Time Series Econometrics

Basic Concepts Lecture

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Day 1

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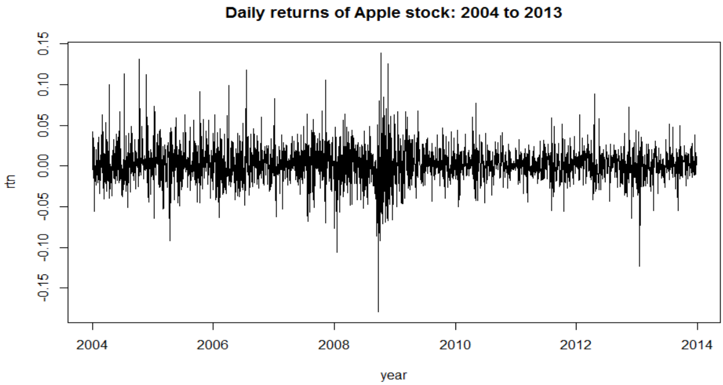
Financial & Economic Time Series

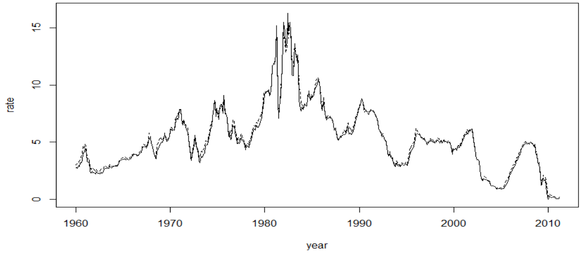
- Financial or Commodities: explicitly focused on valuation of an asset or quantity delivered over time.
- Volatility, higher moments, extreme values get more attention: how risk behaves
  - Asset returns of various kinds
  - Market indices
  - Firm earnings
  - ... measured at different frequencies (high frequency, daily, monthly, quarterly, annually)
  - Spreads (spot vs. futures, term structure of bonds)
- Others:
  - Macroeconomic aggregates (GDP, unemployment)
  - industry specific (hourly electricity load, renewable generation, smart meter data, locational/nodal prices, drilling rig activity)

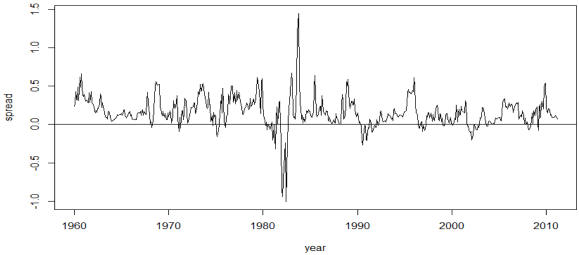
Daily Simple Returns



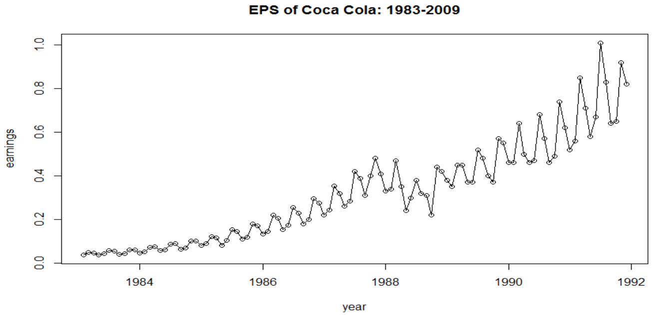
- How stable/dependent is the mean?
- Volatility clustering
- Extreme values/tails

Treasury Bill Spreads





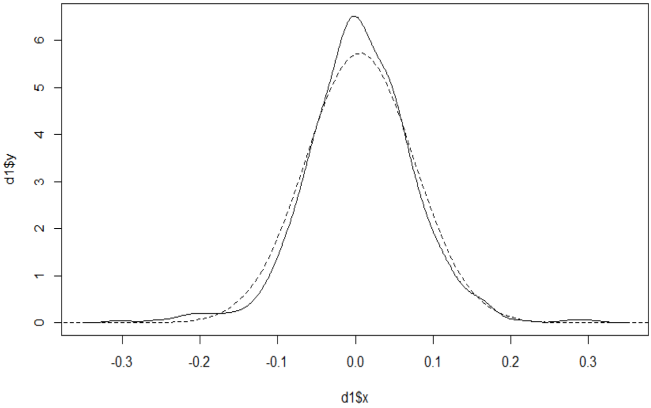
Quarterly Earnings



- Seasonality
- Trend

Compare to Normal Density

Monthly returns of IBM



Review of Returns

- Scale-free summary of investment opportunity
- Some attractive statistical properties (stationarity? Normality?)

One-period, simple return

$$1 + R_t = \frac{P_t}{P_{t-1}}$$

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$$

Review of Returns

Multiperiod or *k*-period return

$$\frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} \times \frac{P_{t-1}}{P_{t-2}} \times \ldots \times \frac{P_{t-k+1}}{P_{t-k}}$$

$$\frac{P_t}{P_{t-k}} = (1 + R_t) \times (1 + R_{t-1}) \times \ldots \times (1 + R_{t-k+1}) = \prod_{j=0}^{k-1} (1 + R_{t-j})$$

Annualized or per-period average (geometric vs arithmetic)

$$\left[\prod_{j=0}^{k-1} (1 + R_{t-j})\right]^{1/k} - 1 \approx \frac{1}{k} \sum_{j=0}^{k-1} R_{t-j}$$

Review of Returns

Log returns

$$r_t = ln(1 + R_t) = ln(\frac{P_t}{P_{t-1}}) = p_t - p_{t-1}$$

$$R_t = e^{R_t} - 1$$

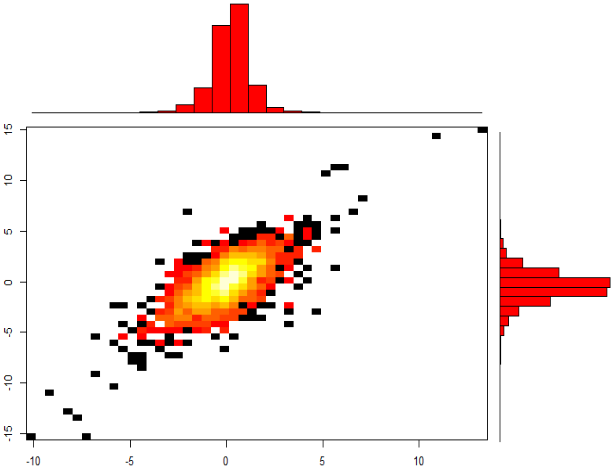
Multiperiod log return

$$ln[(1 + R_t) \times (1 + R_{t-1}) \times ... \times (1 + R_{t-k+1})] = \sum_{j=0}^{k-1} ln(1 + R_{t-j})$$

$$= r_t + r_{t-1} + ... + r_{t-k+1}$$

Joint Distributions

Daily returns of S&P Index (x-axis) and SPDR Energy ETF (y-axis)



Joint Distributions

Say, N assets held in T time periods

$$\{r_{it} | i = 1, ..., N;\ t = 1, ..., T\}$$

Suppose want to describe joint distribution of two random vectors:

$$\underline{\mathbf{X}} = (X_1, ..., X_k),\ \ \underline{\mathbf{Y}} = (Y_1, ..., Y_q)$$

- e.g., S&P and SPDR ETF returns in same time periods ($k = q$),
- same asset held in different time windows of length k and q,
- different assets in only partially overlapping periods, etc.

Joint Distributions

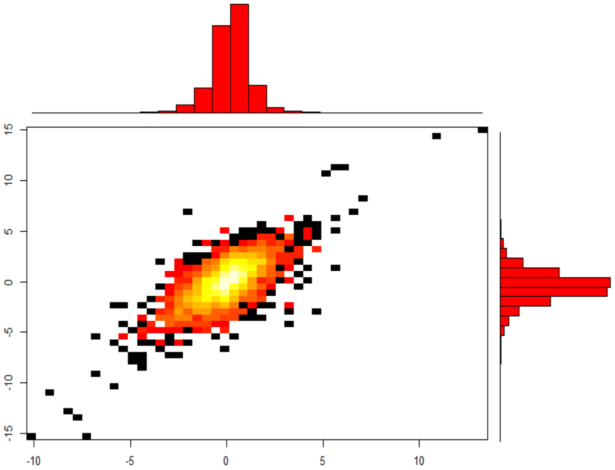
$$F_{\underline{\mathbf{X}},\underline{\mathbf{Y}}}(\underline{\mathbf{x}},\underline{\mathbf{y}};\underline{\theta}) = Pr(\underline{\mathbf{X}} \leq \underline{\mathbf{c_x}}, \underline{\mathbf{Y}} \leq \underline{\mathbf{c_y}}; \underline{\theta})$$

If X and Y are scalar random variables:

$$= \int_{-\infty}^{c_x} \int_{-\infty}^{c_y} f_{X,Y}(x,y;\theta)dydx$$

Joint Distributions

Daily returns of S&P Index (x-axis) and SPDR Energy ETF (y-axis)



Joint Distributions

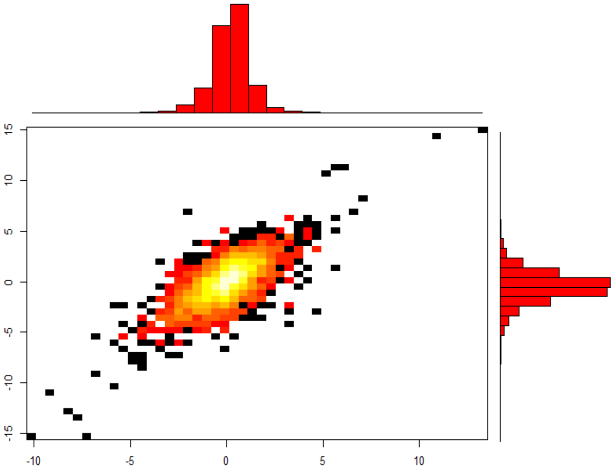
Marginal distribution of $\underline{\mathbf{X}}$ (rotate the picture and look from the side)

$$F_{\underline{\mathbf{X}}}(\underline{\mathbf{x}}; \underline{\theta}) = F_{\underline{\mathbf{X}}, \underline{\mathbf{Y}}}(\underline{\mathbf{x}}, \underline{\infty}; \underline{\theta})$$

Integrate over all the y's, looking at the distribution along the X-axis.

Joint Distributions

Daily returns of S&P Index (x-axis) and SPDR Energy ETF (y-axis)



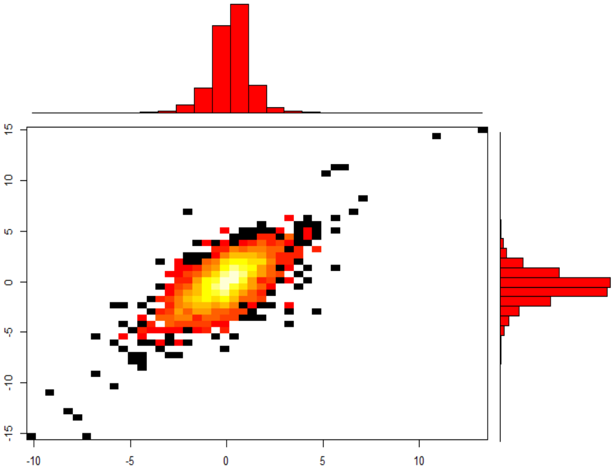
Joint Distributions

Conditional distribution of $\underline{\mathbf{X}}$ given $\underline{\mathbf{Y}} \leq \underline{\mathbf{c_Y}}$

$$F_{\underline{\mathbf{X}}|\underline{\mathbf{Y}}}(\underline{\mathbf{x}};\underline{\theta}) = \frac{Pr(\underline{\mathbf{X}} \leq \underline{\mathbf{c_x}}, \underline{\mathbf{Y}} \leq \underline{\mathbf{c_y}}; \underline{\theta})}{Pr(\underline{\mathbf{Y}} \leq \underline{\mathbf{c_y}}; \underline{\theta})} = \frac{\text{joint}}{\text{marginal of } Y}$$

Joint Distributions

Daily returns of S&P Index (x-axis) and SPDR Energy ETF (y-axis)



What about regressions?

- y_t = return of SPDR ETF at time t
- x_t = return of S&P at time t

A model:

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

More generally:

$$\underline{y} = f(\underline{\mathbf{X}}, \underline{\varepsilon}; \underline{\theta})$$

e.g., $\underline{\theta} = (\beta_0, \beta_1)$, and $f(\cdot) = \beta_0 + \beta_1 x_t + \varepsilon_t$.

- What is ε_t?
  - A measure of our ignorance?
  - A statement about, or a way to estimate, (β_0, β_1)?