbf{w}_k$  and  $\mathbf{v}_k$  are respectively the process noise and measurement noise that are assumed to follow zero-mean multivariate normal distributions with respective covariance matrices  $\mathbf{Q}_k$  and  $\mathbf{R}_k$ .  $\mathbf{h}(\mathbf{x}_k)$  is a nonlinear measurement model as a function of the state vector  $\mathbf{x}_k$  given as

$$\mathbf{h}\left(\mathbf{x}_{k}\right) = \begin{bmatrix} h\left(\mathbf{x}_{k}; f_{1}\right) & h\left(\mathbf{x}_{k}; f_{2}\right) & \dots & h\left(\mathbf{x}_{k}; f_{N_{f}}\right) \end{bmatrix}^{T},\tag{3}$$

where  $h(\mathbf{x}_k; f_i)$  is the measurement model at the  $i^{\text{th}}$  transmission frequency among  $N_f$  discrete frequency bandwidths.  $h(\mathbf{x}_k; f_i)$  is expressed as

$$h(\mathbf{x}_k; f_i) = \overline{\text{TS}}(z_1, H_1, z_2, H_2, r, z_{\text{nb}}) + n_{\text{AdB}},$$
 (4)

where  $\overline{\text{TS}}(z_1, H_1, z_2, H_2, r, z_{\text{nb}})$  is the mean Target Strength (TS) of a swim bladder bearing fish using the Love model with the current state parameters  $\mathbf{x}_k$  of the fish shoal. Note that the measurement model  $h(\mathbf{x}_k; f_i)$  does not include attenuation, and accordingly the measurement  $\mathbf{z}_k$  denotes the attenuation corrected measurement. This attenuation correction can be done based on the history of the estimated state vectors up to k-1 range step. Attenuation factor at each range step calculated using the state estimate at the corresponding range step is accumulated over the range up to the k-1 range. Raw measurement at k range step is then scaled up by this accumulated attenuation.

Prediction step of the current EKF is given as

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$$\hat{\mathbf{x}}_{k|k-1} = \hat{\mathbf{x}}_{k-1|k-1}$$

$$P_{k|k-1} = P_{k-1|k-1} + Q_k,$$
(5)

where  $\hat{\mathbf{x}}_{k|k-1}$  and  $P_{k|k-1}$  are the state estimate and error covariance matrix at range step k given the measurements up to range step k-1.

The state estimate and error covariance matrix are updated by

$$\tilde{\mathbf{y}}_{k} = \mathbf{z}_{k} - \hat{\mathbf{x}}_{k|k-1} 
S_{k} = H_{k} P_{k|k-1} H_{k}^{T} + R_{k} 
K_{k} = P_{k|k-1} H_{k}^{T} S_{k}^{-1} 
\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_{k} \tilde{\mathbf{y}}_{k} 
P_{k|k} = (\mathbf{I} - K_{k} H_{k}) P_{k|k-1},$$
(6)

where  $\tilde{\mathbf{y}}_k$  is the measurement residual,  $\mathbf{S}_k$  is the covariance of the residual and  $\mathbf{K}_k$  is the Kalman gain.  $\hat{\mathbf{x}}_{k|k}$  and  $\mathbf{P}_{k|k}$  are the updated state estimate and estimate covariance.  $\mathbf{H}_k$  is a Jacobian matrix of the measurement model evalated at the predicted state  $\hat{\mathbf{x}}_{k|k-1}$  given as

$$\boldsymbol{H}_{k} = \begin{bmatrix} \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{1})}{\partial z_{1}} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{1})}{\partial H_{1}} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{1})}{\partial z_{2}} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{1})}{\partial H_{2}} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{1})}{\partial r} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{1})}{\partial z_{nb}} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{N_{f}})}{\partial z_{1}} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{N_{f}})}{\partial H_{1}} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{N_{f}})}{\partial z_{2}} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{N_{f}})}{\partial H_{2}} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{N_{f}})}{\partial r} & \frac{\partial h(\hat{\mathbf{x}}_{k|k-1};f_{N_{f}})}{\partial z_{nb}} & 1 \end{bmatrix} . \tag{7}$$

The above scheme is for a single measurement at each range step, however, this scheme can be easily extended for multiple measurements by concatenating multiple measurements and measurement models.

## 2. Results

55 2.1. Fish Shoal State Estimation Accounting for Attenuation using the Extended Kalman Filter

Here, the EKF is used to estimate fish shoal state when attenuation caused by the fish shoal is present in the measurements. First, a synthetic scattering strength is tested to validate the current EKF model. Scattering strength from a fish shoal of 5 km horizontal thickness with two vertical layers is modeled as shown below.

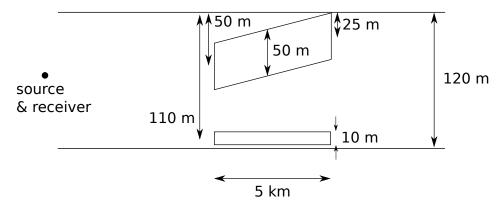


Figure 1. Range-depth configuration of the fish shoal

Neutral buoyancy of the fish shoal is assumed to be 10 m below the sea surface and each vertical layer includes equal fish population. Areal fish number density along the range within the fish shoal is assumed as shown in Figure 2. The maximum areal number density is approximately 3 fish/m<sup>2</sup> at 2.5 km range.

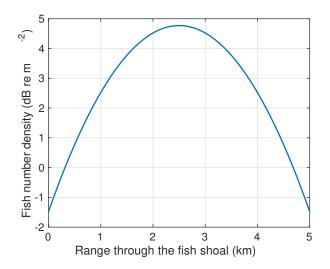
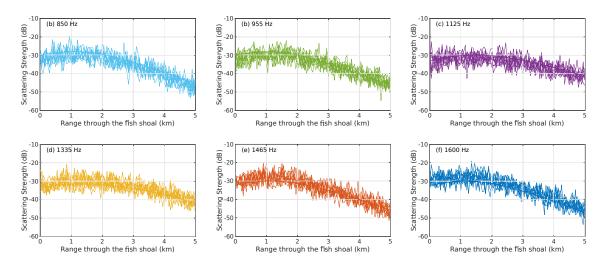


Figure 2. Synthetic areal number density through the fish shoal

Synthetic Scattering Strength (SS) from this fish shoal is shown in Figure 3 and 4, where Figure 3 includes a zero-mean Gaussian white noise with 3 dB standard deviation and Figure 4 shows the SS without any noise. SS at 1465 Hz and 1600 Hz are strong at close ranges but also shows the most attenuation as shown in the in Figure 4 because the resonance scattering frequency is near 1500 Hz as shown in 5.



**Figure 3.** Synthetic Scattering Strength (SS) through the fish shoal with zero-mean Gaussian white noise with 3 dB standard deviation. 10 SS curves are realized at each frequency.

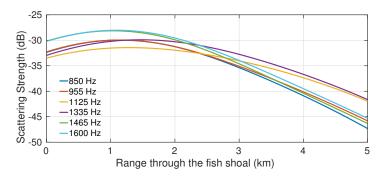


Figure 4. Synthetic Scattering Strength (SS) through the fish shoal without noise

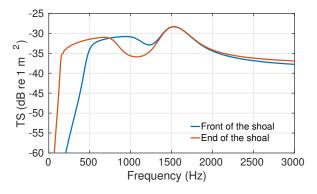
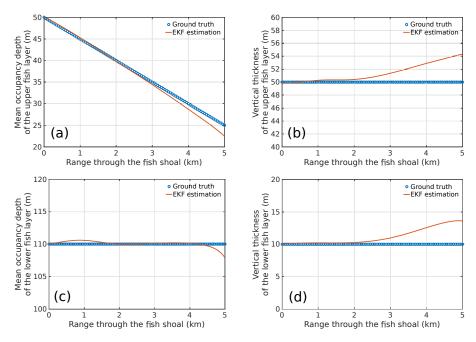


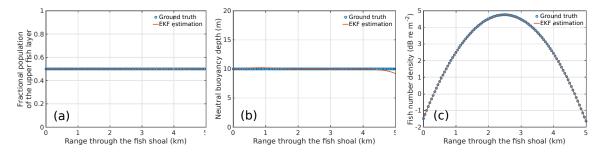
Figure 5. Synthetic Target Strength (TS) at the front and end of the shoal

EKF is applied to this synthetic SS data where 10 measurements at each range step are used to predict and update the fish shoal state. The measurement noise covariance matrix  $Q_k$  is a diagonal matrix since each measurement is assumed uncorrelated and noise at each frequency is assumed to be uncorrelated. Small random perturbation is included in  $Q_k$  to resolve a singularity issue caused when multiple measurements following an identical distribution is used. The process noise covariance matrix  $R_k$  is also a diagonal matrix since each state parameter is assumed uncorrelated to each other.

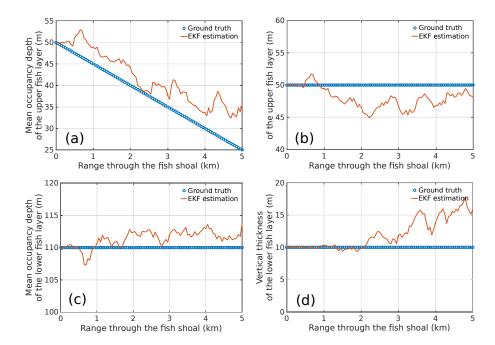
First, the EKF model shows the state estimate using data without any noise. The state estimate is shown in Figures 6 and 7. In general, the estimation is better at closer range steps and deviates from the ground truth as the range steps increases. Figure 6 shows the EKF estimation of the fish shoal vertical distribution. Mean occupancy depth estimation of the upper and lower fish layers shown in Figure 6 (a) and (c) respectively show errors from the ground truth mean occupancy depths to within 4 % and 20 % with respect to their ground truth vertical thicknesses. Shoal vertical thickness estimation of the upper and lower fish layers shown in Figure 6 (b) and (d) respectively show errors from the ground truth vertical thicknesses to within 8 % and 30 % with respect to their ground truth vertical thicknesses. EKF estimation of the fractional population of the upper fish layer, neutral buouancy depth and areal number density are accurate to be within 1% error with respect to their ground truth values.



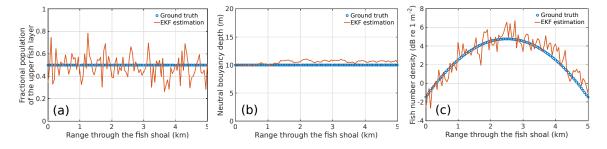
**Figure 6.** Extended Kalman Filter estimation of the fish shoal depth distribution without any noise in the measurements.



**Figure 7.** Extended Kalman Filter estimation of (a) the fractional population of the upper fish layer, (b) neutral buoyancy depth and (c) areal number density without any noise in the measurements.



**Figure 8.** Extended Kalman Filter estimation of the fish shoal depth distribution in the presence of zero-mean Gaussian white measurement noise with 3 dB standard deviation.



**Figure 9.** Extended Kalman Filter estimation of (a) the fractional population of the upper fish layer, (b) neutral buoyancy depth and (c) areal number density in the presence of zero-mean Gaussian white measurement noise with 3 dB standard deviation.