Introduction to the Theory of Computation
Homework #5

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$\boxed{1}$ Proof: By contradiction

- Assume $L = \{a^i b^j | i \neq j\}$ is regular language.
- Let p be the pumping length.
- Now consider the string $s = a^p 1^p$ which is in L.
- Using the pumpig lemma, $s = xyz$ so thate $|xy| = p$
- This means that y consists entirely of a's.
- Considering xz, we have equal the number a's and b's.
- This means the resulting string is still in the language L.
- Hence, L is regular can not be the case.

$\square$

$\boxed{2}$ Let x and y be strings and let L be any language. We say that x and y are distin- guishable by L if some string z exists whereby exactly one of the strings xz and yz is a member of L; otherwise, for every string z, we have $xz \in L$ whenever $yz \in L$ and we say that x and y are indistinguishable by L. If x and y are indistinguishable by L we write $x \equiv_L y$. Show that $\equiv_L$ is an equivalence relation.

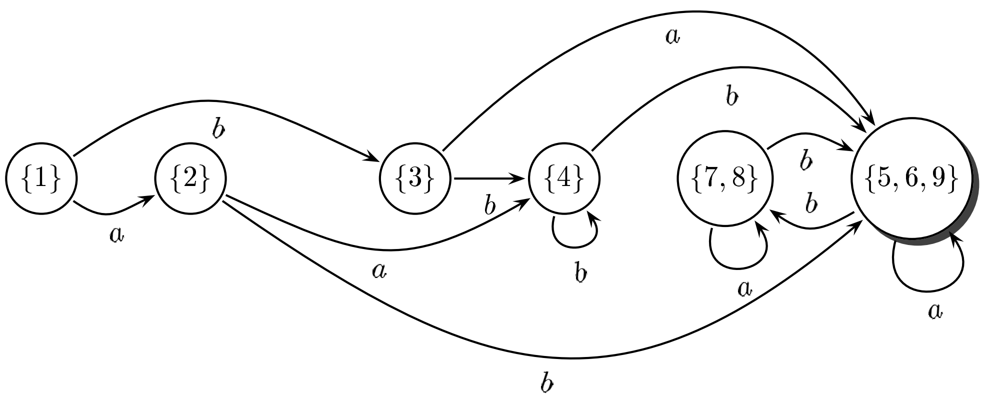
- Let L be a language.
- Let z be a set of strings.
- z is pairwise distinguishable by L if any two strings in z are distinguishable by L.
- The index of L is equal to the number of equivalence classes in L, which can be either finite or infinite.

$\boxed{3}$ We can define a recursive set which infinetly generates all possible strings.

- $\epsilon \in S$
- $S = \{\ ()x \ \text{ or } \ x() \ \text{ or } (x) \mid x \in S\ \}$

$\boxed{4}$ Minimize the DFA.

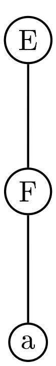
	1	2	3	4	5	6	7	8	9
1	-	-	-	-	-	-	-	-	-
2	x_1	-	-	-	-	-	-	-	-
3	x_1	x_1	-	-	-	-	-	-	-
4	x_1	x_1	x_1	-	-	-	-	-	-
5	x_0	x_0	x_0	x_0	-	-	-	-	-
6	x_0	x_0	x_0	x_0		-	-	-	-
7	x_1	x_1	x_1	x_1	x_0	x_0	-	-	-
8	x_1	x_1	x_1	x_1	x_0	x_0		-	-
9	x_0	x_0	x_0	x_0			x_0	x_0	-



$\boxed{5}$

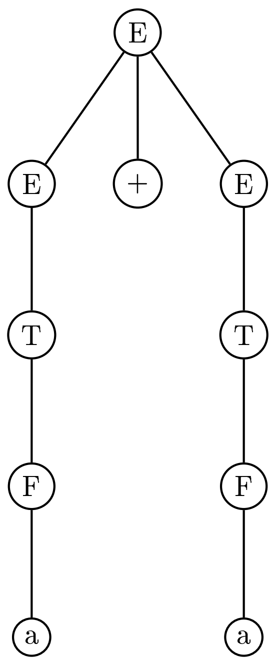
(a) a (left most)

- $E \Rightarrow T$
- $\quad \Rightarrow F$
- $\quad \Rightarrow a$



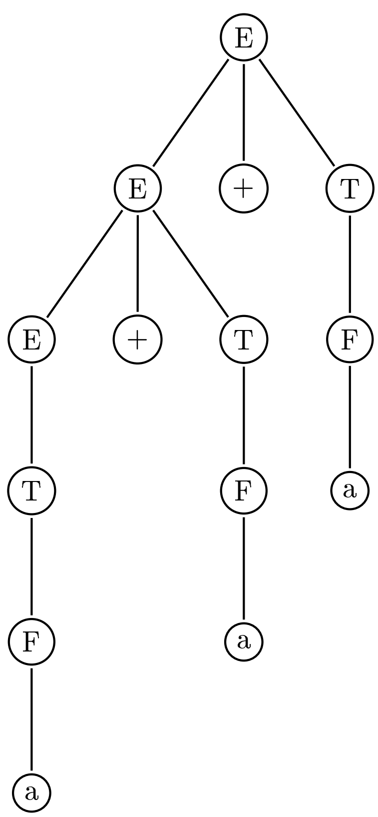
(b) a + a (left most)

- $E \Rightarrow E + T$
- $\quad \Rightarrow T + T$
- $\quad \Rightarrow F + T$
- $\quad \Rightarrow a + T$
- $\quad \Rightarrow a + F$
- $\quad \Rightarrow a + a$



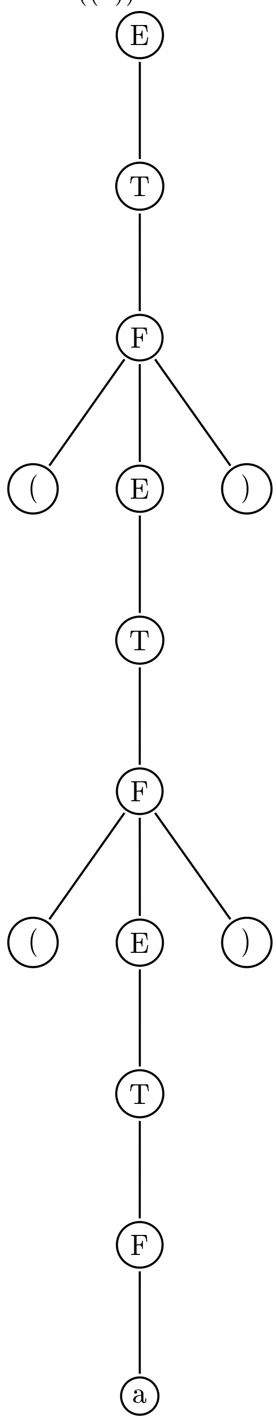
(c) a + a + a (right most)

- $E \Rightarrow E + T$
- $\Rightarrow E + F$
- $\Rightarrow E + a$
- $\Rightarrow E + T + a$
- $\Rightarrow E + T + a$
- $\Rightarrow E + F + a$
- $\Rightarrow E + a + a$
- $\Rightarrow T + a + a$
- $\Rightarrow F + a + a$
- $\Rightarrow a + a + a$



(d) ((a))

$E \Rightarrow T$
$\quad\Rightarrow F$
$\quad\Rightarrow (E)$
$\quad\Rightarrow (T)$
$\quad\Rightarrow (F)$
$\quad\Rightarrow ((E))$
$\quad\Rightarrow ((T))$
$\quad\Rightarrow ((F))$
$\quad\Rightarrow ((a))$



$\boxed{6}$ Sipser 2.4(b): $\{w|w \text{ starts and ends with the same symbol}\}$

$R \to 0R \mid 1R \mid \epsilon$
$S \to 0 \mid 1 \mid 0R0 \mid 1R1$