The Computational Relativity (CoRe) Database

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Introduction

- Current GW interferometers have already observed signals from compact binary coalescences (CBC).
- CBC with neutron stars are promising sources of GRBs and kilonovae, and their GWs can provide information on their neutron star.
- GW data analysis requires waveform templates.

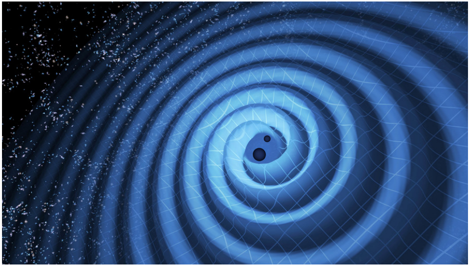

Figure: LIGO/Caltech/MIT.

GW Observations

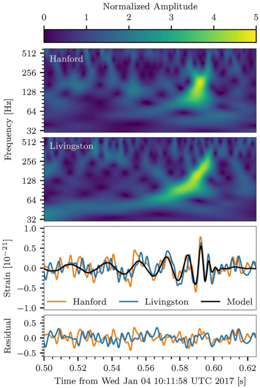

Figure: LIGO/Caltech/MIT.

- On 2015 the first GW coming from two black holes GW150914 was detected by LIGO's Interferometers.
- The first GW from a binary neutron star inspiral GW170817 was observed on 2017 by LIGO and Virgo.

GW data analysis problem

Paramater estimation

With Bayes' theorem:

$$p(\theta|\mathbf{d}, \hat{h}, I) = p(\theta|\hat{h}, I)\frac{p(\mathbf{d}|\theta, \hat{h}, I)}{p(\mathbf{d}|\hat{h}, I)}, \tag{1}$$

where $\hat{h}$ is the GW model dependent on the parameters θ, prior background information I and the observed data $\mathbf{d}$.

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Waveform template

Waveform templates are needed to identify the signal within the recorded data of the detector and measure the source's physical properties.

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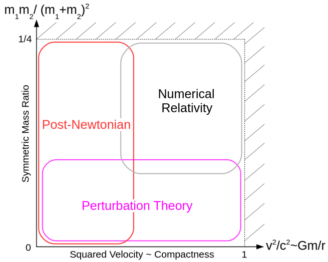
- Analytical techniques: Post-Newtonian theory, Effective-One-Body formalism, etc.
- Both!



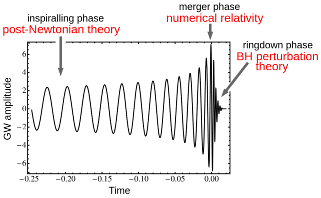


Figure: Images from Blanchet 2019.

NR Simulations

- Uncover physics of the coalescence $\rightarrow$ inform analytical models
- We can extract waveforms from the simulations!
- In fact, several waveform databases exist

[1]Gonzalez+ 2023 Class. Quantum Grav. 40 085011

[2]Kiuchi+ 2017 Phys. Rev. D 96 084060, Kiuchi+, 2020 Phys. Rev. D 101, 084006

[3]Boyle+ 2019 Class. Quantum Grav. 36 195006, Foucart+ 2019 Phys. Rev. D 99 044008

[4]https://stellarcollapse.org/gwcatalog.html, https://bitbucket.org/ciolfir/bns-waveforms

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Existing BNS databases

- CoRe database (254 binaries)[1]
- SACRA-MPI (46 binaries)[2]
- SXS (2 binaries)[3]
- Others[4]

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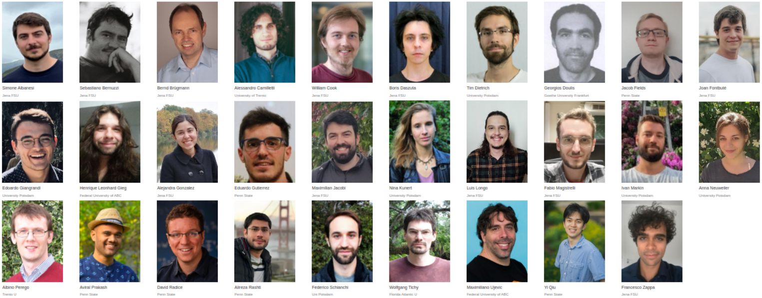
The CoRe Collaboration

We are a collaborative research effort for 3+1 numerical relativity simulations of compact binaries spacetimes from several institutions around the world.



Methods

3+1 NR Simulations Codes

Initial Data: `Lorene`[5], `SGRID` [FAU,Tichy+]
Evolution Code: `BAM` [FSU Jena, Brügmann+], `THC` [PSU, Radice+]

Key Highlights

- 18 different EOS employed, including finite-temperature and non-hadronic EOS.
- Inspiral-Merger simulations with high order schemes.
- Merger and postmerger simulations with microphysics and different neutrino schemes.

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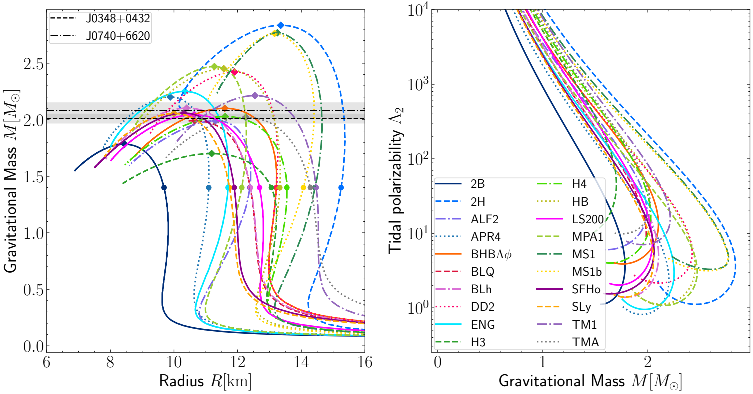
[5] *Gourgoulhon 2001 Phys.Rev. D63 064029, Taniguchi 2001 Phys. Rev. D 64 064012, Taniguchi 2002 Phys. Rev. D 65 044027*

Available EoS



Waveforms

$$h = h_+ - \mathrm{i}h_\times = D_L^{-1} \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h_{\ell m}(t) \, {}_{-2}Y_{\ell m}(\iota, \varphi) \tag{2}$$

$$\kappa_2^{\mathrm{T}} = 3\nu \left[\left(\frac{m_1}{M} \right)^3 \Lambda_1 + (1 \leftrightarrow 2) \right] \tag{3}$$

$$\tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12 m_2) m_1^4 \Lambda_1}{M^5} + (1 \leftrightarrow 2) \tag{4}$$

Note that $\kappa_2^{\mathrm{T}} = \frac{3}{16}\tilde{\Lambda}$ for $q = 1$.

$$\hat{S} = \left(\frac{m_1}{M} \right)^2 \chi_1 + \left(\frac{m_2}{M} \right)^2 \chi_2 \tag{5}$$

Radiated Energy and Angular Momentum

$$\mathcal{E}_{\text{rad}} = \frac{1}{16\pi} \sum_{\ell=2}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \int_0^t dt' |D_L \dot{h}_{\ell m}(t')|^2 \tag{6}$$

$$\mathcal{J}_{\text{rad}} = \frac{1}{16\pi} \sum_{\ell=2}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \int_0^t dt' \, m \left[D_L^2 h_{\ell m}(t') \dot{h}^*_{\ell m}(t') \right] \tag{7}$$

GW luminosity peak

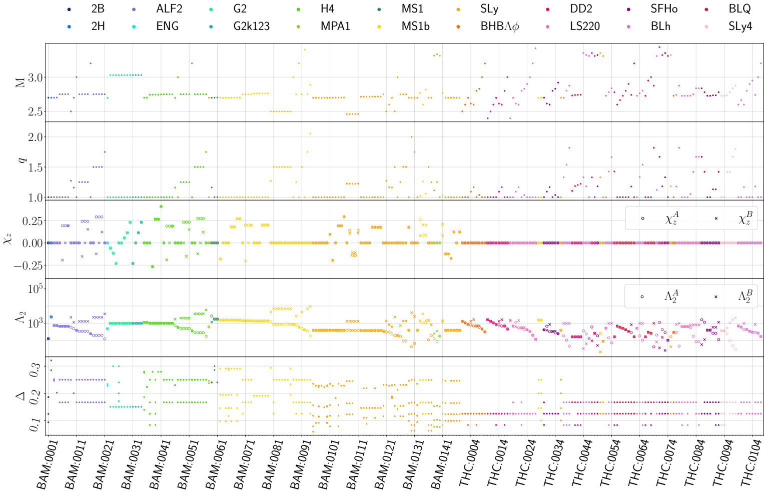
$$L_{\text{peak}} = \max_t \frac{d\mathcal{E}_{\text{rad}}(t)}{dt} \tag{8}$$

Overview

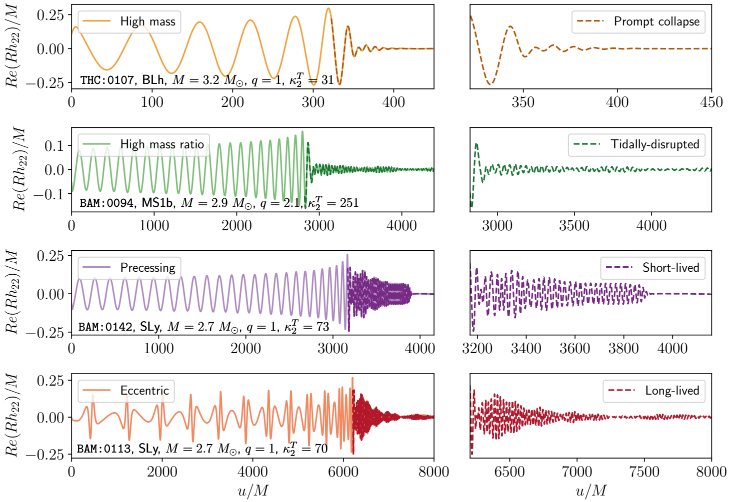
The database:

- Contains 254 distinct configurations, for a total of 590 waveforms
- Includes the strain and Weyl curvature multipoles up to $\ell = m = 4$
- Covers a wide parameter space, including high mass ratios $q \gtrsim 2$ and high spinning NS
- Data consistent with GW170817 and GW190425

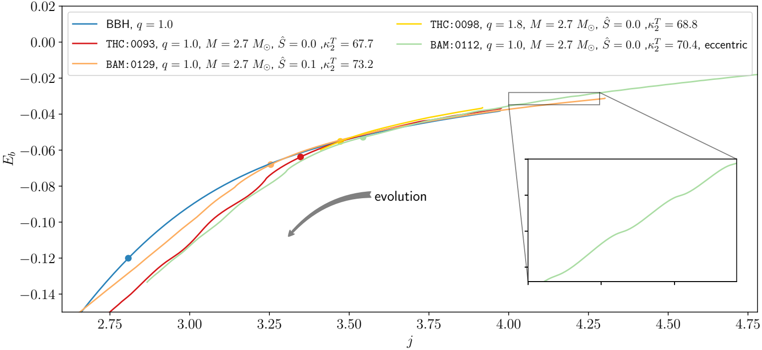
CoRe Simulations



Waveform Zoo



Energy - Ang. Momentum curves



Energy curves allow to analyse the binary dynamics in a gauge invariant way[6].

[6] *Damour et al 2012 Phys.Rev.Lett. 108, 131101*

How to ensure accurate results from our simulations?

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Goal

Build a convergent series to ensure we can trust our data

Uncertainty assessment

The quality of your numerical data depends mainly on two types of errors:

- Truncation errors from the numerical scheme
- Extraction of the GW at a finite radius

Truncation errors

For any finite-differencing algorithm for a quantity f:

$$f^{(h)} = f^{(e)} + \sum_{i=p}^{\infty} A_i h^i \tag{9}$$

The exact value $f^{(e)}$ for $h \to 0$ can't be obtained ..

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$\rightarrow$Use set of data at different resolutions to improve results!
$\rightarrow$Richardson extrapolation method

Richardson extrapolation

Input:

- Dataset $(f^{(h)})$ at different resolutions
- Accurate measure of the convergence order p

Output:

Improved approximation to $f^{(e)}$: $\mathcal{R}[(f^{(h)})]$

Uncertainty

$$\delta f_{(h)} = \mathcal{R}[(f^{(h)})] - f^{(h^{\text{MAX}})} \tag{10}$$

Self convergence

Find the convergence rate r "experimentally" from simulations with grid spacing h at different resolutions:

$$SF = \frac{h_L^r - h_M^r}{h_M^r - h_H^r} \tag{11}$$

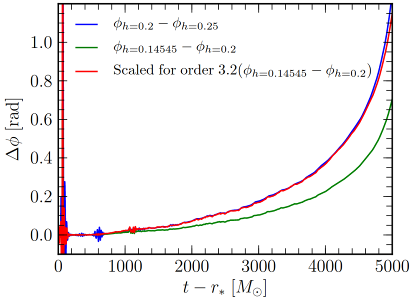

Figure: Plot from Radice+ 2013.

Finite extraction radius uncertainty

- Ideally: Extract waveforms at null infinity $\rightarrow$ numerically impossible

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- In practice: Extract at finite radii $\rightarrow$ generates uncertainty mainly on the amplitude and phase!

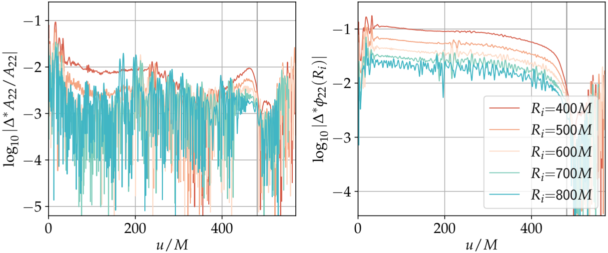


Figure: Plot from ongoing work.

$\rightarrow$ We can approximate the waveform to null infinity using a polynomial of order K

$$f(t, r_j) = f_0(t) + \sum_{k=1}^{K} f_k(t) r_j^{-k} \tag{12}$$

The waveform is evaluated at different radii r_j with $j = 0, .., N$ and extracted using a polynomial of order $K < N$.

Error budget

We can find the total error budget for quantities like the amplitude and phase through the previous methods.

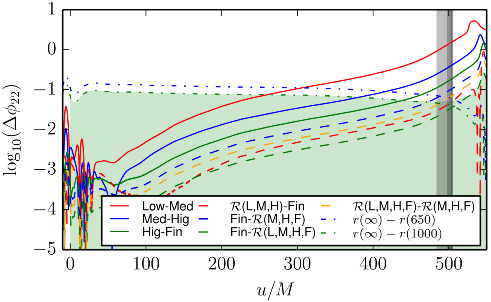


Figure: Plot from Bernuzzi & Dietrich 2016.

Waveform Accuracy

$$\bar{\mathcal{F}} \equiv 1 - \mathcal{F} = 1 - \max_{t_0, \phi_0} \frac{\langle h^{\rm EOB}, h^{\rm NR} \rangle}{||h^{\rm EOB}|| \, ||h^{\rm NR}||}, \tag{13}$$

where t_0 and ϕ_0 denote the initial time and phase, and $||h|| \equiv \sqrt{\langle h, h \rangle}$.
The inner product is defined as

$$\langle h_1, h_2 \rangle \equiv 4\Re \int \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df \tag{14}$$

where $S_n(f)$ is the power spectral density (PSD) of the detector and $\tilde{h}(f)$ the Fourier transform of $h(t)$.

The condition[7]

$$\mathcal{F}_{\text{thr}} > 1 - \frac{\epsilon^2}{2\rho^2} \tag{15}$$

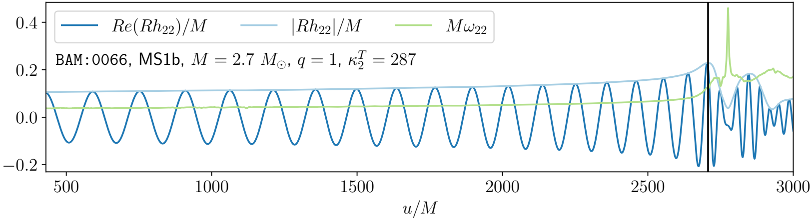
is *necessary* for unbiased parameter estimation (faithful waveforms)[8].
Here ρ is the SNR.

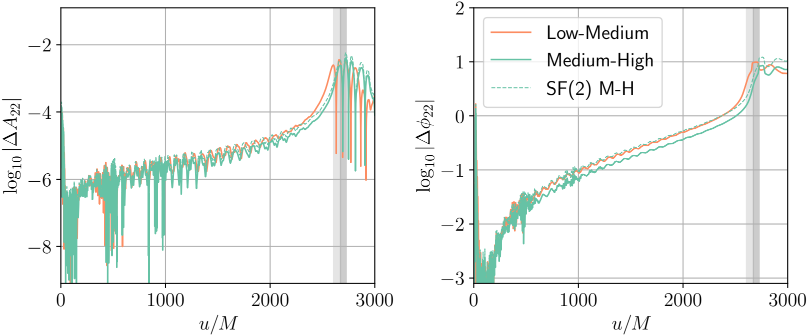
[7] *Damour et al 2011 Phys.Rev. D82 084020*

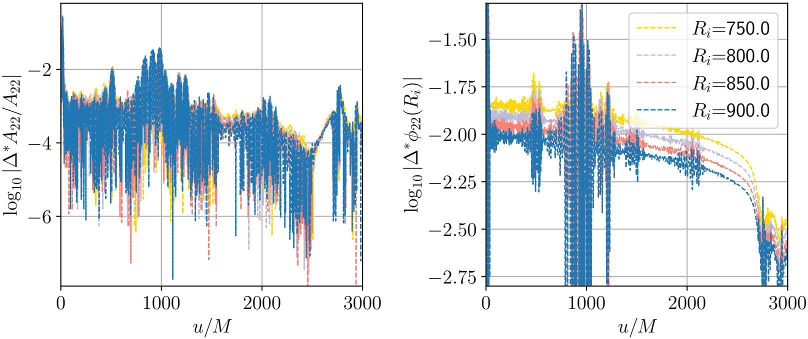
[8] Its violation does not imply that an analysis has biases

NR Waveform Analysis

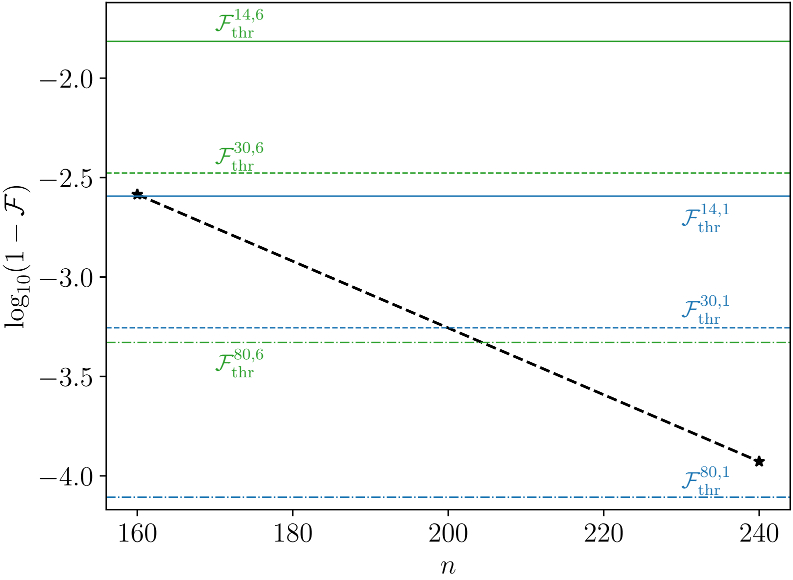
Full numerical analysis of BAM:0066.




$\log_{10}|\Delta A_{22}|$
-2
-4
-6
-8
0
1000
2000
3000
u/M
$\log_{10}|\Delta \phi_{22}|$
2
1
0
-1
-2
-3
Low-Medium
Medium-High
SF(2) M-H

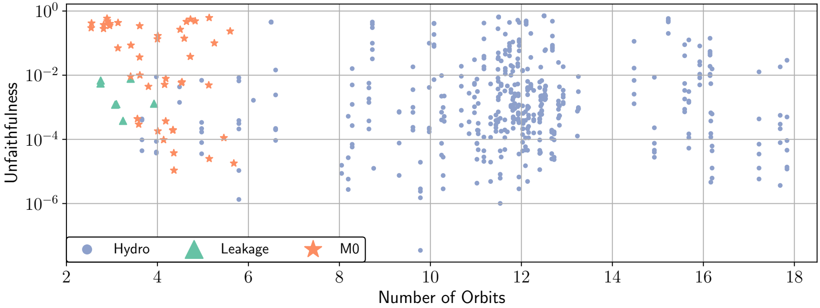


$\log_{10}|\Delta^* A_{22}/A_{22}|$
-2
-4
-6
0
1000
2000
3000
u/M
$\log_{10}|\Delta^* \phi_{22}(R_i)|$
-1.50
-1.75
-2.00
-2.25
-2.50
-2.75
R_i=750.0
R_i=800.0
R_i=850.0
R_i=900.0



$\mathcal{F}_{\mathrm{thr}}^{14,6}$
$\mathcal{F}_{\mathrm{thr}}^{30,6}$
$\mathcal{F}_{\mathrm{thr}}^{14,1}$
$\mathcal{F}_{\mathrm{thr}}^{30,1}$
$\mathcal{F}_{\mathrm{thr}}^{80,6}$
$\mathcal{F}_{\mathrm{thr}}^{80,1}$
-2.0
-2.5
-3.0
-3.5
-4.0
$\log_{10}(1-\mathcal{F})$
160
180
200
220
240
n

Overview of the waveform database accuracy:



- $\bar{\mathcal{F}} < 10^{-2}$ on average $\rightarrow$ useful for PE.
- Long simulations (e.g. 18 orbits) comparable to shorter (e.g. 6 orbits) high resolution simulations.
- Note: Not all low $\bar{\mathcal{F}}$ data is suitable for waveform modelling (short inspiral, focus on PM)

Quasi-Universal Relations

GW Peak Luminosity L_{peak}

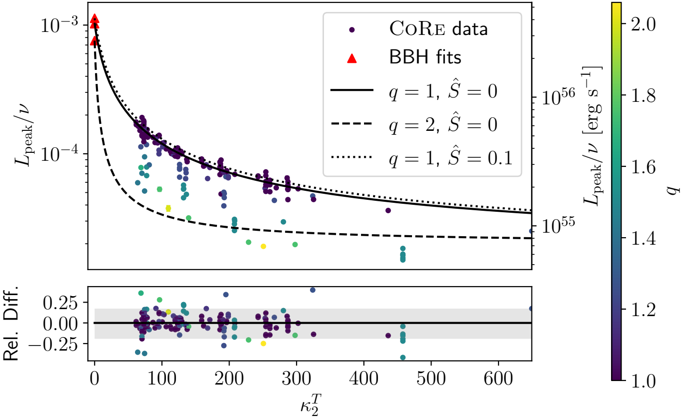
- GW and EM emission dependence on binary's parameters: key to GW and multimessenger astronomy.
- Peak luminosity extracted from the emitted GW energy: $L_{\text{peak}} = \max_t \frac{d\mathcal{E}_{\text{rad}}(t)}{dt}$

Peak luminosity Fits

Based on the BHNS L_{peak}-model[9], where $L^{\text{BBH}}_{\text{peak}}$ are the fits for BBH[10]

$$L_{\text{peak}}(\nu,\ \hat{S},\ \kappa_2^T)/\nu = L^{\text{BBH}}_{\text{peak}} \frac{1 + p_1(\nu,\ \hat{S})\kappa_2^T + p_2(\nu,\ \hat{S}){\kappa_2^T}^2}{(1 + [p_3(\nu,\ \hat{S})]^2\kappa_2^T)^2} \tag{16}$$

[9] *Zappa et al 2019, Phys. Rev. Lett. 123, 041102*
[10] *Keitel et al 2017 Phys. Rev. D96, 024006*



The fit reduces to the BBH case for $\kappa_2^T \to 0$. The average 1σ deviation is about 12% over the entire dataset.

Frequency at merger f_{mrg}

- Merger characterized by a tidal coupling constant universality: dependence on κ_2^T and NS spin[11]
- TD $\ell = m = 2$ waveforms $\rightarrow$ peak in the modulus and in the frequency.
- Use of NR informed models (e.g. EOB) and measurent of $Mf_{\text{mrg}} \rightarrow$ constrain EOS

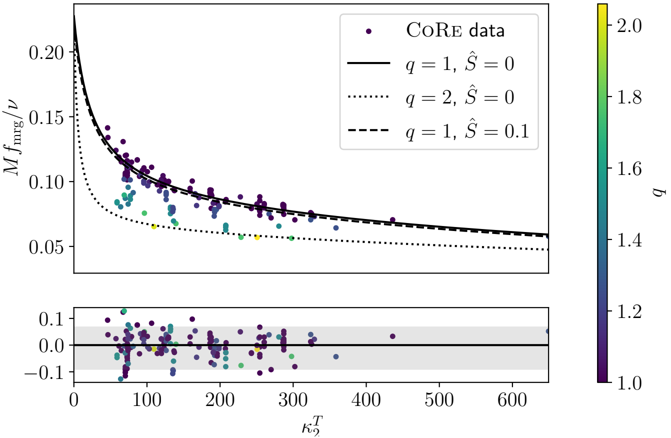
Merger fits

Updated fits for A_{mrg}/M and Mf_{mrg}/ν[12]

$$Q^{\text{fit}} = a_0 Q^M(X) Q^S(\hat{S}, X) Q^T(\kappa_2^T, X) \tag{17}$$

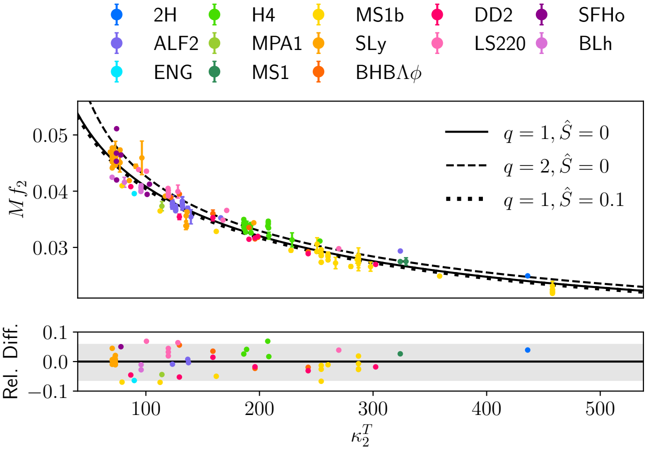
[11] *Bernuzzi et al 2014 Phys.Rev.Lett. 112 201101*

[12] *Breschi et al 2022, arXiv:2205.09112*



Best fit parameters for Mf_{mrg}/ν compared to CoRe data, with a 1σ error below 5% (below 3% for A_{mrg}/M).

PM Frequency peak f_2



Sufficient precision for informative measurements of the NS mass-radius sequence, with a 1σ error below 4%.

Take away

- We hope to detect more CBC $\rightarrow$ need for better waveform templates
- Numerical relativity simulations can provide us with waveforms and information about the coalescence
- However, the main source of errors on the waveforms come from truncation and finite extraction radius errors
- We saw different extrapolation methods to improve our approximations
- The faithfulness functional can be employed to determine whether a waveform template is apt for parameter estimation
- I presented the CoRe DB of BNS waveforms, showing promising results for both PE and QUR fitting.

Useful links

Public CoRe Database

https://core-gitlfs.tpi.uni-jena.de/

`watpy`: Waveform Analysis Tools in Python

https://git.tpi.uni-jena.de/core/watpy

Zenodo

https://zenodo.org/record/7253784

CoRe Website

http://www.computational-relativity.org/

Tutorial

The Database

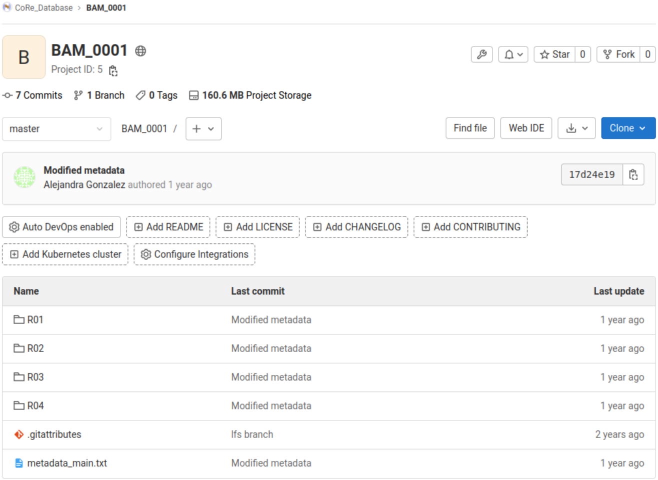
In the public database one can find each simulation as its own repository,

`https://core-gitlfs.tpi.uni-jena.de/`

Additionally, the **core-database-index** repository contains the metadata for all simulations available.

`https://core-gitlfs.tpi.uni-jena.de/core_database/core_database_index`

Simulation repository



Waveform Analysis Tools in Python

`watpy` implements few classes to clone and work with CoRe waveforms.

- `wave()` and `mwaves()` for multipolar waveforms data
- `CoRe_db()` to clone the CoRe DB, add data, etc
- `CoRe_idx()` to work with the CoRe DB index
- `CoRe_sim()` to work with simulation data in a CoRe repository
- `CoRe_run()` to work with one simulation resolution data in a CoRe repository
- `CoRe_h5()` to work with HDF5 data
- `CoRe_md()` to manage the metadata

Installing WATPy

`https://git.tpi.uni-jena.de/core/watpy`

Dependencies

Make sure to have installed: `numpy`, `scipy`, `matplotlib`, `h5py`

Jupyter Notebook

https://github.com/bgiacoma/DTP_TALENT_2024