Exploring Spectral Analysis: PSET 5

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Abstract

This is the paper's abstract ...

1 Introduction

Introduction

Outline The remainder of this article is organized as follows. Section 2 introduces the implications of sampling and aliasing. Our new and exciting results are described in Section 4. Finally, Section 5 gives the conclusions.

2 Sampling and Recovery of Synthetic Signals

Nyquist sampling follows the theorem that for a given signal $f(\mathrm{x})$ composed of frequencies $\underline{\omega}$, $f(\mathrm{x})$ can be recovered using a sampling rate of $\frac{1}{\Delta \mathrm{x}} \geq 2 \times \omega_{\max}$.

3 Recovery of Unknown Signals

The recovery of unknown signals from measurements can be approached in a similar fashion, with the sampling rate significantly affecting the frequencies detected in the signal. For instance, Figure 2 shows a time-series measured annually, while Figure 3 shows a time-series of the same phenomena measured every 10 years. The general process of analysis for each time-series went as follows:

1. The data $\underline{\mathbf{d}}$ were centered about their mean μ, and normalized by their variance σ^2 such that the normalized data $\underline{\mathbf{d}}' = (\underline{\mathbf{d}} - \mu)/\sqrt{\sigma^2}$.
2. The power-spectral-density of the $\underline{\mathbf{d}}'$ signal frequencies $\underline{\omega}$ were estimated by Fourier transform.
3. Periodic structures in the signal were estimated by the element-wise inversion of period $\mathrm{L} = \omega^{-1}$.

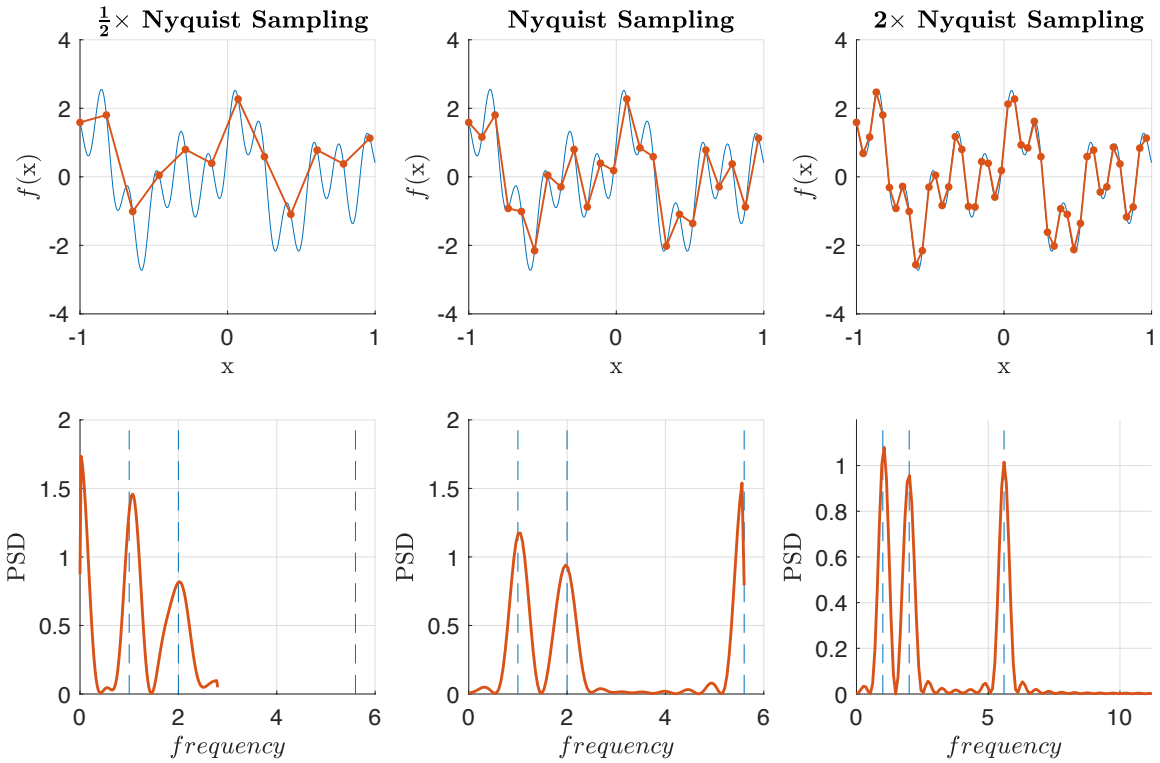

Figure 1: Sampling (in red) of the signal $f(\mathrm{x}) = \cos(2\pi \times \mathrm{x}) + \sin(2\pi \times 2\mathrm{x}) + \sin(2\pi \times 5.6\mathrm{x})$ (in blue) at various sampling rates, where "Nyquist Sampling" is defined as a sampling rate of $\frac{1}{\Delta \mathrm{x}} = 2 \times 5.6$.

4 Results

In this section we describe the results.

5 Conclusions

We worked hard, and achieved very little.

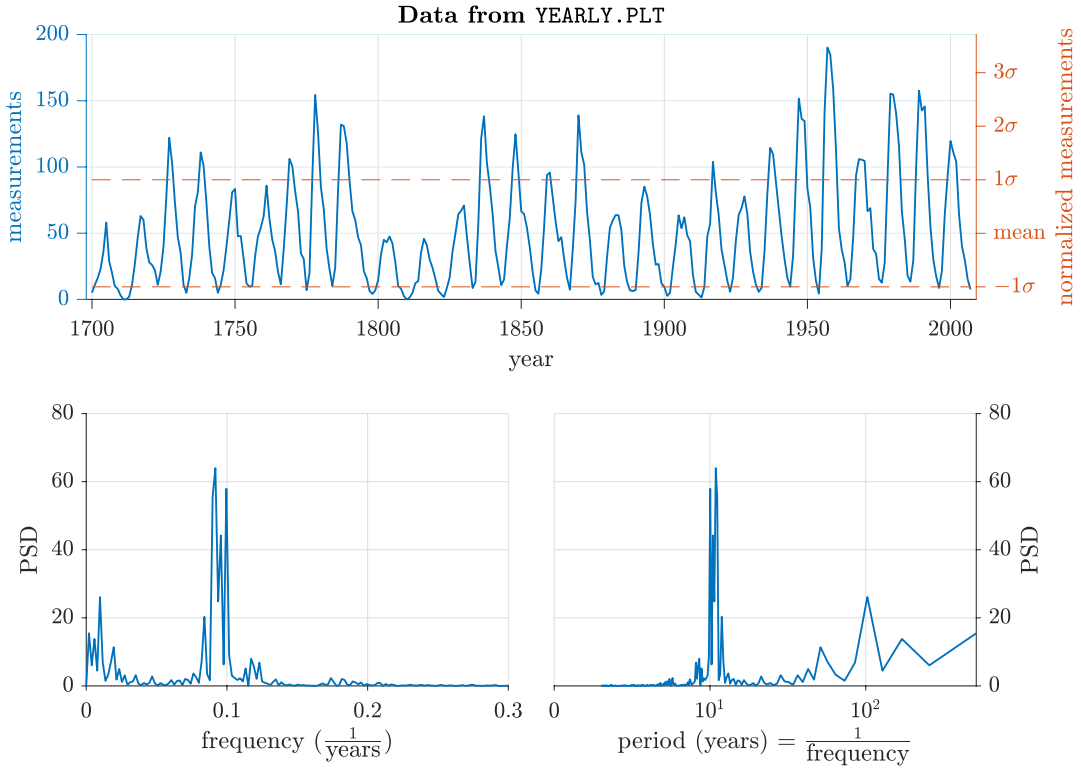

Figure 2: Analysis of time-series given in YEARLY.PLT. On top are the data $\underline{\mathbf{d}}$ with a normalized axis on the right showing the dimensions of $\underline{\mathbf{d}}'$ (see Section 3). On bottom left is the power-spectral-density (PSD) of the $\underline{\mathbf{d}}'$ signal frequencies estimated by PERIODOGRAM, using the default box-car window. Inversion of the frequency shows periodic structure within the data around the 10 and 100 year time-scales (bottom right).

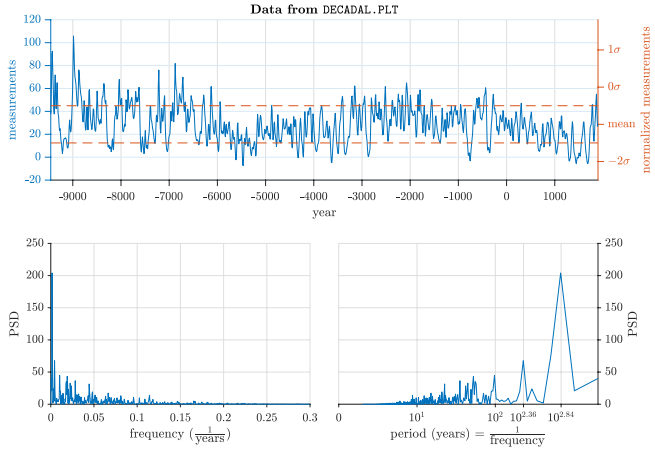

Figure 3: Analysis of time-series given in DECADAL.PLT. On top are the data $\underline{\mathbf{d}}$ with a normalized axis on the right showing the dimensions of $\underline{\mathbf{d}}'$ (see Section 3). On bottom left is the power-spectral-density (PSD) of the $\underline{\mathbf{d}}'$ signal frequencies estimated by PERIODOGRAM, using the default box-car window. Inversion of the frequency shows periodic structure within the data around the 10 and 100 year time-scales (bottom right).

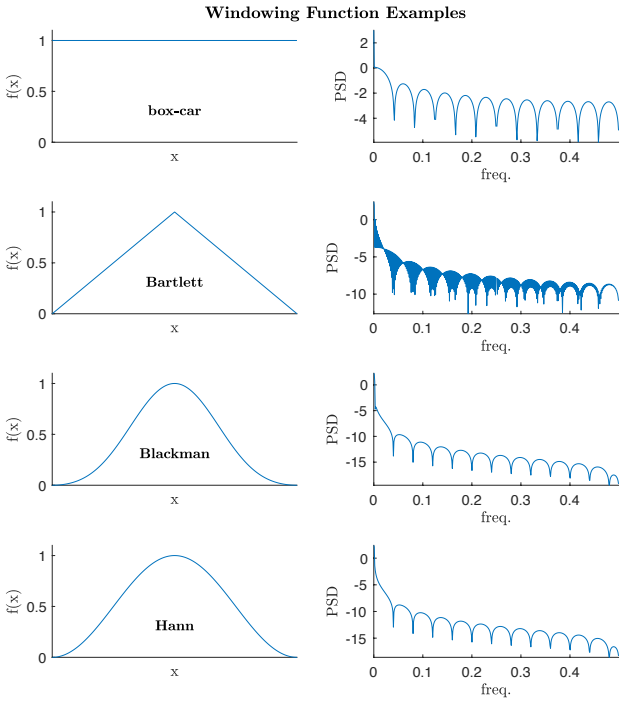

Figure 4: Examples of various windowing functions, over the domain $[-1,1]$, of length 1000.

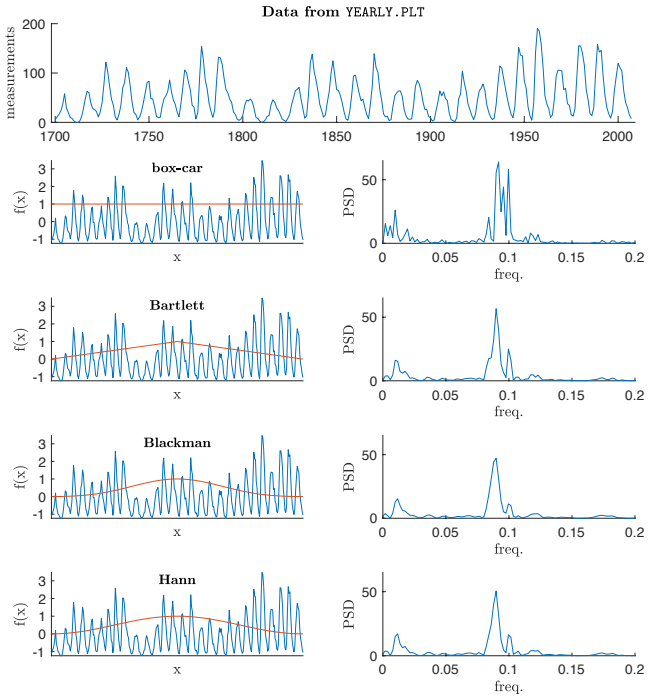

Figure 5: Demonstration of various windowing functions, using `PERIODOGRAM` and the `YEARLY.PLT` dataset. Note that the shape of the estimated PSD is different for each window function, but that all show peaks at frequencies $\omega = 0.1, 0.09, 0.0117$ corresponding to periods of $L = 10, 11.1, 85.5$ years.