Hotelling Confronts CAPM: A Test of the Theory of Exhaustible Resources

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Hotelling confronts CAPM: a test of the theory of exhaustible resources

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Abstract. A model of pricing of natural-resource commodities that integrates financial and product markets is derived and tested. The model unifies two strands of the economic and financial literature: one that builds on the Hotelling model of pricing of exhaustible resources and the other that extends the Capital-Asset-Pricing Model of pricing risky assets. Tests of the predictions make use of a cost function that was estimated from a panel of small Canadian copper mines (Young 1992). Our tests fail to reject the theoretical restrictions that are implied by the Hotelling/CAPM model.

Hotelling se confronte au CAPM (MEDAF): un test de la théorie des ressources épuisables. Un modèle de la formation des prix des ressources naturelles intégrant les marchés financiers et physiques est dérivé et testé. Le modèle unifie deux domaines de la littérature économique et financière, l'un originant avec le modèle d'Hotelling du prix des ressources naturelles non-renouvelables, l'autre dérivant du modèle d'évaluation des actifs financiers (CAPM) risqués. Nous testons les prédictions de ce modèle en utilisant une fonction de coûts estimée avec des données longitudinales provenant des mines de cuivre canadiennes (Young 1992). Les restrictions théoriques implicites à notre modèle d'Hotelling/CAPM ne sont pas rejetées.

I. INTRODUCTION

In 1931 Hotelling published his classic article on the economics of exhaustible resources. Since that time, scores of authors have generalized and extended the basic

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model.[1] Moreover, there is really no competing theory that explains the supply and pricing of nonrenewable resources. By any standard, Hotelling's effort has been successful.

Unfortunately, when we turn to the empirical side, the picture is not as rosy.[2] Indeed, the majority of tests of the Hotelling model produce negative results. In particular, prices and rents do not increase exponentially as predicted; instead they tend to fall. It is interesting, however, that one of the few models that is not routinely rejected by the data originates in the finance literature. Miller and Upton's (1985a) hypothesis concerning the asset value of a mine receives some empirical support.

Our paper contains an empirical test that is based on the Capital-Asset Pricing Model or CAPM (Sharpe 1964; Lintner 1965), which also originates in the finance literature. It builds on the theoretical work of Hartwick and Yeung (1988) and Gaudet and Khadr (1991) that extends the Hotelling model to include risky-resource production and the possibility of diversifying risk by holding a portfolio of other assets.

It is well known that rates of return on real and financial assets are not highly correlated. Indeed, when returns on financial assets fall, investors protect themselves by switching into real assets. Moreover, this behaviour exacerbates the decline in the former and drives up the prices of the latter. If the return to holding a real asset is negatively correlated with the return on the market portfolio, investors will be willing to accept a lower return in order to diversify their risk. Our empirical tests explore this possibility.

The organization of the paper is as follows. In the next section, we describe previous empirical tests of the Hotelling model. We begin with papers that examine the time-series behaviour of prices, p, and thus are not direct tests of the Hotelling model. We move to models that assess the rate of growth of prices, $\dot{p}/p$, which are appropriate when marginal-extraction cost is zero. We then look at models that exploit duality theory to evaluate the rate of growth of shadow prices, $\dot{\lambda}/\lambda$, which are appropriate when marginal cost is non-zero and can depend on the rate of extraction and/or the level of reserves. We conclude with a discussion of tests that involve prices of natural-resource assets rather than commodities.

In section III, we modify the model of Gaudet and Khadr (1991) in two ways. First, we allow for non-constant returns to scale and stock effects in the cost function. In other words, extraction cost is allowed to vary with the rate of extraction and as the reserve base is depleted. This involves changing only the resource-extraction sector of the economy. Second, we derive our estimating equation as a no-arbitrage condition that holds in a somewhat broader context than a competitive equilibrium.

1 Extensions include extraction costs that vary with cumulative extraction (Levhari and Levitan 1977), uncertainty with respect to the size of the reserve base (Loury 1978) or the magnitude and timing of discoveries (Pindyck 1980), and strategic behaviour in resource markets (Slant 1976), to name just a few.

2 Empirical articles include Barnett and Morse (1963), Heal and Barrow (1980), Smith (1981), Slade (1982), Stollery (1983), Halvorsen and Smith (1984, 1991), Farrow (1985), Miller and Upton (1985a, b), and Young (1992).

In section IV, we describe our data and estimation technique. We make use of the cost function for copper mining and milling that was estimated by Young (1992). The basic data consist of a panel of input and output prices and quantities for fourteen small Canadian copper mines.[3] Our econometric model, a stochastic difference equation, relates the rate of growth of shadow prices to technological variables, such as extraction cost, and to financial variables, such as the risk-free rate of return, the rate of return on the market portfolio, and rates of growth of macroeconomic aggregates. Our estimating equation, which nests both the Hotelling model and the CAPM, can be used to assess these theories. Indeed, each model implies restrictions on the parameters of this equation. In addition to the basic CAPM, we consider extensions including the arbitrage-pricing theory of Ross (1976), Merton's (1973) intertemporal CAPM, and Brennan's (1970) after-tax CAPM.

The fourteen copper mines were fairly short lived, mostly less than twenty years, and their years of operation did not coincide perfectly. We thus have an unbalanced panel. Other econometric complications arise due to the presence of variables that were generated in an earlier regression (shadow prices and depletion effects), unobservable expectational variables, and non-constant variances of expected returns. We use a generalized method of moments (GMM) technique for dynamic-panel data to obtain efficient estimates in the presence of these complications.

In section V, we present the empirical results, and in section VI, we summarize and conclude. To anticipate, we cannot reject the joint Hotelling/CAPM model. Moreover, the financial and macroeconomic variables that distinguish our test from previous tests are significant at standard levels of confidence. It therefore seems that the omission of risk and the possibility of risk diversification provides a partial explanation for the poor empirical performance of Hotelling's theory. Nevertheless, as with previous investigations, we do not find strong support for the theory.

II. PREVIOUS TESTS OF THE HOTELLING MODEL

The simplest Hotelling model predicts that, in the absence of extraction cost, the price p of an exhaustible-resource commodity will rise at the rate of interest $r, \dot{p}/p = r$, where a dot denotes a time derivative. When marginal-extraction cost is non-zero, the shadow price, $\lambda = p - C_q$, rises at the rate of interest, $\dot{\lambda}/\lambda = r$, where C is total-cost and a subscript denotes a partial derivative. Finally, when extraction costs depend on the level of reserves remaining in the mine, R, the shadow price rises at a rate that is less than the rate of interest. This lower rate of shadow-price appreciation, which is given by $\dot{\lambda}/\lambda = r + C_R/\lambda$, reflects the user cost associated with deterioration of the quality of ore mined in the future. These are the predictions that have been tested.

1. The behaviour of prices

When one considers a century of data, a striking feature is the decline in the relative prices of the majority of mineral commodities. This stylized fact contrasts with the

3 These data were provided by Denise Young.

prediction of the simplest Hotelling model of exponential increases in the real prices of natural-resource commodities. Perhaps the first to analyse mineral-commodity prices formally were Barnett and Morse (1963), who looked at relative-price trends in an attempt to uncover evidence of natural-resource scarcity. They concluded that, because real prices had fallen over time, scarcity was not a problem.[4]

Other researchers who have examined price trends are not in complete agreement with Barnett and Morse. For example, Smith (1979) looked at the stability of the coefficients of estimated price-trend relationships and decided that the data are too volatile to support definitive conclusions. And Slade (1982) found that, after substantial initial declines, in the 1970s there was some evidence of an upturn in the real-price paths of many mineral commodities. Since that time, however, prices have been increasingly volatile with large run-ups followed by equally large declines (Slade 1991), but there is little evidence of sustained trends.[5]

Given these stylized facts, it is of interest to examine the simple Hotelling model to see if it can be modified to produce prices that fall over substantial periods. There are several modifications that can do the trick. First, in the simple model, the initial stock of reserves is known with certainty as of time zero. In practice, however, extraction of known deposits proceeds simultaneously with exploration for previously unknown ore bodies. Indeed, large discoveries increase the size of the stock and can cause prices to fall. This issue is examined by Pindyck (1980).

Second, the simple Hotelling model does not allow for technical change. The idea that new methods of extraction can lower costs, however, can be developed formally. This issue is explored by Slade (1982), who shows that prices can fall when cost-lowering mining techniques are introduced.

Finally, substitute materials can be found that cause the demand for mineral commodities to shift inward. Several authors (Heal 1976; Hanson 1980) have assessed the behaviour of resource prices and rental rates when a backstop technology is introduced and have shown that rents can fall in the presence of obsolescence. Periods of falling prices are therefore consistent with variants of Hotelling's model. Nevertheless, in the long run prices should rise.

2. *The behaviour of shadow prices*

More formal tests of the Hotelling model rely on estimates of the technology of production of the extractive firm combined with Euler equations associated with dynamic-profit maximization. Examples include Stollery (1983), Farrow (1985), Halvorsen and Smith (1984, 1991), and Young (1992).

These researchers have estimated dual restricted-cost or profit functions for individual mines or mining industries. They assume that the technology of mining involves extracting unprocessed ore, n, which is combined with other inputs to produce metal, q. In other words, both mining and refining are modelled. n, which

4 In an update, Barnett (1979) reached the same conclusion.

5 Tests were also performed by Heal and Barrow (1980) and Smith (1981), who related $\dot{p}/p$ to r and other variables and found little support for the Hotelling model.

is transferred inside a vertically integrated firm, is treated as a fixed factor in the production of q.[6]

Once the firm's technology is known, shadow prices or rental rates, λ, can be approximated by one of two methods. They can be calculated as (i) the difference between price and marginal cost, $p - C_q$ (Stollery 1983; Farrow 1985; Young 1992) or as (ii) the shadow price of the unpriced ore to the vertically integrated metal producer, $-C_n$ (Halvorsen and Smith 1984, 1991). These two estimates of λ do not measure the same thing. The first is the shadow price of one unit of contained metal in situ, whereas the second is the shadow price of one unit of ore of the current grade, also in situ.

The structural approach is able to deal with some of the shortcomings of the reduced-form models. In particular, technical change and demand shifts can be modelled. Nevertheless, although some studies support the Hotelling model, its overall performance has been poor. In particular, estimated shadow prices for some commodities decline at an even faster pace than product prices.

3. The Hotelling-Valuation Principle

A third class of model was developed by Miller and Upton (1985a, b), who based their test on a less widely known implication of Hotelling's analysis of the optimal time pattern of exploitation of an exhaustible resource. Miller and Upton call their implication the Hotelling-Valuation Principle (HVP). Specifically, they show that in a competitive market, the value of reserves in any currently operating, optimally managed mineral deposit should depend solely on the current spot price net of marginal-extraction cost, regardless of when the reserves will be extracted.[7] They tested their model using stock-market valuations of the oil and gas reserves of a sample of U.S. companies and found that the data are consistent with their principle. Subsequent tests, however, have found that the HVP overvalues mineral assets (see, e.g., Adelman 1990).

We thus see that, with a few exceptions, there is little empirical support for the predictions of the Hotelling model. This can mean either that firms do not maximize profits or that the model is too simple to explain observed behaviour. As the second explanation seems more fruitful, it is explored in our empirical tests.

III. THE THEORETICAL MODEL

The standard results from the theory of exhaustible resources apply to resource commodities sold in competitive markets under conditions of certainty. We maintain the competitive-market assumption but relax the certainty or perfect-foresight assumption. When price or extraction cost is a random variable, the mine is a risky asset, and when there are other risky assets in the economy, mine owners can diversify their risk. We therefore have a problem in portfolio choice. Before developing this problem formally, we review standard asset-pricing models.

6 In other words, the market price of n is not relevant. However, n is optimally chosen.

7 The Hotelling Valuation Principle does not hold, however, when there are stock effects in the cost function.

1. The Capital-Asset Pricing Model (CAPM)

In the simplest version of CAPM, risk-averse investors have homogeneous beliefs and two-fund separation holds in financial markers.[8] Under two-fund separation, portfolio decisions can be separated into the choice of an optimal combination of a (single) risky asset and the riskless asset (Tobin 1958). Moreover, a combination of two-fund separation and homogeneous beliefs implies that it is possible to price any risky asset using a convex combination of the risk-free rate of return r and the expected rate of return on the market portfolio r^m. In other words, all investors hold the same portfolio of risky assets.

Formally, the CAPM relationship says that if r^j is the expected rate of return on risky asset j, then

$$r^j = r + \beta^j(r^m - r). \tag{1}$$

In other words, the rate of return on asset j consists of the risk-free rate r plus an asset-specific risk premium $\beta^j(r^m - r)$. Moreover, the 'beta' coefficient, β^j, that determines the risk premium is the ratio of the covariance of r^j and r^m to the variance of $r^m, \beta^j = \sigma_{jm}/\sigma_m^2$.

There are a number of restrictive assumptions that underlie the derivation of the CAPM. Perhaps the most important from our point of view is the static nature of the model, which must be combined with a dynamic resource-extraction problem. We also consider relaxing the assumptions that imply two-fund separation and introducing non-neutral taxes. These extensions are of interest because of their empirical implications.

a. Intertemporal CAPM (ICAPM)

A dynamic version of CAPM in which investors trade continuously was developed by Merton (1973). Asset prices are assumed to evolve according to the relationship

$$\frac{dp_t^j}{p_t^j} = r_t^j dt + \sigma_j dz_j, \qquad j = 1, \ldots, N, \tag{2}$$

where r_t^j is the instantaneous (certain) rate of price appreciation for the jth asset and dz_j is the increment of a stochastic process z_j that obeys Brownian motion. In other words, z_j is a Wiener process. Equation (2) implies that current return is known exactly, but that uncertainty about future returns grows with the time horizon.

If investors maximize the expected utility of their lifetime consumption, and if the investment-opportunity set is constant over time, the ICAPM relationship is

$$r_t^j = r_t + \beta^j(r_t^m - r_t), \qquad \beta^j = \sigma_{jm}/\sigma_m^2. \tag{3}$$

8 Sufficient conditions for two-fund separation are that traders have preferences that are quadratic in wealth or that all risky-asset returns are normally distributed.

Equation (3) is the simplest version of the CAPM that is compatible with a dynamic model of resource extraction. Since it is virtually identical to the static CAPM, in what follows we refer to (3) as CAPM.

More generally, we expect current asset demands to be affected by the possibility of future changes in the investment-opportunity set. In particular, investors will demand assets as vehicles to hedge against unfavorable shifts in future investment possibilities. However, if $K - 1$ state variables are sufficient to describe changes in the opportunity set, then there are K fundamental assets (portfolios or mutual funds) that span the space of asset returns. We assume that K is much less than the total number of assets N.

Unfortunately, ICAPM is silent about not only the identity of the fundamental assets or factors, but also about their number K. The theory does suggest, however, properties that the factors should possess. A hedging asset should be correlated with expected future changes in investment opportunities, which suggests choosing the rate of return on a long-term bond as a factor.

b. Arbitrage Pricing Theory (APT)

The Arbitrage-Pricing Theory due to Ross (1976) was developed to deal with the restrictive assumptions that underlie two-fund separation (normality of returns or quadratic utility). Again we suppose that there are K fundamental assets (portfolios or mutual funds) that span the space of returns. If the unexpected part of each asset's return is a linear combination of the unexpected parts of the returns on the K fundamental assets, then the expected return of each asset is the same linear combination of the expected returns on the fundamental assets. CAPM is therefore a special case of APT with $K = 1$. APT, however, is more general in that it is an arbitrage relationship that holds outside of equilibrium.

ICAPM and APT can be integrated. For example, Chamberlain (1988) develops an intertemporal asset-pricing model that shows that, at each time t, asset prices obey the APT formula. Moreover, this formula is identical to the CAPM when $K = 1$ or to the ICAPM when $K > 1$. In our empirical work, we therefore make no distinction between the two multi-factor models.

As is the case with ICAPM, APT does not identify the factors or their number. Nevertheless, it suggests properties that they should possess. In particular, the time-series movements of the factors should explain a substantial fraction of the movements of the returns on the traded assets. In practice, rates of change of financial and macroeconomic aggregates are often used as factor (see, e.g., Chen, Roll, and Ross 1986).

c. After-tax CAPM

If firms pay dividends, the rate of return on the market portfolio is $r^m = (\dot{p}^m + \text{DIV})/p^m$, where p^m is the price of one share in the market portfolio, and DIV is the dividend that is received by the owner of that share. A complication arises, when dividends are taxed more heavily than capital gains, as was true in Canada and the United States.[9] This complication led to the development of an after-tax CAPM (Brennan 1970) and an empirical literature investigating the existence of tax effects on stock prices.[10] When unfavourable tax treatment causes investors to demand higher pre-tax returns on dividend-paying stocks, capital gains and dividend yields should be separated. Moreover, each should be multiplied by $(1 - T_t^i)$ where T^i is the marginal rate of taxation on asset class i.

In our empirical tests, we extend the Hotelling/CAPM model (HCAPM) that is derived below to include hedging, APT, and tax factors. The empirical debates that surround CAPM, however, are not our primary focus; instead, we wish to see how alternative specifications of the financial model affect HCAPM.

2. *The Hotelling/Capital-Asset-Pricing Model (HCAPM)*

a. Modelling uncertainty

When the assumptions that underlie CAPM hold, a modified version of CAPM determines the equilibrium rate of resource-shadow-price appreciation. We analyse the optimization problem faced by the manager of a mine from which ore is extracted and sold in a competitive market, and we derive the expected rate of return to owning this mine when production is risky and investors are risk averse. In other words, we obtain the appropriate risk-adjusted discount rate for cash flows from mining.

There are many ways to introduce uncertainty into natural-resource markets. In this paper, we focus on continuous technology and demand shocks. In our model, mining cost depends on the rate of extraction, q, the level of reserves remaining in the mine, R,[11] and a random productivity shock, Θ,

$$TC = C(q, R, \Theta). \tag{4}$$

Furthermore, the rate of technical progress consists of a deterministic portion plus stochastic increments. In other words, Θ is an Ito process defined by

$$\frac{d\Theta}{\Theta} = \mu_\Theta dt + \sigma_\Theta dz_\Theta, \tag{5}$$

where μ_Θ is the instantaneous rate of technical change.

With uncertain industry extraction, price p is a random variable. As we are interested in the mine, not the industry, we adopt an exogenous specification of the evolution of p. Gaudet and Khadr (1991), however, show that, when the technology shock Θ is a geometric Brownian motion (GBM) as in (5), in a competitive equilibrium, p is also a GBM. We assume that price evolves according to

$$\frac{dp}{p} = \mu_{pt} dt + \sigma_p dz_p, \tag{6}$$

9 Appendix C contains a brief discussion of Canadian taxation of capital gains, dividends, and resource rents.

10 For example, Blume (1980) tests tax effects using U.S. data, whereas Morgan (1980) uses Canadian data. Both find evidence of dividend aversion.

11 Costs rise when it becomes necessary to extract ores that are in thinner veins, more deeply buried, or of lower grade. It is clearly the reserves that remain *in the mine* that matter here. The evolution of price, in contrast, depends on *industry* reserves (see equation (6)).

where μ_{pt}, the instantaneous rate of price appreciation, varies over time as *industry* reserves change. Industry reserves, however, are exogenous to the firm. The price-taking firm knows the current rate of price appreciation exactly, but uncertainty about future returns grows with the time horizon.

The specification (6) has a number of advantages.[12] When there is no uncertainty ($\sigma_p = 0$), prices obey the standard Hotelling rule, and when the resource is not exhaustible ($\mu_{pt} = 0$), prices obey the rule for commodities that are traded in efficient markets. Furthermore, (6) allows for unforeseen events such as entry and exit or even irrational behaviour on the part of a resource cartel.[13] Indeed, (6) can approximate any continuous price path regardless of the model that underlies its generation.

b. Efficient resource extraction

The manager chooses an extraction path $\{q_t\}, 0 < t < \infty$, to maximize the expected present value of profits, where profits are discounted by the appropriate risk-adjusted discount factor, ρ. This stochastic-control problem has a value function V that is given by

$$\begin{aligned} V(R, p, \Theta) &= \max_{\{q_\tau\}} E_t \left\{ \int_t^{\infty} e^{-\rho(\tau - t)} [p_\tau q_\tau - C(q_\tau, R_\tau, \Theta_\tau)] d\tau \right\} \\ &=: \max_{\{q_\tau\}} E_t \left\{ \int_t^{\infty} e^{-\rho(\tau - t)} \pi_\tau d\tau \right\}, \end{aligned} \tag{7}$$

subject to

$$\dot{R}_\tau = -q_\tau, \qquad R_\tau \geqq 0, \qquad q_\tau \geqq 0, \tag{8}$$

(5), and (6). Instantaneous profit π is assumed to be concave in q. The first-order conditions for this problem can be manipulated to yield

PROPOSITION 1. *When the mine is managed optimally, the evolution of the shadow price, $\lambda = p - C_q$, is governed by*

$$\frac{\frac{1}{dt} E_t d\lambda}{\lambda} =: \frac{\mathring{\lambda}}{\lambda} = r + \frac{C_R}{\lambda} + \beta(r^m - r). \tag{9}$$

Proof. See appendix A.

12 Pindyck (1981) was perhaps the first economist to assume that resource prices follow a GBM.

13 Clearly, a resource cartel violates a competitive-market assumption. In that sense, our partial-equilibrium framework is more general. We required only that the firms of interest are price takers and need not specify how the expected-price trajectory is determined.

Equation (9), which forms the basis for our econometric tests, is the HCAPM relationship. We can compare it with the case where there is no uncertainty in the extraction sector. When $\sigma_\Theta^2 = \sigma_p^2 = 0$, λ is not a random variable, $\beta = 0$, and we have the standard Hotelling relationship, $\mathring{\lambda}/\lambda = r + C_R/\lambda$. When production is risky, in contrast, $\mathring{\lambda}/\lambda$ is the average expected rate of change of the shadow price, which is equal to the risk-free rate of return for natural-resource assets, $r + C_R/\lambda$, plus the risk premium, $\beta(r^m - r)$. When investors are risk neutral, the risk premium is zero. Finally, when $C_R = 0$, the average expected rate of resource-price appreciation obeys the standard CAPM relationship.

If β were negative (i.e., if the expected rate of shadow-price appreciation were negatively correlated with the expected rate of return on the market portfolio), investors would be willing to own mines that yielded less than the risk-free rate of return for natural-resource assets. This is true because holding an asset with a negative beta would reduce the risk associated with holding the market portfolio. If β were positive, in contrast, investors would demand a rate of return that exceeds $r + C_R/\lambda$ as their payment for bearing risk.

Condition (9) is derived as a no-arbitrage relationship; we do not solve for the competitive equilibrium. The assumption of non-constant-marginal cost is therefore crucial. Indeed, the manager's only control variable is extraction. If variations in q did not affect marginal cost, the manager could not ensure that (9) held. In particular, when marginal cost is constant, price must change until, in equilibrium, (9) holds. With our model, in contrast, even when (6) does not result from a competitive equilibrium, (9) is valid.

IV. DATA AND EMPIRICAL SPECIFICATION

1. Data and preliminary data analysis

The application involves copper mines that were open for some portion of the 1956–82 period. The price of copper is extremely volatile, and mines are therefore risky assets. From the point of view of risk diversification, however, *undiversifiable* risk matters. Prior to discussing the data, therefore, we examine the relationship between the rate of change copper price and the return on the market portfolio.

Table 1 shows that the North American producer price of copper appreciated at an average rate of 3.3 per cent a year in nominal terms over the 1956–82 period. It also shows that the variance of copper price is nearly 100 times its mean. Clearly, risk is an important feature of this market. Nevertheless, the rate of copper-price appreciation is virtually uncorrelated with the return on the Toronto Stock Exchange 300 Index (r^m). In addition, when we separate the overall return into capital gains and dividends, we see that $\dot{p}/p$ is positively correlated with capital gains but negatively correlated with dividends. There is some evidence, therefore, that much of the risk associated with owning copper mines is diversifiable. Indeed, it is even possible that the β for copper mines is negative. In other words, it is

TABLE 1
Statistics for $\dot{p}/p$

Mean	Variance	Correlation with r^m	Correlation with K gains	Correlation with dividends
3.3	241.6	0.09	0.31	−0.23

possible that investors are willing to accept a low rate of return on mines in order to improve their risk positions.[14]

The raw mining data, which were collected by Young (1992), are a panel of observations on fourteen small Canadian copper mines. These data consist of prices and quantities of inputs, outputs, and ore grades for each year of operation of each mine. The firms were chosen to conform as closely as possible to the assumptions of the Hotelling model. For example, all firms were small in the domestic as well as the international market and can therefore be considered to be price takers. Moreover, each operated in a small geographic area where reserves were known fairly accurately at the time that the mine opened. Finally, mines were excluded if the data for early years were incomplete.

The fourteen mines jointly span the 1956–82 period. No single mine, however, operated in every year. Observations for each firm end when the firm ceases production, is acquired by another firm, or becomes involved in other ventures. In the latter two instances, the copper-cost data can no longer be separated from other cost data. These data are described in greater detail by Young (1992).

The data were used to estimate translog-cost functions for mining and milling. In other words, Young estimated a function, $C(n, q, R, v)$, where n is extraction, q is the output of the milling process, R is remaining reserves, and v is a vector of input prices.[15] This function is a second-order approximation to an arbitrary technology. Estimation of the HCAPM relationship (equation (9)) requires firm-level values of $C_q = \partial C/\partial q$ and $C_R = \partial C/\partial R$. Young provided us with these partial derivatives for each firm in each year.[16]

We re-emphasize the importance of using marginal rather than average costs. With constant returns to scale, the shadow price λ is $p - c^i$, where c^i is a mine-specific constant. The mine manager, therefore, has no control variable that can ensure efficient extraction. Moreover, when all mines have the same risk charac-

14 The β for a mine, which is a real asset, is not the same as the β for a traded share in the mining firm, which is a financial asset. Clearly the two are correlated. When the stock market crashes, however, the value of *shares in mining companies* can follow. The *price of copper metal*, in contrast, can rise.

15 Young estimated four versions of her model that differ according to whether the capital stock is assumed to be quasi-fixed or in long-run equilibrium and whether the dynamic restrictions associated with the Hotelling model are imposed.

16 We chose to use the data from the cost function in which capital is a variable factor, since the restricted cost function yielded positive estimates of C_R. Moreover, we chose the version that does not impose Hotelling's rule, since this rule will be tested.

teristics, efficiency requires that all mines have the same average costs. The assumption of identical average costs, however, is clearly violated in practice. When marginal costs vary with the rate of extraction, in contrast, the optimizing behaviour of mine managers leads to equality of the marginal profit associated with each unit produced, regardless of how price is determined.

Estimation of (9) also requires data for the rate of return on a risk-free asset, r^f, and on the market portfolio, r^m. For the former we use the Canadian 90-day Treasury Bill rate, converted to an annual rate of return by averaging the monthly numbers. We use the return on the Toronto Stock Exchange (TSE) 300 Index as a measure of r^m. The TSE, which is the largest Canadian stock exchange and the seventh largest in the world, handles about 75 per cent of Canadian trading. Although it might be preferable to use a wider measure of the market return, the TSE 300 is the broadest generally available index for Canadian stocks.[17] Moreover, since the TSE 300 is made up of large firms, the problem of thin trading is reduced. Dividends associated with holding the TSE 300 are separated from capital gains by subtracting the share-price index from the total-return index. This is done so that tax effects can be assessed.

We use a long-term bond rate for the hedging asset required by the intertemporal CAPM. This was chosen as the Canadian Government 10-year bond rate and is denoted r^{lt}. The difference between r^{lt} and r^f is also a proxy for the term structure of interest rates.[18] Finally, we must specify other 'factors' for the Arbitrage Pricing Theory. As Huberman (1987) notes, there are three methods of doing this. The first approach uses factor analysis to estimate the factor loadings. The second uses the estimated covariance of asset returns to choose factors in a somewhat more ad hoc fashion. Finally, the third approach, which is purely judgmental, uses a combination of economic theory and intuition to pick the factors. We follow Chan, Chen, and Hsieh (1985) and Chen, Roll, and Ross (1986) in choosing the third approach and make no clear-cut distinction between APT and ICAPM. In other words, we estimate a multi-factor model.

A priori, one expects general-economic variables to influence systematic or undiversifiable risk. Our additional factors are the growth rate of gross domestic product, the rate of inflation, and the Canadian/U.S. dollar exchange rate.[19] The current rate of economic activity is often regarded as an indicator of the health of

17 It is common to use a share or equally weighted index of the stocks traded on an exchange as a proxy for r^m. Unfortunately, the data needed to construct such an index are not available as far back as 1956. We therefore follow Mehra and Prescott (1985) and use a less comprehensive index.

18 Chan, Chen, and Hsieh (1985) and Chen, Roll, and Ross (1986) use this proxy. In addition to the term structure, the distinction between the bond and the T-Bill rate is that with the former, income is secure, whereas with the latter, the interest rate is secure. The latter is therefore the appropriate proxy for r^f.

19 Chan, Chen, and Hsieh (1985) find that industrial production, time-varying risk premia, the term structure, and to a lesser extent the rate of inflation influence expected stock returns. We use GDP instead of industrial production because no consistent measure of industrial production exists for Canada over the period that we are studying. An exchange-rate variable is not often included in U.S. studies, but it seems more important for Canada.

the economy. The variable that we use in the regression analysis is the deviation of $\dot{GDP}/GDP$ from trend (GDFT). Changes in the rate of inflation influence nominal cash flows as well as the nominal rate of interest. For the regression analysis we use deviations from the trend in the rate of inflation of the CPI (CDFT). Finally, the exchange rate influences foreign investors' willingness to hold Canadian stocks. We again construct deviation-from-trend growth in the Canadian/U.S. dollar exchange rate (EDFT).[20]

Capital gains and dividend data were found in TSE 300 Indices (1991), consumer and producer-price indices were taken from CANSIM,[21] and all other data came from the *Canadian Economic Observer – Historical Supplement* (Statistics Canada Catalogue #11-210).

Trading takes place continuously; our data, however, are annual. We therefore replace the stochastic differential equation (9) with a stochastic difference equation in which rates of growth are calculated as

$$\left(\frac{\dot{x}}{x}\right)_t = \frac{x_t - x_{t-1}}{x_{t-1}}. \tag{10}$$

All rates of growth are calculated from yearly average data. In particular, the capital-gains portion of r^m is calculated as the rate of growth of the average share-price index. We do this to be consistent with the construction of λ and C_R. In addition, we construct the risk-free rate of return and the long-term bond rate for year t by taking averages over years t and $t-1$.[22] Finally, all estimated trends are linear.

Variable mnemonics used in the tables are as follows:

LDOT the growth rate of the shadow price, $\dot{\lambda}/\lambda$,
CRL the depletion effect, C_{R_t}/λ_{t-1},
RF the risk free rate of return, r^f,
RMSPI the rate of return on the share-price index, $\dot{p}^m/p^m$,
DIV the dividend return, DIV_t/p^m_{t-1},
RLT the long-term bond rate, r^{lt},
GDOT the growth rate of nominal GDP,
GDFT the deviation from trend GDOT,
CPIDOT the rate of inflation,
CDFT the deviation from trend CPIDOT,
EDOT the rate of change of the exchange rate,
EDFT the deviation from trend EDOT.

Table 2 shows means, standard deviations, minima, and maxima for each variable. The first two variables are averaged over the 144 observations in the panel,

20 The results that we obtain are insensitive to whether we use rates of change or their deviations from trend. We detrend the macro variables so that they have zero mean.
21 CANSIM is a computerized database that is maintained by Statistics Canada.
22 See Washburn and Binkley (1990) for a discussion of this issue.

TABLE 2
Summary statistics

Variable	Mean per cent	Standard deviation	Minimum	Maximum	Correlation with LDOT
LDOT	5.6	48.3	−137.6	373.2	1.00
CRL	−1.9	2.5	−17.0	−0.1	−0.05
RF	6.1	2.7	3.0	15.8	−0.12
RMSPI	5.1	14.4	−24.0	36.1	0.25*
DIV	5.1	11.3	−14.2	38.2	−0.27*
RLT	7.3	2.1	3.9	14.7	−0.09
GDOT	9.9	4.2	3.5	19.4	0.34*
CPIDOT	5.4	3.7	0.8	12.4	0.02
EDOT	0.9	3.0	−3.3	7.8	0.03

* Denotes significance at 1 per cent.

whereas all others are averaged over the twenty-seven years of the panel. Shadow prices are on average positive, whereas depletion effects are negative, implying that their ratio is negative. Moreover, unlike prices, shadow prices have grown faster than the rate of inflation. In addition, they are extremely volatile, a fact that is not surprising given the volatility of copper prices, which is compounded by technological shocks. Finally, the rate of return on the market portfolio (the sum of RMSPI and DIV) is higher and more variable than RF.

As a prelude to more detailed analysis, the final column in table 2 lists correlation coefficients between each explanatory variable and LDOT. We see that, similar to the results for price, LDOT is positively correlated with RMSPI but negatively correlated with DIV. In other words, the covariances between the rate of growth of the shadow price and the two sources of returns on the market portfolio are similar in magnitude but opposite in sign. LDOT is also positively correlated with GDOT; all other correlation coefficients are statistically insignificant. Nevertheless, the correlation between LDOT and RF is marginally significant (at the 10 per cent level) and negative. A negative relationship is opposite to that predicted by the Hotelling model but consistent with previous empirical findings. Finally, there is little trend in real shadow prices.

2. Empirical specification

The most general model that we have discussed can be written as

$$\text{LDOT}_{it} = \alpha_0 + \alpha_1\text{CRL}_{it} + \alpha_2\text{RF}_{it} + \alpha_3\text{RMSPI}_{it} + \alpha_4\text{DIV}_{it} + \alpha_5\text{RLT}_{it} + \alpha_6\text{GDFT}_{it} + \alpha_7\text{CDFT}_{it} + \alpha_8\text{EDFT}_{it} + \epsilon_{it}, \qquad i = 1\ldots 14,\ t = t_{0i}\ldots T_i. \tag{11}$$

All other models are nested in (11) and can be obtained by setting coefficients equal to zero.

β does not appear directly in the estimating equation; the method that we use to obtain β from (11) is described in appendix B. Equation (11) implies that β is constant over time and across assets. We test for and accept homogeneity across firms. Unfortunately, our time series are too short to allow modelling of time-varying betas.

One can estimate equation (11) by ordinary-least squares (OLS). There are a number of reasons, however, why OLS yields inefficient parameter estimates. These reasons include the panel nature of the data, generated explanatory variables, unobservability, and non-constancy of the variance of LDOT.

To illustrate, t_{0i} and T_i in (11) are the beginning and ending periods for firm i; the unbalanced nature of the panel is reflected in the fact that the sample for firm i is indexed by i. Second, both C_q and C_R were generated from an earlier regression, and the variables LDOT and CRL are non-linear functions of these generated variables. Third, equation (11) contains realized values of unobservable expectational variables (i.e., rates of return on portfolios and rates of growth of macro aggregates). Finally, variances of expected returns are unlikely to be constant.

To circumvent these problems, we use a generalized method of moments (GMM) estimator for unbalanced panels.[23] In addition to lagged values of variables in equation (11), we use the year, the rate of change of the producer-price index, and the rate of growth of aggregate consumption as instruments.

With respect to the unobservables, we assume that investors have rational expectations and use all information efficiently in forming forecasts. GMM minimizes the correlation between any information known at time t and the period-t innovations. Since our variables are rates of growth, variables dated $t-2$ or earlier can act as valid instruments. As a check on the model, we present tests of the overidentifying restrictions that ensure that our instrument set is uncorrelated with the errors.

The use of averaged data induces an MA(1) structure in the error term (Working 1960). Moreover, heteroscedasticity can be introduced by non-constant variances of asset returns. We correct for heteroscedasticity and serial correlation of an unknown form using Newey and West's (1987) procedure.

If we rewrite equation (11) as $y = X\alpha + u$, and let Z be a matrix of instruments, the GMM objective function is

$$\min_{\alpha} \; u'Z(M)^{-1}Z'u, \tag{12}$$

where $Z'u$ is a vector of moment restrictions, and M is the Newey and West (1987) weighting matrix. We use an iterative procedure to minimize (12). In the first iteration, the weighting matrix, M, is replaced by an identity matrix, $M_1 = I$, and in subsequent interactions $j > 1$, the previous-iteration residuals, $\hat{u}_{j-1}$ are used to estimate M_j.[24] This process continues until M converges.

The various models and tests are as follows:

23 An appendix that describes the estimation technique is available from the authors upon request.
24 The standard practice is to iterate only twice. Kocherlakota (1990), however, suggests that the small-sample properties of two-iteration estimators are worse than those of multi-iteration estimators.

i) Hotelling
   A priori coefficient restrictions: $\alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 0$.
   Test: $\alpha_0 = 0$, $\alpha_1 = 1$, $\alpha_2 = 1$.
ii) CAPM
   A priori coefficient restrictions: $\alpha_1 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 0$.
   Test: $\alpha_0 = 0, \alpha_2 + \beta = 1$.
iii) HCAPM
   A priori coefficient restrictions: $\alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 0$.
   Tests: Hotelling: $\alpha_0 = 0$, $\alpha_1 = 1$.
   CAPM: $\alpha_0 = 0$, $\alpha_2 + \beta = 1$.
   Joint: $\alpha_0 = 0$, $\alpha_1 = 1$, $\alpha_2 + \beta = 1$.
iv) K-factor model[25]
   No a priori coefficient restrictions.
   Tests: Hotelling: $\alpha_0 = 0$, $\alpha_1 = 1$.
   CAPM: $\alpha_0 = 0$, $\alpha_2 + \beta + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 = 1$.
   Joint: $\alpha_0 = 0, \alpha_1 = 1, \alpha_2 + \beta + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 = 1$.
v) After-tax CAPM
   No a priori coefficient restrictions.
   RMSPI_t and DIV_t multiplied by $(1 - T_i)$ for year t (see appendix C).
   Tests: Same as for K-factor model.

For models (iv) and (v) we also test two-fund separation. The null hypothesis is $\alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 0$, and rejection of the null is consistent with $K > 1$. In other words, it is evidence that additional factors contain information that is useful for pricing risky assets.

V. EMPIRICAL RESULTS

Table 3 contains the GMM estimates of equation (11). In this table, standard errors are shown in parentheses under their respective coefficients, and $\hat{\beta}$ is the estimate of beta (see appendix B). The equation shown corresponds to the after-tax model. The equation without correction for tax factors, however, is qualitatively similar.

First consider the coefficients of the 'Hotelling' variables, α_1 and α_2. The final estimates of these coefficients are not significantly different from zero at standard levels of confidence. Nevertheless, they have their predicted signs.

Next consider the coefficients of the two components of the rate of return on the market portfolio, α_3 and α_4. These coefficients are negative and significant, as is the calculated beta. A negative β is evidence that these real assets are good hedges against systematic risk, and investors are willing to accept a smaller return as a consequence.

Finally, consider the additional factors. With the exception of RLT, the coeffi-

25 The appropriate test for the K-factor model is discussed by Huberman and Kandel (1987).

TABLE 3
GMM estimates (dependent variable: $\dot{\lambda}/\lambda$)

Model	Constant (α_0)	CRL (α_1)	RF (α_2) Per cent	RMSPI (α_3) Per cent	DIV (α_4) Per cent	RLT (α_5) Per cent	GDFT (α_6) Per cent change	CDFT (α_7) Per cent change	EDFT (α_8) Per cent change	$\hat{\beta}$	J-statistic
After tax											
First step	0.27	5.43*	31.82	−3.27*	−2.67	−58.15	8.37	−44.67**	6.94*		
	(0.22)	(2.73)	(47.9)	(1.58)	(1.37)	(74.7)	(8.27)	(9.82)	(3.50)		
Newey	0.04	3.54	6.72	−0.55*	−1.13*	−15.47	9.30**	−22.20**	4.79**	−0.70	10.25
West	(0.08)	(2.45)	(5.06)	(0.26)	(0.47)	(29.02)	(3.11)	(3.61)	(1.68)		[0.74]

NOTES
Standard errors in parentheses; p-values in brackets.
* denotes significance at 5 per cent.
** denotes significance at 1 per cent.

TABLE 4
GMM hypothesis tests

Test: χ^2

Model	Overall Explanatory power	Hotelling	CAPM	Joint	2-fund separation
After tax	158.2*	1.08	1.01	1.51	89.93*
	(0.00)	(0.58)	(0.60)	(0.68)	(0.00)

NOTES
p-values in parentheses.
* denotes rejection at 1 per cent.

cients of these variables are significant and have reasonable signs.[26] We see that shadow prices grow faster when GDP is growing, when the rate of inflation is falling, and when the Canadian dollar is appreciating. All of these results make intuitive sense. Furthermore, they indicate that financial and macro factors are important determinants of shadow prices.

Table 4 contains hypothesis tests for the various models. All tests are Wald χ^2 statistics, and p-values for each test are shown in parentheses under the corresponding statistics. The table shows that the simplest Hotelling model is not rejected. However, this model has little explanatory power (i.e., one cannot reject the hypothesis α_1 and α_2 are jointly zero). The full model, in contrast, has high explanatory power. It therefore makes sense to test the coefficient restrictions implied by the theory.

The χ^2 test statistics reveal that none of the theoretical restrictions implied by HCAPM (Hotelling, CAPM, and Joint in the table) is rejected. This regularity is encouraging for the empirical relevance of the modified Hotelling model. We see that when financial variables that adjust for risk are added, the model receives empirical support. Moreover, failure to reject the theoretical restrictions is not vacuous, as with the simplest Hotelling model.

The restrictions corresponding to two-fund separation, in contrast, fare less well: they are rejected at any reasonable level of confidence. Rejection implies that investors use information contained in macro aggregates to price assets. This last finding is consistent with other tests of the APT (see Connor and Korajczyk 1992 for a survey). A multi-factor model clearly outperforms a model with $K = 1$.

The J-statistic in table 3 tests the overidentifying restrictions. This statistic, which is distributed χ^2 with degrees of freedom equal to the number of instruments minus the number of parameters, is well within acceptable levels.

The estimate of β is not only negative, but fairly large in absolute value. This

26 The magnitudes of these coefficients may seem large. Recall, however, that these variables, unlike the others, have zero means.

result is very convenient; it is consistent with CAPM, and it explains a number of puzzles concerning the generally poor empirical performance of the Hotelling model. Nevertheless, we are somewhat sceptical. Real assets can be good hedges against disappointing performance of financial assets. The degree of risk diversification that is implied by a β of -0.7, however, seems excessive. In other words, the addition of risk may take us part of the way towards explaining the Hotelling puzzle, but the degree of diversification that is required to obtain reasonable signs and modest significance for the 'Hotelling' coefficients seems too large to be believable. We therefore suggest that there are other factors in addition to risk that can be driving the results.

For example, copper is a storable commodity. Moreover, when inventories are in excess supply, extraction costs are sunk. In a downturn, therefore, price might drop until price net of *marginal storage cost* rises at the rate of interest. Given that storage cost is considerably less than extraction cost, price could fall precipitously and by large amounts.

VI. SUMMARY AND CONCLUSIONS

A model of pricing of a risky natural-resource commodity sold by risk-averse investors is derived and tested. This model, which unifies financial and product markets, differs from previous research in a similar view (e.g., Hartwick and Yeung 1988; Gaudet and Khadr 1991) in two respects. First, we allow for non-constant returns and depletion effects in the cost function, and second, we derive our HCAPM as a no-arbitrage rather than a competitive-equilibrium condition.

The second modification has both strengths and weaknesses. The weaknesses are associated with the fact that we do not derive an equilibrium-price process. The strength is that our equilibrium condition holds in a more general setting. For example, the behaviour of price could result from the actions of producers who have market power. As long as the firms of interest are price takers, our condition is valid.

Our model receives some empirical support. Indeed, none of the restrictions that are implied by the theory is rejected by the data. Moreover, the coefficients of the financial and macro factors tend to be individually significant and often have signs that are consistent with a priori expectations.

Our results can be contrasted with the overall poor performance of the Hotelling model in previous tests. On the one hand, we have reason for optimism; the Hotelling model is not rejected. On the other hand, optimism must be tempered. Whereas financial and macro factors are significant determinants of returns associated with owning a mine, traditional cost factors are not. Nevertheless, the 'Hotelling' coefficients have signs that are consistent with a priori expectations, and depletion effects are marginally significant.

It seems that the addition of risk and the possibility of risk diversification improves the empirical performance of Hotelling's theory. This conclusion should not be very surprising. Uncertainty is pervasive in natural-resource markets, and resource-commodity prices are extremely volatile as a consequence. It therefore seems natural that medium-term risk considerations should be at least as important as long-term depletion factors when investors are making their portfolio and output decisions.

We do not claim to have solved the Hotelling puzzle. In particular, the degree of risk diversification that is implied by our estimate of β is too large to be believable. Nevertheless, we feel that we have taken a step in the direction of understanding the behaviour of resource-commodity prices.

APPENDIX A: PROOF OF PROPOSITION 1

We begin with risk-neutral investors in order to obtain the risk-neutral rate of return for mines when the mine is a risky asset. The risk-adjusted discount factor, ρ, will then equal the risk-neutral rate of return plus the risk premium, $\beta(r^m - r)$.[27] With risk neutrality, the Bellman equation corresponding to (8) is[28]

$$rV = \max_q \left[\pi - V_R q + V_p p\mu_p + V_\Theta \Theta \mu_\Theta + \frac{1}{2} V_{pp} p^2 \sigma_p^2 + \frac{1}{2} V_{\Theta\Theta} \Theta^2 \sigma_\Theta^2 + V_{p\Theta} p\Theta\sigma_{p\Theta}\right]. \tag{A1}$$

We can partially differentiate (A1) with respect to R at the optimal q to obtain

$$rV_R = -C_R - V_{RR} q + V_{Rp} p\mu_p + V_{R\Theta}\Theta\mu_\Theta + \frac{1}{2} V_{Rpp} p^2 \sigma_p^2 + \frac{1}{2} V_{R\Theta\Theta} \Theta^2 \sigma_\Theta^2 + V_{Rp\Theta} p\Theta\sigma_{p\Theta}. \tag{A2}$$

It is possible to simplify (A2) by applying Ito's lemma to the right-hand side, which yields

$$rV_R = -C_R + \frac{1}{dt} E_t dV_R. \tag{A3}$$

Rearranging (A3), the stochastic Euler equation, we have

$$\frac{\frac{1}{dt} E_t dV_R}{V_R} =: \frac{\mathring{V}_R}{V_R} = r + \frac{C_R}{V_R}. \tag{A4}$$

$V_R = \partial V / \partial R$ is the shadow price of R. Substituting into (A4), we have

$$\frac{\frac{1}{dt} E_t d\lambda}{\lambda} = \frac{\mathring{\lambda}}{\lambda} = r + \frac{C_R}{\lambda}. \tag{A5}$$

27 The only difference between our model and that of Gaudet and Khadr (1991) is in the rate of return for risk-neutral investors. They derive the risk premium for natural-resource assets.

28 We are assuming incomplete exhaustion, that is, $R_\infty > 0$. When cost rises as reserves are depleted, this seems natural.

(A5) is Hotelling's rule for risk-neutral investors. It states that the right-hand side, the expected average rate of change of the shadow price, should equal the risk-neutral rate of return for natural-resource assets. Not surprisingly, the rule is the same as the one obtained when there is no uncertainty. When investors are risk averse, in contrast, the expected average rate of change of the shadow price will not equal the risk-neutral rate. Indeed, investors will be indifferent among assets only if their risk-adjusted rates of return are equal. The mine manager will therefore vary q until (9) holds.[29] In other words, returns must reflect undiversifiable risk.

APPENDIX B: CALCULATION OF BETA

Tests of CAPM and HCAPM involve β, the coefficient of $r^m = (\dot{p}^m + \mathrm{div})p^m$, whereas the equation that is estimated involves β_1 and β_2, the coefficients of $\dot{p}^m/p^m$ and div/p^m, respectively. We use the following procedure to obtain an estimate of β. Equation (11) can be rewritten as

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \sum_{j=3}^{k} \beta_j x_j + \epsilon, \qquad \text{(B1)}$$

where $y =$ LDOT, $x_1 =$ RMSPI, $x_2 =$ DIV, and x_j, $j = 3, \ldots, k$, are the other variables in (11). (B1) can be contrasted with

$$y = \tilde{\alpha} + \beta x_0 + \sum_{j=3}^{k} \tilde{\beta}_j x_j + \tilde{\epsilon},$$

where $x_0 = x_1 + x_2$ is the rate of return on the market portfolio, r^m. We need to express β as a function of the coefficients in (B1). Suppose that $\tilde{\beta}_j = \beta_j$, $j = 3, \ldots, k$,[30] and consider the OLS normal equations

$$s_{yi} = \sum_{j=1}^{k} b_j s_{ji}, \qquad \text{(B2)}$$

where

$$s_{yi} = \frac{\Sigma_m (y_m - \bar{y})(x_{im} - \bar{x}_i)}{n}, \quad s_{ij} = \frac{\Sigma_m (x_{im} - \bar{x}_i)(x_{jm} - \bar{x}_j)}{n},$$

and b_j is the OLS estimate of β_j. (B2) can be rearranged to yield

29 A no-arbitrage argument of this sort is used by Hotelling (1931) and Solow (1974).
30 This hypothesis was tested and was not rejected.

$$b_1 = \frac{s_{y1} - b_2 s_{12} - \sum_{j=3}^{k} b_j s_{j1}}{s_{11}}, \quad b_2 = \frac{s_{y2} - b_1 s_{12} - \sum_{j=3}^{k} b_j s_{j2}}{s_{22}},$$

$$\hat{\beta} = b_0 \frac{s_{y0} - \sum_{j=3}^{k} b_j s_{j0}}{s_{00}} = \frac{s_{y1} + s_{y2} - \sum_{j=3}^{k} b_j (s_{j1} + s_{j2})}{s_{00}}$$

$$= \frac{b_1 s_{11} + b_2 s_{22} + (b_1 + b_2) s_{12}}{s_{00}}.$$

In performing hypothesis tests involving β, we use the estimated coefficients b_1 and b_2 and the empirical covariances of the random variables.

APPENDIX C: CALCULATION OF TAX FACTORS

In Canada prior to 1972 capital gains were tax exempt, whereas the dividend-tax credit equalled only 20 per cent of dividend income. The tax reform of 1972, however, reduced the tax disadvantage of dividends by increasing their exemption and taxing capital gains at 50% of the regular income-tax rate. Nevertheless, capital gains still enjoy a more favorable treatment.

Tax factors $(1 - T_i)$ on dividends and capital gains are calculated using $T_i =$ $(1 -$ fraction exempt$) \times$ marginal income-tax rate. A marginal-income tax rate of 0.30 yields table C1.

TABLE C1
Tax factors $(1 - T_i)$

i	$t \leqq 1972$	$t > 1972$
p^m	1.0	0.85
DIV	0.76	0.78

Canadian taxation of mining profit is very complex. Nevertheless, as long as there are no major changes in tax schedules, taxation does not affect the calculation of $\dot{\lambda}/\lambda$. Indeed, the same factor appears in both numerator and denominator. In 1974, however, there were substantial increases in mining-profit taxes at both federal and provincial levels. In the same year there were high profits, owing to record-high prices. These two factors approximately cancelled each other, leaving the rate of after-tax shadow-price appreciation at zero. For this reason, LDOT is multiplied by zero in 1974.

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