Time Series Econometrics
Brief Overview

Day 1 or 2

Econometrics and Time Series

- Two main uses of econometrics
  1. Predict an unknown
     - Forecasting, nowcasting, prediction in-sample vs out-of-sample.
     - Leverage multivariate/conditional correlations, joint distributions of variables, dependence on past values.
  2. Estimate a "true" parameter
     - Causal inference, construct counterfactuals
     - Causal effect of a policy or decision, demand and supply elasticities, shape parameters of production functions, cost functions, utility functions, etc.

Power plant emissions control example

Time Series

- What is special about time series?
  - Data observations have a particular **order** - time.
  - May be **dependence** of each observation on the last one, or the last 10.
    - How many past values? Which ones? How strong is the dependence? How can we use it or control for it?
  - Classical statistics: observations are independent.
    - Sample 100 people about their purchases at different price levels. Then sample 100 more.
    - Randomized control trial of a drug with 100 people. Conduct many trials.

Linear model examples

- Capital Asset Pricing Model
- Forecasting the future from past & present values
- Estimate supply and demand functions
- Estimate the causal impact of a policy change on an industry

General problem

Want to know

1. "true" relationship between time series variables y_t and x_t, or
2. the best prediction of y_t if all we know is x_t.

$$y_t = \alpha + \beta x_t + e_t$$

- α is intercept, β is slope in the x dimension.
- x_t might be a vector of multiple explanatory variables.
- e_t is everything else about y_t not captured in x_t.

General problem

$$y_t = \alpha + \beta x_t + e_t$$

- What is in e_t? Make a list of things that determine y_t, whether observable or not.
- How can **lag dependence within** y_t, x_t, e_t help or hurt the model?
- How can **joint dependence between** y_t, x_t, e_t help or hurt the model?
- *pay attention to subscripts.*

Examples: CAPM

- Market Model: y_t is a stock return, x_t market return, e.g., S&P500, S&P Value-Weighted, sector index, etc.

$$r_{it} = \alpha + \beta r_{mt} + e_t$$

- α is stock i's average return when markets are zero.
- β is stock i's volatility relative to market (i.e., $\beta > 1$ vs. $\beta < 1$).

Examples: CAPM

- Capital Asset Pricing Model: y_t and x_t are *excess returns* over a risk free rate r_{ft}, e.g., interest on 3-month T-bill:

$$(r_{it} - r_{ft}) = \alpha + \beta(r_{mt} - r_{ft}) + e_t$$

  - Fama-French 3-factor model[1]

$$(r_{it} - r_{ft}) = \alpha + \beta(r_{mt} - r_{ft}) + \gamma_1 SMB_t + \gamma_2 HML_t + e_t$$

  - SMB_t (small - big): small cap portfolio return minus large cap.
  - HML_t (high - low): value minus growth portfolio return.

[1] Kenneth French updates factor data at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Examples: Autoregressive (AR) model

$$y_t = \alpha + \beta x_t + e_t$$

- AR(p): x_t is a vector containing p **lags** (past values) of y_t and other variables z_t.

$$y_t = \alpha + \beta_1 y_{t-1} + ... + \beta_p y_{t-p} + \theta_1 z_{1,t-1} + \theta_2 z_{2,t-1} + e_t$$

- What if e_t also depends on its own past values?

Examples: Supply and Demand functions

$$
\begin{aligned}
q_{Dt} &= a - bP_t + e_{Dt} \\
q_{St} &= c + dP_t + e_{St} \\
q_{St} &= q_{Dt}
\end{aligned}
$$

- e_{Dt} and e_{St} are demand and supply shifters/shocks. Some observable, some not observable.
- Algebra shows that in equilibrium,

$$
\begin{aligned}
q_t &= \tfrac{-b\cdot c - a\cdot d}{-b-d} + \tfrac{-b\cdot e_{St} - d\cdot e_{Dt}}{-b-d} \\
P_t &= \tfrac{c-a}{-b-d} + \tfrac{e_{St} - e_{Dt}}{-b-d}
\end{aligned}
$$

- What do you get if you regress q_t on P_t? P_t is determined by the demand and supply shocks.

Supply/Demand example

Examples: Causal Effect of Policy

$$y_t = \alpha + \beta D_t + \theta x_t + e_t$$

- $D_t = 1$ when some policy is in place, $D_t = 0$ otherwise.
- x_t might be a vector of multiple control variables.
- e_t is everything else about y_t not captured in x_t.
- Is e_t correlated with D_t? Who receives the policy? When does it happen?

Other examples

Examples:

- Energy demand: y_t is electricity or natural gas consumption, or fuel in storage, x_t is weather (e.g., heating/cooling degree days, hurricane incidence, etc.)
- Commodity market linkages (cointegration):
  - y_t is copper price, x_t is gold price.
  - y_t is global LNG price, x_t is Brent crude price.

Estimation

- Classical statistics: parameters $\alpha, \beta, \theta, a, b, c, d$, etc. have objectively true but unknowable values.
- We can estimate them from a sample: $\hat{\alpha}, \hat{\beta}, \hat{\theta}$, etc.
- Every sample will produce a slightly different answer.
  - Our estimates are *noisy or uncertain* — they have *variance*.
- What is the optimal way to estimate them so that they are
  - **unbiased**: close to the truth on average across samples.
  - **efficient**: have the lowest variance possible.
- The estimates are the answer to an optimization problem.

Estimation

- Parameter estimates are the answer to an optimization problem.
- Pick parameters that minimize a loss function, e.g., sum of squared residuals.

$$min_{\alpha,\beta} \sum_{t=1}^{T} e_t^2 = min_{\alpha,\beta} \sum_{t=1}^{T} (y_t - \alpha - \beta x_t)^2$$

- Ordinary Least Squares (OLS). Not the only loss function, but the most common and has some nice properties.

Estimation

- Gauss-Markov Theorem: Under some assumptions, OLS gives the Best Linear Unbiased Estimate
  - True model is linear in parameters and residuals.
  - Right hand side variables are not constants or perfectly correlated with each other.
  - Residuals e_t have a constant variance (not more noisy for some values of x than others).
  - Residuals e_t are uncorrelated with each other.
  - All right hand side variables are uncorrelated with the residual e_t.
- How many of these are likely to be met in our examples, with time series data?

Estimation

- Another idea: pick parameters that maximize the likelihood of having observed your data.
- Recall $e_t = y_t - \alpha - \beta x_t = y_t - \hat{y}_t$.
- Suppose $f(e_1, ..., e_t, ..., e_T)$ is the joint probability distribution of the residuals, e.g., $e_t \sim N(0, \sigma^2)$.

$$max_{\alpha,\beta} f(e_1, ..., e_t, ..., e_T)$$

- If all the e_t are **independent** from each other:

$$max_{\alpha,\beta} f(e_1) \cdot f(e_2) \cdot ... \cdot f(e_T)$$

$$max_{\alpha,\beta} \sum_{t=1}^{T} ln\ f(e_t)$$

- Maximum Likelihood Estimation (MLE). Useful in more settings than OLS, shares many similar properties.