Introduction to the Theory of Computation
Homework #6

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$\boxed{1}$ 2.2

(a) Example 2.36 uses the pumping lemma for Context Free Languages that the language $Z = \{a^n b^n c^n | n \geq 0\}$ is not context free. The language $A = \{a^m b^n c^n | m, n \geq 0\}$ given can be generated by the following grammar.
$S \rightarrow XY$
$X \rightarrow aY \mid \epsilon$
$Y \rightarrow bYc \mid \epsilon$
Thus A is obviously context-free. We can similarly generate a grammar for the language given $B = \{a^n b^n c^m | m, n \geq 0\}$ which is also context free.
Finally $A \cap B = Z$ which is not context-free.

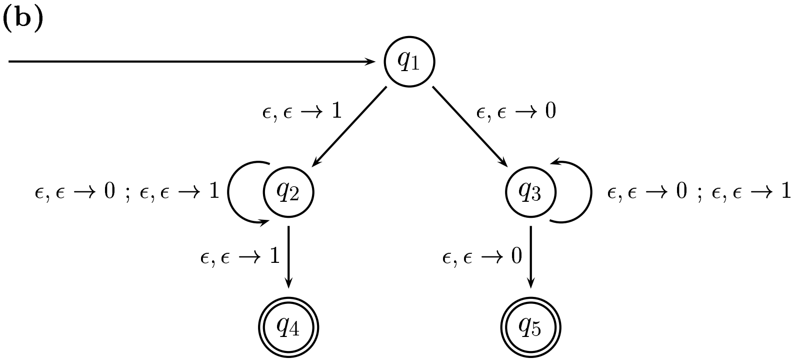
(b)
$C = \overline{\overline{C}} = \overline{\overline{A \cap B}} == \overline{\overline{A} \cap \overline{B}}$
However, then C will be context free.
If $\overline{A}$ and $\overline{B}$ are context free then so are, $\overline{A} \cap \overline{B}$.
Thus $\overline{\overline{A} \cap \overline{B}}$ is context free by these assumptions.
Therefore the class of CFG's are NOT closed under complementation.

$\boxed{2}$ 2.9

$S \rightarrow WY \mid ZX$
$W \rightarrow aW \mid \epsilon$
$X \rightarrow cX \mid \epsilon$
$Y \rightarrow bYc \mid \epsilon$
$Z \rightarrow aZb \mid \epsilon$
Yes the CFG is ambiguous, two different possible derivation's exist for a string like "abc".

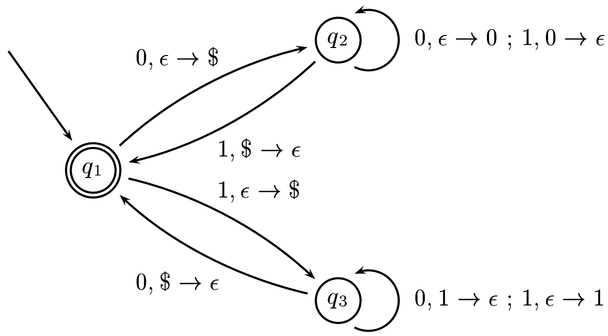
$\boxed{3}$ Sipser: 2.5

(b)



$\boxed{4}$ Give informal descriptions and PDA.

(a) The same number of 0's and 1's no matter what the order.



(b) Any combination of 0's and 1's as long as their are not the same number of each.