

# Optimization Techniques (ENGG 6140)

## Assignment 1

Due time: February 17, at 11:59:59 pm EST time zone. Please upload solutions in Dropbox of CourseLink. Submissions after the due time are not accepted because the solution to the assignment will be released to you after the due time.

### I. QUESTION 1: ABOUT CONVEXITY [20 POINTS]

#### A. *about convexity of sets [10 points]*

Show that the following set is a convex set:

$$\{x \in \mathbb{R}^n | Ax \leq b, Cx = d\},$$

where  $A \in \mathbb{R}^{m \times n}$ ,  $b \in \mathbb{R}^m$ ,  $C \in \mathbb{R}^{k \times n}$ ,  $d \in \mathbb{R}^k$ .

Hint: Take two points in this set and use the definition of the convex set to show it is convex.

#### B. *about convexity of functions [10 points]*

Show that the following function is a convex function:

$$f(x) = x^2,$$

where  $x \in \mathbb{R}$ .

Hint: Use any of the definitions of the convex function to show this function is convex.

### II. QUESTION 2: ABOUT LINEAR PROGRAMMING [25 POINTS]

#### A. *forming the problem as linear programming [5 points]*

There is a company which is producing two products whose profits are \$3 and \$2. However, the company has a fixed \$3 loss of profit in total. The resources for two of the first product and one of the second product are at most \$9. The available budget for one of the first product and two of the second product is at least \$9. Please form this problem as a linear programming problem.

#### B. *solving linear programming by visualization [10 points]*

Answer these questions by solving the problem using visualization:

- What amount of the products can result in the most profit for the company?
- What is the most profit for the company?

Hint: You should calculate the objective function at all the extreme points.

#### C. *solving linear programming by tableau simplex method [10 points]*

Answer the questions of Section II-B using the tableau simplex method.

### III. QUESTION 3: ABOUT SENSITIVITY ANALYSIS IN LINEAR PROGRAMMING [25 POINTS]

Consider three bottom-connected water tanks whose widths are 0.5, 6, and 5 meters and heights are  $x_1$ ,  $x_2$ , and  $x_3$ , respectively. Assume there are the following physical restrictions on the system:

$$4x_1 + 6x_2 + 3x_3 \leq 24,$$

$$x_1 + 1.5x_2 + 3x_3 \leq 12,$$

$$3x_1 + x_2 \leq 12.$$

Assume that the following heights maximize the total height of water in the water tanks:  $x_1^* = 0$ ,  $x_2^* = x_3^* = \frac{8}{3}$ . Let the maximum total water height be  $c^* = \frac{88}{3}$ .

#### A. sensitivity analysis question 1 [15 points]

The factory wants to change the width of the third water tank to 4 hoping it does not change the total height. Do you recommend this change to the manager? What about changing it to 2?

#### B. sensitivity analysis question 2 [10 points]

The factory wants the height of water in the first water tank to be at least one meter. What change does it make to the optimal solution and the total height?

### IV. QUESTION 4: ABOUT INTEGER LINEAR PROGRAMMING [20 POINTS]

Solve the following optimization problem.

$$\begin{aligned} &\underset{x_1, x_2}{\text{maximize}} && c = 5x_1 + 8x_2 \\ &\text{subject to} && x_1 + x_2 \leq 6, \\ & && 5x_1 + 9x_2 \leq 45, \\ & && \frac{2}{3}x_1 + x_2 \geq 2, \\ & && x_1, x_2 \geq 0, \\ & && x_1, x_2 \in \mathbb{Z}. \end{aligned}$$

Hint: Use the branch and bound method and use the visualization method in each step of the branch and bound algorithm.

### V. QUESTION 5: CODING PROBLEM [10 POINTS]

Implement the tableau simplex method for solving linear programming using Python and solve the Problem II-C using your implementation.

Hint: You should use while loop with a termination condition to implement the iterations of the tableau method.