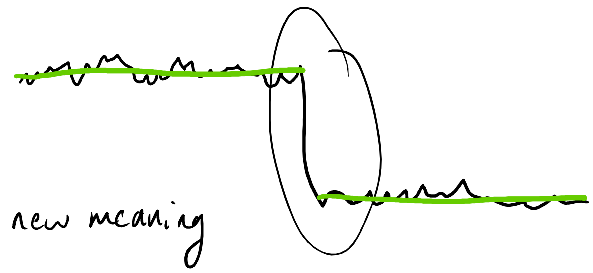
Unit roots/nonstationary data

Wednesday, September 27, 2023 2:38 PM

model nonstationary processes
- unit roots / random walks
- structural breaks $\longrightarrow$



random walks & unit roots

$\rightarrow$ trend & intercept take on new meaning

$p_t = \log(P_t)$ $\qquad a_t \sim$ white noise

$$p_t = p_{t-1} + a_t$$

$$(1 - \phi_1 L) p_t = a_t \qquad \phi_1 = 1$$

repeated substitution

$$p_t = p_{t-2} + a_{t-1} + a_t$$

$$p_t = p_{t-3} + a_{t-2} + a_{t-1} + a_t$$

permanent past shocks

$$= a_t + a_{t-1} + a_{t-2} + a_{t-3} + \dots$$

$$Var(p_t) = Var(a_t + a_{t-1} + a_{t-2} + \dots)$$

$$= \sigma_a^2 + \sigma_a^2 + \sigma_a^2 + \dots \longrightarrow \infty$$

Forecast $\rightarrow$ random walk not mean revert

$$\hat{p}_h(1) = E(p_{h+1} \mid p_h, p_{h-1}, p_{h-2}, \dots) = E(p_h \mid p_h) + E(a_{h+1} \mid p_h)$$

$$= p_h + 0$$

b/c $\quad p_{h+1} = p_h + a_{h+1}$

$$\hat{p}_h(2) = p_h$$

recall $\quad p_{h+2} = p_h + a_{h+1} + a_{h+2}$

$$e_h(\ell) = a_{h+1} + a_{h+2} + \dots + a_{h+\ell}$$

$$Var(e_h(\ell)) = Var(a_{h+1} + a_{h+2} + \dots + a_{h+\ell}) = \ell\,\sigma_a^2$$

forecast error variance is growing with ~~time~~ forecast horizon

Random walk with drift (not trend)

$$p_t = \mu + p_{t-1} + a_t$$

μ ↑ average growth / decline rate of log price

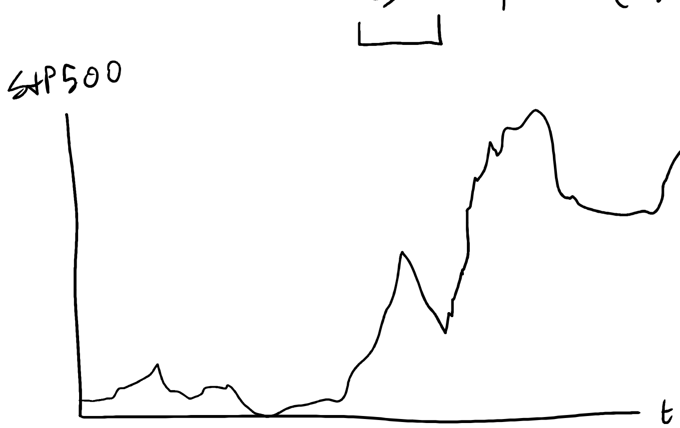
$$p_t - p_{t-1} = r_t = \mu + a_t$$

repeated substitution

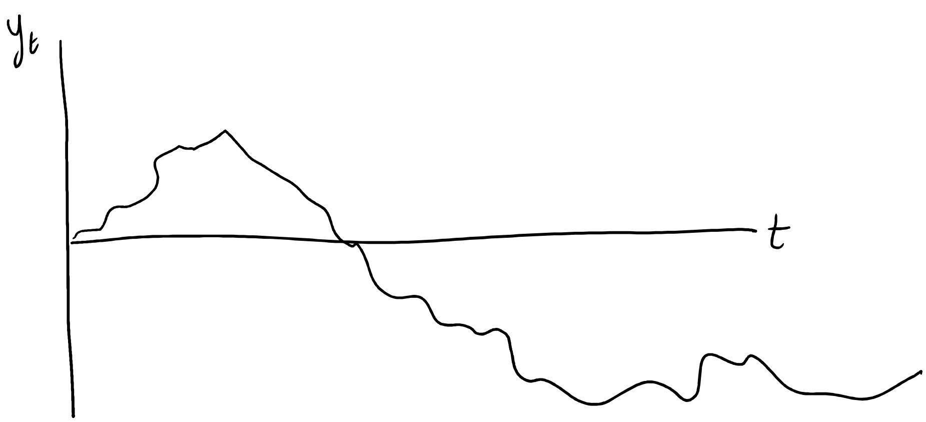
$$p_t = \mu + (\mu + p_{t-2} + a_{t-1}) + a_t$$

$$p_t = \mu + \mu + (\mu + p_{t-3} + a_{t-2}) + a_{t-1} + a_t$$

$$= \underbrace{t\mu} + p_0 + (a_1 + a_2 + \dots + a_t)$$



sometimes hard to tell w/ eyeball test



Recap: MA(q) stationary : intercept = mean

AR(p) stationary or ARMA(p,q) : intercept proportional to mean

Random walk: intercept is average drift rate (growth or decline)

Alternative model for growth/shrinking over time:

$$p_t = \beta_0 + \beta_1 t + r_t \quad \text{where } r_t \text{ is stationary}$$

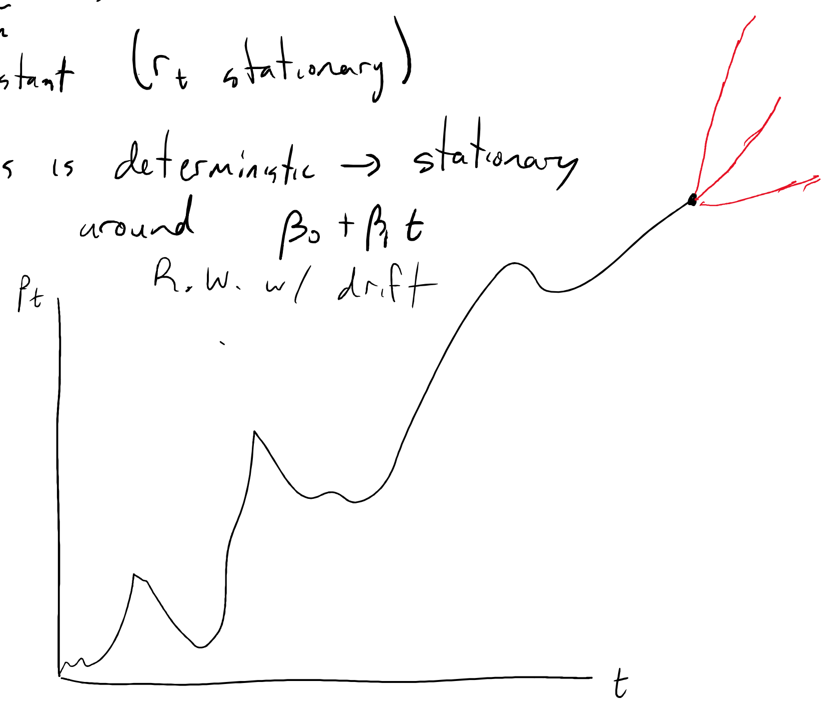
How to distinguish R.W. w/ drift from stationary around a deterministic trend

$$Var(p_t) = Var(\underbrace{\beta_0 + \beta_1 t}_{\text{not random}} + r_t)$$

$$= Var(r_t) = \text{constant} \quad (r_t \text{ stationary})$$

this says progress is deterministic → stationary around $\beta_0 + \beta_1 t$

$p_t = \beta_0 + \beta_1 t + r_t$



$p_t = \mu + p_{t-1} + a_t$

Unit root testing

- want a reasonable description of the data under the null hypothesis <u>and</u> the alternative hypothesis

Consider: a possible R.W. without drift

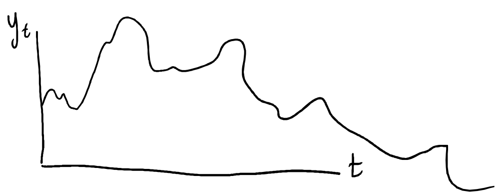
<u>Case 1</u> (in R: type = "none") <u>Case 2</u>: (in R: type = "drift")

$$y_t = \rho y_{t-1} + \varepsilon_t$$

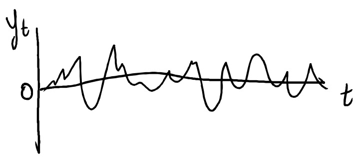
$H_0 : |\rho| = 1$

$H_a : |\rho| < 1$

Null:



Alternative:



$$y_t = \phi_0 + \rho y_{t-1} + \varepsilon_t$$

$H_0 : \phi_0 = 0 , |\rho| = 1$ unit root with no drift

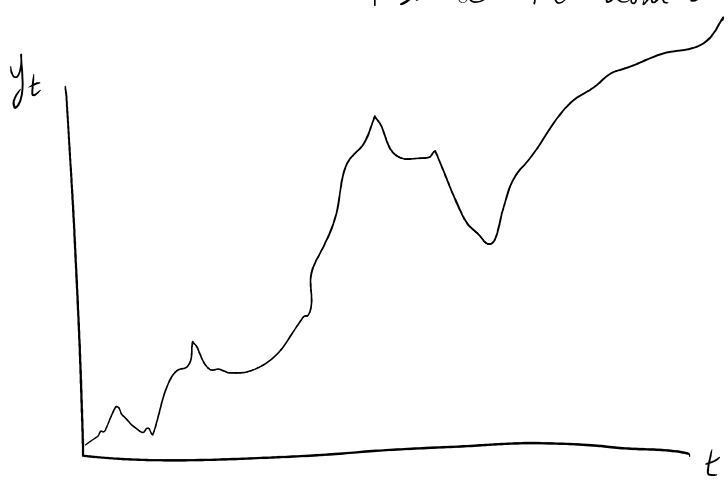
$H_a : \phi_0 \neq 0 , |\rho| < 1$ stationary process w/ potentially non zero mean

Default in stats packages:
Case 1

Don't use that!

Use Case 2 if the data appear not to trend or drift!!!

- Possible Random Walk with drift



Case 3

$$y_t = \phi_0 + \rho y_{t-1} + \varepsilon_t$$

$H_0 : |\rho| = 1$

$H_a : |\rho| < 1$

Impose: $\phi_0 \neq 0$

Case 4

tau phi3 phi2

$$y_t = \phi_0 + \rho y_{t-1} + \beta t + \varepsilon_t$$

$H_0 : \phi_0 > 0 , |\rho| = 1 , \beta = 0$ R: type="trend"

$H_a : \phi_0 \neq 0 , |\rho| < 1 , \beta > 0$

Use Case 4 if your eyes suspect drift/trend

Use Case 4 if your eyes suspect drift/trend

Case 2: we think no drift

$$y_t = \phi_0 + \rho y_{t-1} + \varepsilon_t$$

Rewrite

$$y_t - y_{t-1} = \Delta y_t = \phi_0 + \underbrace{(\rho - 1)}_{\text{test whether this is zero}} y_{t-1} + \varepsilon_t$$

Case 4: we suspect drift or trend:

$$y_t = \phi_0 + \rho y_{t-1} + \beta t + \varepsilon_t$$

Rewrite

$$\Delta y_t = \phi_0 + \underbrace{(\rho - 1)}_{\text{testing whether this is zero}} y_{t-1} + \beta t + \varepsilon_t$$

In R:

redefine

$$\Delta y_t = \phi_0 + \rho y_{t-1} + \varepsilon_t$$

"Dickey Fuller test"

Augmented Dickey Fuller (ADF)

→ add lags to capture AR(p)

→ find AR lag order that minimizes AIC/BIC and eliminates residual autocorrelations

$$\Delta y_t = \phi_0 + \rho y_{t-1} + \beta t + \underbrace{\phi_1 \Delta y_{t-1} + \phi_2 \Delta y_{t-2} + \dots + \phi_p \Delta y_{t-p}}_{\text{stationary regressors under the null hypothesis}} + \varepsilon_t$$

nonstationary under the null

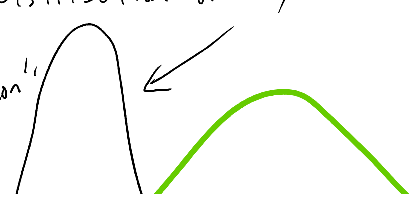
if Case 4

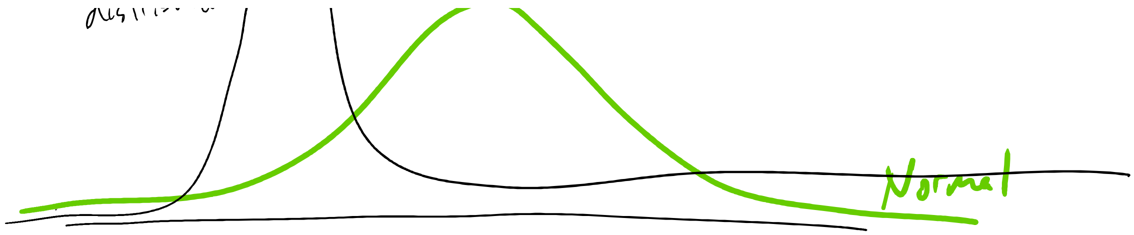
Covariate - Augmented Dickey Fuller (CADF)

$$\Delta y_t = \underbrace{\phi_0 + \rho y_{t-1} + \beta t}_{} + \phi_1 \Delta y_{t-1} + \phi_2 \Delta y_{t-2} + \alpha_1 x_{1t} + \alpha_2 x_{2t} + \varepsilon_t$$

Distribution of ρ across samples

"Nonstandard distribution"




Normal