Appendix A: Quadrotor Design Model

This application covers the optimization of a quadrotor's structural and propulsive platform with respect to performance with respect to a mission. The components considered for optimization were the motor, battery, propellers, support rods, ESCs, landing skid, and mission characteristics. While many other components effect the performance and capabilities of the quadrotor, (the controller, housing, onboard electronics, wiring, etc.), these parts were chosen because they had characteristics which were most easy to quantify and model. For our model, other choices are assumed to be made before optimization based on their unique capabilities. Additionally, this model neglects the most studied problem in quadrotor design–the control problem. While an optimal control system may have a large impact on the overall performance above or below the estimated performance, it is treated as a separate problem from our design application.

The following sections describe how the individual characteristics of each component are modelled and how each of these models are combined into a single black-box model of performance with respect to a mission.

A.1 Motor Characteristics

Motors are considered to be "designed" based on selection from a table or catalog. This decision variable is denoted by x_m, and defines the resulting parameters of the motor ($k_{v,b}, R_{0,b}, I_{0,b}$) the performance constraints ($P_{m_{max}}$ and $I_{m_{max}}$), mass, area, and cost of the motor (M_m, A_m, and C_m).

$$x_m \rightarrow k_{v,b}, R_{0,b}, I_{0,b}, P_{m_{max}}, I_{m_{max}}, M_m, A_m, C_m \tag{1}$$

A.2 Battery Design

The battery is defined based on three decision variables: the cell choice x_{bc}, number of cells used in series x_{bs}, and the number of cells used in parallel x_{bp}. The cell choice x_{bc} determines the capacity mAh, discharge rate C, mass M_c, and cost C_c of each cell used in the battery.

$$x_{bc} \rightarrow mAh, C, M_c, C_c \tag{2}$$

Each cell is assumed to give the nominal 3.7 volts of voltage, which makes it possible to calculate the total voltage of the battery V_b from the number of cells in series x_{bs}. Additionally, the total battery such as mass M_b, cost C_b, current available I_{bmax}, and energy E_b are calculated from the characteristics of the cells and the number in series x_{bs} and parallel x_{bp}.

$$V_b = 3.7v * x_{bs} \tag{3}$$
$$M_b = M_c * N_{bs} * N_{bp} \tag{4}$$
$$C_b = C_c * N_{bs} * N_{bp} \tag{5}$$
$$I_{bmax} = C * 1000 * mAh * N_{bp} \tag{6}$$
$$E_b = V * 1000 * 3600 * mAh * N_{bp} \tag{7}$$

A.3 Propeller Design

The propeller is defined by six decision variables: the diameter x_{pd}, average angle x_{pan}, twist x_{ptw}, average chord x_{pc}, taper x_{pta}, and airfoil x_{pai}. The choice of airfoil gives a number of aerodynamic characteristics c_{l0}, c_{la}, c_{lmin}, c_{lmax}, c_{d0}, c_{d2}, c_{clcd0}, c_{Reref}, c_{Reexp}, and the thickness-to-chord ratio th_p, as shown.

$$x_{pai} \rightarrow c_{l0}, c_{la}, c_{lmin}, c_{lmax}, c_{d0}, c_{d2}, c_{clcd0}, c_{Reref}, c_{Reexp}, th_p \tag{8}$$

Each of these coefficients must be determined based on airfoil data and placed in a table before running the optimization.

The various geometric characteristics can also be estimated, such as the average thickness $t_{p,ave}$, cross-sectional area A_p, volume V_p, chord at the root $c_{p,root}$, thickness at the root $t_{p,root}$, area moment of inertia at the root $I_{p,root}$, mass M_p, and cost

C_p.

$$
\begin{aligned}
t_{p,ave} &= th_p * x_{pc} && (9)\\
A_p &= 0.6 * t_{p,ave} * x_{pc} && (10)\\
V_p &= A_p * x_{pd} && (11)\\
c_{p,root} &= 2 * x_{pc}/(x_{pta}+1) && (12)\\
t_{p,root} &= c_{p,root} * th_p && (13)\\
I_{p,root} &= 0.036 * t_{p,root}^3 * c_{p,root} && (14)\\
M_p &= V_p * \rho_p && (15)\\
C_p &= V_p * \rho_{p,cost} && (16)\\
& && (17)
\end{aligned}
$$

Note that the density ρ_p, cost density $\rho_{p,cost}$, yield strength $S_{p,y}$, and young's modulus E_p must be provided assuming a specific material. For these tests the values of $1190 kg/in$, $17697\$/in$, $55 mPa$, and $2 GPa$, are used, assuming the propeller is made of polycarbonate.

A.4 Support Rod Design

The support rods are defined by the decision variables of thickness x_{rt}, width x_{rw}, height x_{rh}, material x_{rm}, and the variable used to construct the propeller x_{pd} as well as the residual quantity $x_{res,w}$. The length of the rod L_r is:

$$
L_r = max(\frac{1.25 * x_{pd}}{\sqrt{2}} - \frac{x_{res,w}}{2}, 0.01) \quad (18)
$$

The material x_{rm} determines the Young's modulus E_r, ultimate strength $S_{r,ut}$, yield strength $S_{r,y}$, density ρ_r, and cost density $\rho_{r,c}$ of the material.

$$
x_{rm} \rightarrow E_r, S_{r,ut}, S_{r,y}, \rho_r, \rho_{r,c} \quad (19)
$$

The geometry and structural characteristics of the rod can be constructed using the following equations.

$$
\begin{aligned}
W_{r,in} &= x_{rw} - x_{rt} && (20)\\
H_{r,in} &= x_{rh} - x_{rt} && (21)\\
A_{r,xs} &= x_{rw} * x_{rh} - W_{r,in} * A_{r,xs} && (22)\\
V_r &= A_{r,xs} * L_r && (23)\\
M_r &= V_r * \rho_r && (24)\\
C_r &= V_r * \rho_{r,c} && (25)\\
I_{r,x} &= \frac{1}{12} * x_{rw} * x_{rh}^3 - \frac{1}{12} * W_{r,in} * H_{r,in}^3 && (26)\\
I_{r,y} &= \frac{1}{12} * x_{rw}^3 * x_{rh} - \frac{1}{12} * W_{r,in}^3 * H_{r,in} && (27)\\
K_{r,y} &= \frac{3 * I_{r,x} * E_r}{L_r^3} && (28)\\
K_{r,x} &= \frac{3 * I_{r,y} * E_r}{L_r^3} && (29)\\
A_{r,pl} &= x_{rh} * x_{rw} && (30)\\
& && (31)
\end{aligned}
$$

A.5 ESC Characteristics

Electronic speed controls (ESCs) are also chosen from a table or catalog. This decision variable is denoted by x_e, and defines the cost C_e, mass M_e, current limit $I_{max,e}$, and compatibility constraints $V_{max,e}$ and $V_{min,e}$ of the ESC.

$$x_e \rightarrow C_e, M_e, I_{max,e}, V_{max,e}, V_{min,e} \tag{32}$$

A.6 Landing Skid Characteristics

The landing skid is defined by the decision variables of material $x_{s,m}$, leg angle $x_{s,\theta}$, diameter $x_{s,d}$, and thickness $x_{s,t}$, as well as the residual quantities representing the frame width $x_{res,w}$ and payload height $x_{res,plh}$.

As with the support rod, the material choice defines the various material properties of the landing skid.

$$x_{sm} \rightarrow E_s, S_{s,ut}, S_{s,y}, \rho_s, \rho_{s,c} \tag{33}$$

The geometry and structural characteristics of the rod can be constructed using the following equations.

$$D_s = \frac{x_{res,h} + 0.02}{tan(90 - x_{s,\theta})} \tag{34}$$

$$L_{s,ll} = sqrt((x_{res,h} + 0.02)^1 + 2 * D_s^2) \tag{35}$$

$$L_{s,tot} = 4 * L_{s,ll} + 2 * x_{res,w} + 2 * D_s \tag{36}$$

$$A_s = \frac{pi}{2} * x_{s,d}^2 - \frac{pi}{2} * (x_{s,d} - x_{s,t})^2 \tag{37}$$

$$V_s = L_{s,tot} * A_s \tag{38}$$

$$C_s = V_s * \rho_{s,c} \tag{39}$$

$$M_s = V_s * \rho_s \tag{40}$$

$$A_{s,pl} = 2 * x_{s,d} * L_{s,ll} \tag{41}$$

$$I_s = \frac{\pi}{4} * (x_{s,d})^4 - (x_{s,d} - x_{s,t})^4 \tag{42}$$

$$K_s = 2 * \frac{3 * I_s * E_s}{D_s^3} \tag{43}$$

$$\tag{44}$$

A.7 Residual Quantities

A number of quantities of the design are taken as givens. These are shown in the Table 1.

A.8 System Characteristics

The mass, planform area,natural frequency, and cost of the system can be computed from the sum of the attributes of the components.

$$M_{sys} = 4 * M_m + M_b + 4 * M_r + 4 * M_e + M_s + x_{res,m} \tag{45}$$

$$A_{sys,pl} = 4 * A_{r,pl} + 4 * A_m + A_{s,pl} + x_{res,As} \tag{46}$$

$$f_{sys,nx} = sqrt(\frac{K_{r,x}}{0.5 * M_r + M_m + M_p})/2 * \pi \tag{47}$$

$$f_{sys,ny} = sqrt(\frac{K_{r,y}}{0.5 * M_r + M_m + M_p})/2 * \pi \tag{48}$$

$$C_{sys} = 4 * C_r + 4 * C_m + C_b + 4 * C_p + 4 * C_e + C_s + x_{res,c} \tag{49}$$

$$\tag{50}$$

Symbol	Meaning	Value
$x_{res,m}$	Mass	$0.1212kg$
$x_{res,w}$	Frame Width	$0.102m$
$x_{res,h}$	Frame Height	$0.0360m$
$x_{res,c}$	Cost	\$480
$x_{res,P}$	Power Use	$10.4W$
$x_{res,A}$	Plan-form Area	$0.0104m^2$
$x_{res,As}$	Planform Area (side view)	$0.0037m^2$
$x_{res,plh}$	Payload Height	$0.0360m$

Table 1: Residual Quantities

Then the performance of the system at various operational characteristics can be modelled. This is done by entering data into Qprop and reading the results. This is done for a hovering, climbing, and flight characteristic.

A.8.1 Hovering

The hovering characteristic is defined by each motor providing the thrust required to hold the craft at equilibrium with zero velocity. This corresponds to:

$$T_{req} = 9.81 * M_s ys/4 \tag{51}$$
$$vel = 0 \tag{52}$$

which are quantities used in Qprop for this characteristic.

A.8.2 Climbing

The climbing characteristic is defined by each motor providing the thrust required to hold the craft at equilibrium with the climbing velocity $x_{o,vc}$, which is considered an “operational” decision variable controlling the drag. This drag is calculated as:

$$D = 0.5 * 1.225 * 1.5 * A_{sys,pl} * x_{o,vc} \tag{53}$$

This corresponds to:

$$T_{req} = 9.81 * (M_s ys + D)/4 \tag{54}$$
$$vel = x_{m,vc} \tag{55}$$

which are quantities used in Qprop for this characteristic.

A.8.3 Forward Flight

The forward flight characteristic is defined by each motor providing the correct thrust to not move in the vertical direction while at the travel angle of $x_{o,\theta}$. The total thrust required is then:

$$T_{req} = \frac{M_s ys * 9.81}{cos(x_{o,\theta}) * 4} \tag{56}$$

The velocity in the x-direction then:

$$vel_x = \frac{sqrt(4 * T_req * sin(x_{o,\theta}))}{0.5 * 1.5 * 1.225 * A_{sys}} \tag{57}$$

And the velocity used in Qprop is then:

$$vel = vel_x * sin(x_{o,\theta}) \tag{58}$$

T_{req} and vel are used in Qprop for the forward flight characteristic.

A.8.4 Operational Characteristics Modelling

Qprop is used to model the propulsion system at different velocities vel and thrust requirements T_{req}. Using these parameters, and motor and propeller characteristics, QProp finds the power use P, actual thrust T, current I, and rotational speed RPM.

$$\begin{matrix} T_{req}, vel, \\ k_v, R_0, I_0, \\ Cl_0, Cl_a, Cl_{min}, Cl_{max}, \\ Cd_0, Cd_2, Clcd_0, \\ \vec{R}, \vec{\alpha}, \vec{c} \end{matrix} \rightarrow P_{oper}, T_{oper}, I_{oper}, RPMoper, V_{oper}, J_{oper} \tag{59}$$

A.9 Constraints

For the battery, the constraint values are calculated using the operational characteristics I_{oper} and V_{oper}. For the motor, the constraint values are calculated using the operational characteristics I_{oper} and P_{oper}. For the propeller, the constraint values are constructed from the operational characteristic T_{req}. The stress and the maximum allowable stress is then calculated using:

$$M = T_{req}/2 * x_{pd}/2 \tag{60}$$

$$\sigma_p = M * 0.5 * \frac{t_{p,root}}{I_{p,root}} \tag{61}$$

$$\sigma_{p,max} = S_{p,y} * 2 \tag{62}$$

$$\tag{63}$$

For the support rod, the constraint values are calculated using the operational characteristics RPM_{oper} T_{req}, and J_{oper} and . This gives the following constraint quantities:

$$f_f = RPM_{oper}/60 \tag{64}$$

$$\delta_x = T_{req}/K_{r,x} \tag{65}$$

$$\sigma_x = T_{req} * L_r * 0.5 * x_{rh}/I_{r,x} \tag{66}$$

$$\sigma_y = 0.5 * J_{oper} * x_{rw}/I_{r,y} \tag{67}$$

$$\tag{68}$$

The ESC uses the operational value I_{oper} in its constraints. The landing skids constraints are based on force transfer in the event of a fall. This force Fs is calculated by calculating the deflection δ_s.

$$\delta_s = sqrt(\frac{M_{sys}}{(2 * K_s}) * V_s \tag{69}$$

$$F_s = \delta_s * K_s \tag{70}$$

where the impact velocity V_s is given as $4.4m/s$.

A.10 Mission Characteristics

The performance with respect to a mission may then be calculated based on the flight performance at each operational characteristic. For the given mission, the quadrotor must fly to a height of $h_{poi} = 30m$, travel to points of interest $d_{poi} = 30m$ apart with value $V_{poi} = 100$ for $t_{poi} = 60s$. The operational cost is $c_{oper} = 0.01\$/s$.

The time and energy used in climbing and descending is then calculated to be:

$$t_{climb} = h_{poi}/V_{climb} \tag{71}$$
$$E_{climb} = (4 * P_{climb} + x_{res,P}) * t_{climb} \tag{72}$$
$$t_{descend} = t_{climb} \tag{73}$$
$$E_{descent} = E_{climb} \tag{74}$$
$$\tag{75}$$

assuming the same time and energy are used in descent as in climbing.

The energy available for the mission may then be calculated as:

$$E_{avail} = E_b - (E_{climb} + E_{descent}) \tag{76}$$

The energy used to observe each point of interest may be calculated based on the energy of hovering for a fixed amount of time and flying between points:

$$E_{hover} = (4 * P_{hover} + P_{sys}) * t_{poi} \tag{77}$$
$$t_{flight} = d_{poi}/V_{flight} \tag{78}$$
$$E_{flight} = (4 * P_{flight} + P_{sys}) * t_{flight} \tag{79}$$
$$E_{poi} = E_{hover} + E_{flight} \tag{80}$$
$$\tag{81}$$

Finally, the number of points of interest that can be observed, the time taken to perform the mission, and the resulting mission value $R_{mission}$ can be calculated.

$$n_{poi} = \frac{E_{avail}}{E_{poi}} \tag{82}$$
$$t_{mission} = t_{climb} + t_{descend} + n_{poi} * (t_{flight} + t_{poi}) \tag{83}$$
$$R_{mission} = n_{poi} * V_{poi} - t_{mission} * c_{oper} \tag{84}$$
$$\tag{85}$$

Finally, the objective f, the total profit of the quadrotor, may be calculated based on the assumption that ten of these missions are performed in quick succession.

$$f = -10 * R_{mission} + C_{sys} \tag{86}$$