Introduction to the Theory of Computation
Homework #2

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$\boxed{1}$ `Proof:  By structural induction`

- Observe $(\epsilon^{\mathcal{R}})^{\mathcal{R}} = (\epsilon)^{\mathcal{R}} = \epsilon$
- Assume $ax^{\mathcal{R}} = x^{\mathcal{R}}a$
- Suppose $x = a‘y$

  $((a'y)^{\mathcal{R}})^{\mathcal{R}} = ((a(ya'))^{\mathcal{R}})^{\mathcal{R}}$

  $= (((ay)a'))^{\mathcal{R}})^{\mathcal{R}}$

  $= (a'(ay)^{\mathcal{R}})^{\mathcal{R}}$

  $= ((a'y^{\mathcal{R}})a)^{\mathcal{R}}$

  $= ((a'y^{\mathcal{R}})a)^{\mathcal{R}}$

  $= (y^{\mathcal{R}}a')^{\mathcal{R}}$

  $= (y^{\mathcal{R}}a')^{\mathcal{R}}$

  $= (a'y^{\mathcal{R}})$

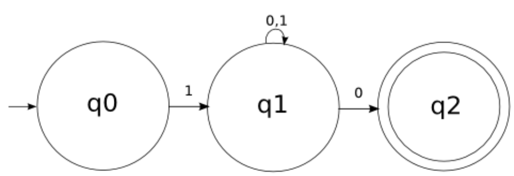
  $= (a'y^{\mathcal{R}})$

  $= a'y$

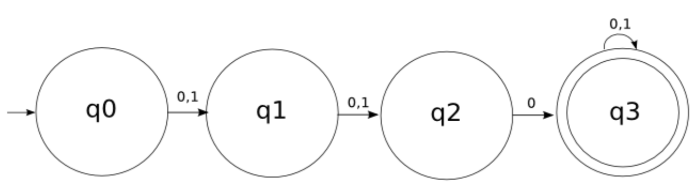
  $\square$

$\boxed{2}$ `Diagrams the DFAs recognizing the following languages.  The alphabet is` $\{0, 1\}$

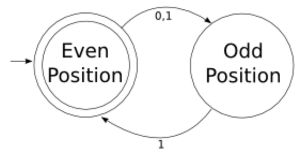
(a) $\{w|w$ `begins with a 1 and ends with a 0`$\}$



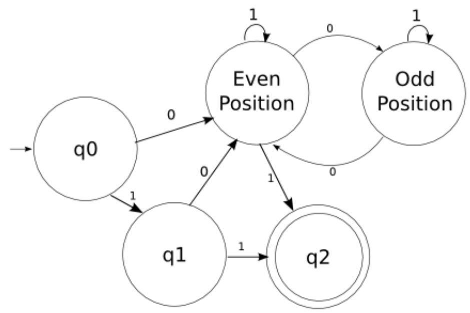
(d) $\{w|w$ `has length at least 3 and its third symbol is a 0`$\}$



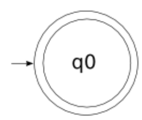
(i) $\{w|$every odd position of w is a 1$\}$



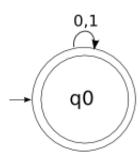
(l) $\{w|w$contains an even number of 0s, or contains exactly two 1s $\}$



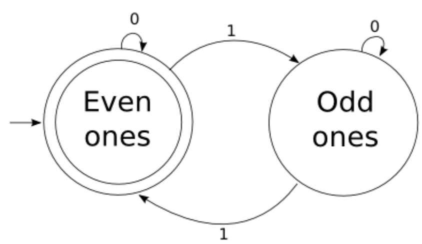
(m) The empty set



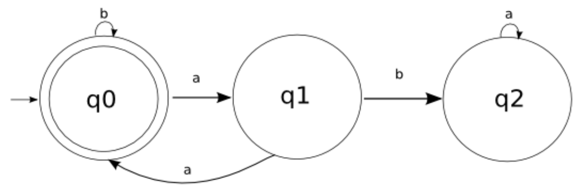
(n) All strings except the empty string



$\boxed{3}$



$\boxed{4}$ **(a)**

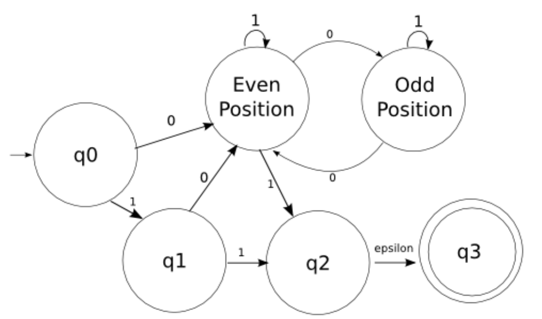


(b)

$$
\begin{gathered}
Q_k = \{q_0, q_1, q_2\} \\
\Sigma_k = \{a, b\} \\
\delta_k(q_0, a) = q_1 \\
\delta_k(q_0, b) = q_0 \\
\delta_k(q_1, a) = q_0 \\
\delta_k(q_1, b) = q_2 \\
\delta_k(q_2, a) = q_2 \\
F_k = \{q_0\}
\end{gathered}
$$

$\boxed{6}$ `Exercise 1.7, Give NFAs with specified number of states`

(c)

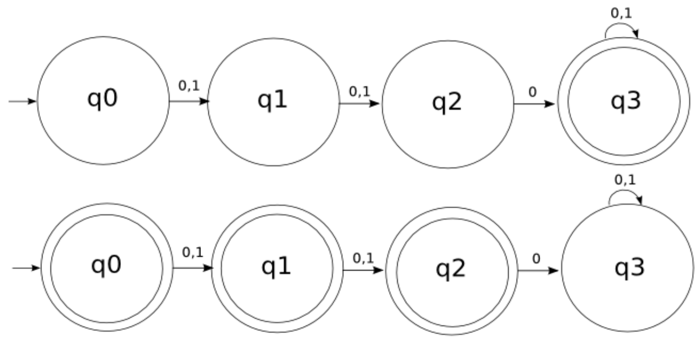


$\boxed{7}$ `Calculate` $\delta^*(q_0, 10)$.

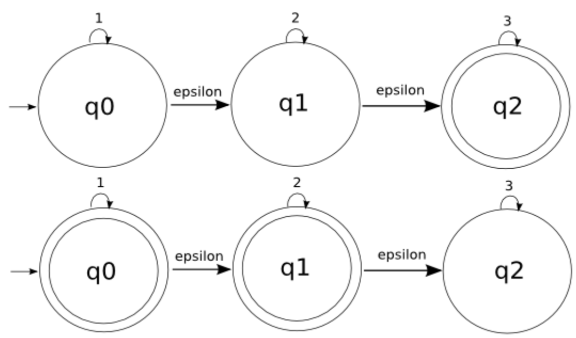
- $\delta^*(q_0, 10) = \{q_3\}$
- $\delta^*(q_3, 10) = \{q_6\}$
- $\delta^*(q_6, 0) = \{q_5\}$
- $\delta^*(q_5, \epsilon) = \{q_3\}$
- $\delta^*(q_3, \epsilon) = \{q_2\}$

$\boxed{8}$

(a)
$\Sigma = \{0, 1\}$



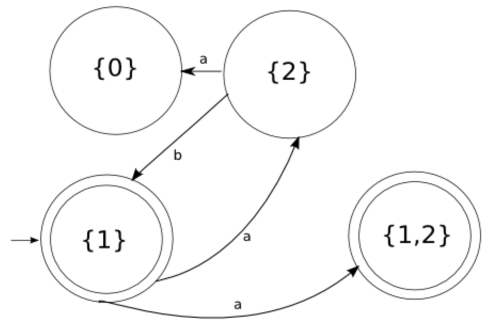
(b)
$\Sigma = \{1, 2, 3\}$



`Yes, the languages a NFA regonizes are closed under complement. The complement language will be recognized because of what we can do with epsilon transitions.`

$\boxed{9}$ Convert the following two NFA's to equivalent DFA's

(a)



(b)