Introduction to the Theory of Computation
Homework 2

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Due December, 16th

1. Use structural induction to prove that given an alphabet Σ, for any $x \in \Sigma^*$, $(x^{\mathcal{R}})^{\mathcal{R}} = x$. You may use the following lemma (which you do not have to prove). Given an alphabet Σ, for any $a \in \Sigma$ and $x \in \Sigma^*$, $ax^{\mathcal{R}} = x^{\mathcal{R}}a$.

2. Exercise 1.6(a),(d),(i),(l),(m),(n).

3. Show that the set of *binary* integers (given as strings over $\{\mathtt{0}, \mathtt{1}\}$) that are divisible by 3 is regular, by giving a DFA that recognizes it. Leading `0`s are allowed. The empty string should be accepted. Briefly explain your answer.

4. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, and let k be a positive integer constant. Let L_k be the language defined as follows: $L_k = \{x \in \Sigma^* \mid \text{ the number of } \mathtt{a}\text{s in } x \text{ is divisible by } k\}$.

   For example, $L_2 = \{x \in \Sigma^* \mid x \text{ contains an even number of } \mathtt{a}\text{s}\}$.

   (a) Draw the transition diagram of a DFA that accepts L_3.

   (b) Give a 5-tuple specifying the DFA M_k such that $L(M_k) = L_k$.
   $M_k = (Q_k, \Sigma, \delta_k, q_k, F_k)$ such that ....

5. Exercise 1.32.

6. Exercise 1.7(c).

7. Let $\Sigma = \{\mathtt{0}, \mathtt{1}\}$, and consider the transition table for an NFA below.

q	$\delta(q, \varepsilon)$	$\delta(q, \mathtt{0})$	$\delta(q, \mathtt{1})$
q_0	$\{q_3\}$	$\{q_4\}$	$\emptyset$
q_1	$\emptyset$	$\{q_0\}$	$\emptyset$
q_2	$\emptyset$	$\emptyset$	$\{q_1\}$
q_3	$\{q_2\}$	$\emptyset$	$\{q_6\}$
q_4	$\{q_0\}$	$\emptyset$	$\emptyset$
q_5	$\{q_3\}$	$\emptyset$	$\{q_4\}$
q_6	$\emptyset$	$\{q_5\}$	$\emptyset$

   Calculate $\delta^*(q_0, \mathtt{10})$.

8. Exercise 1.14.

9. Exercise 1.16.