Introduction to the Theory of Computation
Homework #1

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$\boxed{1}$ Write formal descriptions of the following sets:

(a) $\{x \in \mathcal{N} | x = 1 \wedge mod(x, 10) = 0\}$

(b) $\{x \in \mathcal{N} | x > 5\}$

(c) $\{x \in \mathcal{N} | x < 5\}$

(d) The set containing the string aba

(e) $\{x \notin \Sigma^*\}$

(f) $\{\emptyset\}$

$\boxed{2}$ Suppose S is a set with n elements:

(a) $|S| = n$, The number of relations on S $= 2^{n^2}$

(b) $|S| = n$, The number of reflexive relations on S $= 2^{n^2 - n}$

(c) $|S| = n$, The number of symmetric relations on S $= 2^{\frac{n(n+1)}{2}}$

(d) $|S| = n$, The number of reflexive and symmetric relations on S $= 3^{\frac{n(n-1)}{2}}$

$\boxed{3}$ Find the error in the proof:

- The proof mentions the sets H1 and H2 which are not necessaraly groups of multiple hourses. Therefore the claims made in the induction step are not necessarily valid.

$\boxed{4}$ Give a simple non-recursive definition in each case:

(a) Append all $y \in \Sigma$ onto the end of any $x \in L$

(b) Generate all strings with one 'a', and zero or more 'b's.

$\boxed{5}$ Give recursive definitions of each of the following sets.

(a) The set N of all natural numbers.

- $1 \in \mathcal{N}$
- $(x + 1) \in \mathcal{N}$ if $(x + 1) > 0$

(b) The set S of all natural numbers divisible by 7.

- $7 \in \mathcal{S}$
- $(x+1) \in \mathcal{S}$ if $mod(x, 7) > 0$

(c) The set A of all strings in $\{a, b\}^*$ containing the substring 'aa'.

- $'aa' \in \mathcal{A}$
- $x\sigma, \sigma x \in \mathcal{A}$ if $x \in \{a, b\}^*$ and $\sigma \in \mathcal{A}$

$\boxed{6}$ Recursive definition for the number of characters in a string

- $n_b(\epsilon) = 0$
- $n_b('b') = 1$
- $n_b('a') = 0$
- $n_b(x) = n_b(\sigma) + n_b(c)$ if $c \in \Sigma^*$

$\boxed{7}$ Proof:

- Observe that $|\epsilon| = 0$
- Assume a definition for length $|x\sigma| = 1 + |x|$ if $x \in \Sigma^*$ and $\sigma \in \Sigma$

$$|xy| = |x| + |y|$$

$$|(\sigma z)y| = |\sigma z| + |y|$$

$$|\sigma(zy)| = |\sigma z| + |y|$$

$$1 + |zy| = |\sigma z| + |y|$$

$$1 + |zy| = 1 + |z| + |y|$$

$$|zy| = |z| + |y|$$

$\square$