

Package ‘statcomp’

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Title Statistical Complexity and Information measures for time series analysis

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Description An implementation of local and global statistical complexity measures for time series analysis in R. The package provides functions to compute statistical complexity and information measures for any given time series based on ordinal statistics (Bandt and Pompe 2002). The ordinal pattern statistics are used to calculate a variety of global (Permutation Entropy, MPR complexity) and local (Fisher Information) complexity and information measures (for further information, see Martin, Plastino and Rosso 2007; Olivares et al 2012). In addition, methods to derive weighted ordinal pattern distributions are supplied, where a user-specified weights-generating function can be selected (e.g. variance-weighted, Fadlallah et al 2013). Complexity and information measures constitute a simple, quick and powerful tool to classify and cluster (a large number of) time series, including for model-data comparisons.

Depends R (>= 2.7.0)

License GPL-2

LazyData true

Imports zoo

RoxygenNote 5.0.1

NeedsCompilation yes

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adjust_pattern	<i>A function to create new pattern-coding schemes for the Fisher Information.</i>
----------------	--

Description

Adjusts and reorders a pattern ordering matrix.

Usage

adjust_pattern(pattern_matrix, adjustment)

Arguments

- | | |
|----------------|---|
| pattern_matrix | A numeric matrix that specifies the pattern to be transformed into the position vector. ATTENTION: Pattern should be in the ranks permutation notation, otherwise does not really make sense. |
| adjustment | A character vector, either adjustment = "jumps" or adjustment = "bitflips" that denotes the sorting type |

Details

This function reorders permutations based on "jumps" or based on "bitflips".

Value

A numeric matrix that contains the permutation matrix.

Author(s)

Sebastian Sippel

References

Sebastian Sippel (2014). Master Thesis. University of Bayreuth.

fis

A (low-level) function to compute the Fisher-information

Description

The function computes the Fisher information, i.e. a local information measure based on two different discretizations.

Usage

```
fis(opd, discretization)
```

Arguments

opd A numeric vector that details an ordinal pattern distribution in a user-specified permutation coding scheme.

discretization The discretization scheme to use, either 'Olivares.2012' or 'Ferri.2009'

Details

The Fisher information is a local information and complexity measure, computed based on the ordinal pattern distribution. The Fisher information is based on local gradients, hence it is sensitive to the permutation coding scheme. Options for discretization: 'Olivares.2012' or 'Ferri.2009', following Fisher Information discretization schemes in the respective publications.

Value

The normalized Fisher information measure in the range [0, 1].

Author(s)

Sebastian Sippel

References

Olivares et al (2012): Physica A 391 (2012) 2518-2526, Olivares et al (2012): Physics Letters A 376 (2012) 1577-1583, Ferri et al (2009): Phys. Lett. A 373 (2009) 2210-2214.

Examples

```
x = arima.sim(model=list(ar = 0.3), n = 10^4)
opd = ordinal_pattern_distribution(x = x, ndemb = 6)
fis(opd = opd)
```

```
generate_lehmerperm_matrix
```

A function to generate the Lehmer permutation ordering.

Description

Generates all permutations of a given embedding dimension, ordered according to the Lehmer coding scheme.

Usage

```
generate_lehmerperm_matrix(ndemb)
```

Arguments

ndemb The embedding dimension.

Details

This function converts ranks to indices and back.

Value

A numeric matrix that contains the Lehmer permutation pattern.

Author(s)

Sebastian Sippel

References

<http://www.keithschwarz.com/interesting/code/?dir=factoradic-permutation>

```
global_complexity
```

A function to compute global information and complexity measures for time series

Description

This is a high-level function that calculates global complexity measures directly from a given time series or ordinal pattern distribution.

Usage

```
global_complexity(x = NA, opd = NA, ndemb)
```

Arguments

x	(OPTIONAL) If opd is not specified, a time series vector x must be specified
opd	A numeric vector that details an ordinal pattern distribution in a user-specified permutation coding scheme.
ndemb	(OPTIONAL) If x is given, the embedding dimension (ndemb) is required.

Details

This function calculates the following global measures of complexity and information:

- Permutation Entropy (PE, cf. Bandt and Pompe, 2002)
- Permutation Statistical complexity (MPR complexity, cf. Martin, Plastino and Rosso, 2006)
- Number of "forbidden patterns" (cf. Amigo 2010)

Value

A named vector containing the three global complexity measures.

Author(s)

Sebastian Sippel

References

Bandt and Pompe (2002): Physical Review Letters 88 (2002), 174102-1-174102-4. Martin, Plastino and Rosse (2006): Physica A 369 (2006) 439-462 Amigo (2010): Permutation Complexity in Dynamical Systems. Springer. ISBN 978-3-642-04083-2

Examples

```
x = arima.sim(model=list(ar = 0.3), n = 10^4)
global_complexity(x = x, ndemb = 6)
# or:
opd = ordinal_pattern_distribution(x = x, ndemb = 6)
global_complexity(opd = opd, ndemb = 6)
```

hellinger_distance	<i>Distance measure between ordinal pattern distributions: Hellinger distance</i>
--------------------	---

Description

Compute the Hellinger Distance

Usage

```
hellinger_distance(p, q)
```

Arguments

p	An ordinal pattern distribution
q	A second ordinal pattern distribution to compare against p.

Details

This function returns a distance measure.

Value

A vector of length 1.

Author(s)

Sebastian Sippel

References

none

Examples

```
hellinger_distance(p=ordinal_pattern_distribution(rnorm(10000), ndemb = 5), q= ordinal_pattern_distribution
```

henon_map	<i>A function to generate a time series from the Henon Map</i>
-----------	--

Description

Generates a time series from the Henon map

Usage

```
henon_map(N, a, b, startx="rand", starty="rand", disregard_N=0)
```

Arguments

N	length of the time series that is to be generated
a	Henon map parameter a
b	Henon map parameter b
startx	start value in x direction. Default is to random.
starty	start value in y direction. Default is to random.
disregard_N	Number of values at the beginning of the series to disregard

Value

A vector of length N

Author(s)

Sebastian Sippel

References

Olivares et al., 2012

Examples

```
henon_map(N = 10^4, a=1.4, b=0.3)
```

limit_curves*Limit curves in the Entropy-Complexity plane*

Description

Compute the limit curves in the Entropy Complexity plane

Usage

```
limit_curves(ndemb, fun = "min")
```

Arguments

ndemb	Embedding dimension
fun	Whether the upper (max) or lower (min) limit curve should be computed

Details

This function returns the respective limit curve.

Value

A list with two entries

Author(s)

Sebastian Sippel

References

none

logistic_map*A function to generate a time series from the logistic map*

Description

Generates a time series from the logistic map

Usage

```
logistic_map(N, r, start="rand", disregard_N=0)
```

Arguments

N	length of the time series that is to be generated
r	logistic map parameter, must be in the range [0,4]
start	start value. Default is to random.
disregard_N	Number of values at the beginning of the series to disregard

Value

A vector of length N

Author(s)

Sebastian Sippel

References

Rosso et al, 2007, Physical Review Letters.

Examples

```
logistic_map(N = 10^4, r=4)
```

maxd3

Maximum curve of time-causal entropy-complexity plane at ndemb=3

Description

Maximum curve of time-causal entropy-complexity plane at ndemb=3

Usage

```
maxd3
```

Format

A data frame with 494 rows and 2 columns:

x x-values of minimum curve if ndemb==3

y y-values of minimum curve if ndemb==3 ...

Source

Computed based on Martin, M. T., A. Plastino, and O. A. Rosso. "Generalized statistical complexity measures: Geometrical and analytical properties." *Physica A: Statistical Mechanics and its Applications* 369.2 (2006): 439-462.

maxd4

Maximum curve of time-causal entropy-complexity plane at ndemb=4

Description

Maximum curve of time-causal entropy-complexity plane at ndemb=4

Usage

maxd4

Format

A data frame with 2139 rows and 2 columns:

x x-values of minimum curve if ndemb==4

y y-values of minimum curve if ndemb==4 ...

Source

Computed based on Martin, M. T., A. Plastino, and O. A. Rosso. "Generalized statistical complexity measures: Geometrical and analytical properties." *Physica A: Statistical Mechanics and its Applications* 369.2 (2006): 439-462.

maxd5

Maximum curve of time-causal entropy-complexity plane at ndemb=5

Description

Maximum curve of time-causal entropy-complexity plane at ndemb=5

Usage

maxd5

Format

A data frame with 4151 rows and 2 columns:

x x-values of minimum curve if ndemb==5

y y-values of minimum curve if ndemb==5 ...

Source

Computed based on Martin, M. T., A. Plastino, and O. A. Rosso. "Generalized statistical complexity measures: Geometrical and analytical properties." *Physica A: Statistical Mechanics and its Applications* 369.2 (2006): 439-462.

maxd6

Maximum curve of time-causal entropy-complexity plane at ndemb=6

Description

Maximum curve of time-causal entropy-complexity plane at ndemb=6

Usage

maxd6

Format

A data frame with 3438 rows and 2 columns:

x x-values of minimum curve if ndemb==6

y y-values of minimum curve if ndemb==6 ...

Source

Computed based on Martin, M. T., A. Plastino, and O. A. Rosso. "Generalized statistical complexity measures: Geometrical and analytical properties." *Physica A: Statistical Mechanics and its Applications* 369.2 (2006): 439-462.

mind3

Minimum curve of time-causal entropy-complexity plane at ndemb=3

Description

Minimum curve of time-causal entropy-complexity plane at ndemb=3

Usage

mind3

Format

A data frame with 500 rows and 2 columns:

x x-values of minimum curve if ndemb==3

y y-values of minimum curve if ndemb==3 ...

Source

Computed based on Martin, M. T., A. Plastino, and O. A. Rosso. "Generalized statistical complexity measures: Geometrical and analytical properties." *Physica A: Statistical Mechanics and its Applications* 369.2 (2006): 439-462.

mind4

*Minimum curve of time-causal entropy-complexity plane at ndemb=4***Description**

Minimum curve of time-causal entropy-complexity plane at ndemb=4

Usage

mind4

Format

A data frame with 500 rows and 2 columns:

x x-values of minimum curve if ndemb==4

y y-values of minimum curve if ndemb==4 ...

Source

Computed based on Martin, M. T., A. Plastino, and O. A. Rosso. "Generalized statistical complexity measures: Geometrical and analytical properties." *Physica A: Statistical Mechanics and its Applications* 369.2 (2006): 439-462.

mind5

*Minimum curve of time-causal entropy-complexity plane at ndemb=5***Description**

Minimum curve of time-causal entropy-complexity plane at ndemb=5

Usage

mind5

Format

A data frame with 500 rows and 2 columns:

x x-values of minimum curve if ndemb==5

y y-values of minimum curve if ndemb==5 ...

Source

Computed based on Martin, M. T., A. Plastino, and O. A. Rosso. "Generalized statistical complexity measures: Geometrical and analytical properties." *Physica A: Statistical Mechanics and its Applications* 369.2 (2006): 439-462.

mind6	<i>Minimum curve of time-causal entropy-complexity plane at ndemb=6</i>
-------	---

Description

Minimum curve of time-causal entropy-complexity plane at ndemb=6

Usage

mind6

Format

A data frame with 500 rows and 2 columns:

x x-values of minimum curve if ndemb==6

y y-values of minimum curve if ndemb==6 ...

Source

Computed based on Martin, M. T., A. Plastino, and O. A. Rosso. "Generalized statistical complexity measures: Geometrical and analytical properties." *Physica A: Statistical Mechanics and its Applications* 369.2 (2006): 439-462.

MPR_complexity	<i>A function to compute the MPR-complexity</i>
----------------	---

Description

The function computes the MPR complexity, i.e. a generalized (global) complexity measure based on the Jenson-Shannon divergence.

Usage

MPR_complexity(opd)

Arguments

opd A numeric vector that details an ordinal pattern distribution.

Details

Generalized complexity measures combine an information measure (i.e. entropy) with the distance of the distribution from the uniform distribution ("disequilibrium"). As a global measure, MPR-complexity is insensitive to the permutation coding scheme.

Value

The normalized MPR complexity measure in the range [0, 1].

Author(s)

Sebastian Sippel

References

Martin, M. T., A. Plastino, and O. A. Rosso. "Generalized statistical complexity measures: Geometrical and analytical properties." *Physica A: Statistical Mechanics and its Applications* 369.2 (2006): 439-462.

Examples

```
x = arima.sim(model=list(ar = 0.3), n = 10^4)
opd = ordinal_pattern_distribution(x = x, ndemb = 6)
MPR_complexity(opd)
```

nbitflips

A function to compute bitflip statistics and time series

Description

Computation of bitflip statistics of a time series

Usage

```
nbitflips(x, ndemb)
```

Arguments

x	A numeric vector (e.g. a time series), from which the ordinal pattern distribution is to be calculated
ndemb	Embedding dimension of the ordinal patterns (i.e. sliding window size) for which bitflips are to be calculated. Should be chosen such as $\text{length}(x) \gg \text{ndemb}$

Details

This function returns a histogram and time series of the number of bitflips occurring in the associated ordinal patterns. NA values are allowed, and any pattern that contains at least one NA value will be ignored. **WARNING:** Can be slow with very long time series ($n > 10^7$).

Value

A list with two entries is returned.

Author(s)

Sebastian Sippel

References

Sippel, S., Master Thesis, University of Bayreuth, 2014.

Examples

```
x = arima.sim(model=list(ar = 0.3), n = 10^6)
nbitflips(x = x, ndemb = 6)
```

ordinal_pattern_distribution

A function to compute ordinal pattern statistics

Description

Computation of the ordinal patterns of a time series (see e.g. Bandt and Pompe 2002)

Usage

```
ordinal_pattern_distribution(x, ndemb)
```

Arguments

x	A numeric vector (e.g. a time series), from which the ordinal pattern distribution is to be calculated
ndemb	Embedding dimension of the ordinal patterns (i.e. sliding window size). Should be chosen such as $\text{length}(x) \gg \text{ndemb}$

Details

This function returns the distribution of ordinal patterns using the Keller coding scheme, detailed in Physica A 356 (2005) 114-120. NA values are allowed, and any pattern that contains at least one NA value will be ignored. (Fast) C routines are used for computing ordinal patterns.

Value

A character vector of length $\text{factorial}(\text{ndemb})$ is returned.

Author(s)

Sebastian Sippel

References

Bandt and Pompe, 2002.

Examples

```
x = arima.sim(model=list(ar = 0.3), n = 10^4)
ordinal_pattern_distribution(x = x, ndemb = 6)
```

`ordinal_pattern_time_series`*A function to compute time series of ordinal patterns*

Description

Computation of the ordinal patterns of a time series (see e.g. Bandt and Pompe 2002)

Usage

```
ordinal_pattern_time_series(x, ndemb)
```

Arguments

<code>x</code>	A numeric vector (e.g. a time series), from which the ordinal pattern time series is to be calculated
<code>ndemb</code>	Embedding dimension of the ordinal patterns (i.e. sliding window size). Should be chosen such as $\text{length}(x) \gg \text{ndemb}$

Details

This function returns the distribution of ordinal patterns using the Keller coding scheme, detailed in Physica A 356 (2005) 114-120. NA values are allowed, and any pattern that contains at least one NA value will be ignored. (Fast) C routines are used for computing ordinal patterns.

Value

A character vector of $\text{length}(x)$ is returned.

Author(s)

Sebastian Sippel

References

Bandt and Pompe, 2002.

Examples

```
x = arima.sim(model=list(ar = 0.3), n = 10^4)
ordinal_pattern_time_series(x = x, ndemb = 6)
```

permutation_entropy	<i>A function to compute the permutation entropy</i>
---------------------	--

Description

Computation of the permutation entropy of a time series based on its ordinal pattern distribution (see Bandt and Pompe 2002). Permutation entropy is a global information measure, hence insensitive to the permutation ordering scheme.

Usage

```
permutation_entropy(opd)
```

Arguments

opd	A numeric vector that details an ordinal pattern distribution.
-----	--

Details

This function calculates the permutation entropy as described in Bandt and Pompe 2002.

Value

The normalized permutation entropy as a numeric value in the range [0,1].

Author(s)

Sebastian Sippel

References

Bandt and Pompe, 2002.

Examples

```
x = arima.sim(model=list(ar = 0.3), n = 10^4)
opd = ordinal_pattern_distribution(x = x, ndemb = 6)
permutation_entropy(opd)
```

powernoise	<i>A function to generate k-noise</i>
------------	---------------------------------------

Description

Generates samples of power law noise.

Usage

```
powernoise(k, N)
```


Arguments

k	Power law scaling exponent
N	number of samples to generate

Details

Generates samples of power law noise. The power spectrum of the signal scales as f^{-k} . The R function uses `fft()`, similarly to the `knoise_fft` Matlab function.

Value

A named list with three entries is returned. `x` - $N \times 1$ vector of power law samples

Author(s)

Sebastian Sippel and Holger Lange

Examples

```
powernoise_series = powernoise(k=2, N=10000)
```

quadratic_map

A function to generate a time series from the Quadratic map

Description

Generates a time series from the Quadratic map

Usage

```
quadratic_map(N, k, start="rand", disregard_N=0)
```

Arguments

N	length of the time series that is to be generated
k	Quadratic map parameter
start	start value. Default is to random.
disregard_N	Number of values at the beginning of the series to disregard

Value

A vector of length N

Author(s)

Sebastian Sippel

References

Grebogi, Celso, Edward Ott, and James A. Yorke. "Crises, sudden changes in chaotic attractors, and transient chaos." *Physica D: Nonlinear Phenomena* 7.1-3 (1983): 181-200.

Examples

```
quadratic_map(N = 10^4, k=1.4)
```

rank_to_permutation	<i>A function to convert a "ranks-based" permutation notation to an "index-based" permutation scheme.</i>
---------------------	---

Description

Converts permutations denoted by ranks to permutations denoted by indices and back.

Usage

```
rank_to_permutation(pattern, permutation.notation)
```

Arguments

pattern	A numeric vector that denotes a permutation pattern.
permutation.notation	The permutation notation that should be used. Could be "Olivares.2012" or "Keller.2005".

Details

This function converts ranks to indices and back.

Value

A numeric vector, which contains the transformed permutation.

Author(s)

Sebastian Sippel

References

Sebastian Sippel (2014). Master Thesis. University of Bayreuth.

schuster_map	<i>A function to generate a time series from the Schuster Map</i>
--------------	---

Description

Generates a time series from the Schuster map

Usage

```
schuster_map(N, z, start="rand", disregard_N=0)
```

Arguments

N	length of the time series that is to be generated
z	Schuster map parameter
start	start value. Default is to random.
disregard_N	Number of values at the beginning of the series to disregard

Value

A vector of length N

Author(s)

Sebastian Sippel

References

Rosso et al., 2007, Physical Review Letters

Examples

```
schuster_map(N = 10^4, z=2)
```

skew_tent_map	<i>A function to generate a time series from the logistic map</i>
---------------	---

Description

Generates a time series from the Skew-Tent map

Usage

```
skew_tent_map(N, a, start="rand", disregard_N=0)
```

Arguments

N	length of the time series that is to be generated
a	Skew-Tent map parameter, must be in the range [0,1]
start	start value. Default is to random.
disregard_N	Number of values at the beginning of the series to disregard

Value

A vector of length N

Author(s)

Sebastian Sippel

References

Rosso et al, 2007, Physical Review Letters.

Examples

```
skew_tent_map(N = 10^4, a=0.1847)
```

tent_map

A function to generate a time series from the logistic map

Description

Generates a time series from the logistic map

Usage

```
tent_map(N, mu, start="rand", disregard_N=0)
```

Arguments

N	length of the time series that is to be generated
mu	Tent map parameter, must be in the range [0,2]
start	start value. Default is to random.
disregard_N	Number of values at the beginning of the series to disregard

Value

A vector of length N

Author(s)

Sebastian Sippel

References

Feldman, D. P., McTague, C. S., & Crutchfield, J. P. (2008). The organization of intrinsic computation: Complexity-entropy diagrams and the diversity of natural information processing. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 18(4), 043106.

Examples

```
tent_map(N = 10^4, mu=1.8)
```

transformPermCoding	<i>A function to generate a vector from an index-transformation vector from a permutation coding scheme</i>
---------------------	---

Description

Generates a position vector to change the ordinal pattern distribution in the default permutation coding scheme (i.e. generated by `ordinal_pattern_distribution(x, ndemb)`) into a user-specified coding scheme. This is a required input for the function `changePermCodingOPD`.

Usage

```
transformPermCoding(target_pattern, ndemb)
```

Arguments

target_pattern	A numeric matrix that specifies the pattern to be transformed into the position vector.
ndemb	Embedding dimension of the ordinal patterns (i.e. sliding window size). Should be chosen such as $\text{length}(x) \gg \text{ndemb}$

Details

This function returns a character vector to transform the output of `ordinal_pattern_distribution` (permutation coding as of Keller and Sinn, 2005) into a user-specified permutation coding scheme. For example, pattern #5 in "lehmerperm" ($\text{ndemb} = 5$) is given by the ranks $c(0, 1, 4, 2, 3)$. This corresponds to pattern #41 in the (original) Keller coding scheme, as given by `transformPermCoding(target_pattern = "lehmerperm", ndemb = 5)[5]`.

Value

A numeric vector of length $\text{factorial}(\text{ndemb})$, which contains the positions of the corresponding patterns in the Keller Coding scheme.

Author(s)

Sebastian Sippel

References

see e.g. Olivares et al. 2012

Examples

```
transformPermCoding(target_pattern = "lehmerperm", ndemb = 4)
```

```
weighted_ordinal_pattern_distribution
```

A function to compute weighted ordinal pattern statistics

Description

Computation of weighted ordinal patterns of a time series. Weights can be generated by a user-specified function (e.g. variance-weighted, see Fadlallah et al 2013).

Usage

```
weighted_ordinal_pattern_distribution(x, ndemb)
```

Arguments

x	A numeric vector (e.g. a time series), from which the weighted ordinal pattern distribution is to be calculated
ndemb	Embedding dimension of the ordinal patterns (i.e. sliding window size). Should be chosen such as $\text{length}(x) \gg \text{ndemb}$

Details

This function returns the distribution of weighted ordinal patterns using the Keller coding scheme, detailed in Physica A 356 (2005) 114-120. NA values are allowed. The function uses old and slow R routines and is only maintained for comparability. For faster routines, see [weighted_ordinal_pattern_distribution](#).

Value

A character vector of length $\text{factorial}(\text{ndemb})$ is returned.

Author(s)

Sebastian Sippel

References

Fadlallah et al (2013). PHYSICAL REVIEW E 87, 022911 (2013)

See Also

[weighted_ordinal_pattern_distribution](#)

Examples

```
x = arima.sim(model=list(ar = 0.3), n = 10^4)
weighted_ordinal_pattern_distribution(x = x, ndemb = 6)
```

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