A Appendix: Quadrotor Design Problem Formulation

A.1 Problem Definition

This application covers the optimization of a quadrotor's structural and propulsive platform with respect to flight performance and cost. The main components considered were the battery, motors, propellers, and support rods. These components were chosen because they are the primary drivers of the potential flight performance of the craft and the primary intersecting structural consideration.

To simplify quadrotor design to the conceptual level, many interrelated design problems were not considered. The effect of several components (the controller, ESCs, housing, wiring, landing skid, etc.) were simply estimated under the assumption that different choices would have similar impact, or that those parts could be designed in conjunction with the rest of the craft without a large impact on the performance of the platform with respect to the objectives considered. Notably, the most often studied problem in quadrotor design has to do with the control system of the craft. While an optimal control system may have a large impact on the overall performance above or below the estimated performance, it is still treated as a separate problem from our design application.

A.2 Motor Design

For the motor, only one choice is made–a choice of several motors from a catalog (taken from the T-Motor Professional Series). This choice, A_m, gives the defining parameters of the motor model (k_v, R_0, I_0) as well as the performance constraints ($P_{m_{max}}$ and $I_{m_{max}}$), mass, area, and cost of the motor (M_m, A_m, and C_m).

$$A_m \rightarrow k_v, R_0, I_0, P_{m_{max}}, I_{m_{max}}, M_m, A_m, C_m \tag{1}$$

These parameters are then entered into the performance and constraint calculations in Section A.7.

A.3 Battery Design

For the battery, we assume several choices are made: the cells to be used and the number of cells to be used in parallel and series. From these choices, the total performance characteristics of the battery can be constructed. The cell choice A_c determines the capacity mAh, discharge rate C, mass M_c, and cost C_c of each cell used in the battery.

$$A_c \rightarrow mAh, C, M_c, C_c \tag{2}$$

Since each cell is designed to give (nominally) 3.7 volts of voltage, the total voltage V of the battery can be calculated from the number of cells in series N_s. The total mass and cost of the battery M_b and C_b can be calculated from the mass M_c and C_c of each cell multiplied by the number of cells $N_s * N_p$. The current available I_{bmax} can be calculated from the discharge rate of each cell

mAh, the C rating of each cell C, and the number of cells used in parallel N_s. Finally, the energy E available may be calculated from the voltage V available, the capacity of each cell mAh, and the number of cells in parallel N_p. Each of these performance characteristics are then entered into the performance and constraint calculations in Section A.7.

$$V = 3.7v * N_s \tag{3}$$
$$M_b = M_c * N_s * N_p \tag{4}$$
$$C_b = C_c * N_s * N_p \tag{5}$$
$$I_{bmax} = C * 1000 * mAh * N_p \tag{6}$$
$$E = V * 1000 * 3600 * mAh * N_p \tag{7}$$

A.4 Propeller Design

For the propeller, the choice of airfoil A_a, diameter D_p, blade angles α_r and α_t and chords C_r and C_t are used to construct the properties of the propeller. The choice of airfoil A_a determines the lift and drag characteristics of the blade at each angle, as shown in equation 8. The coefficients for these relationships (obtained from xfoil), are then used in QProp. It also specifies the airfoil's ratio of thickness to chord tc. Additionally, the radius vector $\vec{R}$ and the chord $\vec{c}$ and angle vectors $\vec{\alpha}$ needed by qprop are constructed from the diameter of the propeller D_p and the dimensions of each at the root (C_r and α_r) and tip of the propeller blade (C_t and α_t), assuming they change linearly across the blade.

$$A_a \rightarrow Cl_0, Cl_a, Cl_{min}, Cl_{max}, Cd_0, Cd_2, Clcd_0, tc \tag{8}$$
$$\vec{R} = [0.02, ..., D_p/2] \tag{9}$$
$$\vec{\alpha} = \alpha_r + \vec{R} * (\alpha_t - \alpha_r)/(.5D_p) \tag{10}$$
$$\vec{c} = c_r + \vec{R} * (c_t - c_r)/(.5D_p) \tag{11}$$

These vectors are entered into Qprop at each iteration to calculate the flight characteristics of the propulsion system.

The mass M_p and cost C_p may also be calculated by estimating the volume V_p of the propeller. To find this, the average chord c_{avg} and thickness t_{avg} of the propeller must be calculated from the root and tip dimensions, as well as the thickness ot chord ratio tc. Then the cross-sectional area A_{xs} may be estimated (assuming the airfoil is a roughly triangular section), as well as the volume V_p, cost C_p, and mass M_p:

$$c_{avg} = (c_r - c_t)/2 \tag{12}$$
$$t_{avg} = tc * c_{avg} \tag{13}$$
$$A_{xs} = 0.5 * t_{avg} * c_{avg} \tag{14}$$
$$V_p = A_{xs} * D_p \tag{15}$$
$$C_p = cd * V_p \tag{16}$$
$$M_p = \rho * V_p \tag{17}$$

where cd is the cost density of the propeller assumed to be $17700\$/m^3$ (or $.29\$/in^3$), and ρ is the density of the propeller assumed to be $1190kg/m^3$. This is based on the assumption that the propeller is made out of polycarbonate.

A.5 Support Rod Design

For the support rod, the design is defined by the choice of material, wall thickness, and diameter. Similar to the rest of the "black box" model used in the formulation (where equality constraints between components are considered a part of the model, rather than constraints which must be enforced) the length L_r is determined to the value required to keep the propellers a safe distance from each other. It can then be calculated using the propeller diameter D_p and frame width W_{res}.

$$L_r = max(\frac{1.25 * D_p}{\sqrt{2}} - \frac{W_{res}}{2}, 0.01) \tag{18}$$

The material choice A_{mat} (Aluminum, Titanium, Polycarbonate, or Nylon) determines the Young's modulus E_y, density ρ, ultimate strength S_{ut}, and cost density cd of the material used in the rod. Note that while carbon fiber is a commonly used material in quadrotor design, it was was not considered in our application because the strength of a carbon fiber rod is not necessarily dictated by the wall thickness as it is with other materials. Then the cross-sectional area A_r, area moment of inertia I, and bending stiffness k may be calculated. Each of these quantities help define the structural performance of the rod, and are entered into the constraint calculation. Additionally, the cost C_r and mass M_r may be calculated from the length L, cross-sectional area A_r, and mass/cost density (ρ or cd) of the material.

$$A_{mat} \rightarrow E_y, \rho, S_{ut}, cd \tag{19}$$
$$A_r = \pi * 0.5 * (d^2 - (d - t)^2) \tag{20}$$
$$I = \pi * (d^4 - (d - t)^4)/64 \tag{21}$$
$$k = (3 * I * E_y)/L^3 \tag{22}$$
$$M_r = A_r * L * \rho \tag{23}$$
$$C_r = A_r * L * cd \tag{24}$$

A.6 Residual Quantities

To capture the impact of the components not being designed, a series of residuals have been added to simulate the impact of the rest of the system, including those listed in Table 1.

A.7 System Modelling

These component characteristics may then be used to find the overall system characteristics. The system mass M_{sys}, planform area A_{sys}, and cost C_{sys} are

Residual Quantity	Value
Residual mass	$M_{res} = 0.3kg$
Residual power use	$P_{res} = 5W$
Frame width	$W_{res} = 7.5cm$
Residual planform area	$A_{res} = W_{res}^2 = 56.25cm^2$
Residual cost	$C_{res} = 50$

Table 1: Residual quantities in the design.

all the sums of the components and residuals mentioned in the previous sections.

$$M_{sys} = 4 * (M_m + M_p + M_r) + M_b + M_{res} \tag{25}$$

$$A_{sys} = 4 * (A_m + A_r) + A_{res} \tag{26}$$

$$C_{sys} = 4 * (C_m + C_p + C_r) + C_b + C_{res} \tag{27}$$

Qprop is used to model the performance of the propulsion system. For the purposes of this paper, it will be used to find the performance of the propulsion system at different velocities vel and thrust requirements T_{req}. Using these parameters, as well as the characteristics of the motor and propeller, QProp finds the power use P, actual thrust T, current I, and rotational speed RPM.

$$\begin{matrix} T_{req}, vel, \\ k_v, R_0, I_0, \\ Cl_0, Cl_a, Cl_{min}, Cl_{max}, \\ Cd_0, Cd_2, Clcd_0, \\ \vec{R}, \vec{\alpha}, \vec{c} \end{matrix} \quad \rightarrow P, T, I, RPM \tag{28}$$

A.7.1 Hovering Characteristics

The hovering characteristic is modelled in order to determine the value of the “maximize hover time” objective. This characteristic is defined by a flight velocity of $vel_h = 0$ and a thrust requirement $T_{req,h}$ defined by the fraction of the total mass of the system the propeller must hold up.

$$T_{req,h} = M_{sys} * 9.81/4 \tag{29}$$

Note that if the thrust requirement cannot be met, this method reports a failure which corresponds to a specific low reward value less than or equal to Q_{init}, the lowest possible learned value.

A.7.2 Climbing Characteristics

The climbing characteristic is modelled in order to determine the value of the ”minimize power use in a climb” objective. This characteristic is defined by $vel_c = 10m/s$. Since the quadrotor is a blunt object, it generates drag D_c estimated by the equation:

$$D_c = \rho * C_d * A_{sys} * vel_c^2 \tag{30}$$

where ρ is the density of air ($1.225kg/m^3$) and $C_d = 1.5$ is the assumed coefficient of drag. The thrust requirement may then be calculated as the combination of the mass M_{sys} and drag D_c held by each individual motor:

$$T_{req,h} = (D_c + M_{sys} * 9.81)/4 \tag{31}$$

A.8 Structural Characteristics

Using the data calculated previously, the performance of the support rod may be modelled. First, the RPM value may be used to calculate the frequency f_{motor} of the motor. Then the natural frequency f_n of the rod-mass system (assuming the rod acts as a cantilever beam with stiffness k and masses $0.5 * M_r + M_m$) may be calculated, as well as the deflection of the rod δ when thrust T_{oper} is applied on the beam with stiffness k.

$$f_{motor} = RPM_h/60 \tag{32}$$

$$f_n = \frac{1}{2 * \pi} * \sqrt{\frac{k}{0.5 * M_r + M_m}} \tag{33}$$

$$\delta = T_{oper}/k \tag{34}$$