Multivariate Time Series Regression: Forecasting and Granger Causality

This Lecture

- Forecasting with one equation
- Lag length in multivariate model
- Granger causality

Forecasting with one equation

- Suppose we want to forecast y_{1t} based on
  - $(y_{1,t-1}, ..., y_{1,t-p})$
  - $(y_{2,t-1}, ..., y_{2,t-p})$
  - ...
  - $(y_{n,t-1}, ..., y_{n,t-p})$
  - and maybe some deterministic functions of the trend (time, time-squared, a sinusoid in time, seasonal dummies, etc.)
- Let $\mathbf{y_t} = (y_{1t}, y_{2t}, ..., y_{nt})'$ be an (n x 1) vector
  - $\mathbf{x_t} = (1, \mathbf{y}'_{\mathbf{t-1}}, ..., \mathbf{y}'_{\mathbf{t-p}})'$ is a (k x 1) vector, k=np+1
  - $\mathbf{y_t}, \mathbf{x_t}$ are covariance stationary

Forecasting with one equation

- Our forecast is $\hat{y}_{1t|t-1} = \hat{\beta}'\mathbf{x_t} = \hat{\beta}'(1, \mathbf{y'_{t-1}}, ..., \mathbf{y'_{t-p}})'$
- For example: natural gas prices, oil prices, and drilling activity. Trying to forecast drilling activity.
  - y_{1t} =oil & gas drilling activity
  - y_{2t} =oil prices return
  - y_{3t} =natural gas price return

$$\begin{aligned}\hat{y}_{1t|t-1} \quad &= \quad \beta_0 + \beta_{1,dr} y_{1,t-1} + ... + \beta_{p,dr} y_{1,t-p} \\ &\quad +\beta_{1,oil} y_{2,t-1} + ... + \beta_{p,oil} y_{2,t-p} + \beta_{1,ng} y_{3,t-1} + ... + \beta_{p,ng} y_{3,t-p}\end{aligned}$$

Forecasting farther ahead

- What is our forecast of $\hat{y}_{1,t+2|t}$?
- We need predictions of $y_{1,t+1}, y_{2,t+1}, y_{3,t+1}$
- We have a forecast of $\hat{y}_{1,t+1|t}$.
- For $\hat{y}_{2,t+1|t}, \hat{y}_{3,t+1|t}$:
    1. Plug in several scenarios we want to evaluate
    2. Build another forecasting model for y_2, y_3. VAR (vector autoregression)

Lag Length in Multivariate Model

- How to determine lag length
  - AIC/BIC as before, checking for white noise residuals, etc.
  - Another option:
    - "Test down the model": Successive F-tests of the p-th lag
    - Ho: coefficients on $y_{1,t-p}, \ldots, y_{n,t-p}$ are all jointly zero
  - All variables don't necessarily need same lag length.
- How to deal with MA terms
  - We could model MA terms explicitly, but
  - Tend to be captured by lags of y_2, y_3, etc.
  - Typical practice to add more lags of x variables.

Multivariate Lag Length Selection Example

Packages and data:

```
require(quantmod)
require(forecast)
require(fBasics)
require(CADFtest)
require(urca)
require(sandwich)
require(lmtest)
require(nlme)
require(car)
require(vars)
require(texreg)
getSymbols("MCOILWTICO",src="FRED") # Monthly WTI oil price
getSymbols("IPN213111N",src="FRED") # Monthly Oil & gas drilling index
getSymbols("MHHNGSP",src="FRED")    # Monthly Henry Hub natural gas
```

Multivariate Lag Length Selection Example

Data preparation

- In practice, test each for unit root. For this example, just work with returns/first differences.
- Convert to ts() object in order to use dynlm().

```
data = merge.xts(MHHNGSP,MCOILWTICO,IPN213111N,join="inner")
plot(data)

dgas = ts(na.omit(diff(log(data$MHHNGSP))),freq=12,start=1997+1/12)
doil = ts(na.omit(diff(log(data$MCOILWTICO))),freq=12,start=1997+1/12)
dwell = ts(na.omit(diff(data$IPN213111N)),freq=12,start=1997+1/12)
```

Multivariate Lag Length Selection Example

AIC and sequential F-test for lag length selection.

- Sequentially estimate the model with 6 lags, then 5, then 4, etc.
- For each, capture AIC and p-value from F-test of last lag.

```
x6 = dynlm(dwell ~ L(dgas,c(1:6)) + L(doil,c(1:6)) + L(dwell,c(1:6)))
a6 = AIC(x6)

# F-test of the 6th lag of each variable:
F6 = linearHypothesis(x6,
            c("L(dgas, c(1:6))6=0","L(doil, c(1:6))6=0" ,"L(dwell, c(1:6))6=0"),
            vcov=vcovHAC(x6),test="F",data=x6)
f6p = F6$`Pr(>F)`   # p-value of F-test
```

Repeat with 5 lags (replace 6 with a 5 everywhere), then 4, etc.

Multivariate Lag Length Selection Example

Using a loop for this instead:

```
fp = list()        # empty list for F-test p-values
a = list()         # empty list for AIC's
for (i in 2:6)
{
  x = dynlm(dwell ~ L(dgas,c(1:i)) + L(doil,c(1:i)) + L(dwell,c(1:i)))
  a[i] = AIC(x)
  Ft = linearHypothesis(x,c(paste("L(dgas, c(1:i))",i,"=0",sep="")
                              ,paste("L(doil, c(1:i))",i,"=0",sep="")
                              ,paste("L(dwell, c(1:i))",i,"=0",sep=""))
                         ,vcov=vcovHAC(x)
                         ,test="F",data=x)
  fp[i] = Ft$`Pr(>F)`[2]
}
fp
a
```

Multivariate Lag Length Selection Example

Results

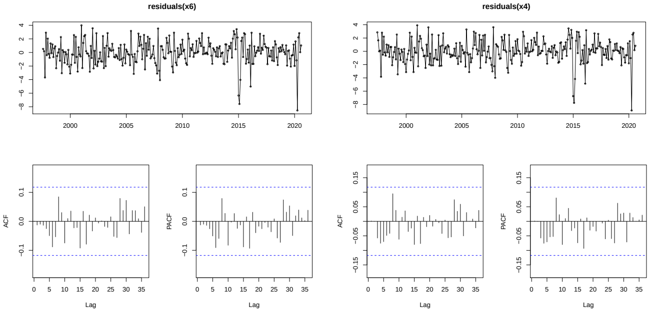
- 4 lags is the last to reject the null with F-test
- 6 lags has lowest AIC
- Check both for residual autocorrelation

```
F.results = cbind(f1p,f2p,f3p,f4p,f5p,f6p)
AIC.results = cbind(a1,a2,a3,a4,a5,a6)
F.results

##                f1p         f2p        f3p          f4p       f5p       f6p
## [1,]            NA          NA         NA           NA        NA        NA
## [2,] 1.940533e-33 0.001926831 0.02703708 4.935449e-05 0.3737614 0.2505741

AIC.results

##             a1       a2       a3      a4       a5       a6
## [1,] 1201.069 1158.691 1152.322 1136.48 1133.331 1132.875
```

Multivariate Lag Length Selection Example

Both remove residual autocorrelation: either model is adequate

```
tsdisplay(residuals(x6))
tsdisplay(residuals(x4))
```



Multivariate Lag Length Selection Example

```
texreg(x4,fontsize="scriptsize",include.loglik=FALSE,single.row=TRUE,
       caption="Results with regular standard errors")
```

	Model 1
(Intercept)	−0.09 (0.11)
L(dgas, c(1:4))1	0.97 (0.87)
L(dgas, c(1:4))2	0.82 (0.87)
L(dgas, c(1:4))3	1.96 (0.86)*
L(dgas, c(1:4))4	1.50 (0.85)
L(doil, c(1:4))1	8.47 (1.15)***
L(doil, c(1:4))2	5.18 (1.27)***
L(doil, c(1:4))3	0.94 (1.32)
L(doil, c(1:4))4	4.25 (1.32)**
L(dwell, c(1:4))1	0.60 (0.06)***
L(dwell, c(1:4))2	−0.07 (0.07)
L(dwell, c(1:4))3	0.11 (0.07)
L(dwell, c(1:4))4	−0.06 (0.06)
R^2	0.69
Adj. R^2	0.68
Num. obs.	279

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Multivariate Lag Length Selection Example

Results with HAC standard errors.

```
coeftest(x4,vcov=vcovHAC(x4))


##
## t test of coefficients:
##
##                     Estimate Std. Error t value  Pr(>|t|)
## (Intercept)        -0.087234   0.109754 -0.7948 0.4274283
## L(dgas, c(1:4))1    0.969660   1.021311  0.9494 0.3432656
## L(dgas, c(1:4))2    0.818415   0.810234  1.0101 0.3133669
## L(dgas, c(1:4))3    1.955124   0.824754  2.3706 0.0184747 *
## L(dgas, c(1:4))4    1.504894   0.880803  1.7085 0.0887012 .
## L(doil, c(1:4))1    8.471846   2.125468  3.9859 8.690e-05 ***
## L(doil, c(1:4))2    5.181937   1.351992  3.8328 0.0001582 ***
## L(doil, c(1:4))3    0.942940   1.179315  0.7996 0.4246756
## L(doil, c(1:4))4    4.249355   1.173040  3.6225 0.0003494 ***
## L(dwell, c(1:4))1   0.596625   0.103005  5.7922 1.956e-08 ***
## L(dwell, c(1:4))2  -0.068104   0.092271 -0.7381 0.4611130
## L(dwell, c(1:4))3   0.108995   0.080822  1.3486 0.1786181
## L(dwell, c(1:4))4  -0.062351   0.056299 -1.1075 0.2690768
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Granger Causality

- Ho: coefficients on $y_{2,t-1}, ..., y_{2,t-p}$ are all zero (Granger causality of y_2)
- We would say that oil price changes “Granger cause” drilling activity
- This does not necessarily mean that oil price changes **cause** drilling activity. It means they have *forecasting information* about drilling activity.

Granger Causality

- In some cases, Granger causality can be the opposite of true causality.
  - Stock returns Granger-cause (predict) GDP growth, but *are caused by it*.
- Likely no variable would Granger-cause oil prices, for example, but oil prices Granger-cause (and actually cause) many things.

Granger causality of WTI price on drilling activity

```
linearHypothesis(x4,c("L(doil, c(1:4))1=0","L(doil, c(1:4))2=0" ,
                  "L(doil, c(1:4))3=0","L(doil, c(1:4))4=0"),
                vcov=vcovHAC(x4),test="F",data=x4)


## Linear hypothesis test
##
## Hypothesis:
## L(doil, c(1:4))1 = 0
## L(doil, c(1:4))2 = 0
## L(doil, c(1:4))3 = 0
## L(doil, c(1:4))4 = 0
##
## Model 1: restricted model
## Model 2: dwell ~ L(dgas, c(1:4)) + L(doil, c(1:4)) + L(dwell, c(1:4))
##
## Note: Coefficient covariance matrix supplied.
##
##   Res.Df Df     F   Pr(>F)
## 1    270
## 2    266  4 14.21 1.59e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Granger causality of HH price on drilling activity

```
linearHypothesis(x4,c("L(dgas, c(1:4))1=0","L(dgas, c(1:4))2=0",
                   "L(dgas, c(1:4))3=0","L(dgas, c(1:4))4=0"),
                 vcov=vcovHAC(x4),test="F",data=x4)


## Linear hypothesis test
##
## Hypothesis:
## L(dgas, c(1:4))1 = 0
## L(dgas, c(1:4))2 = 0
## L(dgas, c(1:4))3 = 0
## L(dgas, c(1:4))4 = 0
##
## Model 1: restricted model
## Model 2: dwell ~ L(dgas, c(1:4)) + L(doil, c(1:4)) + L(dwell, c(1:4))
##
## Note: Coefficient covariance matrix supplied.
##
##   Res.Df Df      F  Pr(>F)
## 1    270
## 2    266  4 2.4944 0.04336 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```