Estimating Models

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linear Model $y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \varepsilon_t$

OLS $\min_{\hat{\beta}} \sum_{t=1}^{T} \varepsilon_t^2$

MLE : assume $\varepsilon_t \sim iid\ N(0, \sigma^2)$

Method of Moments:

from Gauss-Markov, $E(\varepsilon_t | \underline{x}_1, \underline{x}_2) = 0$

equivalently ① $E(\varepsilon_t \cdot x_{1t}) = 0 = Cov(\varepsilon_t, x_{1t})$

Population Moments ② $E(\varepsilon_t \cdot x_{2t}) = 0 = Cov(\varepsilon_t, x_{2t})$

③ $E(\varepsilon_t \cdot 1) = 0$

claim

Plug in sample analogue

① $\frac{1}{T}\sum_{t=1}^{T}(y_t - \beta_0 - \beta_1 x_{1t} - \beta_2 x_{2t}) x_{1t} = 0$

② $\frac{1}{T}\sum_{t=1}^{T}(y_t - \beta_0 - \beta_1 x_{1t} - \beta_2 x_{2t}) \cdot x_{2t} = 0$

③ $\frac{1}{T}\sum_{t=1}^{T}(y_t - \beta_0 - \beta_1 x_{1t} - \beta_2 x_{2t}) \cdot 1 = 0$

solve system of equations for $\beta_0, \beta_1, \beta_2$

Generalized Method of Moments – more conditions than unknowns

worried $E(x_{2t} \cdot \varepsilon_t) \neq 0$

But z_{1t}, z_{2t} meet condition

$E(\varepsilon_t \cdot z_{1t}) = 0$

$E(\varepsilon_t \cdot z_{2t}) = 0$

I have 4 conditions, 3 unknowns

Sample version is –

Overidentified system → no unique solution

$$\begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t \cdot 1 = 0 \\ \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t \cdot x_{1t} = 0 \\ \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t \cdot z_{1t} = 0 \\ \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t \cdot z_{2t} = 0 \end{bmatrix} = \underset{4\times 1}{m}$$

$\min_{\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2} \underset{1\times 4}{m^T} \cdot \underset{4\times 4}{\underset{\sim}{W}} \cdot \underset{4\times 1}{m}$

optimal $\underset{\sim}{W} = (\underset{4\times 1}{m} \cdot \underset{1\times 4}{m^T})^{-1}$