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Competing contact processes on Watts-Strogatz network

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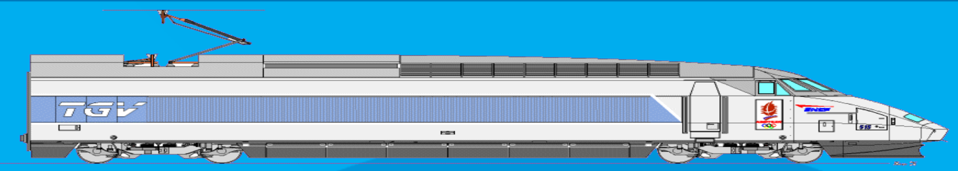
Timetable



- Contact processes – introduction and examples
- Voter and invasion models
- Sznajd model
- Watts-Strogatz network with regulated clustering coefficient
- Sznajd model vs invasion model – simulation algorithm and results discussion
- “With neigbourhood” model vs voter model – simulation algorithm and results discussion
- Conclusions

What is a contact process?

- Stochastic process having the Markov property
- Important role of computer simulation
- Many scientific disciplines:

  - biology (epiemics spread)
  - sociology (opinions, gossips spread)
  - computer science (computer viruses spread)

Competing epidemics*

- Network of interactions
- SIR model

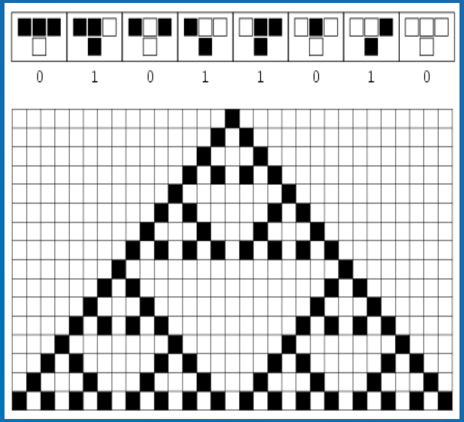
- When recovered, agent becomes resistant to both diseases
- Diseases differ with infectivity
- Results: phase diagram
  - The overwhelming dominance of one disease
  - There is an area in which both infections coexist
- A strong dependence on the initial state

*Competing epidemics on complex networks, Brian Karrer, M. E. J. Newman, 2011

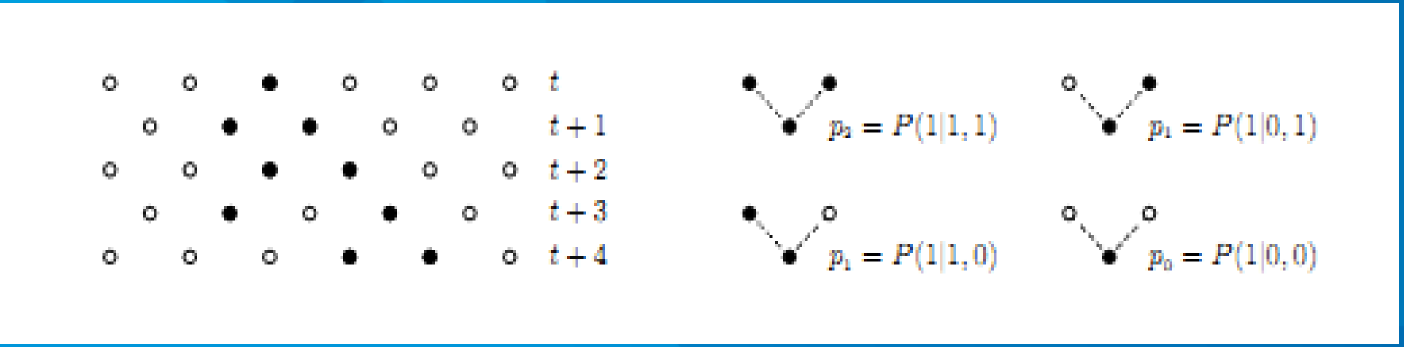
How to model CP? - cellular automata

- Mathematical models, described by:
  - network of cells
  - a set of states of a single cell
  - transition rules

They can be deterministic:

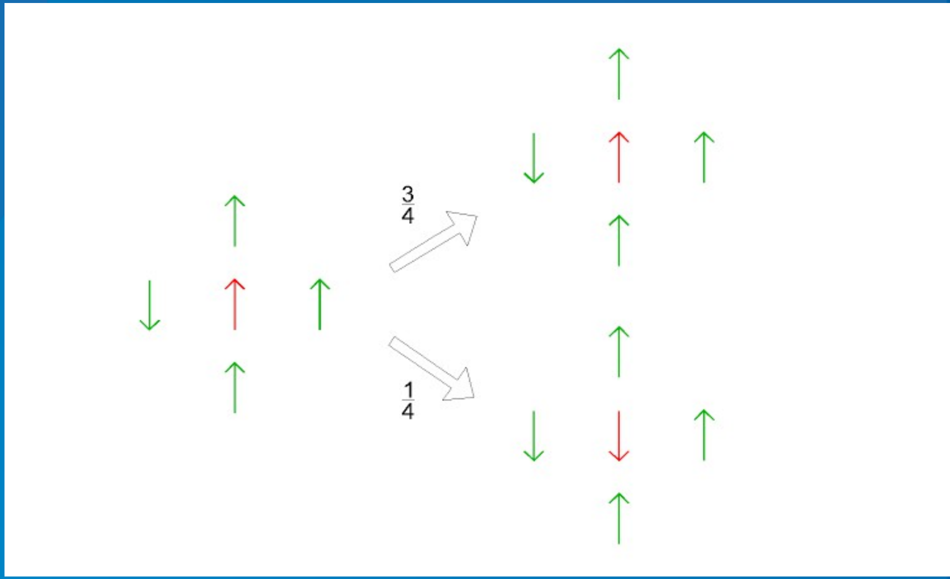


or probabilistic - eg. Domany-Kinzel automata:



Voter model

- agents-voters placed in the graph nodes
- an agent is randomly selected
  - selected agent accepts the opinion of a randomly selected neighbour

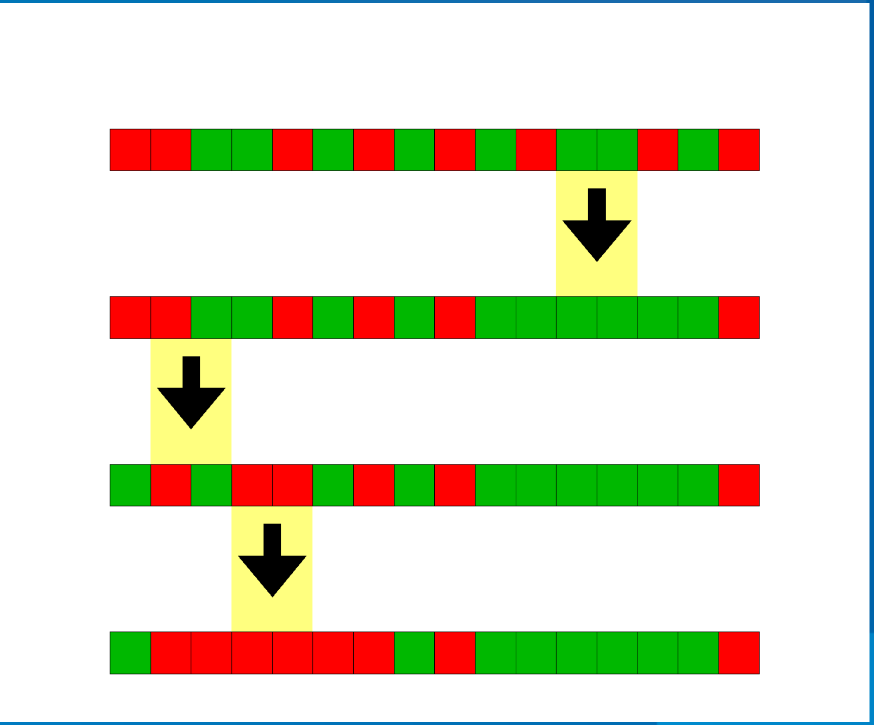


Invasion model

- agents are again placed in the graph nodes
- an agent is randomly selected
- selected agent imposes its opinion on a randomly selected neighbour
- adapted in sociophycics to study the dynamics of reaching consensus among a group of agents who impose their opinion to neighbours

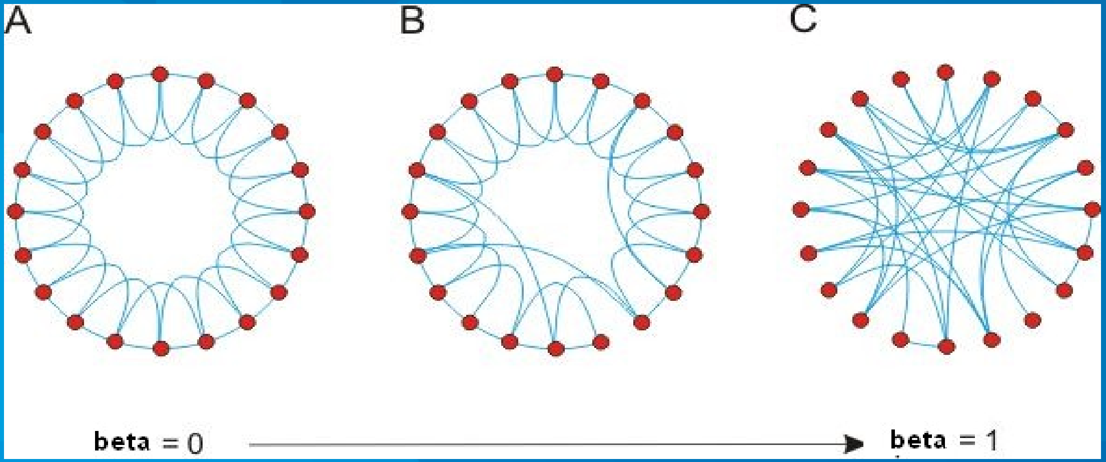
Sznajd model

- a random pair of neighboring nodes is chosen
- if both nodes are in the same state, both neighbours will accept this state also
- if the nodes are in different states, neighbours accept different states also
- Result: 3 possible stationary states



Watts-Strogatz network

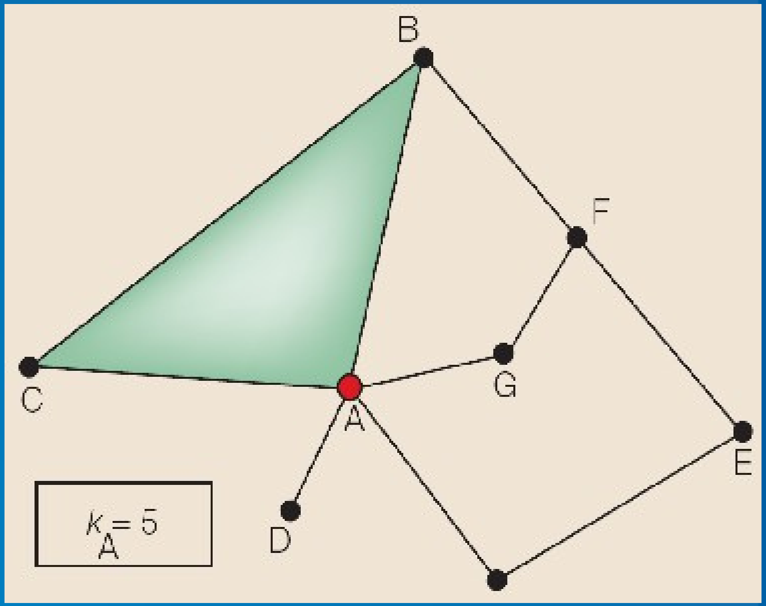
- Sparse but highly clustered network
- Construction:
  - 1. regular network with periodic boundary conditions
  - 2. with a specified probability β, one end of each edge is switched to a random, another vertex in the network



What is clustering coefficient?

Local clustering coefficient for the i-th vertex:

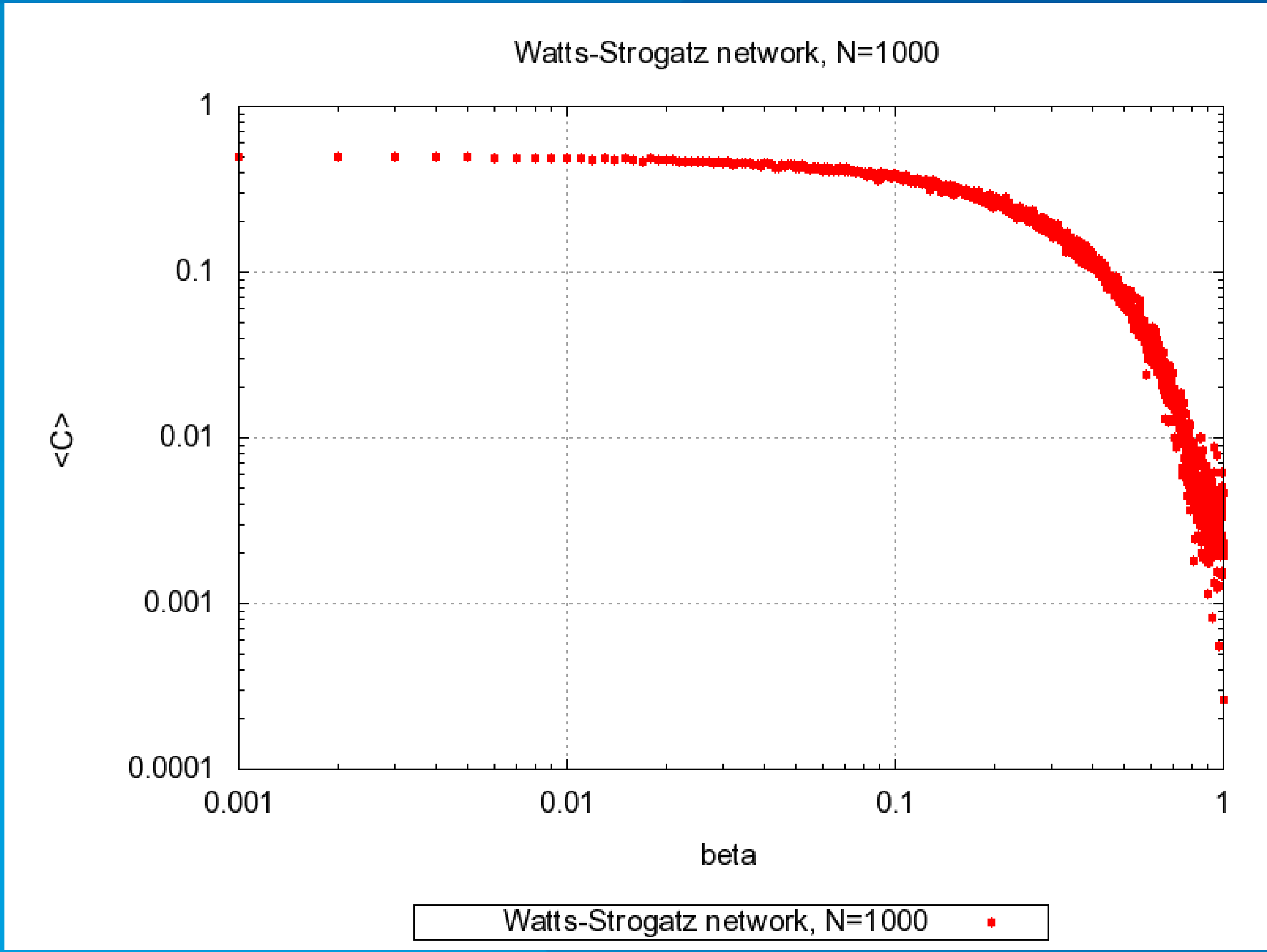
$$C_i = \frac{2n_i}{k_i(k_i - 1)}$$



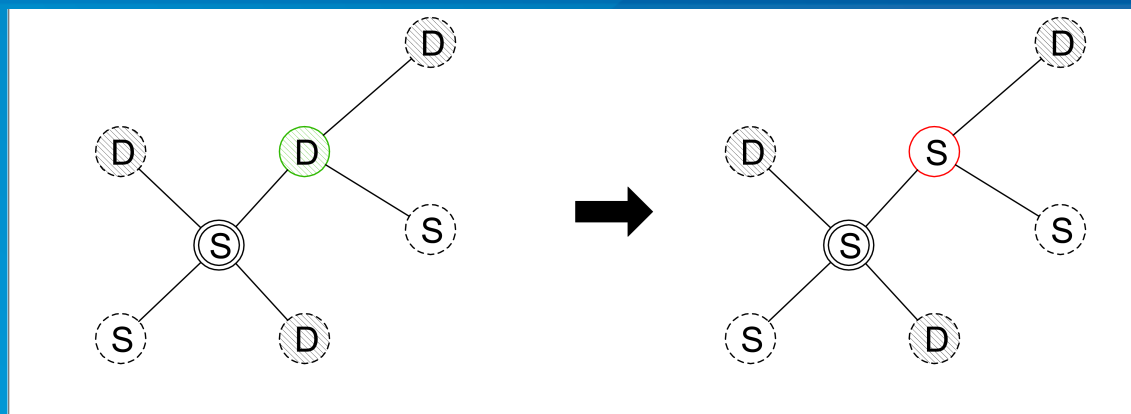
The average clustering coefficient for the network:

$$< C > = \frac{\sum_i^N C_i}{N}$$

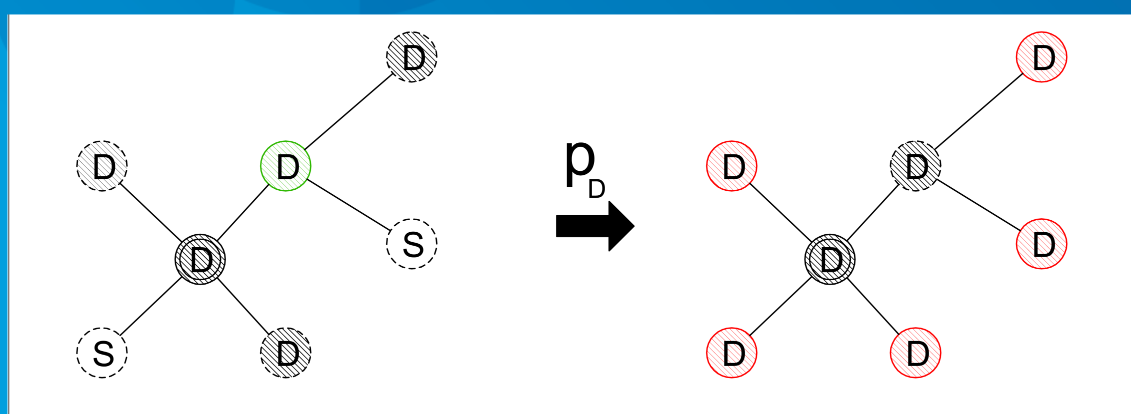
Switching bonds influences clustering coefficient



Sznajd vs invasion model



“S” and “D” transition rules



Simulation algorithm

Generate family of networks of size N having identical <k> and various <C>

Randomly assign S or D states to nodes

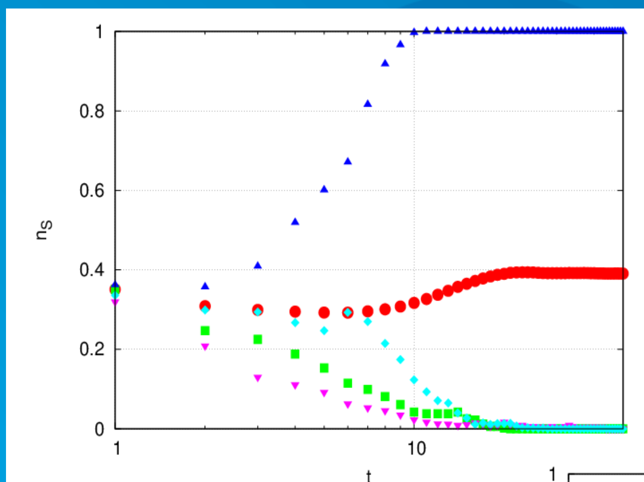
In each iteration t, perform N draws with replacement on the set of N nodes

Then, depending on the type of the drawn node, apply an appropriate transition rule

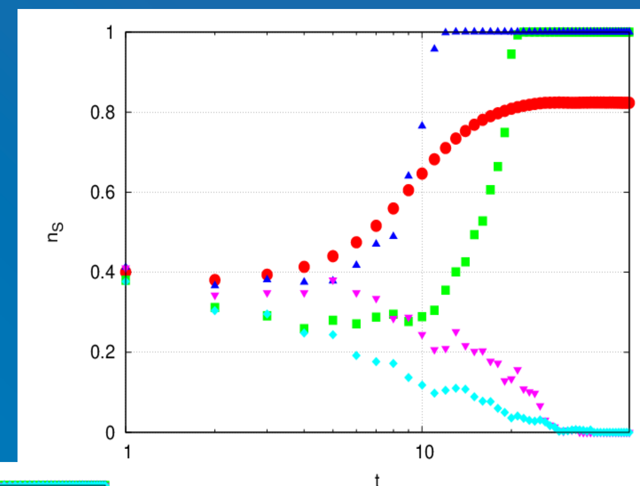
End simulation after the specified number of iterations

Singular time series

<C>=0.1, p=0.3

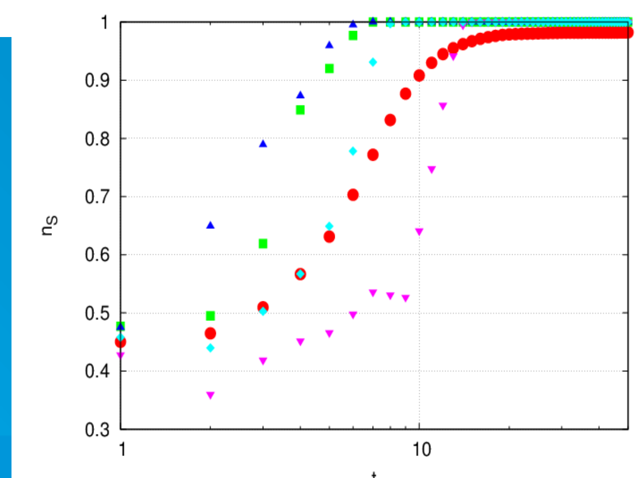


$n_s(t=0)=0.35$



$n_s(t=0)=0.4$

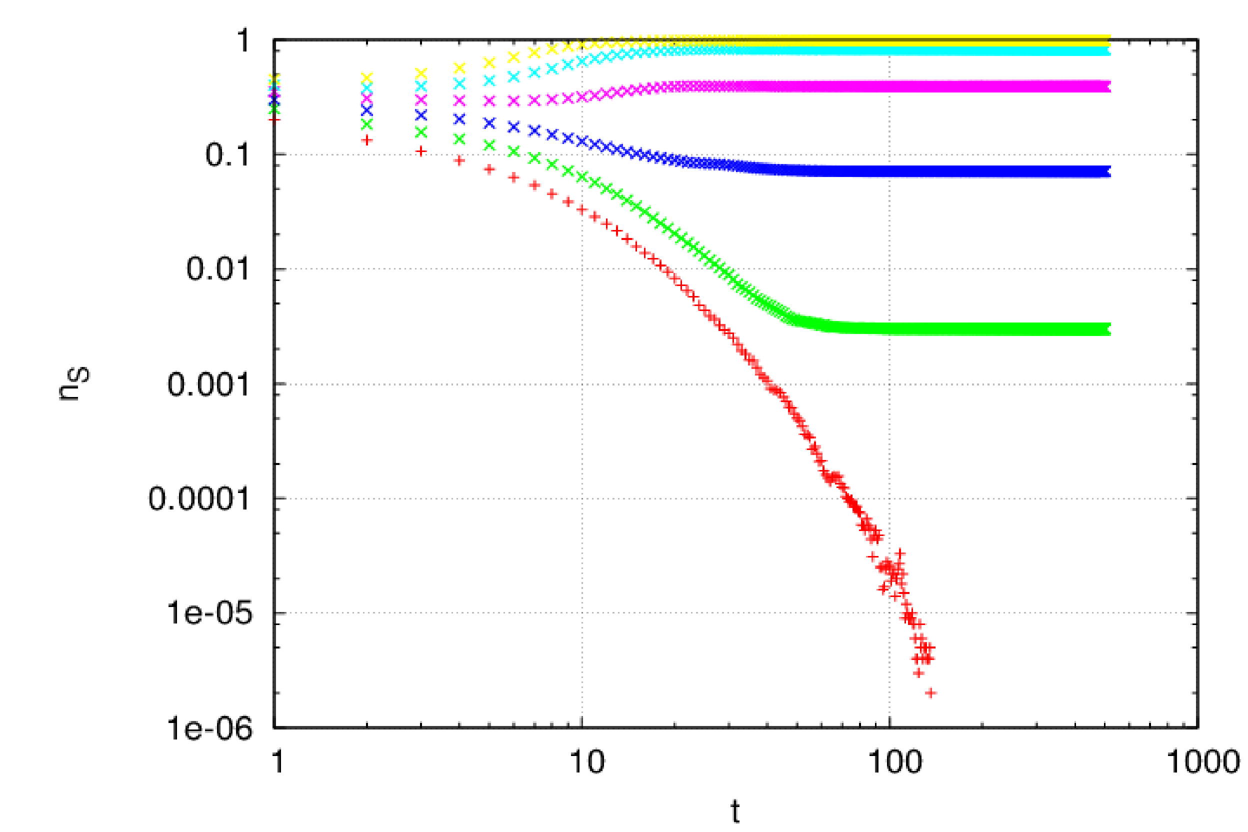
$n_s(t=0)=0.45$



- Bold red line is an average time series

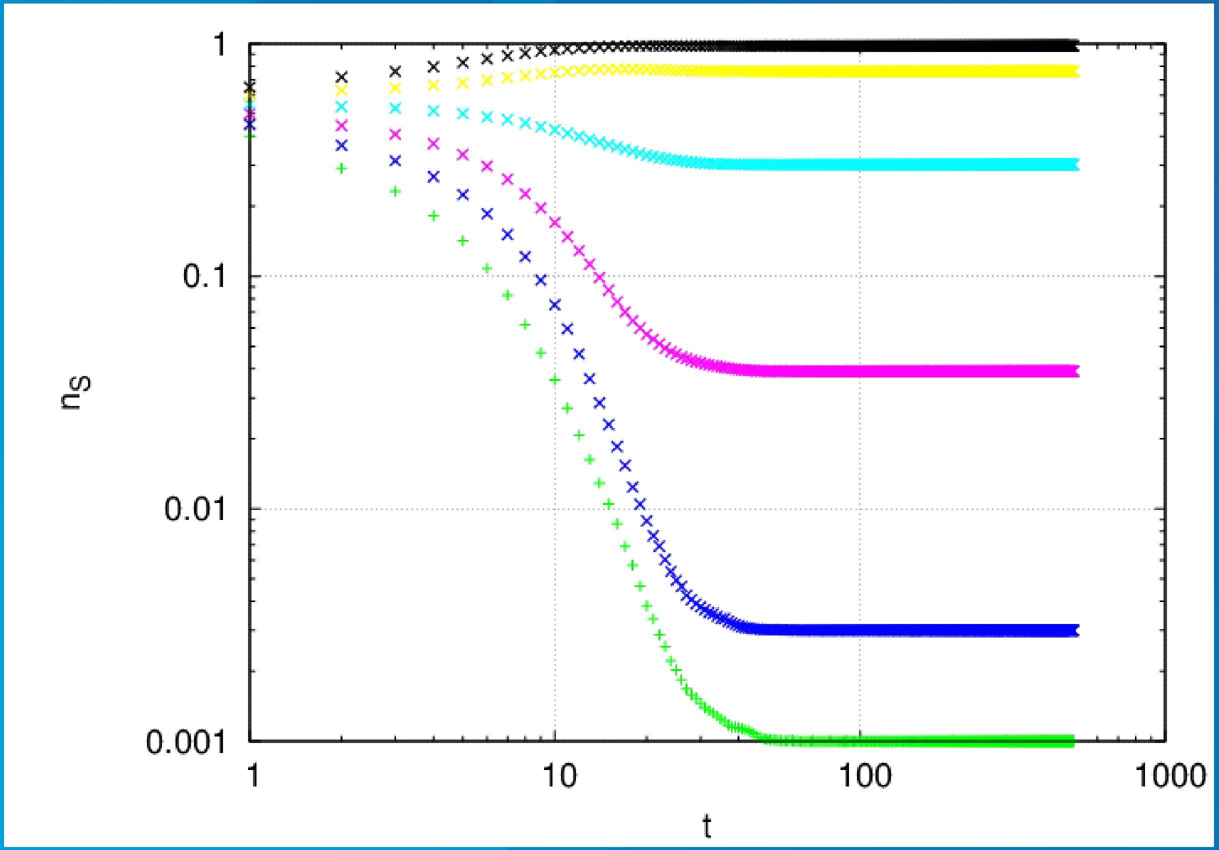
Average time series

$\langle C\rangle=0.1$, $p=0.3$, $n_s(t=0)=\{0.2, 0.25, 0.3, 0.35, 0.4, 0.45\}$



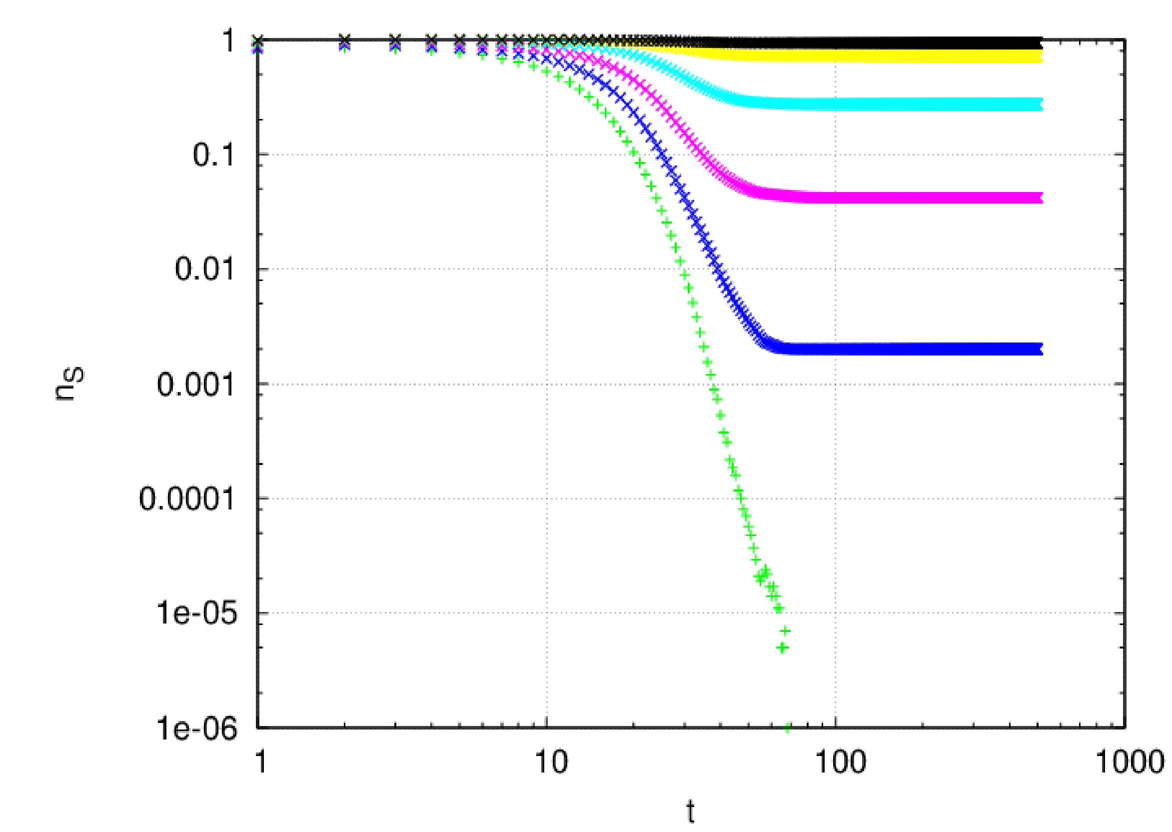
Average time series - continued

<C>=0.2, p=0.4, $n_s(t=0)=\{0.4, 0.45, 0.5, 0.55, 0.6, 0.65\}$



Average time series - continued

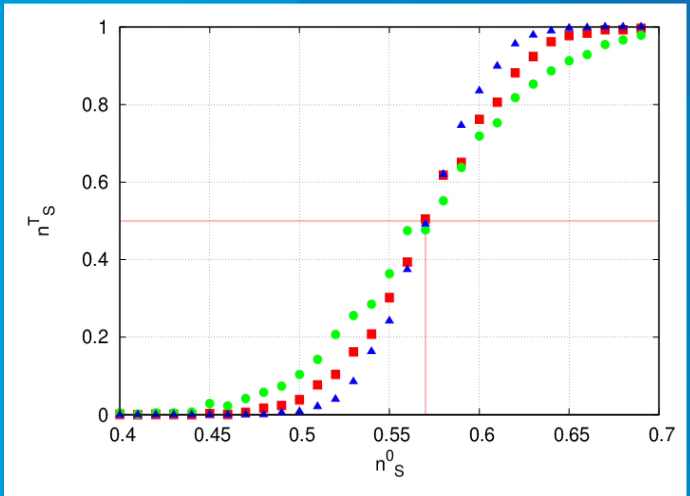
<C>=0.4, p=0.6, $n_s(t=0)$={0.8, 0.84, 0.88, 0.92, 0.96}



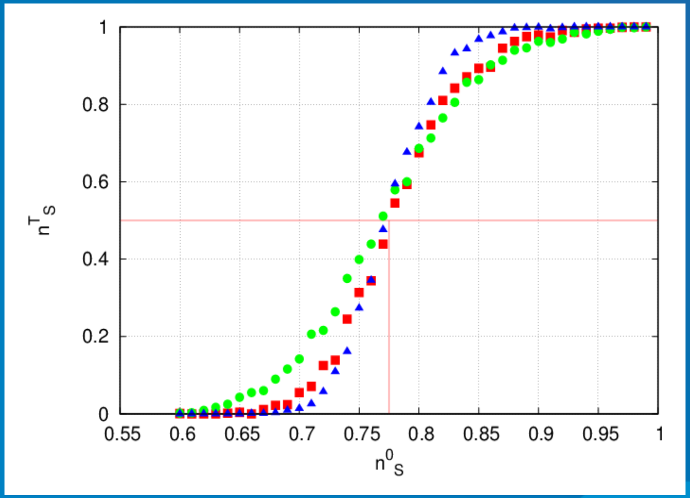
Dependence on an initial state

- Introduced **n^*_s** value to measure this dependence:
  - an average value of the initial share of nodes S for which:
  - half the simulation ends with the dominance of process S
  - the other half – with the dominance of process D

<C>=0.2, p=0.4



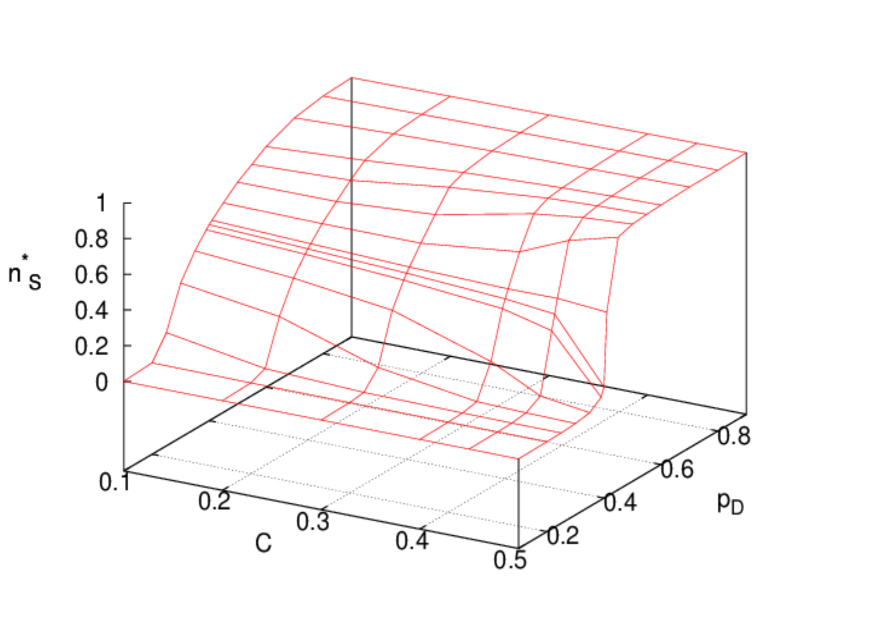
<C>=0.3, p=0.5



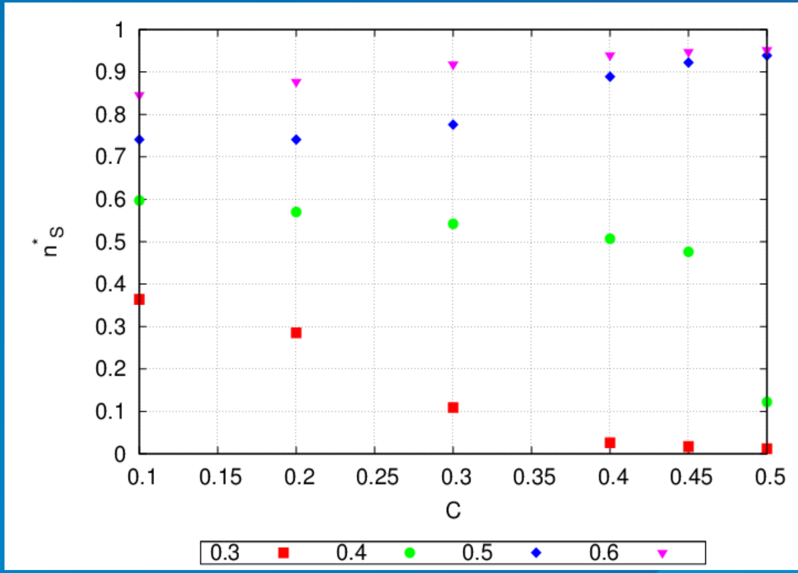
N={500, 1000, 2000}

Dependence on clustering coefficient and transition probability

N=1000



- Complex dependence on clustering coefficient



- Straight dependence on transition probability

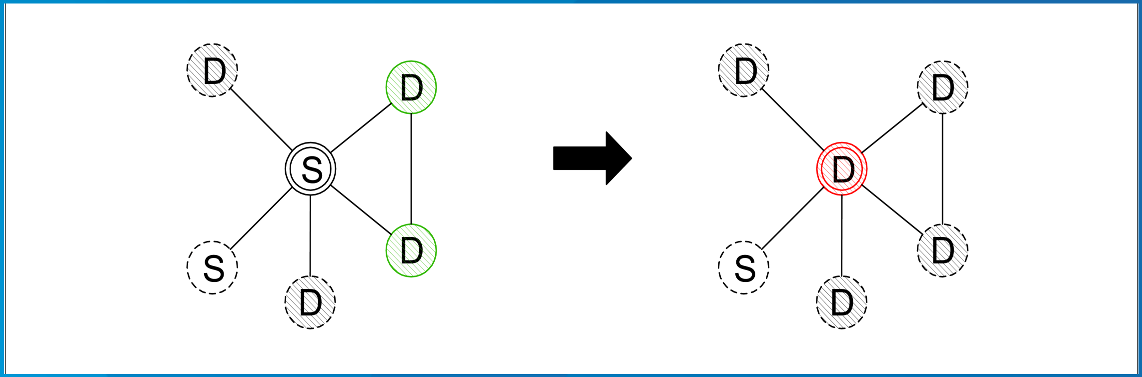
“With neighbourhood” model

Research motivation:

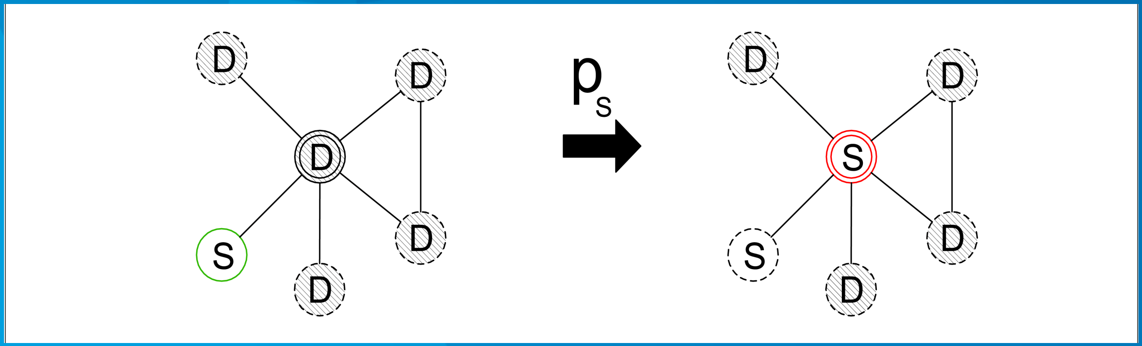
- investigate the mechanism, which wouldn't be as active as Sznajd model, where a pair of nodes in consistent states infects all nodes around
- preserve the unique feature of the Sznajd model: copying the state of the pair

- Real life examples:
  - Paxos family of consensus protocols
  - inheriting some genetic ilnesses, as hemophilia

"With neighbourhood" vs voter model



"S" and "D" transition rules



Simulation algorithm

Generate family of networks of size N having identical <k> and various <C>

Randomly assign S or D states to nodes

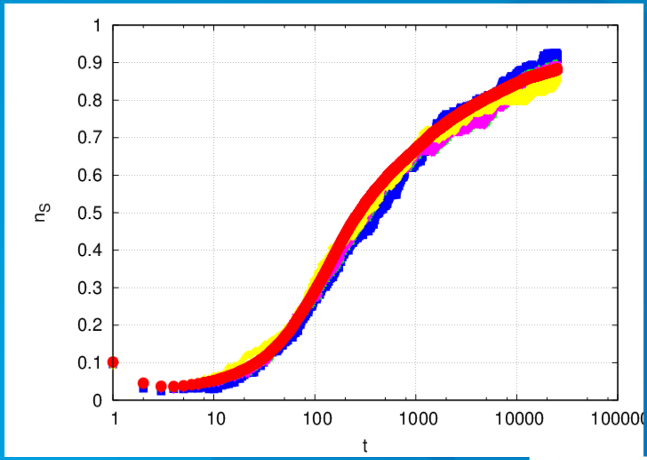
In each iteration t, generate a random N-element permutation of the set of all nodes to be visited

Then, depending on the type of the visited node, apply an appropriate transition rule

End simulation after the specified number of iterations

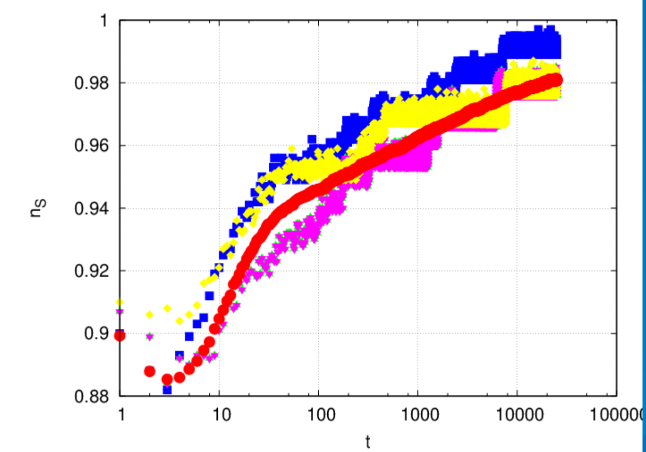
Singular time series – reduced <C>

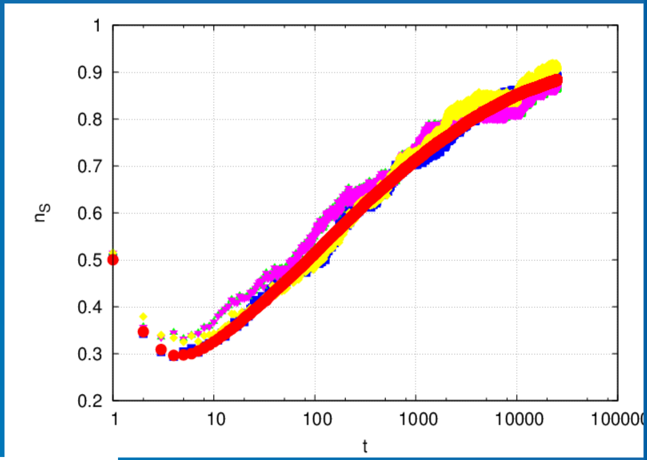
<C>=0.3, p=0.1



$n_s(t=0)=0.1$

$n_s(t=0)=0.9$

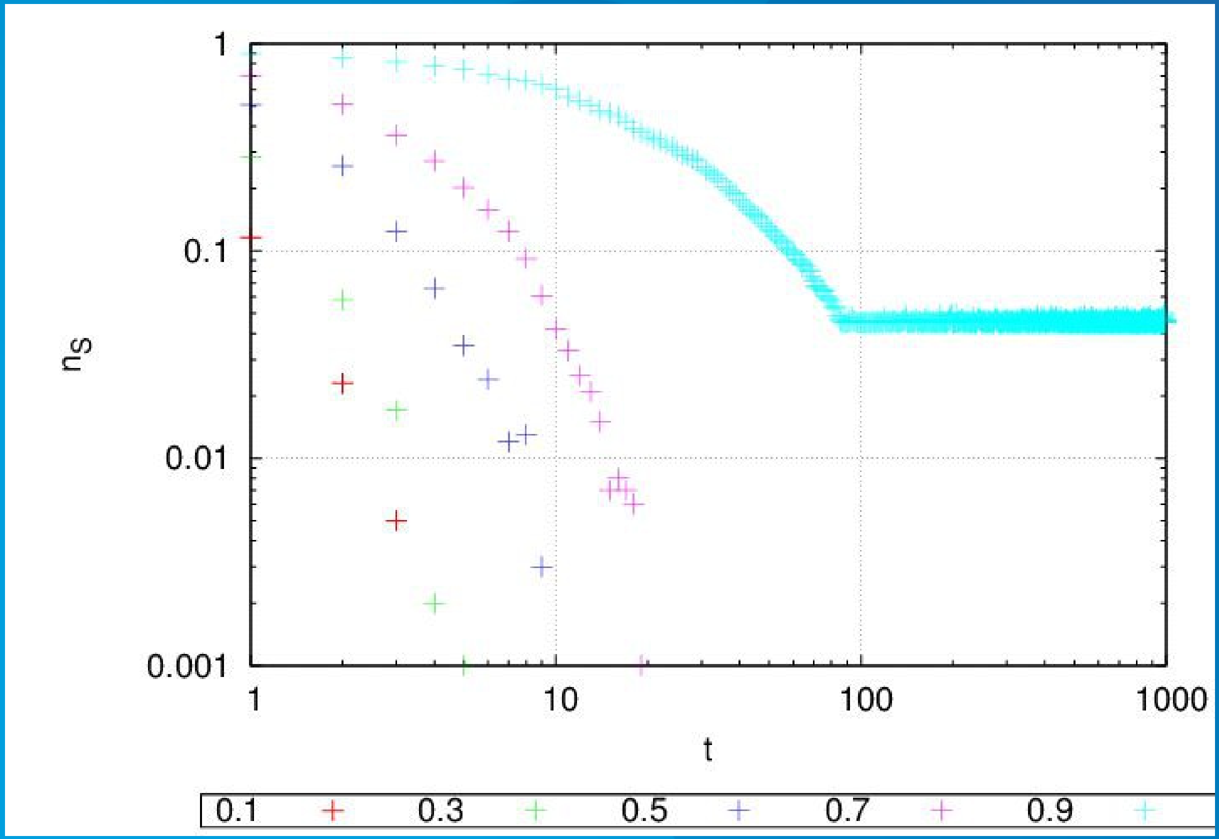




$n_s(t=0)=0.5$

Singular time series – slightly reduced <C>

<C>=0.49, p=0.1

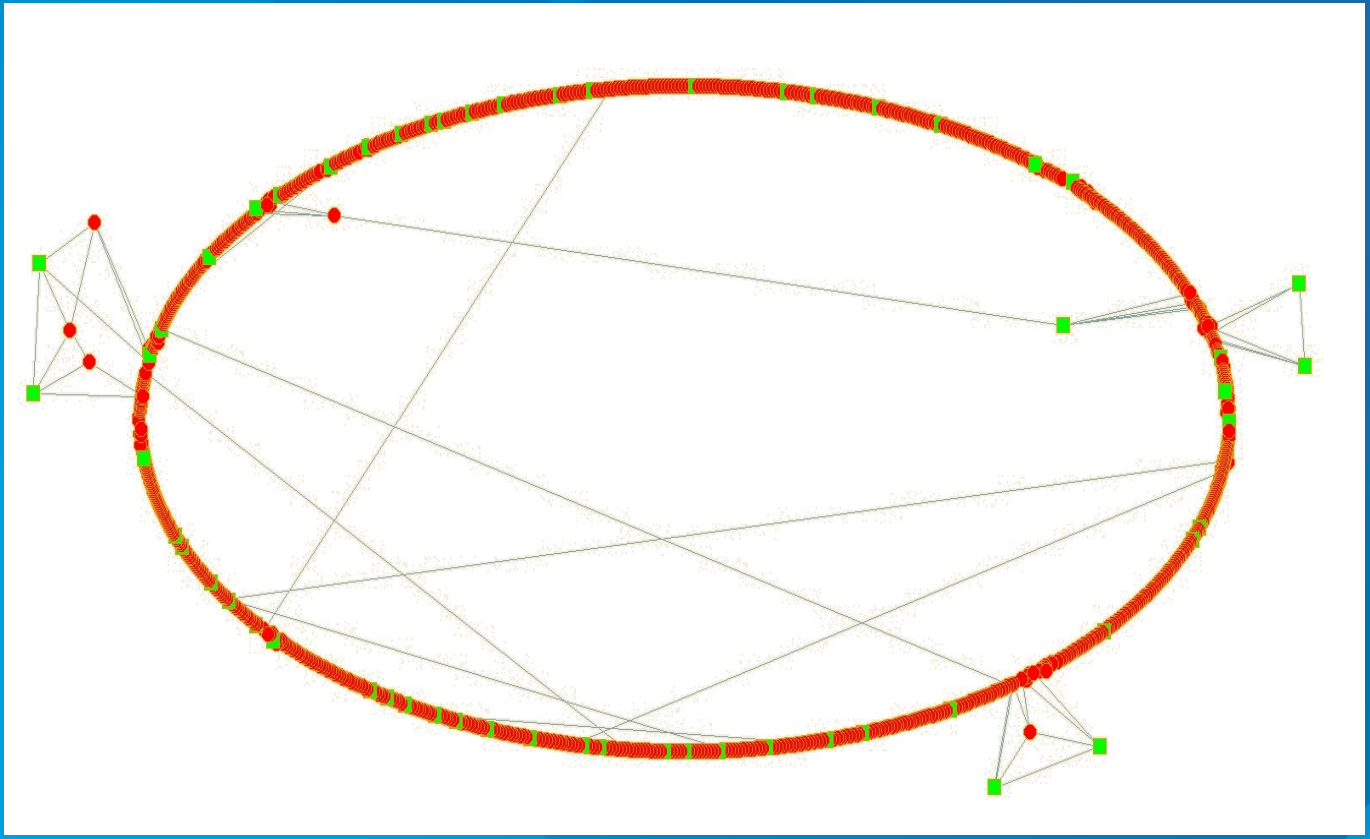


$n_s(t=0)=\{0.1, 0.3, 0.5, 0.7, 0.9\}$

- heterogenous absorbing state

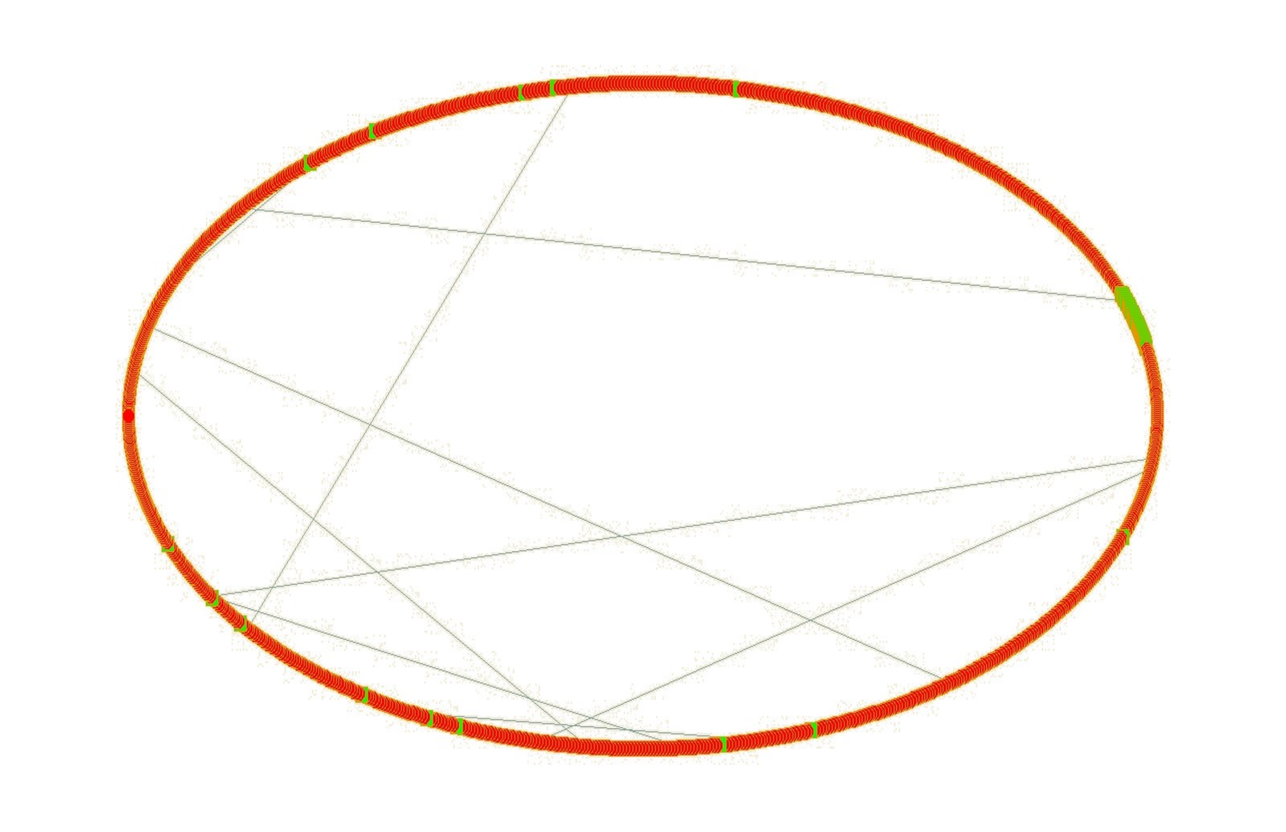
Why is D process “stuck” on nodes?

t=0



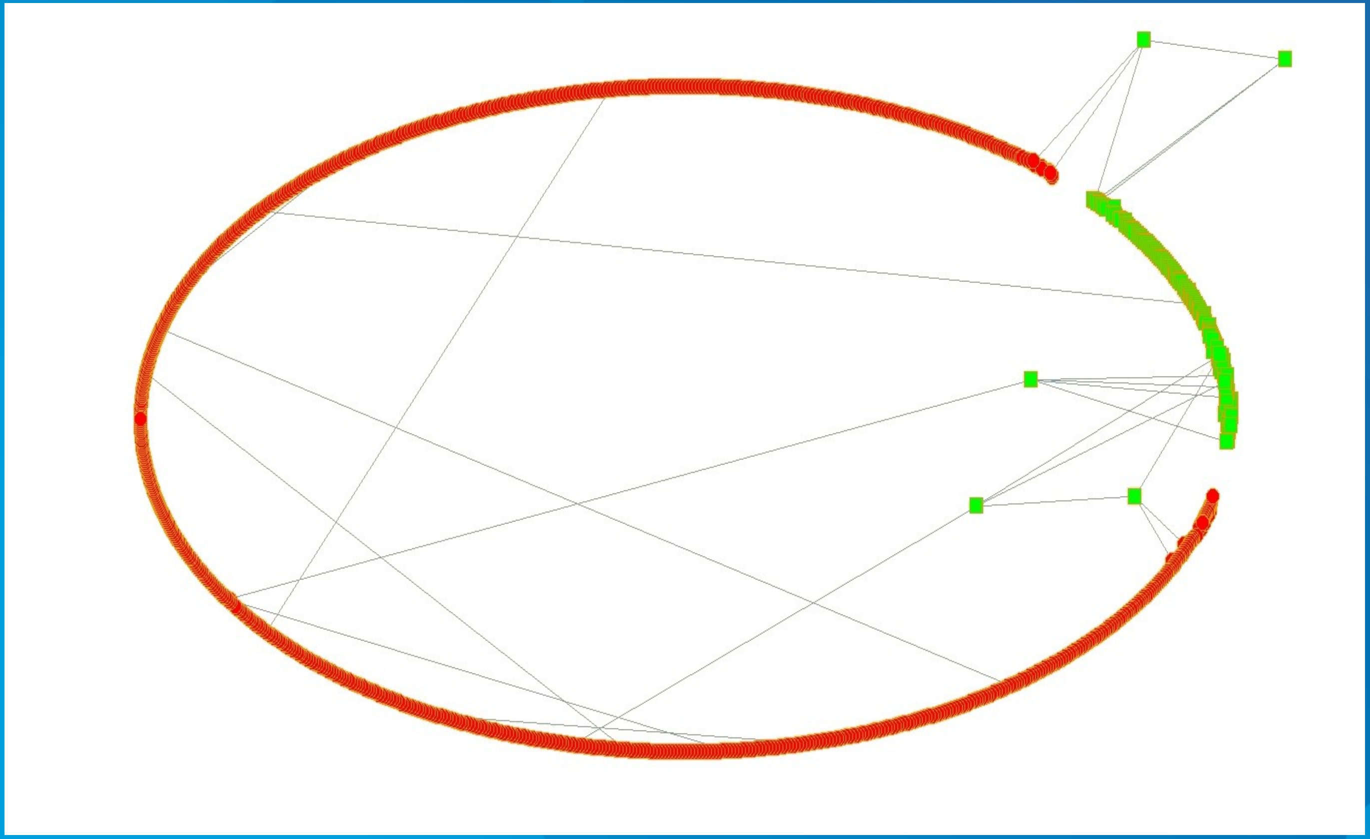
Why is D process “stuck” on nodes?

t=10



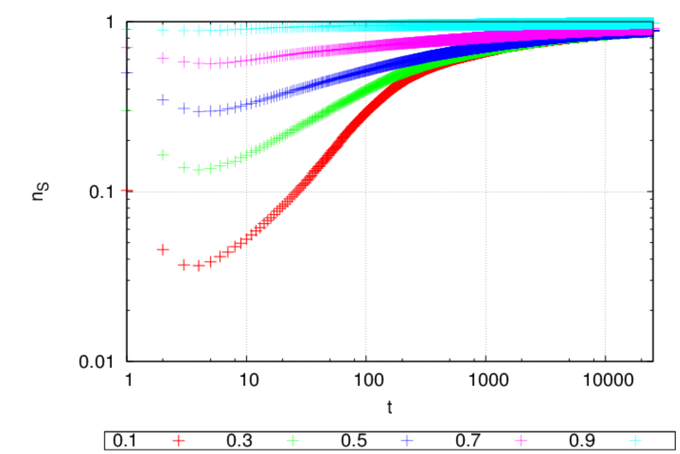
Why is D process “stuck” on nodes?

t=120

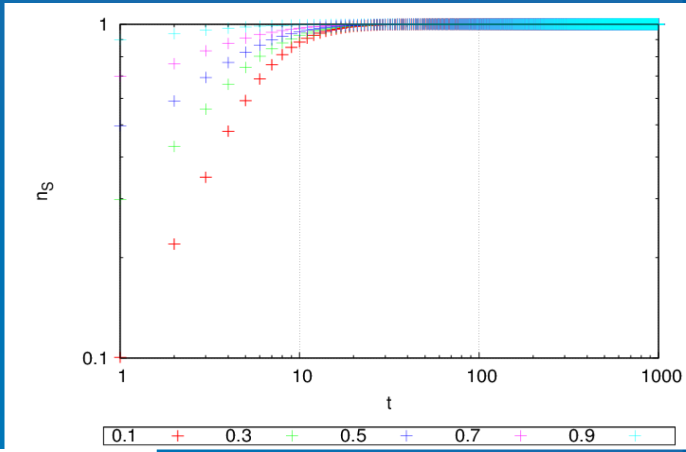


Average time series -reduced <C>

<C>=0.3

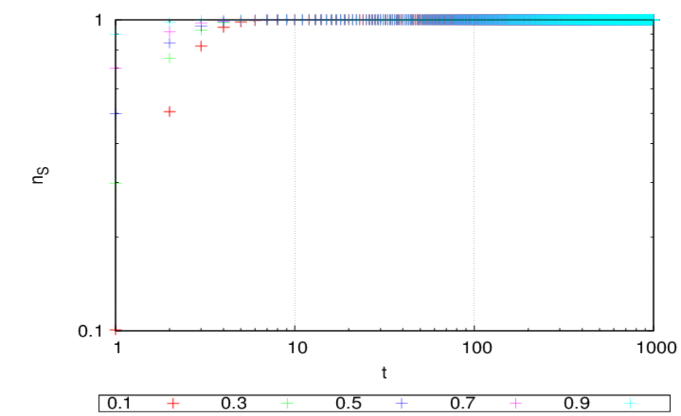


p=0.1



p=0.5

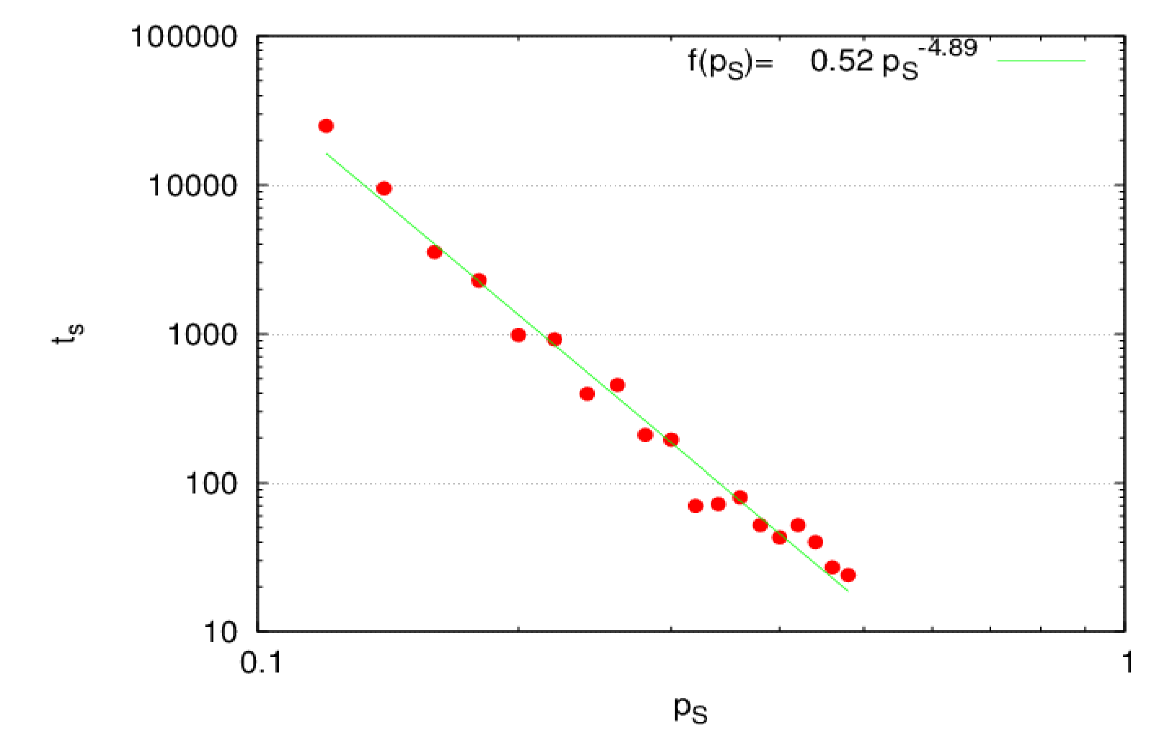
p=0.9



- Longer simulation times are required for the small S-transition probabilities

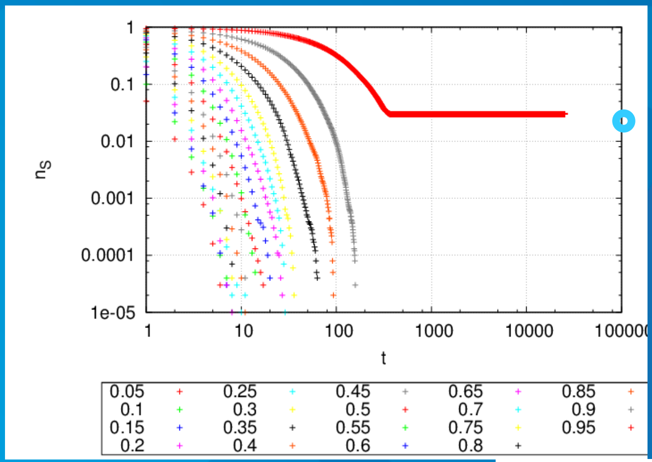
Very long simulation times required

<C>=0.1, $n_s(t=0)=0.75$



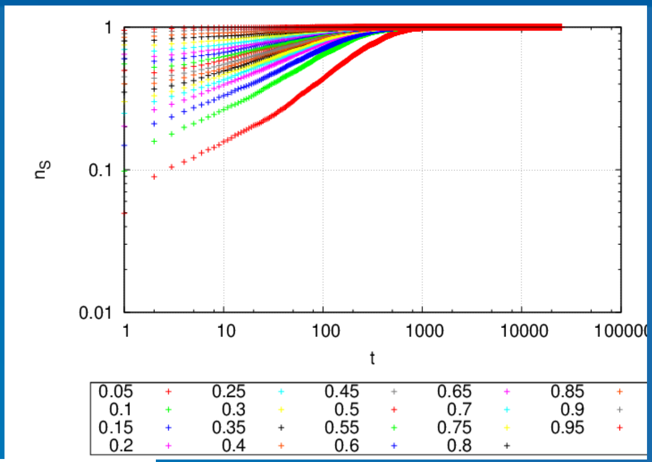
- Maximum simulation time (25 000 iterations) is not sufficient to reach stationary state for small p

Average time series - regular network



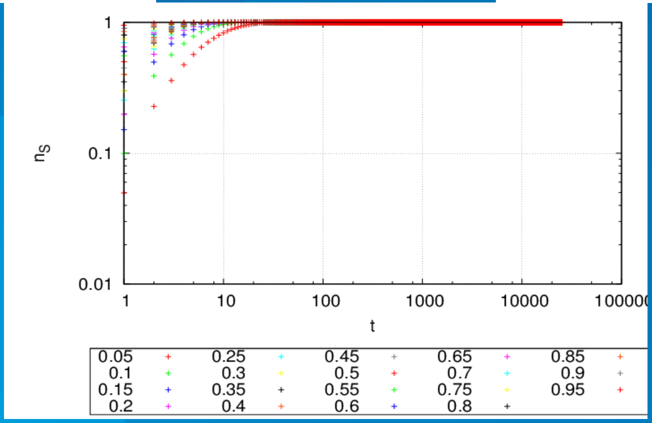
absorbing configuration

p=0.1



p=0.5

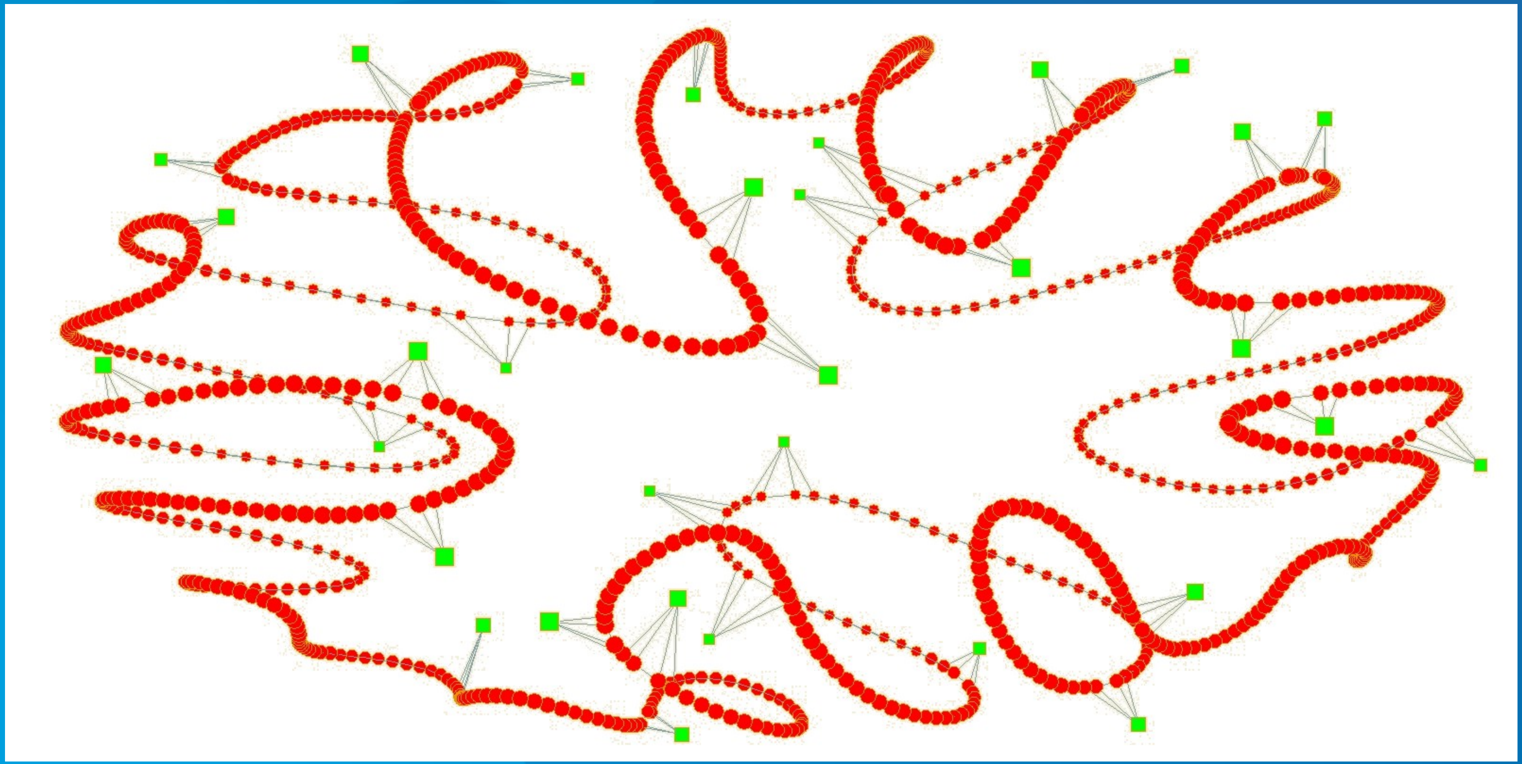
p=0.9



- Much shorter simulation times than for reduced <C>

Absorbing S-configuration effect

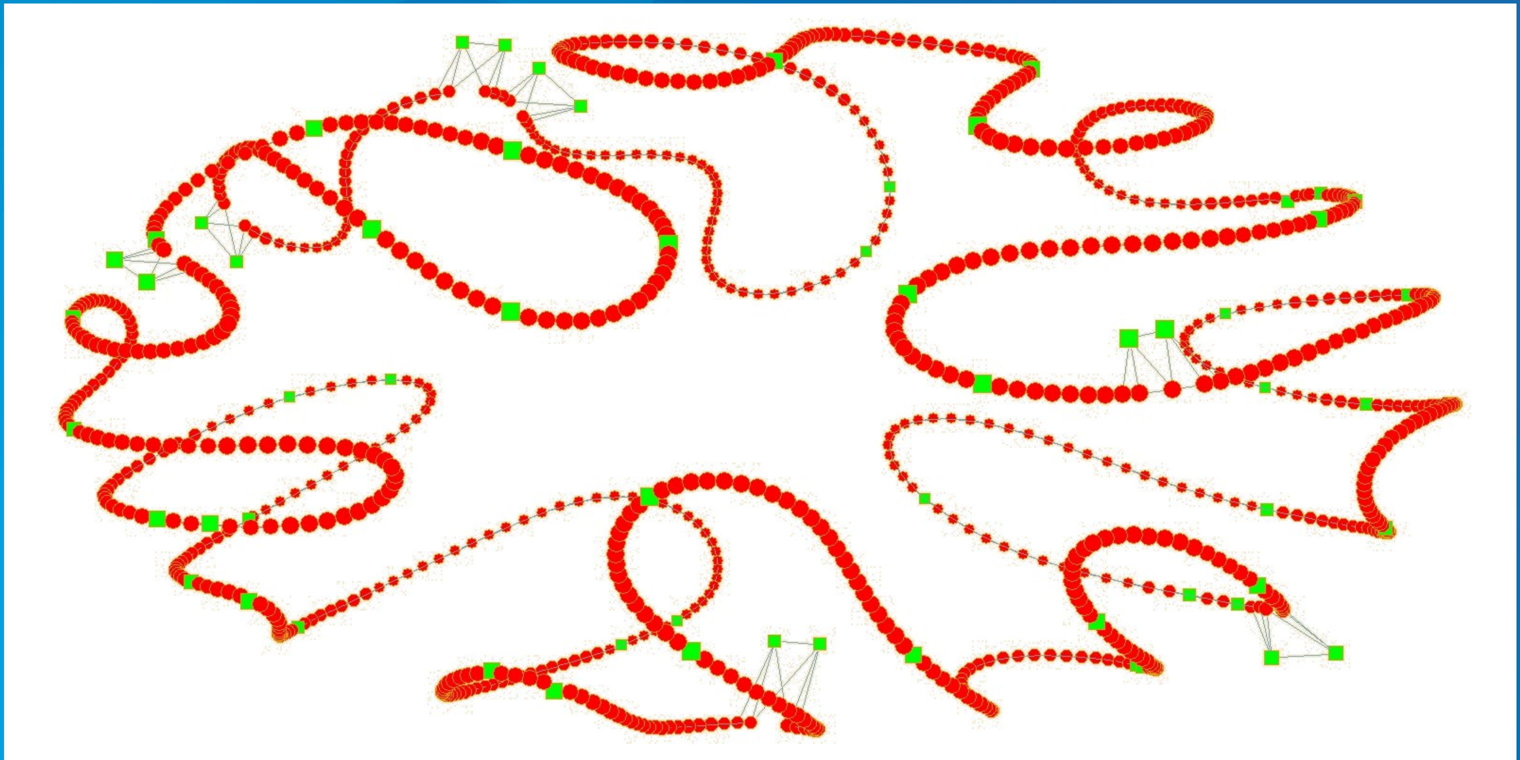
t=0



- No single connected D-D pair – D process cannot be activated

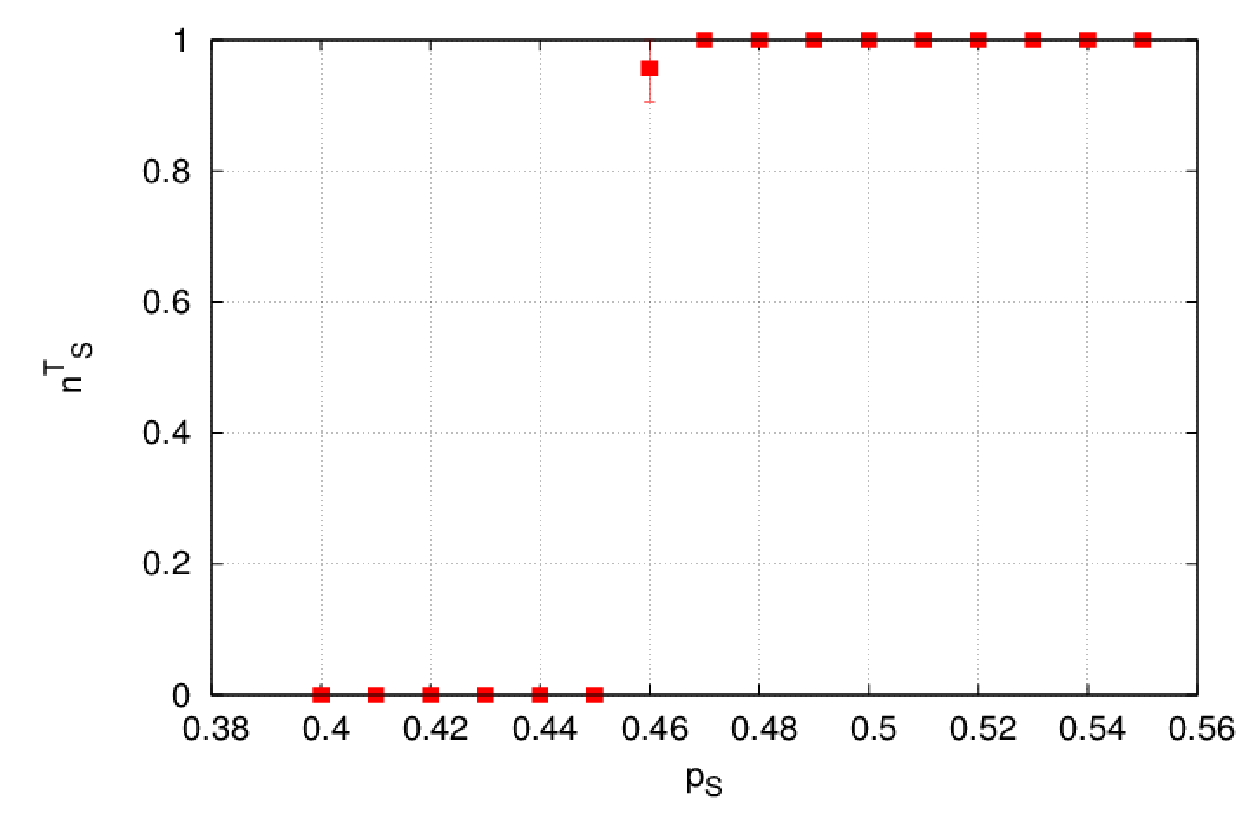
Absorbing S-configuration effect

t=0



- Connected D-D pairs exist – D process can be activated

Phase diagram for regular network



- Every point is an average from time series for $n_S(t=0)=0.25$, 0.5 and 0.75

Conclusions

Sznajd vs invasion	“With neighbourhood” vs voter
• homogenous absorbing state observed • dependence between the final and initial share of S nodes • n^*_S does not depend on the size of the network • n^*_S increases with the probability of D process	• no dependence between the final and initial share of S nodes • observed the stationary and non-stationary phases - very long simulation time is required • The final part of S nodes increases with the probability of the S process and decreases with increasing clustering coeff.

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