Introduction to the Theory of Computation
Homework 1

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Before you start on the homework, please read the rules on collaboration and submission in the syllabus.

1. Exercise 0.2.
2. Suppose S is a set with n elements.
   (a) How many relations are there on S?
   (b) How many reflexive relations are there on S?
   (c) How many symmetric relations are there on S?
   (d) How many relations are there on S that are both reflexive and symmetric?
3. Exercise 0.11.
4. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. In each part below, a recursive definition is given of $L \subseteq \Sigma^*$. Give a simple non-recursive definition in each case. (Regular expressions as answers are *not* allowed.)
   (a) $\mathtt{a} \in L$; $x\mathtt{a} \in L$ if $x \in L$; $x\mathtt{b} \in L$ if $x \in L$.
   (b) $\mathtt{a} \in L$; $\mathtt{b}x \in L$ if $x \in L$; $x\mathtt{b} \in L$ if $x \in L$.
5. Give recursive definitions of each of the following sets.
   (a) The set $\mathcal{N}$ of all natural numbers.
   (b) The set S of all natural numbers divisible by 7.
   (c) The set A of all strings in $\{\mathtt{a}, \mathtt{b}\}^*$ containing the substring $\mathtt{aa}$.
6. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Give a recursive definition of function $n_{\mathtt{b}}$ on Σ^* such that for all strings $x \in \Sigma^*$, $n_{\mathtt{b}}(x)$ is the number of $\mathtt{b}$s in x.
7. Use structural induction to show that given an alphabet Σ, for any $x, y \in \Sigma^*$, $|xy| = |x| + |y|$.