Non-Renewable Resources, Extraction Technology and Endogenous Growth*

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Abstract

We document increasing extraction but constant real price trends for 65 non-renewable resources from 1700 to 2018. Why have resources not become scarcer with global economic growth? Resource stocks are not fixed but a function of geology and endogenous innovation in extraction technology. Rising resource demand incentivizes firms to innovate and allows extraction from lower grade deposits. Prices stay constant because resource quantities increase exponentially with lower grade deposits, which follows from a geological law. This offsets diminishing returns in innovation. As a result, the interaction between geology and innovation determines the long-run growth rate of aggregate output. There is no depletion effect. If innovation in extraction continues, a flat long-run supply curve of fossil fuels is a reasonable assumption. A rising carbon tax could discourage such innovation, limiting fossil fuel extraction and greenhouse gas emissions. (JEL codes: O30, O41, Q30, Q43, Q54)

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1 Introduction

Economic intuition suggests that non-renewable resources, such as copper or crude oil, become scarcer with economic growth. However, our new data set for 65 resources provides evidence to the contrary. As the global economy grew over the past three centuries, not only has extraction increased, but most real prices have exhibited non-increasing trends. We propose an explanation for these facts: Increased resource demand incentivices firms to invest in innovation in extraction technology. Innovation exploits a geological law which states that greater resource quantities are found in progressively lower grade deposits. The result is rising extraction at non-increasing prices to meet growing global demand. The interaction between innovation and geology co-determines the long-run growth rate of aggregate output.

Three stylized facts support this mechanism. First, resources are abundant in the Earth's crust, but only a small fraction is economically recoverable with current technology. Geologists call this fraction "reserves." Second, resources are log-normally distributed according to the Fundamental Law of Geochemistry (Ahrens, 1953). Greater resource quantities are in harder-to-extract lower grade deposits. Third, firms innovate to convert previously inaccessible lower grade deposits into reserves.

We integrate a more realistic extraction sector into a standard model of endogenous growth with three sectors (Acemoglu, 2002). The aggregate output sector uses the resource and an intermediate good as inputs. In the extraction sector firms observe the aggregate resource demand and invest in new extraction technology, which allows them to increase their reserves and to extract the resource from lower grade deposits. Extractive firms buy technology from technology firms. These firms innovate because technology is specific to deposits of particular grades and hence rivalrous.[1] Firms in the intermediate good sector produce intermediate goods.

The interaction between geology and endogenous innovation implies constant growth rates of extraction and aggregate output, as well as a constant real resource price on the balanced growth path. Although it becomes more costly to develop technologies for extracting from lower grade deposits, the resource quantities in lower grade deposits increase exponentially. Thus, the higher marginal cost is offset. The geological distribution affects the rate of aggregate output growth. For example, a higher geological resource abundance leads to a lower price and higher growth rates for extraction and aggregate output. Our results contrast with conventional models in which a "depletion effect" drags down growth (see Nordhaus et al., 1992; Weitzman, 1999; Jones and Vollrath, 2002).

In our model, a higher level of aggregate output and a larger population lead to more innovation in extraction, and ultimately, a larger flow of resources. If the resource and the intermediate good are complements, a higher resource abundance lowers innovation in extraction. If the two goods are substitutes, a higher abundance fosters innovation in the extraction sector.

[1] This interpretation of innovation is close to Desmet and Rossi-Hansberg (2014), where non-replicable production factors, like land, incentivize innovation despite perfectly competitive markets. See also Hellwig and Irmen (2001).

Our paper addresses the puzzling mismatch of theory and empirical evidence on growth and resource scarcity (see Jones and Vollrath, 2002; Hassler et al., 2019). Growth theory predicts greater resource scarcity with economic growth based on the optimal depletion of a finite resource: While the economy grows at a constant rate, extraction declines at a constant rate and resource prices rise at the real rate of interest. Substitution and increased resource efficiency allow continued growth (see e.g. Dasgupta and Heal, 1974; Nordhaus et al., 1992; Aghion and Howitt, 1998; Jones and Vollrath, 2002; Groth, 2007). The literature, however, acknowledges that these predictions are in contradiction to empirical evidence on resource price and output trends (see von Hagen, 1989; Krautkraemer, 1998; Livernois, 2009; Slade and Thille, 2009).

Our paper and its microfoundations provide a building block to address a variety of research questions. For example, our results suggest that if innovation in extraction continues at a sufficient rate, a flat long-run supply curve of fossil fuels is a reasonable assumption for models of climate change. We show in an extensions that an increasing carbon tax would discourage such innovation, potentially limiting fossil fuel extraction and greenhouse gas emissions. In contrast to models of the "Green Paradox," which assume a finite stock of fossil fuels (Sinn, 2008; van der Ploeg and Withagen, 2010), demand side policies to curb greenhouse gases are effective in our setup.

1.1 Literature

We formalize a key idea by Nordhaus (1974, p. 24) that "higher resource consumption levels in the future will lead to mining of lower and lower ore grades. Whether or not this leads to continuing decline of the resource/labor price ratio depends on whether technological progress continues to outstrip the movement to lower grade ores."[2] We include a resource sector with extraction technology in an endogenous growth model. We add the idea of the resource distribution in the Earth's crust, which is essential for overcoming decreasing returns to scale and leads to non-increasing prices.

We build our analysis on three strings of the literature. First, the resource economics literature based on Hotelling (1931) has long relaxed scarcity to certain extent to account for periods of non-increasing resource prices and non-declining resource production. These extensions include exploration (Pindyck, 1978), drilling (Anderson et al., 2018), cost-reducing exogenous technological change (Slade, 1982; Cynthia-Lin and Wagner, 2007), the availability of a backstop technology (Heal, 1976), learning by doing (Rausser, 1974), and firms' limited planning horizons (Spiro, 2014).[3] We add to this literature by introducing the geological distribution of resources and by integrating a resource sector with extraction technology into a general equilibrium endogenous growth model.

Second, while the literature on exogenous growth with non-renewable resources

[2]Note that Barnett and Morse (1963, p. 240) formulated a related idea.

[3]See Slade and Thille (2009) and Krautkraemer (1998) for excellent overviews of the theoretical but also empirical literature building on Hotelling (1931)

mostly focuses on resource augmenting technological change (see e.g. Nordhaus et al., 1992), Hart (2016) presents a growth model including a resource in heterogeneous deposits with exogenous labor productivity increases. After a temporary "frontier phase" with a constant resource price and extraction rising at the rate of aggregate output, the firms needs to extract resources from greater depths. Subsequently, a long-run balanced growth path is reached with constant resource extraction and prices that rise in line with wages. This paper is close to ours in that it recognizes the role of the resource distribution in the Earth's crust. Another paper is Acemoglu et al. (2019), who study the role of fracking in the energy transition, postulating exogenous innovation augments a constant flow of natural gas leading to a constant price. Our paper adds to this literature by endogenizing technological change in extraction and provide geological microfoundations, which allows us to study how output growth affects technological change in extraction.

Finally, endogenous growth models with non-renewable resources mostly focus on resource augmenting technological change (see Groth, 2007; Andre and Smulders, 2014; van der Meijden and Smulders, 2017; Acemoglu et al., 2012, e.g.). However, Tahvonen and Salo (2001) examines the transition from non-renewable to renewable energy in a growth model with learning-by-doing in extraction, implying an inverted U-shaped extraction and a U-shaped price path. We add to this study by providing economic and geological microfoundations to the innovation process and by stressing the role of demand for final goods in causing innovation.

2 Long-Run Trends in Resource Markets

Our data set includes all major non-renewable resources, namely fossil fuels (coal, crude oil and natural gas), 37 metals (e.g. copper, rare earths) and 25 non-metals (e.g. cement, phosphate rock). The data and a supplementary appendix, with sources and descriptions, are downloadable at https://sites.google.com/site/mstuermer1/.

2.1 Extraction Increased Strongly

Figure 1) illustrates that global resource extraction rose strongly with global real GDP over the last three centuries. Extraction increased from about 3.3 million metric tons in 1700 to 21 billion metric tons in 2018. Per capita extraction rose from roughly 5 to 3,000 kilograms. Regression results confirm that the individual series exhibit significantly positive growth rates (see table 2 in the supplementary appendix).

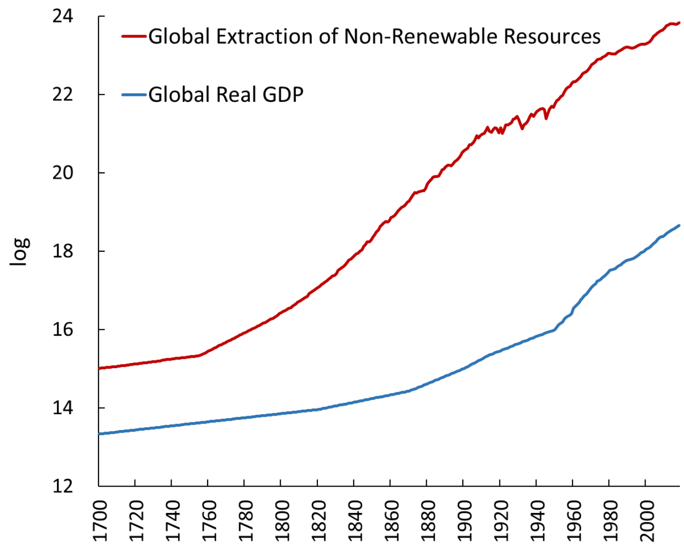

Figure 1: World Extraction of 65 Resources (left axis) and World Real GDP (right axis), both in logs, 1700-2018. The total quantity of extracted resources increased roughly in line with world real GDP. See figure 1 in the supplementary appendix for individual series.

2.2 Resource Prices Exhibit Non-Increasing Trends

At the same time, real prices have remained the same or declined over the long term (see figure 2). Regression results in table 3 of the supplementary appendix confirm these trends for individual series. Based on a broader and longer data set, we reconfirm earlier evidence in Krautkraemer (1998); von Hagen (1989); Cynthia-Lin and Wagner (2007) and others. Although the literature is certainly not definitive on price trends (see Pindyck, 1999; Lee et al., 2006; Harvey et al., 2010; Jacks, 2013), we conclude that inflation adjusted prices generally do not increase over the long term.

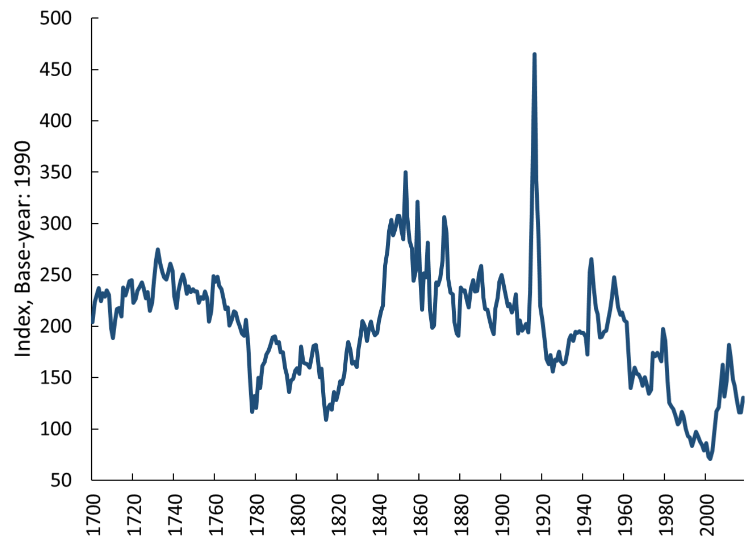

Figure 2: Inflation Adjusted Price Index for 65 Resources (equally weighted), 1700-2018. See figure 2 in the supplementary appendix for individual series.

3 Stylized Facts

To accommodate these long-run trends in resource markets, build a model with a more realistic extraction sector. We base the model on three key stylized facts about geology and extraction technology, which we lay out in the following.

3.1 Resources are Abundant in the Earth's Crust

The physical quantities of resources are abundant in the Earth's crust (see Nordhaus, 1974). For example, the crust consists of 0.0068 percent copper on average, which is about 1,500,000 billion metric tons (see table 1). Hydrocarbons such as coal or crude oil are also plentiful (see Aguilera et al. (2012) and Rogner (1997)). As the crust makes up less than one percent of the planet, more resources exist in other layers.

	Crustal Abundance (Bil. mt)	**Reserves** (Bil. mt)	**Annual Output** (Bil. mt)	**Crustal Abundance/ Annual Output** (Years)	**Reserves/ Annual Output** (Years)
Aluminum	1,990,000,000[d]	30[b1]	0.06[a]	491	42[1]
Copper	1,500,000[d]	0.8[b]	0.02[b]	483	26
Iron	1,390,000,000[d]	83[b2]	1.2[a]	580	39[2]
Lead	290,000[d]	0.1[b]	0.005[b]	1,099	16
Tin	40,000[d]	0.005[b]	0.0003[b]	1,405	14
Zinc	2,250,000[d]	0.23[b]	0.013[b]	668	14
Gold	70[d]	0.00005[b]	0.000003[b]	925	15
Coal[3]		511[c]	3.9[c]		63[c]
Crude Oil[4]	15,000,000[e]	241[c]	4.4[c]	558	41[c]
Nat. Gas[5]		179[c]	3.3[c]		34[c]

Table 1: Availability of selected resources in years of production left in the crustal mass and in reserves assuming exponentially increasing annual mine production (based on average historical growth rates).

Notes: Average annual growth rates of production from 1990 to 2010: Aluminum: 2.5%, Iron: 2.3%, Copper: 2%, Lead: 0.7%, Tin: 0.4%, Zinc: 1.6%, Gold: 0.6%, Crude oil: 0.7%, Natural gas: 1.7%, Coal: 1.9%, Hydrocarbons: 1.4%. [1]Data for bauxite, [2]data for iron ore, [3]includes lignite and hard coal, [4]includes conventional and unconventional oil, [5]includes conventional and unconventional gas, [6]all organic carbon in the earth's crust. Sources: [a]U.S. Geological Survey (2016), [b]U.S. Geological Survey (2018), [c]Federal Institute for Geosciences and Natural Resources (2017), [d]Perman et al. (2003), [e]Littke and Welte (1992).

Of course, extracting most of these quantities is impossible or extremely costly with current technology. Only a small fraction called "reserves" is economically extractable (see column 3). For these resources firms have established that extraction is profitable under defined investment assumptions and with reasonable certainty at current prices.[4]

[4]We leave out the "reserve base" to ease exposition. The reserve base encompasses those resources that have a reasonable potential for becoming economically available within planning horizons (U.S. Geological Survey, 2018).

The improvement of extraction technology increases reserves, as it makes exploration and extraction of lower grade deposits profitable. If firms continue to develop technology, extraction could grow exponentially for a couple of hundred to a thousand years before exhaustion (column 4). For example, if all copper in the crust were accessible and copper extraction continued to grow two percent per year, it would take roughly 480 years to reach exhaustion.[5]

The reality is more complicated due to factors such as recycling and environmental externalities. We conclude, however, that for all practical purposes, non-renewable resources can be considered as inexhaustible for the foreseeable future.

3.2 Resources are Log-Normally Distributed

Variations in geochemical processes have shaped the characteristics of resource deposits over billions of years. Deposits differ along dimensions such as ore grades, thickness and depths. We focus on ore grade as the most important. Grade is the concentration of the resource in the surrounding rock of a deposit. For example, a gold nugget is a high-grade deposit (100 percent), while specks of gold in a stream are a low grade deposit (close to zero percent).

Ahrens (1953, 1954)) postulates in his Fundamental Law of Geochemistry that each chemical element exhibits a log-normal grade-quantity distribution in the Earth's crust, implying a decided positive skewness. The log-normal distribution has become

[5] If we assume extraction at current levels, humans could sustain resource production for millions of years if there is continuous innovation. See table 1 in the supplementary appendix.

the standard assumption in assessing mineral resources (see e.g. Wellmer, 1998; Singer, 2010) based on ample evidence for metals and hydrocarbons (see Ahrens, 1954; Kaufman et al., 1975; Schuenemeyer and Drew, 1983; Mudd, 2007; Singer, 2013; Gerst, 2008).

Many natural phenomena are approximated by log-normal distributions, due to the salient role of multiplicative effects and the lower bounding at zero (see Blackwood, 1992; Limpert et al., 2001). For example, the amount of petroleum in a reservoir is the mathematical product of rock volume, porosity and pore petroleum saturation. Each parameter is normally distributed, but their multiplicative effects leads to a log-normal distribution (Blackwood, 1992).

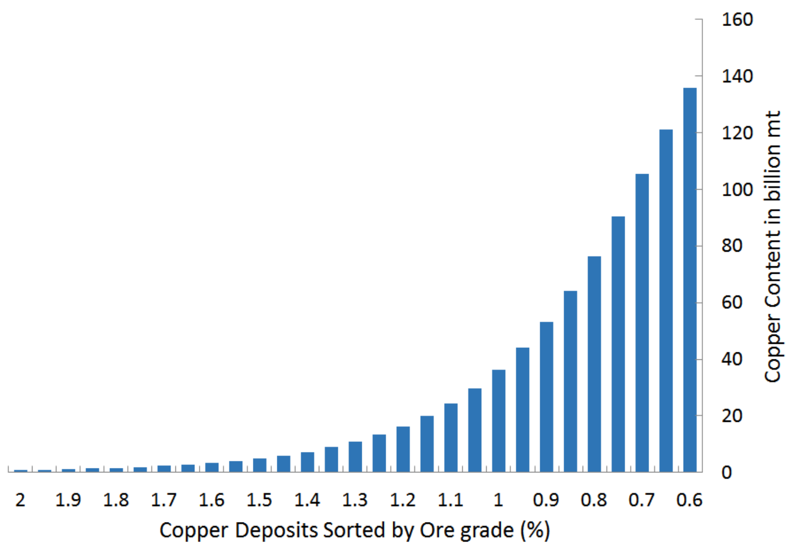

Figure 3: Grade-quantity distribution of copper in the Earth's crust. The copper content increases, as the ore grades of deposits decline. The x-axis has been reversed for illustrative purposes. Source: Gerst (2008).

The literature proposes different approximations for the tail of the distribution, which represents near-future production (see Lasky, 1950; Krige, 1962; Musgrove, 1971; DeYoung, 1981). There might also be a discontinuity in the distribution due to the mineralogical barrier, the point below which atomic substitution traps metal atoms (Skinner, 1979; Gordon et al., 2007). However, this hypothesis has not been empirically confirmed (see Gerst, 2008). We conclude that a log-normal distribution is a reasonable first approximation.

3.3 Innovation in Extraction Technology

Empirical evidence suggests that innovation in extraction technology is a major margin for expanding reserves, as it makes extraction from lower grade deposits profitable (see Lasserre and Ouellette, 1991; Simpson, 1999; Mudd, 2007; Wellmer, 2008).

For example, Radetzki (2009) describes how innovation in mining equipment, prospecting and metallurgy have gradually lowered the cut-off grade of copper mines over the last 5,000 years (see figure 4). Another example, is the development of hydraulic fracturing and horizontal drilling, which unlocked vast amounts of previously unexploitable hydrocarbon resources in low grade deposits. New extraction technologies are often specific to certain characteristics or types of deposits.

There is evidence that innovation affects the cut-off grade with diminishing returns. The example of copper shows that under the assumption that global real R&D spending has stayed constant (or increased), there are diminishing returns in terms of making deposits of lower grades extractable (see figure 4).

Of course, exploration (or discovery) is another margin for increasing reserves. However, we can look at extraction technology as a a pre-condition for exploration. For example, geologists always knew about unconventional, low grade oil deposits. Once fracking made extraction from these deposits profitable, firms started exploring for more of these low grade deposits to apply the new technology.

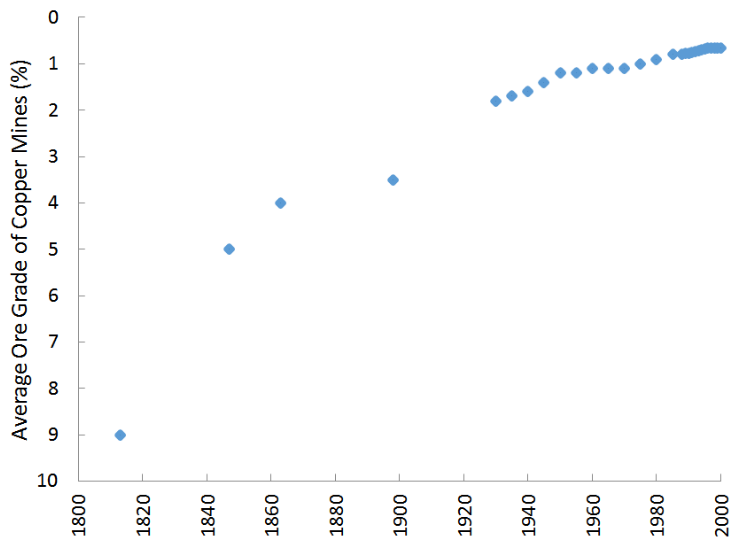

Figure 4: The historical development of average ore grades of world copper mines suggests diminishing returns of innovation in terms of lowering the cut-off grade. The y-axis has been reversed for illustrative purposes. Source: Gerst (2008).

4 Model

Based on these stylized facts, we model resource flows as a function of geology and endogenous innovation in extraction technology. We develop a micro-founded extraction sector and integrate it into a standard endogenous growth model with directed technological change (see Acemoglu, 2002) to study its effects on economic growth.

4.1 Preferences, Endowments and Final Good Production

In our highly stylized model of the global economy we first start with a representative household that has constant relative risk aversion preferences:

$$\int_0^\infty \frac{C_t^{1-\theta}-1}{1-\theta} e^{-\rho t} dt \ . \tag{1}$$

The variable C denotes aggregate consumption, ρ is the discount rate, θ is the coefficient of relative risk aversion, and t the time subscript, which we drop whenever possible. A representative firm produces a final output good

$$Y = \left[(1-\gamma) R_{Extr}^{\frac{\varepsilon-1}{\varepsilon}} + \gamma Z^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \ , \tag{2}$$

combining inputs from two sectors, namely a non-renewable resource R^{Extr} and an intermediate good Z, with a constant elasticity of substitution $\varepsilon \in (0, \infty)$. The final good is the numeraire. The two inputs are substitutes when $\varepsilon > 1$ and complements when $\varepsilon < 1$. Throughout we ignore the Cobb-Douglas case $\varepsilon = 1$. The distribution parameter $\gamma \in (0, 1)$ indicates the importance of the two inputs in producing output.

Each of the two sectors consists of production firms and technology firms. The representative household owns all firms. Her budget constraint is $C + I + M \leq Y$.

Production firms' aggregate spending on machines is $I = I_Z + I_R$ and technology firms' R&D spending is $M = M_Z + M_R$. The usual no-Ponzi game conditions apply.

4.2 The Case without Extraction Technology

Let's assume there is no innovation in extraction and the economy starts with zero resource stocks. In this case there will be no extraction and the resource price will be infinitely high. Following Dasgupta and Heal (1979), this implies that aggregate growth is impossible, if the intermediate good and the resource are complements. If the two good are substitutes, aggregate output growth is still possible based on the intermediate good as sole input.

If there is no innovation in extraction but we allow for a positive resource stock at the start, our model reverts back to standard growth models with resources (see Jones and Vollrath, 2002; Groth, 2007). Firms' resource stocks S will decline at the same constant rate as extraction R^{Extr}, $\dot{S}_t = -R_t^{Extr}$, with $S_t \geq 0, R_t^{Extr} \geq 0$ and $S_0 > 0$. The over-dot denotes the time-derivative. The resource price will increase at the rate of interest following (Hotelling, 1931). The depletion rate would drag down the equilibrium growth rate due to a so-called "depletion effect." Aggregate output growth would depend on the elasticity of substitution among others.

4.3 Extraction Technology and Reserves

In our model, resource stocks are not fixed but a function of geology and firms' purposeful innovation in extraction technology. Based on the stylized facts, we establish three principles for the micro-foundations of the sector. First, firms extract resources from their reserves, which are a function of innovation in extraction technology. Second, innovation affects the cut-off grade, and finally, the cut-off grade determines the amount of new reserves based on the geological distribution.

We start with the first principle. We assume that firms extract the resource from a continuum of heterogeneous deposits that differ along one characteristic, which is the grade O.[6] Grade is the concentration of the resource in the surrounding rock of a deposit. Deposits range from high grade to low grade ($O \in (0, 1)$).

Reserves are deposits that are extractable with grade-specific technology. The cut-off grade O^* is the minimum extractable grade. A machine of variety j allows firms to lower the cut-off grade to claim one period exclusive property rights on the corresponding deposits and to increase their reserves. We denote this flow of new reserves by R^{Tech}. Firms can extract the resource from their reserves at zero cost, a flow which we denote as R_t^{Extr}. The cost to extract from deposits below the cut-off

[6] Firms are infinitely small and operate in a perfectly competitive environment. Firm level production functions exhibit constant returns to scale, so there is no loss of generality in focusing on aggregate production functions. We assume a fully competitive sector, because we model long-run trends. Historically, producer efforts to raise prices were only successful in some non-oil commodity markets in the short run, as longer-run price elasticities proved to be high (see Radetzki, 2008; Herfindahl, 1959; Rausser and Stuermer, 2016). Similarly, a number of academic studies doubt OPEC's ability to raise prices over the long term (see Aguilera and Radetzki, 2016, for an overview).

grade is infinitely high. Firms' reserves evolve according to:

$$\dot{S}_t = -R_t^{Extr} + R_t^{Tech}, \quad S_t \geq 0, R_t^{Tech} \geq 0, R_t^{Extr} \geq 0. \tag{3}$$

The use of a machine type j is rivalrous, because machine varieties are specific to deposits of certain grades. This simplifying assumption picks up the idea that mining firms need to adjust their technology to the specific geological characteristics of deposits to extract the resource. Furthermore, machines depreciate after one period. Firms have full knowledge about the geology. There is no search (or exploration) of new deposits.

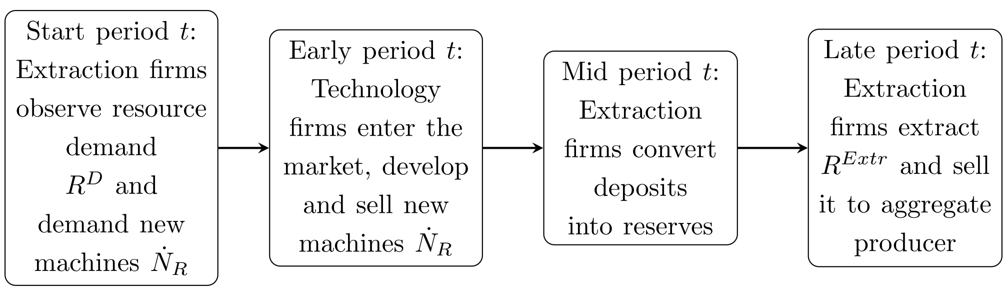

Figure 5: Firms' Problem and Timing

4.4 Innovation

The second principle is the effect of innovation on the cut-off grade. Sector-specific technology firms innovate and develop new varieties of extraction technology (or machines). Each new machine variety makes a range of deposits down to the new cut-off grade extractable. The innovation possibilities frontier, which determines the creation of new technologies, takes the form:[7]

$$\dot{N}_R = \eta_R M_R \ . \tag{4}$$

Technology firms can freely enter into research and invest into the invention of one new machine variety at a time. The cost of a new machine variety is $\frac{1}{\eta_R}$, as one unit of the final good generates a flow rate $\eta_R > 0$ of perpetual patents. While the number of machines per variety is constant (at one), the number of new varieties in a given time interval increases over time. The knowledge diffuses immediately to other technology firms and can be used to invent other machine varieties. The patent grants the firm the right to build machines of variety j at a fixed marginal cost $\psi_R > 0$. Technology firms sell the machine to extraction firms at price χ_R. Given that new ore grades are exhausted after one use, the economic value of the patent vanishes after the immediate exploitation.

Our technology function maps the technology level N_R onto the cut-off grade $O^\star$:

$$O^\star(N_R) = e^{-\mu N_R}, \ \mu \in \mathbb{R}_+ \ \ N_R \in (0, \infty) \ . \tag{5}$$

Figure 6, Panel A, shows how two equal advances in technology from 0 to N_R and from N_R to N_R', lead to diminishing returns in terms of cut-off grades $O^\star$ and $O^{\star\prime}$ and hence also in terms of R&D investment. The curvature parameter is μ. If μ is high,

[7] In line with Acemoglu (2002) there is no aggregate uncertainty in the innovation process. There is idiosyncratic uncertainty, but with many different technology firms undertaking research, equation 4 holds deterministically at the aggregate level.

the average effect of new technology on the cut-off grade is relatively high.

4.5 Geological Function

After establishing how innovation affects the cut-off grade, we show how the cut-off grade relates to the amount of reserves in the ground. We approximate the tail of the log-normal geological distribution by an increasing relationship between the cut-off grade O^* and the resource quantity in reserves (see figure 6):

$$S(O^\star) = -\delta ln(O^\star), \quad \delta \in \mathbb{R}_+ \ , \ O^\star \in (0,1) \tag{6}$$

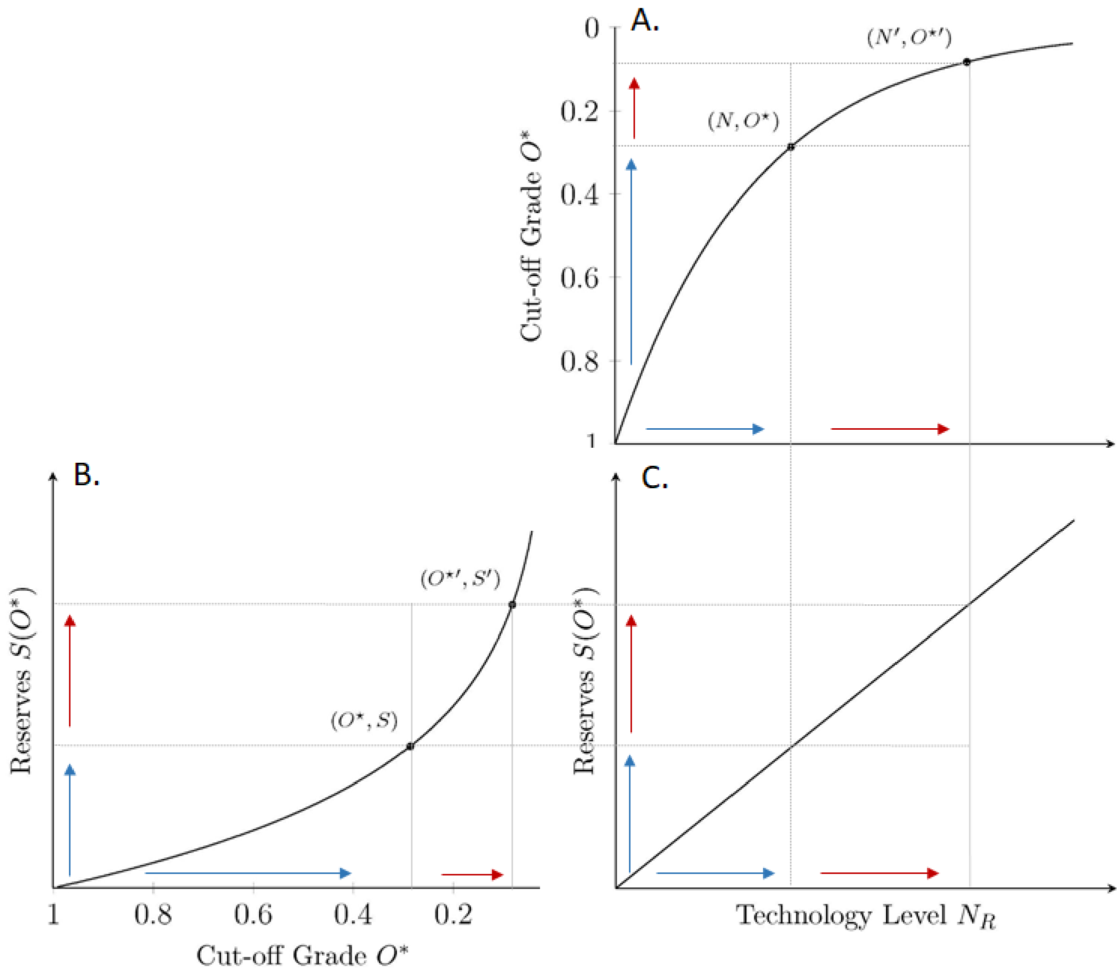

Figure 6: The interaction between the extraction technology function (Panel A) and the geological function (Panel B) leads to a linear relationship between technology level N_R and reserves S (Panel C). The y-axis in panel A and the x-axis in panel B have been reversed for illustrative purposes.

Panel B shows the geological function. Decreasing advances in the cut-off grades $O^\star$ and $O^{\star\prime}$ map into equal increases in the reserve level S and S' due to increasing returns in the geological distribution. Parameter δ controls the curvature. If δ is high, the average concentration of the resource is high.

The functional form implies that the resource quantity approaches infinity as the cut-off grade approaches zero. Although we recognize that resources are ultimately finite, we assume that the physical quantities in the Earth's crust are not limited for any economically relevant horizon (see also Nordhaus, 1974). This compares to firms maximizing profits over an infinite horizon in the macroeconomic literature.

4.6 Marginal Effect of Extraction Technology on Reserves

Diminishing returns in innovation and increasing returns in the geology produce a linear relationship between technology level and reserves, implying a constant technology cost per unit of new reserves.

Panel C summarizes how the two functions offset each other. As the natural log in the geological function and the exponential in the technology function cancel out, the relationship between the technology level N_R and quantity of deposits converted into reserves S becomes linear. See Appendix 1.1 for the proof.

Lemma 1 Reserves S increase proportionally to the technology level N_R:

$$S(O^\star(N_R)) = \delta\mu N_R \; .$$

The marginal effect of new extraction technology on reserves equals:

$$\frac{dS(O^\star(N_R))}{dN_R} = \delta\mu \ .$$

These constant returns of innovation in terms of new reserves and the assumption of grade-specific technology lead to the production function for new reserves:[8]

$$R^{Tech} = \delta\mu\dot{N}_R. \tag{7}$$

New reserves are a function of technological change $\dot{N}_R$, the geological parameter δ and the technological parameter μ. If the parameters δ and μ are high, the marginal return of new technology in terms of new reserves will also be high. This result is a first approximation. We discuss other functional forms in section 6.

4.7 Intermediate Good Sector

Firms produce an intermediate good $Z = \frac{1}{1-\beta_Z}\left(\int_0^{N_z} x_z(j)^{1-\beta_Z} dj\right) L_Z^{\beta_Z}$. Labor L is in fixed supply and $x_Z(j)$ is the number of machines of variety j. Varieties are partial complements as implied by $\beta_Z \in (0,1)$. New varieties make labor more productive. The innovation possibilities frontier $\dot{N}_Z = \eta_Z M_Z$. Firms can freely enter the market, invent a machine variety j and produce machines at cost $\psi_Z > 0$. Firms set machine prices $\chi_Z(j)$ with some degree of market power. See Acemoglu (2002) for details.

[8]Please see Appendix 1.2 for the derivation of this equation.

5 Equilibrium

Endogenizing the supply of the resource through innovation in extraction technology leads to an equilibrium that speaks to the long-run trends in resource markets and aggregate global output. In such an equilibrium, the evolution of technology in both sectors is determined by free entry, the factor price of labor is consistent with market clearing, and the time paths of $[C_t, I_t, M_t]_{t=0}^{\infty}$ are based on household maximization.[9]

5.1 Aggregate Resource Demand

We derive the resource demand of the representative firm by taking the first-order condition in the aggregate production function, equation (2):

$$R^D = \frac{Y(1-\gamma)^{\varepsilon}}{p_R^{\varepsilon}} \, . \tag{8}$$

The resource demand rises with higher aggregate output Y, but decreases with a higher resource price p_R.

5.2 Demand for Extraction Technology

The extractive firms observe the aggregate resource demand and determine how much resource they extract from their reserves and how much new extraction technology

[9]Our focus is on the derivations for the extractive sector. Those for the intermediate good sector follow Acemoglu (2002) and are in section 3.2. of the supplementary appendix.

they buy from the technology firms.

As extractive firms are perfectly competitive and face no uncertainty about geology, we assume that their flow of extracted resources R_t^{Extr} is equal to their flow of new reserves R_t^{Tech}. We hold the stock of reserves constant and set it to zero. As a result, firms resource extraction is a function of new technology $\dot{N}$ and the parameters from the geological function δ and the technological function μ:

$$R_t^{Extr} = R_t^{Tech} = \delta \mu \dot{N}_{Rt}. \tag{9}$$

Extraction firms face a marginal cost curve and produce what is demanded at a given price. The firms' optimization problem is static since machines depreciate fully after use. Firms maximize current profits

$$\pi_R^E = p_R R^{Extr} - \chi_R \dot{N}_{Rt}, \tag{10}$$

which are a function of the revenue from selling the resource, the cost of extraction and investment in new technology. The inverse supply function of the resource is constant and we obtain a market equilibrium at resource price:

$$p_R = \chi_R \frac{1}{\delta \mu};. \tag{11}$$

We obtain the isoelastic demand of extraction firms for new machine varieties from the aggregate resource demand, equation (8), the extraction function, equation (9), and the derived resource price, equation (11):

$$\dot{N}_R = \frac{1}{\delta\mu}\frac{Y(1-\gamma)^{\varepsilon}}{(\chi_R \frac{1}{\delta\mu})^{\varepsilon}} \ . \tag{12}$$

Higher aggregate output Y has a positive effect on the demand for new machine varieties, while a higher machine price χ_R has a negative effect. The effects of the parameters from the geological and the technological functions δ and μ depend on the elasticity of substitution ε.

5.3 Extraction Technology Price

Technology firms sell new machine varieties to the extraction firms at price χ_R. Technology firms enter the market and develop new machine varieties until the value of entering, namely profits π_R, equals market entry cost, which is the cost to develop a new variety η_R. This free-entry condition is:

$$\pi_R = \frac{1}{\eta_R} \text{ if } M_R > 0 \ . \tag{13}$$

Because each machine variety is specific to deposits of certain grades, the present value of a patent for a new variety depends only on instantaneous profits

$$V_R(j) = \pi_R(j) = (\chi_R(j) - \psi_R)x_R(j) \ , \tag{14}$$

which are the difference between the machine price $\chi_R(j)$ and the machine production cost ψ_R multiplied by the number of machines x_R. Due to the grade-specific nature of technology, each firm builds one machine per machine variety, $x_R(j) = 1$, sells the machine to an extraction firm and stays in the market for one time period.

Technology firms do not have market power, because machine varieties are perfect substitutes in producing the homogeneous resource and there is free entry. Free entry implies that firms cannot block other firms from developing more advanced machine, varieties so that the resource can be extracted from deposits of lower grades.. We derive the machine price from the resource demand, equation (8), and the equilibrium resource price, equation (11):

$$\chi_R(j) = \left(Y/R^{Extr}\right)^{\frac{1}{\varepsilon}} (1-\gamma)\delta\mu. \tag{15}$$

5.4 Extraction

We obtain firms' equilibrium extraction from the machine price equation, the present value of profits, equation (14), and the free entry condition, equation (13):

Proposition 1 *Extraction increases with aggregate output Y, but also with parameter δ in the geological function and μ in the technology function in equilibrium.*

$$R^{Extr} = \frac{Y(1-\gamma)^{\varepsilon}}{\left(\left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\delta\mu}\right)^{\varepsilon}}\ , \tag{16}$$

This result speaks to the historical trends in extraction for many resources. It contrasts with standard growth models, in which the extraction of the resource declines at a constant rate following Hotelling (1931).

5.5 Resource Price

To derive the equilibrium real resource price, we insert the equilibrium resource extraction equation into the technology price equation (15) and obtain:

$$\chi_R(j) = \frac{1}{\eta_R} + \psi_R \ , \tag{17}$$

The machine price reflects the markup $\frac{1}{\eta_R}$ to finance R&D and the cost to produce machines ψ_R. Using the equilibrium machine price and the pricing equation (11), we compute the equilibrium resource price.

Proposition 2 *The real resource price level has an inverse relationship with the parameters from the geology function δ and the technology function μ:*

$$p_R = \left(\frac{1}{\eta_R} + \psi_R \right) \frac{1}{\delta \mu} \ . \tag{18}$$

For example, if the average crustal concentration of a resource δ is low, the price level of the resource will be high. One can think about gold in this case. In contrast, if δ is high, like in the case of aluminum, the price level will be relatively low.

The resource price in our model contrasts with that of traditional models, in which the price increases at the real rate of interest due to a scarcity rent based on the Hotelling (1931) rule of optimal extraction of a finite resource (see e.g. Jones and Vollrath, 2002; Groth, 2007). Instead of being limited by a fixed stock, firms can expand their reserves (or resource stocks) by innovation in our model. The resource price becomes equals to the marginal cost of production due to perfect competition. Marginal extraction costs are constant as geology and technology have offsetting effects (see Lemma 1). The supply curve is flat in the long run.

5.6 Aggregate Output Growth

The interaction of extraction technology and geology determine the growth rate of aggregate output on the balanced growth path among the usual factors. We define the balanced growth path (BGP) equilibrium as a consumption path that grows at the constant rate g^* with a constant relative price p.

Proposition 3 *There exists an unique BGP equilibrium in which the relative technologies are given by equation* (28) *in the appendix, and consumption and output grow at rate:*[10]

$$g^* = \theta^{-1}\left(\beta\eta_Z L\left[\gamma^{-\varepsilon}-\left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}p_R^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}\frac{1}{\beta}}-\rho\right). \tag{19}$$

The growth rate depends positively on the population (number of workers L) and negatively on the resource price p_R. The price is ultimately defined by the parameters

[10]This is under the assumption that $\beta\left[(1-\gamma)_R^{\varepsilon}(\eta_R R^{Extr})^{\sigma-1}+\gamma_Z^{\varepsilon}(\eta_Z L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}>\rho$ and $(1-\theta)\beta\left[\gamma_R^{\varepsilon}(\eta_R R^{Extr})^{\sigma-1}+\gamma_Z^{\varepsilon}(\eta_Z L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}<\rho$. There are no transitional dynamics in the model, as $\dot{N}_R$ is a jump variable.

δ in the geological and μ in extraction technology function (see equation (18)). For example, a lower crustal concentration δ leads to a higher resource price but a lower aggregate growth rate in equilibrium.

This outcome contrasts with that of standard models, where the depletion rate of the finite resource and the rate of population growth drag down the growth rate through a so called "depletion effect" (see Jones and Vollrath, 2002; Groth, 2007). The result also shows in conjunction with Propositions 1 and 2 that our model speaks to the historical trends of increasing extraction, constant real resource prices and increasing global economic output.

Adding the extractive sector changes the interest part of the Euler equation, $g^* = \theta^{-1}(r^* - \rho)$ compared to the model by Acemoglu (2002). Instead of two exogenous production factors, the real interest rate r^* only includes laor in our model, but adds the resource price, as the intermediate good price p_Z depends on the resource price p_R (see equation (24)). The intermediate good sector exhibits increasing aggregate returns to scale and is the engine of growth. The extraction sector has only constant aggregate returns to scale due to the rivalrous nature of extraction technology.

Under certain conditions there is a "limits to growth" effect that is common in conventional models. Economic growth is impossible if the resource cannot be substituted by other production factors (see Dasgupta and Heal, 1979). If $(1-\gamma)^{\varepsilon}\left(\frac{1}{\eta_R \delta \mu} + \frac{\psi_R}{\delta \mu}\right)^{1-\varepsilon} > 1$ holds, substitution between the intermediate good and the resource is low. R&D investment in extraction technology has a small yield in terms of new reserves.

5.7 Growth in Extraction Technology

We can show that a higher aggregate output level leads to a higher growth rate of innovation in extraction technology and a larger resource flow. Using the extraction function, equation (9), the resource demand, equation (8), and the resource price, equation (18) we obtain:

Proposition 4 *The growth rate of extraction technology develops proportionally to the level of aggregate output.*

$$\dot{N}_R^* = (1-\gamma)^{\varepsilon} Y \left(\frac{1}{\eta_R} + \psi_R \right)^{-\varepsilon} (\delta\mu)^{\varepsilon - 1} \ .$$

The incentive to innovate, thus, grows in line with the size of the economy as more resources are demanded from the aggregate output sector.[11]. The increased demand leads to more innovation in extraction technology and hence a larger flow of the resource.

The resource abundance, controlled by the the parameters of the geological function δ and the technology function μ, affects the rate of technological change in the extractive sector depending on the elasticity of substitution. A higher abundance (δ or μ up) leads to a lower resource price. If the resource and the intermediate good are complements ($\varepsilon < 1$), this leads to decelerating technology growth in the extraction

[11] Note that an increase in population raises the growth rate of the economy - as common in endogenous growth models - and hence leads to a higher resource supply in our context

sector. If the resource and the intermediate good are substitutes ($\varepsilon > 1$), a higher abundance causes an acceleration in technological change in the resource sector.[12]

Proposition 4 stands in contrast to the intermediate good sector, in which technology grows at the same rate as aggregate output. The reason for this difference is that extraction firms can only use the flow of new technology. Previously developed technology cannot be employed because it is grade-specific and the deposits of the corresponding grades have been depleted.

6 Extensions

6.1 Different Functional Forms

We can generalize our model to different forms of the geological and the extraction technology function. If the functional forms are such that increasing returns in the geology more than offset the decreasing returns in extraction technology, the resource price declines and growth rates of aggregate output and extraction increase.

If increasing returns in geology do not fully offset decreasing returns in technology, prices rise and growth rates of aggregate output and extraction are lower. As property rights hold for only one period, firms will increase extraction until the price equals production cost.[13] There will be no scarcity rent like in Hotelling (1931) and no social

[12] For an analysis of the relative changes in extraction and intermediate good technologies please see appendix Appendix 2.

[13] In the real world, firms typically do not hold property rights for resources over the long term. They mostly lease rights from private owners or the government for a definite period of time and for a constant share of their revenue. These leases typically require firms to start production at some

cost reflecting that present extraction pushes up future production cost (Heal, 1976).

6.2 The Effect of a Carbon Tax

Our model shows that an increasing carbon tax would discourage firms from innovation in extraction technology, potentially limiting fossil fuel extraction and greenhouse gas emissions going forward. Introducing a carbon tax T drives up the equilibrium resource price:

$$p_R = \left(\frac{1}{\eta_R} + \psi_R + T \right) \frac{1}{\delta\mu} \ . \tag{20}$$

The tax needs to steadily increase to more than offset the positive effect of the growing level of output on innovation in extraction technology:

$$\dot{N}_R^* = (1-\gamma)^{\varepsilon} Y \left(1/\eta_R + \psi_R + T\right)^{-\varepsilon} (\delta\mu)^{\varepsilon-1} \ .$$

As we assume that the resource and the intermediate good are not perfect substitutes, some innovation in fossil fuel extraction is necessary to keep the economy growing. We can think about crude oil that would still be needed for the production of plastics, for example.

A carbon tax negatively affects the aggregate growth rate on the BGP in our model. This is under the assumption that the tax revenue is not redistributed but

time or the lease is terminated early. In addition, there is a substantial risk of expropriation in many countries (see e.g. Stroebel and Van Benthem, 2013).

disappears. If the tax revenue was redistributed and, for example, subsidized innovation in the intermediate good sector, intuition suggests that this may partially offset the negative effect on aggregate output growth. For certain parameter sets such redistribution could even lead to a higher rate of aggregate output growth than without the tax, as innovation in the intermediate good sector leads to increasing aggregate returns in contrast to the extractive sector with its constant aggregate returns.

6.3 The Effect of New Resources on Economic Growth

Our data-set shows that many new types of resources have been discovered and came into use during the last three centuries. For example, aluminum was discovered and inventions made its extraction possible in the 19th century. It has become one of the most widely used metals since then.

Extending the model to a set of distinct resources allows us to study how such developments affect existing resource markets and aggregate output growth. We first define extraction R^{Extr}, prices p_R and investments M_R as aggregates of aluminum a and the related metal copper c, $i \in [a, c]$,

$$
\begin{aligned}
R^{Extr} &= \left(\sum_i R_i^{Extr \frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \\
p_R &= \left(\sum_i p_{R_i}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \\
M_R &= \sum_i M_{R_i}.
\end{aligned}
$$

Let's assume that aluminum and copper are substitutes ($\sigma > 1$) as they were both used for the production of containers, construction materials and house-ware items. If the aluminum price drops immediately to its steady-state and the steady state price of copper is higher, the price of the resource aggregate will immediately decline. This will lead to a permanent increase in the growth rate of the economy, equation (19).

The empirical data shows that aluminum prices initially declined over a couple of decades and then stabilized. To model this transition, we adjust the innovation possibilities frontier, equation (4), $\dot{N}_R^a = \eta_R^a \min\ (\bar{N}_R/N_R^a, 1)\ M_R^a$, in which $\bar{N}_R$ is the average technology level. Innovation for a newly discovered resource may benefit from the existing stock of generalextraction technology. Technology catches up to the average and then grows at the normal steady state rate.

7 Conclusion

Implementing geology and endogenous innovation in extraction technology into a standard growth model predicts that economic growth leads to constant resource prices and exponentially increasing extraction. Rising resource demand due to aggregate output growth incentivizes firms to invest in new extraction technology to convert lower grade deposits into reserves. Prices are constant as increasing returns from the geological resource distribution offset diminishing returns in innovation. In contrast to traditional growth models with resources, there is no depletion effect that drags down aggregate output growth.

Our model provides a building block for future research. It would be desirable to introduce a more complex extraction cost curve and to study the trade-offs that firms face between R&D investment and higher cost more closely. We also observe positive reserve holdings. A model with stochastic R&D could generate this phenomenon as firms would hold reserves to insure against risk. Furthermore, some of the factors that we omitted, such as recycling, other geological characteristics, resource efficiency and environmental policies could account for additional resource-specific dynamics.

Finally, our model helps to explain historical trends and it seems reasonable that these tends could possibly continue in the foreseeable future. However, the cost of extraction could increase over the very long run despite continuous innovation in extraction technology. As deposits of very low grades need to be extracted, energy requirements could increase in a non-linear way. The concave part of the log-normal geological distribution could also be reached. Interdisciplinary studies including economics and Earth sciences are needed to foster our understanding of these dynamics.

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Appendix 1 Proofs and Model Details

Appendix 1.1 Proof of Proposition 1

$$\begin{aligned} S(O^{\star}(N_{Rt})) &= -\delta \ln(O^{\star}(N_{Rt})) \\ &= -\delta \ln(e^{-\mu N_{Rt}}) \\ &= \mu\delta N_{Rt} \end{aligned}$$

Appendix 1.2 Derivation of Extraction Firms' New Reserves

The production function for new reserves can be written as:

$$R_t^{Tech} = \delta\mu \lim_{h \to 0} \frac{1}{h} \int_{N_R(t-h)}^{N_R(t)} x_R(j)^{(1-\beta)} dj \ , \tag{21}$$

Extraction firms produce reserves by employing machines x_R of different varieties j. Firms can only use varieties j to extract the resource from deposits of certain grades. As a consequence, only one machine is produced per variety, $x_R(j) = 1$ and only new machine varieties can be used $\dot{N}_R(t)$. In contrast, the intermediate good sector can use infinitely many machines from the full range of machine varieties $[0, N_Z(t)]$. As the resource is a homogenous good that can be produced from heterogeneous deposits, machine varieties are substitutes in the sense that they produce the same resource ($\beta = 0$). Under these assumptions, we obtain:

$$
\begin{aligned}
R_t^{Tech} &= \delta\mu \lim_{h\to 0} \frac{1}{h} \int_{N_R(t-h)}^{N_R(t)} 1 dj \\
&= \delta\mu \dot{N}_R \,.
\end{aligned}
$$

Appendix 1.2.1 Resource and Intermediate Good Prices

The relative price of the resource and the intermediate good from the marginal product condition of the aggregate production function is:

$$
\begin{aligned}
p &\equiv \frac{p_R}{p_Z} = \frac{1-\gamma}{\gamma} \left(\frac{R^{Extr}}{Z} \right)^{-\frac{1}{\varepsilon}} && (22) \\
&= \frac{1-\gamma}{\gamma} \left(\frac{\delta\mu \dot{N}_R}{\frac{1}{1-\beta} p_Z^{\frac{1-\beta}{\beta}} N_Z L} \right)^{-\frac{1}{\varepsilon}} && (23)
\end{aligned}
$$

The second line substitutes from the derived production functions of the extractive sector, equation (9), and of the intermediate good sector (see suppl. appendix).

Appendix 1.2.2 Proof of the Balanced Growth Path

Setting the price of the final good as the numeraire and rearranging gives:

$$
p_Z = \left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} p_R^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \,. \qquad (24)
$$

Intertemporal prices of the intermediate good are given by the real interest rate $[r_t]_{T=0}^{\infty}$. This implies constant intermediate good and resource prices p_Z and p_R respectively.

Household optimization implies $\lim_{t\to\infty}\left[\exp\left(-\int_0^t r(s)ds\right)(N_{Zt}V_{Zt}+\dot{N}_{Rt}V_{Rt})\right]=0$ and $\frac{\dot{C}_t}{C_t}=\frac{1}{\theta}(r_t-\rho)$. The total value of corporate assets is $N_{Zt}V_{Zt}+\dot{N}_{Rt}V_{Rt}$. Maximizing utility in equation (1) with respect to consumption and investment yields the first-order conditions $C^{-\theta}e^{-\rho t}=\lambda$ and $\dot{\lambda}=-r\lambda$. The consumption growth rate is:

$$g_c^*=\theta^{-1}(r^*-\rho)\ . \tag{25}$$

We solve for the BGP interest rate: $r^*=\theta g^*+\rho$. The free entry condition for the technology firms imposes that profits from investing in patents must be zero. Revenue per unit of R&D investment is given by V_Z, cost is equal to $\frac{1}{\eta_Z}$. So, $\eta_Z V_Z=1$. From equation (26), we obtain $\frac{\eta_Z\beta p_Z^{\frac{1}{\beta}}L}{r^*}=1$. Solving for r^* and using equation (25) we get:

$$g^*=\theta^{-1}(\beta\eta_Z L p_Z^{\frac{1}{\beta}}-\rho)\ .$$

We obtain the BGP growth rate substituting p_Z by equations (24) and (18).

Appendix 2 Directed Technological Change

Our setup allows firms to endogenously allocate investment in innovation across the two sectors. However, while Acemoglu (2002) includes two fixed factors, our model only includes labor as a fixed factor. The second factor, the reserves, are endogenous.

The two sectors are also not symmetrical. This changes the way directed technological change works.

Let V_Z and V_R be the balanced growth path net present discounted values of new innovations in the two sectors. Then the Hamilton-Jacobi-Bellman Equation version of the value function for the intermediate good sector $r_t V_Z(j) - \dot{V}_Z(j) = \pi_Z(j)$ and the free entry condition of extraction technology firms imply that:

$$V_Z = \frac{\beta p_Z^{1/\beta} L}{r^*} \text{ and } V_R = \chi_R(j) - \psi_R \; . \tag{26}$$

If V_R is greater than V_Z, there is a higher incentive to develop machine varieties in the extractive sector. Using equations (26) and including the equilibrium machine price (17) yields the relative profitability of technology firms in the two sectors:

$$\frac{V_R}{V_Z} = \frac{\chi_R(j) - \psi_R}{\frac{1}{r^*} \beta p_Z^{\frac{1}{\beta}} L} = \frac{\frac{1}{\eta_R}}{\frac{1}{r^*} \beta p_Z^{\frac{1}{\beta}} L} \; . \tag{27}$$

This expression highlights the effects on the direction of technological change: First, the price effect manifests itself in V_R/V_Z in a decreasing function of p_Z. A higher intermediate good price leads to a lower relative profitability of the extractive sector and incentivizes invention of labor-complementing technology. Second, the market size effect is due to V_R/V_Z decreasing in L. An increase in the labor supply leads to a greater market for technology complementing labor, incentivizing such inventions.

Using the free-entry conditions and assuming that both of them hold as equalities, we obtain the BGP technology market clearing condition $\eta_Z V_Z = \eta_R V_R$. Combining this condition with relative prices, equation (23), and relative profitability, equation (27), we obtain the BGP ratio of relative technologies:

$$\begin{aligned}\left(\frac{\dot{N}_R}{N_Z}\right)^* &= \left(\left(\frac{r^*}{\eta_Z \beta L}\right)^{\beta} \frac{1-\gamma}{\gamma p_R}\right)^{\varepsilon} \frac{L p_Z^{\frac{1-\beta}{\beta}}}{(1-\beta)\delta\mu} \\ &= \left(\frac{1-\gamma}{\gamma(1/\eta_R + \psi_R)}\right)^{\varepsilon} \frac{L}{(1-\beta)} * \\ &\quad (\delta\mu)^{\varepsilon-1}\left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\delta\mu}\right)^{1-\varepsilon}\right)^{(1-\varepsilon)\frac{\beta\varepsilon+1-\beta}{\beta}} . \end{aligned} \tag{28}$$

The second line is obtained by substituting in r^* from below equation (25) (and using (19)), p_R from equation (18) and p_Z from equation (24).

The supply of labor L has a positive effect on the optimal ratio of innovation in the two technologies. For parameters η_R, ψ_R, δ and μ both a positive and a negative effect on the direction of technological change are possible.