# Introduction to the Theory of Computation
## Homework #4
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$\boxed{1}$ Sipser: 1.15

**(a)** The states of N are the states of N1.

- $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1 = \{q_1\})$

**(b)** The start state of N is the same as the start state of N1.

- $\delta_1(q1, \epsilon) = q1$
- $\delta_1(q1, x \in \Sigma) = q1$
- $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$

**(c)** $F = \{q\} \cup F_1$. The accept states F are the old accept states plus its start state.

- $\delta_1(q2, \epsilon) = q2$
- $\delta_1(q2, x \in \Sigma) = q2$
- $N_1 = (Q_1, \Sigma, \delta_1, q1, F_1 = \{q2\})$

**(d)** Define $\delta$ so that for any $q \in Q$ and any $a \in \Sigma$,

- $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$

$\boxed{2}$ Proof: By Contradiction

- Suppose $A_2 = \{\ www | w \in \{a, b\}^*\ \}$ is regular.
- Then the pumping lemma applies, and there exists a number n so that any $x \in A_2$ can be pumped.
- Pick $x = a^n b a^n b a^n b$ and $y = b^m$
- Since $|x| \geq n$, it follows that $y = uvw$, $|v| \geq 1$, $x(0) \in A_2$.
- Clearly v is a string of a's.
- Hence $x(0) = a^{n-|v|} b a^n b a^n b$
- But, since $n \neq n - |v|$, $x(0) \notin A_2$.
- Since $x(0) \in A_2$ and $x(0) \notin A_2$ are contradictory,
- L regular can not be the case.

$\square$

$\boxed{3}$ Proof: By Contradiction

- Suppose $L = \{\ a^i b^k | 2k \leq i \leq 3k\ \}$ is regular.
- Then the pumping lemma applies, and there exists a number n so that any $x \in L$ can be pumped.
- Pick $x = a^n$ and $y = b^m$
- Since $|x| \geq n$, it follows that $y = uvw$, $|v| \geq 1$, $x(0) \in L$.
- Clearly v is a string of a's.
- Hence $x(0) = a^n b^{m-|v|}$
- But, since $m \neq m - |v|$, $x(0) \notin L$.
- Since $x(0) \in L$ and $x(0) \notin L$ are contradictory,
- L regular can not be the case.

□

$\boxed{4}$

**(a)** False, $a, b^*$ is regular, but it contains a non-regular subset $\{a^n b^n | n \geq 0\}$

**(b)** False, Non-regular languages have finite subsets, and finite languages are regular.

**(c)** Flase, the union of of a language and it's compliment is $\Sigma^*$ which is regular.

**(d)** Flase, A non-regular languages intersected with it's complement is empty, which is regular.

**(e)** False, $L_2$ could just be a subset of $L_1$.