Systems Biology Markup Language (SBML) Level 2: Structures and Facilities for Model Definitions

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1 Introduction

We present the Systems Biology Markup Language (SBML) Level 2, a model representation formalism for systems biology. SBML is oriented towards describing systems of biochemical reactions common in research on a number of topics, including cell signaling pathways, metabolic pathways, biochemical reactions, gene regulation, and many others. SBML is defined in a neutral fashion with respect to programming languages and software encoding; however, it is primarily oriented towards allowing models to be encoded using XML, the eXtensible Markup Language (Bosak and Bray, 1999; Bray et al., 2000). This document contains many examples of SBML models written in XML, as well as an XML Schema (Biron and Malhotra, 2000; Fallside, 2000; Thompson et al., 2000) that defines SBML Level 2.

Major releases of SBML are termed *levels*. SBML Level 2 evolved out of SBML Level 1 (Hucka et al., 2001). All of the structures of Level 1 can be mapped in a straightforward fashion to Level 2. In addition, a large subset of the structures in Level 2 can be mapped to Level 1. However, a valid SBML Level 1 document is not a valid SBML Level 2 document or vice versa.

SBML Level 2 was created in part studying the modeling facilities provided by the following software systems: BioSpice (Arkin, 2001), Cellerator (Shapiro et al., 2000, 2001, 2003), COPASI (Mendes, 2000), DBSolve (Goryanin, 2001; Goryanin et al., 1999), E-Cell (Tomita et al., 1999, 2001), Gepasi (Mendes, 1997, 2001), Jarnac (Sauro, 2000; Sauro and Fell, 1991), JDesigner (Sauro, 2001), JigCell (Vass et al., 2003), NetBuilder (Schilstra and Bolouri, 2002), ProMot/DIVA (Stelling et al., 2001), StochSim (Bray et al., 2001; Morton-Firth and Bray, 1998), and Virtual Cell (Schaff et al., 2000, 2001). SBML was developed with the help of the authors of these packages and with help and collaboration from the authors of CellML (Hedley et al., 2001).

This SBML Level 2 specification document, the XML Schema corresponding to SBML Level 2, and other related documents are openly available from the SBML project web site, http://www.sbml.org/.

1.1 Scope and Limitations

SBML Level 2 is meant to support non-spatial biochemical models and the kinds of operations that are possible in existing analysis/simulation tools. Future software tools will undoubtedly require further evolution of SBML; we expect that subsequent levels will add additional structures and facilities currently missing from Level 2, once the simulation community gains experience with the current language definition. In Section 6.1, we discuss extensions that will likely be included in SBML Level 3.

The definition of the model description language presented here does not specify *how* programs should communicate or read/write SBML. We assume that for a simulation program to communicate a model encoded in SBML, the program will have to translate its internal data structures to and from SBML, use a suitable transmission medium and protocol, etc., but these issues are outside of the scope of this document.

1.2 Differences between Level 1 Version 1 and Level 2

Compared to SBML Level 1 Version 1, SBML Level 2 introduces the following changes:

- SBML Level 2 supports the inclusion of metadata using the same approach as CellML (Cuellar et al., 2002). All structures in SBML can be annotated with optional content in RDF (Resource Description Format; Lassila and Swick, 1999) following the guidelines put forward by Cuellar et al. See Section 3.1.
- A new field called id replaces the name field previously defined for most SBML structures to identify components in a model. The new id field has a type of SId, whose definition is similar to SName in Level 1; see Section 3.3. In SBML Level 2, the name field has become optional and has been redefined to allow any Unicode characters allowed by the string type of XML Schema (Biron and Malhotra, 2000). The combination of id and name allows software tools to use meaningful names for components in a model, while simultaneously preserving the ability of software tools that cannot display special characters to display and manipulate some sort of labels for those components.

- Formulas in Level 2 are expressed using MathML (W3C, 2000b). The field named formula previously available on the KineticLaw and Rule structures has been replaced by a MathML element named math containing MathML content; see Sections 3.6, 4.8 and 4.9.3. The subset of MathML used in SBML includes logical operators enabling the expression of discontinuous functions, something that was not possible in SBML Level 1.
- All attributes which contain initial conditions or parameter values, including compartment volumes and species concentrations, are optional in Level 2 rather than, as in Level 1, either being mandatory or having default values. A missing attribute value for one of these attributes implies that the value is either unknown, not required for analysis, or should be obtained from an external source; see Sections 4.5, 4.6 and 4.7.
- The top-level Model structure can contain an optional list of global function definitions expressed in MathML and organized in new structures of type FunctionDefinition. See Sections 3.6 and 4.3.
- The top-level Model structure can contain an optional list of events organized in structures of type Event which specify the a discrete event can occur during a simulation. See Section 4.10.
- The namespace for identifiers in a model does not contain any built-in symbols; gone, for example, are the predefined rate laws of SBML Level 1. The approach taken in SBML Level 2 is that each model must itself define whatever functions it uses using the new FunctionDefinition mechanism. Although SBML Level 2 uses a feature of MathML to allow models to refer to two particular built-in entities (a symbol representing time and another symbol representing delay functions), even these entities are not in the same namespace as the model's identifiers. See Section 3.6.2.
- Species, compartments and reactions are each optional in a model; thus, a model does not need to contain species, compartments or reactions to be valid. See section 4.2.
- The Compartment, Species and Parameter structures each have a new boolean field named constant. This field specifies whether the variables represented by these structures can be changed by rules and reactions. See Sections 4.5, 4.6 and 4.7.
- SBML Level 2 specifies certain constraints on the form and use of scalar rules in order to prevent such things as algebraic loops. See Section 4.8.
- Rule structures have been changed and simplified. Instances of AssignmentRule replace instances of ParameterRule, SpeciesConcentrationRule and CompartmentVolumeRule. See Section 4.8.
- A rule is not a substitute for a component definition. For example, a Compartment structure for a given identifier must precede an AssignmentRule structure for the same identifier.
- A new listOfModifiers element has been added to the Reaction structure. This list enumerates species that affect a reaction but are neither created nor destroyed by the reaction. Each species occurring in a kinetic law must appear in the list of reactants, list of products or list of modifiers. See Section 4.9.
- A reaction may have no products or no reactants but must have at least one reactant or product. See Section 4.9.
- The word *specie* has been replaced in all instances by *species*.
- Unlike in SBML Level 1, unit identifiers in Level 2 are in a separate namespace from the namespace used for models, functions, species, compartments, reactions and parameters.
- All structures, including Sbml and listOf_____ elements, are derived from the base type SBase.

1.3 Notational Conventions

We define SBML using a graphical notation based upon UML, the Unified Modeling Language (Eriksson and Penker, 1998; Oestereich, 1999). This UML-based definition in turn is used to define an XML Schema (Biron and Malhotra, 2000; Fallside, 2000; Thompson et al., 2000) for SBML. There are three main advantages to using UML as a basis for defining SBML data structures. First, compared to using other notations or a programming language, the UML visual representations are generally easier to grasp by readers who are not computer scientists. Second, the visual notation is implementation-neutral: the defined structures can be encoded in any concrete implementation language—not just XML, but C or Java as well. Third, UML is a de facto industry standard that is documented in many sources. Readers are therefore more likely to be familiar with it than other notations.

Our notation and our approach for mapping it to XML Schemas is explained in a separate document (Hucka, 2000). A summary of the essential points is presented in Appendix A, and examples throughout this document illustrate the approach. We also follow certain naming and typographical conventions throughout this document. Specifically, the names of data structure attributes or fields begin with a lowercase letter, and the names of data structures and types begin with an uppercase letter. Keywords (names of types, XML elements, etc.) are written in a typewriter-style font; for example, Compartment is a type name and compartment is a field name. Likewise, literal XML examples are also written in a typewriter-style font.

2 Overview of SBML

The following is an example of a simple, hypothetical network of biochemical reactions that can be represented in SBML:

$$S_1 \xrightarrow{k_1[S_1]/([S_1]+k_2)} S_2$$

$$S_2 \xrightarrow{k_3[S_2]} S_3 + S_4$$

Broken down into its constituents, this model contains a number of components: reactant species, product species, reactions, rate laws, and parameters in the rate laws. To analyze or simulate this network, additional components must be made explicit, including compartments for the species, and units on the various quantities. The top level of an SBML model definition simply consists of lists of these components:

beginning of model definition
list of function definitions (optional)
list of unit definitions (optional)
list of compartments (optional)
list of species (optional)
list of parameters (optional)
list of rules (optional)
list of reactions (optional)
list of events (optional)
end of model definition

The meaning of each component is as follows:

Function definition: A named function that may be used throughout the rest of the model.

Unit definition: A name for a unit used in the expression of quantities in a model. Units may be supplied in a number of contexts in an SBML model, and it is convenient to have a facility for both setting default units and for allowing combinations of units to be given abbreviated names.

Compartment: A container of finite volume for substances. In SBML Level 2, a compartment is primarily a topological structure with a volume but no geometric qualities.

Species: A substance or entity that takes part in a reaction. Some example species are ions such as Ca²⁺ and molecules such as glucose or ATP. The primary qualities associated with a chemical species in SBML Level 2 are its initial amount and the compartment in which it is located.

Parameter: A quantity that has a symbolic name. SBML Level 2 provides the ability to define parameters that are global to a model as well as parameters that are local to a single reaction.

Rule: In SBML, a mathematical expression that is used in combination with the differential equations constructed based on the set of reactions; it can be used to establish constraints between variables in a model, define how a variable can be calculated from other variables, or used to define the rate of change of a variable.

Reaction: A statement describing some transformation, transport or binding process that can change the amount of one or more species. For example, a reaction may describe how certain entities (reactants) are transformed into certain other entities (products). Reactions have associated rate laws describing how quickly they take place.

Event: A statement describing an instantaneous, discontinuous transformation of a set of variables of any type (species concentration, compartment volume or parameter value) when some triggering condition is satisfied.

A software package can read an SBML model description and translate it into its own internal format for model analysis. For example, a package might provide the ability to simulate the model by constructing differential equations representing the network and then performing numerical time integration on the equations to explore the model's dynamic behavior.

SBML allows models of arbitrary complexity to be represented. Each type of component in a model is described using a specific type of data structure that organizes the relevant information. The data structures determine how the resulting model is encoded in XML.

In the sections that follow, the various constructs in SBML and their uses are described in detail. Section 3 first introduces a few basic structures that are used throughout SBML Level 2, then Section 4 provides details on each of the main components. Section 5 provides a number of complete examples of models encoded in XML using SBML Level 2. Section 6 contains a list of anticipated enhancements that will be made in SBML Level 3 and a discussion of other efforts related to SBML. Appendix B provides the complete XML Schema for SBML Level 2.

3 Preliminary Definitions

This section covers certain concepts and constructs that are used repeatedly in the rest of SBML Level 2 and are useful to discuss before diving into the details of the components provided in SBML Level 2.

3.1 Type SBase

Each of the main structures composing an SBML Level 2 model definition has a specific data type that is derived directly or indirectly from a single abstract type called SBase. This inheritance hierarchy is depicted in Figure 1 on the following page. In addition to this, all the listOf_____s elements in SBML are derived from SBase.

The type SBase is designed to allow a modeler or a software package to attach arbitrary information to each major component in an SBML model. The definition of SBase is presented in Figure 2 on the next page. SBase contains three fields, all of which are optional: metaid, notes and annotation.

The metaid field is present for supporting metadata annotations using RDF. It has a data type of ID (the XML identifier type), and serves as anchors for metadata references. Metadata expressed using RDF can be placed anywhere within an sbml element and its subelements, except within MathML elements. The metadata elements can include RDF description elements in which the RDF describes attributes contain

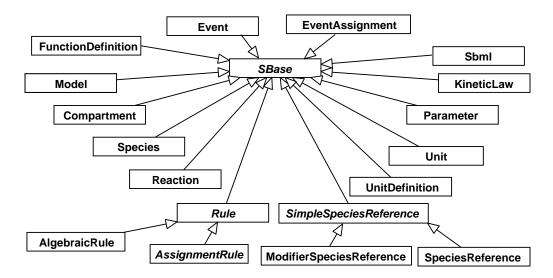


Figure 1: A UML diagram of the inheritance hierarchy of major data types in SBML. Open arrows indicate inheritance, pointing from inheritors to their parents (Eriksson and Penker, 1998; Oestereich, 1999).

SBase

metaid: ID {use="optional"}
notes: (ANY: {namespace="http://www.w3.org/1999/xhtml"}) {minOccurs="0"}
annotation: (ANY) {minOccurs="0"}

Figure 2: The definition of SBase. Text enclosed in braces next to attribute types (e.g., {minOccurs="0"}) indicates constraints on the possible attribute values. We use XML Schema language to express constraints since we are primarily interested in the XML encoding of SBML. The constraint expression use="optional" means that the indicated field is optional and may be omitted in a particular instance in a model. The constraint expression minOccurs="0" likewise means that the indicated field is optional; the alternate form must be used in XML Schema for those fields that are containers (i.e., fields that are encoded as subelements in XML).

the values of the metaid fields of SBML elements in the model. The form of the RDF element content in SBML should follow the form described in the CellML Metadata Specification (Cuellar et al., 2002).

The field notes in SBase is a container for XHTML content. It is intended to serve as a place for storing optional information intended to be seen by humans. An example of the kind of information that would be appropriate to place inside notes is user comments about a particular component of the model. Every data object derived directly or indirectly from type SBase can have a separate value for notes, allowing users considerable freedom for annotating their models. A software tool that reads and manipulates SBML is expected to provide some mechanism for displaying the contents of notes fields in a model.

Finally, SBase includes the field called annotation to provide a container for software-generated annotations that are *not* intended to be seen by humans. This field is a container for arbitrary data (XML type any). As with the user-visible notes field, every data object can have its own value for annotation. Guidelines for using this field are given in the next section.

In the other SBML type definitions presented below, we follow the UML convention of hiding the attributes derived from a parent type such as SBase. It should be kept in mind that these attributes are always available.

3.2 Guidelines for the Use of the annotation Field in SBase

The annotation field in the definition of SBase is formally unconstrained in order that software developers may attach any information they need to different components in an SBML model. However, it is important that this facility not be misused accidentally. In particular, it is critical that information essential to a model definition is *not* stored in annotation. Parameter values, functional dependencies between model components, etc., should not be recorded as annotations.

Here are examples of the kinds of data that may be appropriately stored in annotation: (a) Information about graphical layout of model components; (b) application-specific processing instructions that do not change the essence of a model; (c) identification information for cross-referencing components in a model with items in a database.

Different applications may use XML Namespaces (Bray et al., 1999) to specify the intended vocabulary of a particular annotation. Here is an example. Suppose a particular application needs to annotate data structures in an SBML model definition with screen layout information and a time stamp. The application's developers should choose a URI (*Universal Resource Identifier*; Harold and Means 2001; W3C 2000a) reference that uniquely identifies the vocabulary that the application will use for such annotations, and a prefix string to be used in the annotations. For illustration purposes, let us say the URI reference is "http://www.mysim.org/ns" and the prefix is mysim. An example of an annotation might then be:

The namespace prefix mysim is used to qualify the XML elements mysim:nodecolors and mysim:timestamp; presumably these symbols have meaning to the application. This example places the XML Namespace information on annotation itself rather than on a higher-level enclosing construct or the enclosing document level, but other placements would be valid as well (Bray et al., 1999).

The use of XML Namespaces permits multiple applications to place annotations on SBML elements without risking interference or element name collisions. Annotations stored by different simulation packages can thus coexist in the same model definition. Although XML Namespace names ("http://www.mysim.org/" in the example above) must be URI references, an XML Namespace name is not required to be directly usable in the sense of identifying an actual, retrieval document or resource on the Internet (Bray et al., 1999). The name is simply intended to enable unique identification of constructs, and using URIs is a common and simple way of creating a unique name string. For the convenience of the simulation community, we reserve certain namespace names for use with annotations in SBML. These reserved names are listed in Table 1.

```
http://www.sbml.org/2001/ns/basis
                                           http://www.sbml.org/2001/ns/jdesigner
http://www.sbml.org/2001/ns/biocharon
                                           http://www.sbml.org/2001/ns/jigcell
http://www.sbml.org/2001/ns/bioreactor
                                           http://www.sbml.org/2001/ns/jsim
http://www.sbml.org/2001/ns/biosketchpad
                                           http://www.sbml.org/2001/ns/mcell
http://www.sbml.org/2001/ns/biospice
                                           http://www.sbml.org/2001/ns/moma
http://www.sbml.org/2001/ns/cellerator
                                           http://www.sbml.org/2001/ns/netbuilder
http://www.sbml.org/2001/ns/copasi
                                           http://www.sbml.org/2001/ns/pathdb
http://www.sbml.org/2001/ns/cytoscape
                                           http://www.sbml.org/2001/ns/promot
http://www.sbml.org/2001/ns/dbsolve
                                           http://www.sbml.org/2001/ns/sbedit
http://www.sbml.org/2001/ns/ecell
                                           http://www.sbml.org/2001/ns/stochsim
http://www.sbml.org/2001/ns/gepasi
                                           http://www.sbml.org/2001/ns/vcell
http://www.sbml.org/2001/ns/isys
                                           http://www.sbml.org/2001/ns/winscamp
http://www.sbml.org/2001/ns/jarnac
```

Table 1: Reserved XML Namespace names in SBML Level 2.

Note that the namespaces being referred to here are XML Namespaces specifically in the context of the annotation field on SBase. The namespace issue here is unrelated to the namespaces discussed in Section 3.5 in the context of SName and symbols in SBML.

3.3 id and name attributes on SBML components

As will become apparent below, nearly all structures in SBML include two particular fields: id and name. The id field is required and is used to identify a component within the model definition. Other SBML structures refer to the component using this identifier. The next section defines the data type SId used for the id field, and Section 3.5 describes the scoping and namespace rules for these identifiers.

In contrast to the id field, the name field is optional. Its data type is the type string defined in XML Schema (Biron and Malhotra, 2000; Thompson et al., 2000), which includes all Unicode characters (Unicode Consortium, 1996) except for two delimiter characters, 0xFFFE and 0xFFFF (Biron and Malhotra, 2000). The purpose of the name field is to provide a human-readable label for the component. No restrictions as to its contents are imposed by SBML beyond those defined by the string type of XML Schema.

The recommended practice for handling name is as follows. If a software tool has the capability for displaying the content of name fields, it should display this content to the user as a component's label instead of the component's id field. If the user interface does not have this capability (e.g., because it cannot display special characters), or if the name field is missing on a given component, then the user interface should display the value of the id field. (Script language interpreters are especially likely to display id fields instead of name fields.)

As a consequence of the above, authors of systems that automatically generate the values of id fields should be aware some systems may display the id's to the user. Authors may wish to take some care to have their software create id values that are easy for humans to type and read.

3.4 Type SId

The type SId is the type of the id field found on the majority of SBML components. SId is a data type derived from the basic XML type string, but with restrictions about the types of characters permitted and the sequence in which they may appear. Its definition is shown in Figure 3.

```
letter ::= 'a'..'z','A'..'Z'
digit ::= '0'..'9'
nameChar ::= letter | digit | '_'
name ::= ( letter | '_' ) nameChar*
```

Figure 3: The definition of the type SId expressed in the variant of XML used by the XML 1.0 specification (Bray et al., 2000). The characters (and) are used for grouping, and the character * indicates "zero or more times".

The SId is purposefully not derived from the XML ID type. Doing so would force all SBML identifiers to exist in a single global namespace, which would affect not only the form of local parameter definitions but also future extensions for supporting model/submodel composition. Further, the use of the ID type for SBML identifiers would have limited utility because MathML ci elements are not of the type IDREF (see Section 3.6). If the IDREF-ID linkage cannot be exploited in MathML constructs, the utility of the XML ID type is greatly reduced.

3.5 Component Identifiers and Namespaces in SBML

A biochemical network model can contain a large number of components representing different parts of a model. This leads to a problem in deciding the scope of an identifier: in what contexts does a given identifier X represent the same thing? The approaches used in existing simulation packages tend to fall into two categories that we may call global and local. The global approach places all identifiers into a single global namespace, so that an identifier X represents the same thing wherever it appears in a given model definition.

The local approach places symbols in different namespaces depending on the context, where the context may be, for example, individual rate laws. The latter approach means that a user may use the same identifier X in different rate laws and have each instance represent a different quantity.

The fact that different simulation programs may use different rules for identifier resolution poses a problem for the exchange of models between simulation tools. Without careful consideration, a model written out in SBML format by one program may be misinterpreted by another program. SBML Level 2 must therefore include a specific set of rules for treating identifier and namespaces.

The namespace rules in SBML Level 2 are relatively straightforward and are intended to avoid this problem with a minimum of requirements on the implementation of software tools:

- The identifiers of functions, compartments, species, reactions, events and model-level parameters reside in the same global namespace. This means, for example, that a reaction and a species definition cannot both have the same identifier.
- Each reaction definition (see Section 4.9) establishes a private local namespace for local parameter identifiers. Within the definition of a given reaction, local parameter identifiers introduced in that reaction override (shadow) identical identifiers in the global namespace.
- Unit names exist in a separate global namespace from other identifiers.

The set of rules above can enable software packages using either local or global namespaces for parameters to exchange SBML model definitions. In particular, software environments using local namespaces for parameters internally should be able to accept SBML model definitions without needing to change component identifiers. Environments using a global namespace for parameters internally can perform a simple manipulation of the identifiers of local parameter elements within reaction definitions to avoid name collisions. (An example approach for the latter would be the following: when receiving an SBML-encoded model, prefix each parameter identifier inside each reaction with a string constructed from the reaction's identifier; when writing an SBML-encoded model, strip off the prefix.)

The namespace rules described here provide a clean transition path to future levels of SBML, when submodels are introduced (Section 6.1). Submodels will provide the ability to compose one model from a collection of other models. This capability will have to be built on top of SBML Level 2's namespace organization. A straightforward approach to handling namespaces is to make each submodel's space be private. The rules governing namespaces within a submodel can simply be the Level 2 namespace rule described here, with each submodel having its own (to itself, global) namespace.

3.6 Mathematical Formulas in SBML Level 2

Math in SBML Level 2 is expressed using MathML (W3C, 2000b). MathML is used in the definitions of functions (Section 4.3), kinetic laws (Section 4.9.3), and rules (Section 4.8). The KineticLaw and Rule structures each have a single MathML math subelement, and a function definition has a single lambda subelement as well. The XML namespace for all of these elements is the URI "http://www.w3.org/1998/Math/MathML". [See the W3C document by Bray et al. (1999) for more information about using XML namespaces.]

3.6.1 Subset of MathML Used in SBML Level 2

Only the elements contained in the CellML subset of MathML, with the addition of csymbol, can be used within the MathML math and lambda elements. The subset of MathML used in SBML Level 2 is as follows:

- token: cn, ci, csymbol
- basic content: apply, piecewise, piece, otherwise
- relational operators: eq, neq, gt, lt, geq, leq
- arithmetic operators: plus, minus, times, divide, power, root, abs, exp, ln, log, floor, ceiling, factorial

- logical operators: and, or, xor, not
- calculus: diff
- qualifiers: degree, bvar, logbase
- trigonometric operators: sin, cos, tan, sec, csc, cot, sinh, cosh, tanh, sech, csch, coth, arcsin, arccos, arctan, arccosh, arccot, arccoth, arccsc, arcsec, arcsec, arcsech, arcsinh, arctanh
- constants: true, false, notanumber, pi, infinity, exponentiale
- annotation: semantics, annotation, annotation-xml

The inclusion of logical operators, relational operators, piecewise, piece, and otherwise elements facilitates the encoding of discontinuous expressions. Elements for representing partial differential calculus are not included. Its is anticipated that the requirement for partial differential calculus will be addressed in proposals for SBML Level 3 geometry representations (see Section 6.1).

3.6.2 Use of Token Elements in MathML

MathML whitespace rules apply to the content of ci elements. The content of ci should always be a declared identifier. The set of possible identifiers depends on the containing structure. In the case of math function definitions, the content of ci elements is restricted to the declared arguments and previously declared functions. In all other cases, the content of ci elements can be identifiers of math functions, parameters, compartments or species; i.e., the content should match the value of an id field of a component. When a specie identifier occurs in a ci element, it represents the concentration (i.e., substance/volume) of the specie. When a compartment identifier occurs in a ci element, it represents the volume of the compartment. The units of substance and volume are determined from the built-in substance and volume of Table 3 on page 14.

SBML Level 2 uses the MathML csymbol element to denote standardized math entities without introducing reserved names into the component identifier namespace. The encoding field of csymbol should be set to SBML. The definitionURL should be set to one of the following predefined SBML symbol URLs:

- http://www.sbml.org/sbml/symbols/time. This represents the current simulation time. The units of the current time entity are determined from the built-in time of Table 3 on page 14.
- http://www.sbml.org/sbml/symbols/delay. This represents a delay function. The delay function has the form delay(x, d), taking two arguments. Its value is the value of argument x at d time units before the current time. The units of the d parameter are determined from the built-in time. The delay function is useful for representing biological processes having a delayed response, but where the detail of the processes and delay mechanism is not relevant to the operation of a given model.

It is not necessary for a parser to access the resource pointed to by the URL: in this context the URL should be interpreted as a URI. The content of the csymbol element is for rendering purposes only and can be ignored by a parser.

The following examples demonstrates these concepts. The XML fragment below encodes the formula x + t, where t is the built-in symbol for time.

As a further example, the following XML fragment encodes the equation k + delay(x, 0.1) or alternatively $k_t + x_{t-0.1}$:

Section 5.7 contains a complete model which uses a delay function.

4 SBML Components

In this section, we define each of the major data structures in SBML. To provide illustrations of their use, we give partial model definitions in XML. Section 5 provides many full examples of SBML in XML.

4.1 The SBML Container

An SBML Level 2 model definition consists of a single Sbml structure enclosing a single Model structure (see next Section). The definition of Sbml is shown in Figure 4.

```
Sbml

xmlns: string { value="http://www.sbml.org/sbml/level2" }
level: positiiveInteger { value = "2"}
version: positiiveInteger { value = "1"}
```

Figure 4: The definition of Sbm1. Additional fields are inherited from SBase but are not shown here.

The XML namespace URI for SBML Level 2 is http://www.sbml.org/sbml/level2. In the transformation of UML to XML used in this document, the Sbml structure is turned into an element named sbml. The element has two required attributes: version and level. For SBML Level 2 Version 1, these attributes must be set to "1" and "2", respectively. (The version attribute is present in case SBML Level 2 must be revised in the future to correct errors.)

The following is an abbreviated example of the outermost content of an SBML model definition in XML:

4.2 Models

The Model structure is the highest-level construct in an SBML data stream or document. It defines a grouping of components—the list of function definitions, compartments, species, reactions, parameters, rules and unit definitions that define a given model. Only one component of type Model is allowed per instance of an SBML Level 2 document or data stream, although it does not necessarily need to represent a single biological entity. The UML definition of the Model structure is shown in Figure 5 on the next page.

Model id : SId {use="optional"} name : string {use="optional"} functionDefinition : functionDefinition[0..*] unitDefinition : UnitDefinition[0..*] compartment : Compartment[0..*] species : Species[0..*] parameter : Parameter[0..*] rule : Rule[0..*] reaction : Reaction[0..*] event : Event[0..*]

Figure 5: The definition of Model. Additional fields are inherited from SBase.

The Model structure has an optional field, id, used to give the model an identifier. The identifier must be a text string conforming to the syntax permitted by the SId data type described in Section 3.4. Model also has an optional name field, of type string. The name and id fields should be used as described in Section 3.3.

Model serves as a container for Species, Compartment, FunctionDefinition, UnitDefinition, Parameter, Reaction, Rule and Event components. All of these components are optional (the lists in each of the respective fields are permitted to have zero length). In the XML encoding of an SBML model, the lists of species, compartments, unit definitions, parameters, reactions, function definitions, rules and events are translated into lists of XML elements that each have headings of the form listOf_____s, where the blank is replaced by the name of the component type (e.g., "Reaction"). The resulting XML data object has the form illustrated by the following skeletal model:

```
<model id="My_Model">
    <listOfFunctionDefinitions>
    </listOfFunctionDefintions>
    <listOfUnitDefinitions>
    </listOfUnitDefinitions>
    <listOfCompartments>
    </listOfCompartments>
    t0fSpecies>
    </listOfSpecies>
    <listOfParameters>
    </listOfParameters>
    tofRules>
    </listOfRules>
    <listOfReactions>
    </listOfReactions>
    <listOfEvents>
    </listOfEvents>
</model>
```

Readers may wonder about the motivations for the listOf____s notation. A simpler approach to creating the lists of components would be to place them all directly at the top level under <model> ... </model>. ... </model>. ... because we believe this helps organize the components and makes visual reading of model definitions easier.

4.3 Function Definitions

The FunctionDefinition structure associates an identifier with a function definition. The identifier can then be used in any subsequent MathML apply elements. FunctionDefinition is shown in Figure 6.

Figure 6: The definition of FunctionDefinition. Fields inherited from SBase are omitted here but are assumed.

The FunctionDefinition structure has three fields, id, name and math. The id and name fields have types SId and string, respectively, and operate in the manner described in Section 3.3. MathML ci elements can refer to the function defined by a FunctionDefinition using the value of its id field.

The math field is a container for MathML content that defines the function. The content of this field can only be a MathML lambda element. The function is only available for use in other MathML elements that follow the place of its definition in an SBML model. (This restriction is to prevent recursive and mutually-recursive functions from being expressed.)

The following is an abbreviated SBML example. It shows a FunctionDefinition structure defining pow3(x) as representing x^3 :

4.4 Unit Definitions

Units may be supplied in a number of contexts in an SBML model. A facility for defining units is convenient to have so that combinations of units can be given abbreviated names. This is the motivation behind the UnitDefinition data structure, whose definition is shown in Figure 7.

```
UnitDefinition

id: SId
name: string
unit: Unit[0..*]

Linit
kind: UnitKind
exponent: integer {use="default" value="1"}
scale: integer {use="default" value="0"}
```

Figure 7: The definition of UnitDefinition.

A unit definition consists of a id field of type SId, an optional string field name and an optional list of structures of type Unit. The identifiers defined in the id field are in a separate global namespace from identifiers for species, compartments, reactions, events, etc.

The approach to defining units in SBML is compositional; for example, $meter\ second^{-2}$ is constructed by combining a Unit-type element representing meter with a Unit-type element representing $second^{-2}$. The Unit data structure has a required field, kind, whose value must be taken from UnitKind, an enumeration of units. The possible values of UnitKind are listed in Table 2. The exponent field on Unit represents an exponent on the unit. Its default value is "1" (one). In the example just mentioned, $second^{-2}$ is obtained by using kind="second" and exponent="-2". Finally, the scale field in Unit is an integer attribute that scales the unit. For example, a unit that has a kind value of "gram" and a scale value of "-3" signifies $10^{-3} * gram$, or milligrams.

ampere	farad	joule	lumen	ohm	steradian
becquerel	${\tt gram}$	katal	lux	pascal	tesla
candela	gray	kelvin	meter	radian	volt
Celsius	henry	kilogram	metre	second	watt
coulomb	hertz	liter	mole	siemens	weber
dimensionless	item	litre	newton	sievert	

Table 2: The possible values of kind in a UnitKind structure. All are names of base or derived SI units, except for "dimensionless" and "item", which are SBML additions important for handling certain common cases. "Dimensionless" is intended for cases where a quantity does not have units, and "item" is needed in certain contexts to express such things as "N items" (e.g., "100 molecules"). Although "Celsius" is capitalized, for simplicity, SBML requires that these names be treated in a case-insensitive manner. Also, note that the gram and liter/litre are not strictly part of SI (Bureau International des Poids et Mesures, 2000); however, they are so commonly used in SBML's areas of application that they are included as predefined unit names. (The standard SI unit of mass is in fact the kilogram, and volume is defined in terms of cubic meters.)

Unit combinations are constructed by listing several Unit structures inside a UnitDefinition-type structure. The following example illustrates the definition of an abbreviation named "mmls" for the units $mmol\ l^{-1}\ s^{-1}$:

There are three special unit names in SBML, listed in Table 3, corresponding to the three types of quantities that play roles in biochemical reactions: amount of substance, volume and time. SBML defines default units for these quantities, all with a default scale value of 0. The various components of a model, such as parameters, can use only the predefined units from Table 2, new units defined in unit definitions, or the three predefined names "substance", "time", and "volume" from Table 3. The latter usage signifies that the units to be used should be the designated defaults.

Name	Allowable Units	Default Units
substance	moles or no. of molecules	moles
volume	liters	liters
time	seconds	seconds

Table 3: SBML's built-in quantities. Each of these units has a default scale value of 0.

A model may change the default scales by reassigning the keywords "substance", "time", and "volume" in a unit definition. This takes advantage of the UnitDefinition structure's facility for defining scales on units. The following example changes the default units of volume to be milliliters:

If the definition above appeared in a model, the volume scale on all components that did not explicitly use different units would be changed to milliliters.

4.5 Compartments

A Compartment represents a bounded container in which species are located. The definition of Compartment is shown in Figure 8.

```
id: SId
name: string {use="optional"}
volume: double {use="optional"}
units: SId {use="optional"}
outside: SId {use="optional"}
constant: boolean {use="default" value="true"}
```

Figure 8: The definition of Compartment. Fields inherited from SBase are omitted here but are assumed.

A Compartment data object has an id field of type SId and an optional name field of type string. A compartment also has a floating-point field called volume, representing the total volume of the compartment in the default units of volume. (See Table 3 on the preceding page.) This enables concentrations of species to be calculated in the absence of geometry information. The volume field is optional. A missing volume value implies that the value is either unknown, not required for analysis, or available from an external data source. A missing volume value does not imply that the compartment volume is 1.

A Compartment structure has an optional boolean field called constant which indicates whether the compartment's volume stays constant or can vary during a simulation. A value of false indicates that the compartment's volume can be determined by rules, and the value of the volume field should be taken to be the initial volume of the compartment. The default value for the constant field is true because in typical modeling scenarios, compartment volumes do not change.

The units of volume may be explicitly set using the optional field units in Compartment; the named units must be either one of the base units from Table 2 on the page before, the built-in default named volume, or a new unit defined by a unit definition in the enclosing model. If absent, the units default to the value set by the built-in volume of Table 3.

In an XML data stream containing an SBML model, compartments are listed inside an XML element called listOfCompartments within a Model-type data structure. (See the discussion of Model in Section 4.2.) The following example illustrates two compartments in an abbreviated SBML example of a model definition:

On the Compartment structure, the optional field outside of type SId can be used to express containment relationships between compartments. If present, the value of outside for a given compartment should be the identifier of the compartment enclosing it, or in other words, the compartment that is "outside" of it. This facility can be used to model cell membranes. For example, to express that a compartment B has a membrane that is modeled as another compartment M, which in turn is located within another compartment A, one would write:

In the absence of a value for outside, compartment definitions in SBML Level 2 do not have any implied spatial relationships between each other. For many modeling applications, the transfer of substances described by the reactions in a model sufficiently express the relationships between the compartments. (SBML Level 2 currently does not provide for spatial characteristics aside from compartment volume and containment. As discussed in Section 6.1, we expect that SBML Level 3 will introduce the ability to define geometries and spatial qualities.)

4.6 Species

The term *species* refers to chemical entities that take part in reactions. These include simple ions (e.g., protons, calcium), simple molecules (e.g., glucose, ATP), and large molecules (e.g., RNA, polysaccharides, and proteins). The **Species** data structure is intended to represent these entities. Its definition is shown in Figure 9.

```
id: SId
name: string {use="optional"}
compartment: SId
initialAmount: double {use="optional"}
units: SName {use="optional"}
boundaryCondition: boolean {use="default" value="false"}
charge: integer {use="optional"}
constant: boolean {use="default" value="false"}
```

Figure 9: The definition of Species. As usual, fields inherited from SBase are omitted here but are assumed.

Species has an id field of type SId and optional name field of type string. The field compartment, also of type SId, is used to identify the compartment in which the species is located. The optional field initialAmount, of type double, is used to set the initial amount of the species in the named compartment. A missing initialAmount value implies that the value is either unknown, not required for analysis or available from an external data source. The units of the substance quantity may be explicitly set using the optional field units. The value assigned to units must be chosen from one of the following possibilities: one of the unit names from Table 2 on page 14, the name "substance", or a new unit name defined by a unit definition in the enclosing model. If absent, the units default to the value set by the built-in "substance" of Table 3 on page 14.

The Species structure has an optional boolean field named constant which indicates whether the concentration of that species can vary during a simulation. The default value is false, indicating that the species' concentration can be determined by rules and reactions.

Species also has another optional boolean field, boundaryCondition. By default, when a species is a product or reactant of one or more reactions, the concentration of that species is determined by those reactions. In SBML, it is possible to indicate that a given species' concentration is not determined by the set of reactions even when that species occurs as a product or reactant; i.e., the species is on the boundary of the reaction system but is a component of the rest of the model. The optional boolean field boundaryCondition indicates that the given species is on the boundary of the reaction system. The value of the field defaults to "false", indicating that by default, the species is part of the reaction system. Table 4 shows how to interpret the combined values of the boundaryCondition and constant fields. In practice, the boundaryCondition attribute means that a differential equation derived from the reaction definitions should not be generated for the species. The example model in section 5.5 contains all 4 possible states of the boundaryCondition and constant attributes on species elements.

constant value	boundaryCondition value		can be reactant or product	concentration is changed by
true	true	no	yes	never changes
false	true	yes	yes	rule
true	false	no	no	never changes
false	false	yes	yes	reactions or rule but not both

Table 4: How to interpret the values of the constant and boundary Condition fields of the Species structure.

The optional field charge on Species takes an integer indicating the charge on the species (in terms of electrons, not the SI unit Coulombs). This may be useful when the species involved is a charged ion such as calcium (Ca^{2+}) .

The following example shows two species definitions within an abbreviated SBML model definition. The example shows that species are listed under the heading listOfSpecies in the model:

4.7 Parameters

A Parameter structure is used to declare a variable for use in mathematical formulas in an SBML model definition. By default, parameters have constant value for the duration of a simulation and are therefore called "parameters" instead of variables in SBML. The definition of Parameter is shown in Figure 10.

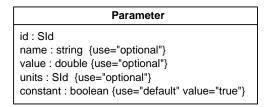


Figure 10: The definition of Parameter. Fields inherited from SBase are omitted here but are assumed.

Parameter has an id field of type SId and an optional name field of type string. The symbol in the id field identifies the parameter. The optional field value determines the value (of type double) assigned to the identifier. A missing value implies that the value is either unknown, not required for analysis or available from an external data source. The units of the parameter value are specified by the optional field units. The value assigned to units must be chosen from one of the following possibilities: one of base unit names from Table 2 on page 14; one of the three names "substance", "time", or "volume" (see Table 3); or the name of a new unit defined in the list of unit definitions in the enclosing Model structure.

The Parameter structure has an optional boolean field named constant which indicates whether the parameter's value can vary during the simulation. The default value is true; a value of false indicates that the parameter's value can be changed by rules (see Section 4.8) and that the value is actually intended to be the initial value of the parameter.

Parameters are used in two places in SBML: in lists of parameters defined at the top level in a Model-type structure, and within individual reaction definitions. Parameters defined at the top level are *global* to the whole model; parameters that are defined within a reaction are local to the particular reaction and (within that reaction) *override* any global parameters having the same names. (See Section 3.5 for further details.)

The following is an example of parameters defined at the Model level:

4.8 Rules

In SBML, rules provide a way to create constraints on variables for cases in which the constraints cannot be expressed using the reaction components (Section 4.9). There are three different possible functional forms of rules, corresponding to the following three general cases (where x is a variable, f is some arbitrary function, and X is the vector of variables that may include species, compartments and parameters):

```
Algebraic rules, left-hand side is zero: 0 = f(X)

Scalar rules, left-hand side is a scalar: x = f(X)

Rate rules, left-hand side is a rate-of-change: dx/dt = f(X)
```

In their general form given above, there is little to distinguish between *scalar* and *algebraic* rules. They are treated as separate cases for the following reasons:

- Scalar rules can simply be evaluated to calculate intermediate values for use in numerical methods;
- Some simulators do not contain numerical solvers capable of solving unconstrained algebraic equations;
- Those simulators that *can* solve these *algebraic* equations make a distinction between the different categories listed above; and
- Some specialized numerical analyses of models may only be applicable to models that do not contain *algebraic* rules.

In SBML, Scalar and rate rules are collectively referred to as assignment rules. SBML uses an abstract Rule structure that contains only one field, math, to hold the right-hand side expression, then derives subtypes of Rule that add fields to distinguish the case of algebraic, scalar and rate rules. Figure 11 on the next page gives the definitions of Rule and the subtypes derived from it. The figure shows there are two subtypes, AlgebraicRule and AssignmentRule, defined directly from Rule.

The type field introduced in AssignmentRule is an enumeration of type RuleType that determines whether a rule falls into the scalar or rate categories. In SBML Level 2, the enumeration has two possible values:

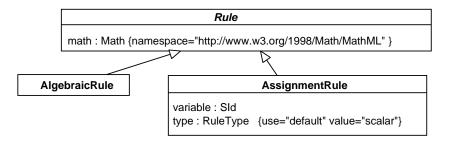


Figure 11: The definition of Rule and derived types.

"scalar" and "rate". The former means that the expression has a scalar value on the left-hand side [i.e., x = f(X)]; the latter means that the expression has a rate of change differential on the left-hand side [i.e. dx/dt = f(X)]. Future releases of SBML may add to the possible values of RuleType.

4.8.1 AlgebraicRule

The rule type AlgebraicRule is used to express equations whose left-hand sides are zero. AlgebraicRule does not add any fields to the basic Rule; its role is simply to distinguish this case from the assignment rule case. An example of the use of AlgebraicRule structures is given in Section 5.4.

4.8.2 AssignmentRule

The left-hand sides of assignment rules can be the identifiers of species, compartments, or parameters. The effects are in general terms the same, but have different effects depending on what variable is being set:

• In the case of a species: if the value of type is "scalar", the rule sets the referenced species's concentration to the value determined by the formula in math; if the value is "rate", the rule sets the rate of change of the species's concentration to the value determined by the formula. The units are in terms of substance/volume, where the substance units are those that are declared on the referenced Species element, and the volume units are those declared on the compartment element that contains the Species.

Restrictions: A rule and a SpeciesReference structure (see Section 4.9) cannot both have the same species attribute value. This means that a rule cannot be defined for a species that is created or destroyed in a reaction. The only exception is when the given species is a boundary condition; i.e., on the Species structure that defines the specie the boundaryCondition field is set to "true".

- In the case of a compartment: if the value of type is "scalar", the rule sets the referenced compartment's volume to the volume determined by the formula in math; if the type is "rate", the rule sets the rate of change of the compartment's volume to the volume determined by the formula.
 - Restrictions: No more than one AssignmentRule can refer to a given compartment in an SBML model definition.
- In the case of a parameter: if the value of type is "scalar", the rule sets the referenced parameter's value to that determined by the formula in math; if the type is "rate", the rule sets the rate of change of the parameter's value to that determined by the formula.

Restrictions: No more than one AssignmentRule can refer to a given parameter.

4.8.3 Constraints on rules

SBML specifically does not stipulate the form of the algorithms that can be applied to rules and reactions. For example, SBML does not specify when or how often rules should be evaluated. The constraints described by rules and kinetic rate laws are meant to apply collectively to the set of variable values for a specific time.

In SBML, no more than one assignment rule can be defined for a given identifier. No assignment rule can be defined for an identifier whose corresponding structure has the field **constant** set to **true**.

A scalar rule for a given identifier overrides the initial value of that identifier; i.e., the initial value should be ignored. This does not mean that any structure declaring an identifier can be omitted if there is a scalar rule for that identifier. For example, there must be a Parameter structure for a given parameter if there is a rule for that parameter.

The ordering of scalar rules is significant: they are always evaluated in the order given in SBML. The math field of a scalar rule structure can contain any identifier in a MathML ci element except for the following:

(a) identifiers for which there exists a subsequent scalar rule, and (b) the identifier for which the rule is defined. These constraints are designed to eliminate algebraic loops among the scalar rules. As an example, consider the following equations, in the order shown:

$$x = x + 1$$
, $y = z + 200$, $z = y + 100$

If this set of equations were interpreted as a set of scalar rules, it would be invalid because the rule for x refers to x and the rule for y refers to z before z is defined.

Eliminating these algebraic loops ensures that scalar rules can be evaluated any number of times without the result of those evaluations changing.

4.8.4 Example of Rule Use

This section contains an example set of rules. Consider the following set of equations:

$$k = \frac{k_3}{k_2}, \quad s_2 = \frac{kt}{1+k_2}, \quad A = 0.10t$$

This math can be encoded by the following scalar rule set:

```
<model>
    tOfRules>
        <assignmentRule id="k">
            <notes>
                <xhtml:p>
                    k = k3/k2
                </xhtml:p>
            </notes>
            <math xmlns="http://www.w3.org/1998/Math/MathML">
                <apply>
                    <divide/>
                    <ci> k3 </ci>
                    <ci> k2 </ci>
                </apply>
            </assignmentRule>
        <assignmentRule variable="s2">
            <notes>
                <xhtml:p>
                    s2 = (k * t)/(1 + k2)
                </xhtml:p>
            </notes>
            <math xmlns="http://www.w3.org/1998/Math/MathML">
                <apply>
                    <divide/>
                    <apply>
                         <times/>
                        <ci> k </ci>
                        <ci> t </ci>
                    </apply>
                    <apply>
                        <plus/>
                        <n> 1 </cn>
                        <ci> k2 </ci>
                    </apply>
                </apply>
```

```
</assignmentRule>
        <assignmentRule variable="A">
            <notes>
                <xhtml:p>
                   A = 0.10 * t
                </xhtml:p>
            </notes>
            <math xmlns="http://www.w3.org/1998/Math/MathML">
                <apply>
                    <times/>
                    <cn> 0.10 </cn>
                    <ci> t </ci>
                </apply>
            </assignmentRule>
    </listOfRules>
</model>
```

4.9 Reactions

A reaction represents any transformation, transport or binding process, typically a chemical reaction, that can transform or change the amount of one or more species. In SBML, a reaction is defined primarily in terms of the participating reactants and products (and their corresponding stoichiometries), along with optional modifier species, an optional kinetic law describing the rate at which the reaction takes place, and optional parameters entering into the kinetic law. These various parts of a reaction are recorded in the SBML Reaction type defined in Figure 12 on the following page.

As with the other main structures in SBML, the Reaction data structure includes a required id field and an optional name. These must be used according to the guidelines described in Section 3.3. The reactant species, product species and modifier species in a reaction are described using the fields reactant, product and modifiers. These fields are optional lists of either SpeciesReference or ModifierSpeciesReference structures, as shown in Figure 12 on the next page. They are described in more detail in Sections 4.9.1 and 4.9.2 below. The kineticLaw is an optional field of type KineticLaw, described in more detail in Section 4.9.3 below.

In addition to the above, the Reaction structure also has two optional boolean fields. The first field, reversible, indicates whether the reaction is reversible. The field is optional, and if left unspecified in a model, it defaults to a value of "true". Although the reversibility of a reaction is determined by the rate law, the need to allow rate laws to be optional leads to the need for a flag indicating reversibility. Information about reversibility in the absence of a kinetic laws is useful in certain kinds of structural analyses such as elementary mode analysis. It is true that the presence of this information in two places (i.e., the rate law and the flag reversible) leaves open the possibility of a model containing contradictory information, but the creation of such a model would indicate an error on the part of the software generating it. Software developers must take care to ensure against logical contradictions in the definitions of reactions.

The field fast is another boolean attribute in the Reaction data structure; a value of "true" signifies that the given reaction is a "fast" one. This may be relevant when computing equilibrium concentrations of rapidly equilibrating reactions. Simulation/analysis packages may chose to use this information to reduce the number of ODEs required and thereby optimize such computations. The default value of fast is "false". (A simulator/analysis package that has no facilities for dealing with fast reactions can ignore this attribute. In theory, if the choice of which reactions are fast is correctly made, then a simulation performed with them should give the same results as a simulation performed without fast reactions. However, currently there appears to be no single unambiguous method for designating which reactions should be considered fast, and some users may designate a reaction as fast when in fact it is not.)

Reaction id: SId name: string {use="optional"} reactant: SpeciesReference[0..*] product: SpeciesReference[0..*] modifier: ModifierSpeciesReference[0..*] kineticLaw: KineticLaw {minOccurs="0"} reversible: boolean {use="default" value="true"} fast: boolean {use="default" value="false"}

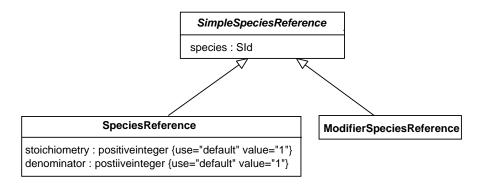


Figure 12: The definitions of Reaction, KineticLaw, SpeciesReference and ModiferSpeciesReference.

4.9.1 SpeciesReference

Every species that enters into a given reaction must appear in that reaction's lists of reactants, products or modifiers. In an SBML model, all species that participate in any reaction are listed in the listOfSpecies field of the top-level Model data structure (see Section 4.2). Lists of products, reactants and modifiers in Reaction type structures do not introduce new species, but rather, they refer back to those listed in the model's listOfSpecies. For reactants and products, the connection is made using the SpeciesReference type data structure defined in Figure 12.

In SpeciesReference, the field species of type SId must refer to the name of an existing species defined in the enclosing Model-type structure. Stoichiometric numbers for the products and reactants can be specified using two optional attributes on the speciesReference element: stoichiometry and denominator. Both attributes take positive integers as values, and both have default values of "1" (one). The absolute value of the stoichiometric number is the value of stoichiometry divided by denominator, and the sign is implicit from the role of the species (i.e., negative for reactants and positive for products). The use of separate numerator and denominator terms allows a simulator to employ rational arithmetic if it is capable of it, potentially reducing round-off errors and other problems during computations.

The following is a simple example of a species reference in a list of reactants within a reaction named "J1":

```
<model>
    ...
    stOfReactions>
         <reaction id="J1">
```

A reaction can contain an empty list of reactants or an empty list of products but must have at least one reactant or product. Also note that whether a given species is allowed to appear as a reactant or product is dictated by certain flags on the structure describing the species in the Model; see Table 4 for more information.

4.9.2 ModifierSpeciesReference

In some cases, a species may act as a catalyst or inhibitor of a reaction, and may not appear in the list of reactants or products because it is neither created nor destroyed in that particular reaction. In that case, the species is known as a *modifier*. (That same species may still be a reactant or product of another reaction.)

The Reaction structure provides a way to express which species act as modifiers in a given reaction. This is the purpose of the modifier field in Reaction; this field is a list of ModifierSpeciesReference structures defined in Figure 12 on the page before. The ModifierSpeciesReference structure has only one field, species, of type SId; its value must be the identifier of a species defined in the enclosing Model.

The following is a simple example of a modifier species reference in a list of reactants within a reaction named "J1":

4.9.3 KineticLaw

A kineticLaw structure describes the rate at which the reaction takes place. The inclusion of a KineticLaw structure in an instance of a Reaction component is optional; however, in general there is no useful default that can be substituted in place of a missing rate law definition in a reaction.

The field math is a MathML element and contains an expression that sets the rate of the reaction, in substance/time units. (Section 3.6 discusses the use of MathML in SBML Level 2). The optional fields substanceUnits and timeUnits determine the units of substance and time. If not set, the units are taken from the defaults defined by the built-in "substance" and "time" of Table 3 on page 14. The only species identifiers that can be used in math are those listed in the reactant, product and modifier fields of the Reaction structure.

An instance of a KineticLaw type structure can contain zero or more parameter structures (Section 4.7) that define symbols that can be used in the math element. As discussed in Section 3.5, reactions introduce local namespaces for parameter identifiers. Within a KineticLaw structure inside a reaction definition, a local parameter whose identifier is identical to a global parameter defined in the enclosing Model-type structure takes precedence over that global parameter.

The following is an example of a complete Reaction structure that defines a reaction named J_1 , in which

 $X_0 \longrightarrow S_1$ at a rate given by $k_1 X_0 S_2$, and in which S_2 is a catalyst and k_1 is a parameter. It demonstrates the use of species references and the KineticLaw structure:

```
<model>
   <listOfReactions>
       <reaction id="J1">
           <listOfReactants>
               <speciesReference species="X0" stoichiometry="1"/>
           <speciesReference species="S1" stoichiometry="1"/>
           </listOfProducts>
           <listOfModifiers>
               <modifierSpeciesReference species="S2"/>
           </listOfModifiers>
           <kineticLaw>
               <math xmlns="http://www.w3.org/1998/Math/MathML">
                   <apply>
                       <times/>
                       <ci> k1 </ci>
                       <ci> X0 </ci>
                       <ci> S2 </ci>
                   </apply>
               <listOfParameters>
                   <parameter id="k1" value="0.1"/>
               </listOfParameters>
           </kineticLaw>
       </reaction>
   </listOfReactions>
</model>
```

4.10 Events

Model has an optional list of Event structures that describe the time and form of explicit instantaneous discontinuous state changes in the model. For example, an event may describe that one species concentration is halved when another species concentration exceeds a given threshold value.

An Event structure defines when the event can occur, the variables that are affected by the event, and how the variables are affected. The effect of the event can optionally be delayed after the occurrence of the condition which invokes it. The operation of an Event structure is divided into two phases (even when the event is not delayed): one when the event is *fired* and the other when the event is *executed*. The Event type is defined in Figure 13. Both Event and EventAssignment are derived from SBase (see Section 3.1). An example of a model which uses events is given below.

```
trigger: (math: Math {namespace="http://www.w3.org/1998/Math/MathML"})
delay: (math: Math {namespace="http://www.w3.org/1998/Math/MathML"})
timeUnits: SId { use="optional" }
eventAssignment: EventAssignment[1..n]
```

```
EventAssignment

variable : SId

math : Math {namespace="http://www.w3.org/1998/Math/MathML"}
```

Figure 13: The definitions of Event and EventAssignment

The following sections describe the fields of the Event structure.

4.10.1 trigger

The trigger field defines when the Event structure has an effect on the model. The trigger field contains a MathML boolean expression. The exact instant that the expression evaluates to true is the time point when the Event is *fired*. The event only fires when the trigger makes the transition from false to true. The event will fire at any further time points when the trigger make this transition.

4.10.2 delay

The optional delay field defines the length of time after the event has *fired* that the event is *executed*. The delay field is another MathML expression. This expression should be evaluated when the rule is *fired*. The default value for the delay field is 0. The value of the delay field should always be positive.

4.10.3 timeUnits

The optional field timeUnits determines the units of time that apply to the delay field. If not set, the units are taken from the defaults defined by the built-in "time" of Table 3 on page 14.

4.10.4 eventAssignment

The eventAssignment field consists of a non-empty list of eventAssignment structures. This field is implemented as a listOfEventAssignments element containing one or more eventAssignment elements. The EventAssignment structures represent variable assignments which have effect when the event is executed. The Assignment structure is shown in Figure 13. The variable field is of type SId and contains the identifier of a variable i.e. a compartment, species or parameter. The structures referenced by the variable field must have their constant fields set to "false". The math field contains a MathML expression which defines the new value of the variable. This expression is evaluated when the Event is fired but the variable only acquires the result or new value when the Event is executed. The order of the EventAssignment structures is not significant (unlike scalar rules), the effect of one assignment cannot affect the result of another assignment. The identifiers occurring in the MathML ci fields of the EventAssignment structures represent the value of the identifier at the point when the Event is fired.

A example of an Event structure follows. This structure makes the assignment $k_2 = 0$ at the point when $P_1 \leq t$:

```
<event>
   <trigger>
        <math xmlns="http://www.w3.org/1998/Math/MathML">
            <apply>
                <leq/>
                <ci>P1 </ci>
                <ci> t </ci>
            </apply>
        </trigger>
    <listOfEventAssignments>
        <eventAssignment variable="k2">
            <math:cn> 0 </math:cn>
        </eventAssignment>
    <listOfEventAssignments>
</event>
```

A complete example model that uses events is given in Section 5.8

5 Example Models Expressed in XML Using SBML

In this section, we present several examples of complete models encoded in XML using SBML Level 2.

5.1 A Simple Example Application of SBML

Consider the following hypothetical branched system:

$$\begin{array}{ccc} X_0 & \underbrace{k_1 X_0} & S_1 \\ \\ S_1 & \underbrace{k_2 S_1} & X_1 \\ \\ S_1 & \underbrace{k_3 S_1} & X_2 \end{array}$$

The following is an XML document that encodes the model shown above:

```
<?xml version="1.0"?>
<sbml xmlns="http://www.sbml.org/sbml/level2" version="1" level="2">
   <model id="Branch">
       <notes>
           <body xmlns="http://www.w3.org/1999/xhtml">
               Simple branch system.
               The reaction looks like this:
               </body>
       </notes>
       <listOfCompartments>
           <compartment id="compartmentOne" volume="1"/>
       </listOfCompartments>
       <listOfSpecies>
           <species id="S1" initialAmount="0" compartment="compartmentOne"</pre>
                    boundaryCondition="false"/>
           <species id="X0" initialAmount="0" compartment="compartmentOne"</pre>
                    boundaryCondition="true"/>
           <species id="X1" initialAmount="0" compartment="compartmentOne"</pre>
                   boundaryCondition="true"/>
           <species id="X2" initialAmount="0" compartment="compartmentOne"</pre>
                   boundaryCondition="true"/>
       </listOfSpecies>
       <listOfReactions>
           <reaction id="reaction_1" reversible="false">
               <listOfReactants>
                   <speciesReference species="X0" stoichiometry="1"/>
               t0fProducts>
                   <speciesReference species="S1" stoichiometry="1"/>
               </listOfProducts>
               <kineticLaw>
                   <math xmlns="http://www.w3.org/1998/Math/MathML">
                       <apply>
                          <times/>
                          <ci> k1 </ci>
                          <ci> X0 </ci>
                       </apply>
                   <listOfParameters>
                       <parameter id="k1" value="0"/>
                   </listOfParameters>
               </kineticLaw>
           </reaction>
           <reaction id="reaction_2" reversible="false">
               <listOfReactants>
                   <speciesReference species="S1" stoichiometry="1"/>
               t0fProducts>
                   <speciesReference species="X1" stoichiometry="1"/>
               </listOfProducts>
               <kineticLaw>
                   <math xmlns="http://www.w3.org/1998/Math/MathML">
```

```
<apply>
                           <times/>
                           <ci> k2 </ci>
                           <ci> S1 </ci>
                       </apply>
                   <listOfParameters>
                       <parameter id="k2" value="0"/>
                   </listOfParameters>
               </kineticLaw>
           </reaction>
           <reaction id="reaction_3" reversible="false">
               <listOfReactants>
                   <speciesReference species="S1" stoichiometry="1"/>
               t0fProducts>
                   <speciesReference species="X2" stoichiometry="1"/>
               </listOfProducts>
               <kineticLaw>
                   <math xmlns="http://www.w3.org/1998/Math/MathML">
                       <apply>
                           <times/>
                           <ci> k3 </ci>
                           <ci> S1 </ci>
                       </apply>
                   <listOfParameters>
                       <parameter id="k3" value="0"/>
                   </listOfParameters>
               </kineticLaw>
           </reaction>
       </listOfReactions>
   </model>
</sbml>
```

In this example, the model has the identifier "Branch". The model contains one compartment, four species, and three reactions. The elements in the st0fReactants> and to the names of elements listed in the to the correspondences between the various elements is explicitly stated by the <speciesReference> elements.

The model also includes a <notes> annotation that summarizes the model in text form, with formatting encoded in XHTML. This may be useful for a software package that is able to read such annotations and, for example, render them in HTML in a graphical user interface.

5.2 Simple Use of Units Feature in a Model

The following model uses the units features of SBML Level 2. In this model, the default value of substance is changed in the list of unit definitions to be mole units with a scale factor of -3, or millimoles. This sets the default substance units in the model; components can override this scale locally. The volume and time built-ins are left to their defaults, ensuring that volume is in liters and time is in seconds. The result is that, in this model, kinetic law formulas define rates in millimoles per second and the species symbols in them represent concentration values in millimoles per liter. All the species elements set the initial amount of every given species to 1 millimole. The parameters Vm and Vm are defined to be in millimoles per liter per second, and millimolar, respectively.

```
<unitDefinition id="mls">
       stOfUnits>
           <unit kind="mole"</pre>
                               scale="-3"/>
           <unit kind="liter" exponent="-1"/>
           <unit kind="second" exponent="-1"/>
       </listOfUnits>
   </unitDefinition>
<listOfCompartments>
    <compartment id="cell"/>
</listOfCompartments>
<listOfSpecies>
   <species id="x0" compartment="cell" initialAmount="1"/>
    <species id="x1" compartment="cell" initialAmount="1"/>
   <species id="s1" compartment="cell" initialAmount="1"/>
    <species id="s2" compartment="cell" initialAmount="1"/>
</listOfSpecies>
<listOfParameters>
    <parameter id="vm" value="2" units="mls"/>
   <parameter id="km" value="2"/>
<listOfReactions>
   <reaction id="v1">
       <listOfReactants>
           <speciesReference species="x0"/>
       <listOfProducts>
           <speciesReference species="s1"/>
       </listOfProducts>
       <kineticLaw>
           <notes>
               <html:p>(vm * s1)/(km + s1)</html:p>
           <math xmlns="http://www.w3.org/1998/Math/MathML">
               <apply>
                   <divide/>
                   <apply>
                       <times/>
                       <ci> vm </ci>
                       <ci> s1 </ci>
                   </apply>
                   <apply>
                       <plus/>
                       <ci> km </ci>
                       <ci> s1 </ci>
                   </apply>
               </apply>
           </kineticLaw>
   </reaction>
   <reaction id="v2">
       <listOfReactants>
           <speciesReference species="s1"/>
       </listOfReactants>
       <listOfProducts>
           <speciesReference species="s2"/>
       </listOfProducts>
       <kineticLaw>
           <notes>
               <html:p>(vm * s2)/(km + s2)</html:p>
           <math xmlns="http://www.w3.org/1998/Math/MathML">
               <apply>
                   <divide/>
                   <apply>
                       <times/>
                       <ci> vm </ci>
                       <ci> s2 </ci>
                   </apply>
```

```
<apply>
                                <plus/>
                                <ci> km </ci>
                                <ci> s2 </ci>
                            </apply>
                        </apply>
                    </kineticLaw>
            </reaction>
            <reaction id="v3">
               <listOfReactants>
                    <speciesReference species="s2"/>
                </listOfReactants>
                tOfProducts>
                    <speciesReference species="x1"/>
                </listOfProducts>
                <kineticLaw>
                    <notes>
                        <html:p>(vm * x1)/(km + x1)</html:p>
                    <math xmlns="http://www.w3.org/1998/Math/MathML">
                        <apply>
                            <divide/>
                            <apply>
                                <times/>
                                <ci> vm </ci>
                                <ci> x1 </ci>
                            </apply>
                            <apply>
                                <plus/>
                                <ci> km </ci>
                                <ci> x1 </ci>
                            </apply>
                        </apply>
                    </kineticLaw>
            </reaction>
        </listOfReactions>
    </model>
</sbml>
```

5.3 Use of Assignment Rules Feature in a Model

This section contains a model which simulates a system containing a fast reaction. This model uses rules to express the mathematics of the fast reaction explicitly rather than using the implicit fast field on a reaction element.

The system modeled is

$$X_0 \qquad \underbrace{k_1 X_0}_{S_1} \qquad S_1$$

$$S_1 \qquad \underbrace{k_f S_1 - k_r S_2}_{S_2} \qquad S_2$$

$$S_2 \qquad \underbrace{k_2 S_1}_{X_1} \qquad X_1$$

$$k_1=0.1, \quad k_2=0.15, \quad k_f=K_{eq}10000, \quad k_r=10000, \quad K_{eq}=2.5$$

this can be approximated with the following system:

$$\begin{array}{ccc} X_0 & \underbrace{k_1 X_0} & T \\ & & & \\ T & \underbrace{k_2 S_1} & X_1 \end{array}$$

$$S_1 = \frac{T}{1 + K_{eq}}, \quad S_2 = K_{eq}S_1$$

The following example SBML model encodes the approximate form.

```
<?xml version="1.0"?>
<sbml xmlns="http://www.sbml.org/sbml/level2" version="1" level="2"</pre>
      xmlns:math="http://www.w3.org/1998/Math/MathML">
        <listOfCompartments>
            <compartment id="cell"/>
        </listOfCompartments>
        <listOfSpecies>
            <species id="X0" compartment="cell" initialAmount="1"/>
            <species id="X1" compartment="cell" initialAmount="0"/>
<species id="T" compartment="cell" initialAmount="0"/>
            <species id="S1" compartment="cell" initialAmount="0"/>
            <species id="S2" compartment="cell" initialAmount="0"/>
        </listOfSpecies>
        <listOfParameters>
            <parameter id="Keq" value="2.5"/>
        tOfRules>
            <assignmentRule variable="S1">
                <math xmlns="http://www.w3.org/1998/Math/MathML">
                    <apply>
                        <divide/>
                        <ci> T </ci>
                        <apply>
                            <plus/>
                            <cn> 1 </cn>
                            <ci> Keq </ci>
                        </apply>
                    </apply>
                </assignmentRule>
            <assignmentRule variable="S2">
                <math xmlns="http://www.w3.org/1998/Math/MathML">
                    <apply>
                        <times/>
                        <ci> Keq </ci>
                        <ci> S1 </ci>
                    </apply>
                </assignmentRule>
        </listOfRules>
        <listOfReactions>
            <reaction id="in">
                <listOfReactants>
                    <speciesReference species="X0"/>
                <listOfProducts>
                    <speciesReference species="T"/>
                </listOfProducts>
                <kineticLaw>
                    <math xmlns="http://www.w3.org/1998/Math/MathML">
                        <apply>
                            <times/>
                            <ci> k1 </ci>
                            <ci> X0 </ci>
                        </apply>
                    <listOfParameters>
                        <parameter id="k1" value="0.1"/>
                    </listOfParameters>
                </kineticLaw>
            </reaction>
            <reaction id="out">
                <listOfReactants>
```

```
<speciesReference species="T"/>
               t0fProducts>
                   <speciesReference species="X1"/>
               </listOfProducts>
               <kineticLaw>
                   <math xmlns="http://www.w3.org/1998/Math/MathML">
                       <apply>
                          <times/>
                          <ci> k2 </ci>
                          <ci> S2 </ci>
                       </apply>
                   <listOfParameters>
                       <parameter id="k2" value="0.15"/>
                   </listOfParameters>
               </kineticLaw>
           </reaction>
       </listOfReactions>
   </model>
</sbml>
```

5.4 Use of Algebraic Rules Feature in a Model

This section contains an example model which contains an AlgebraicRule structure. The model contains a different formulation of the fast reaction described in Section 5.3.

The system described in Section 5.3 can be approximated with the following system:

$$X_0 \xrightarrow{k_1 X_0} T$$

$$T \xrightarrow{k_2 S_1} X_1$$

$$S_2 = K_{eq} S_1$$

 $S_1 + S_2 - T = 0$

with the constraint:

The following example SBML model encodes this approximate form.

```
<?xml version="1.0"?>
<sbml xmlns="http://www.sbml.org/sbml/level2" version="1" level="2">
    <model>
        <listOfCompartments>
             <compartment id="cell"/>
        </listOfCompartments>
        <listOfSpecies>
             <species id="X0" compartment="cell" initialAmount="1"/>
<species id="X1" compartment="cell" initialAmount="0"/>
             <species id="T" compartment="cell" initialAmount="0"/>
             <species id="S1" compartment="cell" initialAmount="0"/>
             <species id="S2" compartment="cell" initialAmount="0"/>
        </listOfSpecies>
        <listOfParameters>
             <parameter id="Keq" value="2.5"/>
        </listOfParameters>
        tOfRules>
             <assignmentRule variable="S2">
                 <math xmlns="http://www.w3.org/1998/Math/MathML">
                      <apply>
                          <times/>
                          <ci> Keq </ci>
                          <ci> S1 </ci>
                      </apply>
```

```
</assignmentRule>
            <algebraicRule>
                <math xmlns="http://www.w3.org/1998/Math/MathML">
                    <apply>
                        <minus/>
                       <apply>
                           <plus/>
                           <cin> S2 </cin>
                           <cin> S1 </cin>
                        </apply>
                        <cin> T </cin>
                   </apply>
               </algebraicRule>
        </listOfRules>
        <listOfReactions>
            <reaction id="in">
               <listOfReactants>
                    <speciesReference species="X0"/>
               </listOfReactants>
               t0fProducts>
                    <speciesReference species="T"/>
               </listOfProducts>
               <kineticLaw>
                   <math xmlns="http://www.w3.org/1998/Math/MathML">
                        <apply>
                           <times/>
                           <ci> k1 </ci>
                           <ci> X0 </ci>
                       </apply>
                    <listOfParameters>
                       <parameter id="k1" value="0.1"/>
                    </listOfParameters>
               </kineticLaw>
            </reaction>
            <reaction id="out">
               <listOfReactants>
                    <speciesReference species="T"/>
               t0fProducts>
                    <speciesReference species="X1"/>
               </listOfProducts>
               <kineticLaw>
                    <math xmlns="http://www.w3.org/1998/Math/MathML">
                       <apply>
                            <times/>
                           <ci> k2 </ci>
                           <ci> S2 </ci>
                       </apply>
                   <listOfParameters>
                        <parameter id="k2" value="0.15"/>
                    </listOfParameters>
               </kineticLaw>
            </reaction>
        </listOfReactions>
    </model>
</sbml>
```

5.5 Use of boundaryCondition and constant attributes on species elements in a Model

This section contains a model which includes four species each with a different combination of values of for the boundaryCondition and constant attributes.

Consider the following hypothetical system:

$$S_1 + S_2 \quad \underline{k_1 S_1 S_2 S_3} \quad S_4$$

where values over time are determined by:

$$\frac{dS_1}{dt} = k_2$$

$$S_2 = 1$$

$$S_3 = 2$$

$$k_1 = 0.5$$

$$k_2 = 0.1$$

and initial values are:

$$S_1 = 0$$

$$S_4 = 0$$

 S_1 and S_2 are reactants but their values are not determined by a kinetic law thus they are both on the boundary of the reaction system and in SBML have boundaryCondition attribute values of true. The value of S_1 varies over time so in SBML S_1 has a constant attribute with a default value of false. The values of S_2 and S_3 are fixed so in SBML they have a constant attribute values of true. S_3 only occurs as a modifier so the value of its boundaryCondition attribute can default to false. S_4 is a product whose value is determined by a kinetic law and therefore in the SBML representation has false values, the default values, for both boundaryCondition and constant attributes.

The following is the XML document that encodes the model shown above:

```
<?xml version="1.0"?>
<sbml xmlns="http://www.sbml.org/sbml/level2" version="1" level="2">
    <model id="x">
        <listOfCompartments>
            <compartment id="compartmentOne" volume="1"/>
        </listOfCompartments>
        <listOfSpecies>
            <species id="S1" initialAmount="0" compartment="compartmentOne"</pre>
                boundaryCondition="true" />
            <species id="S2" initialAmount="1" compartment="compartmentOne"</pre>
                boundaryCondition="true" constant="true" />
            <species id="S3" initialAmount="3" compartment="compartmentOne"</pre>
                constant="true"/>
            <species id="S4" initialAmount="0" compartment="compartmentOne"/>
        </listOfSpecies>
        <listOfParameters>
            <parameter id="k1" value="0.5"/>
            <parameter id="k2" value="0.1"/>
        </listOfParameters>
        tOfRules>
            <assignmentRule variable="S1" type="rate">
                <math xmlns="http://www.w3.org/1998/Math/MathML">
                    <apply>
                        <ci> k2 </ci>
                    </apply>
                </assignmentRule>
        </listOfRules>
        <listOfReactions>
            <reaction id="reaction_1" reversible="false">
                <listOfReactants>
```

```
<speciesReference species="S1" stoichiometry="1"/>
                    <speciesReference species="S2" stoichiometry="1"/>
                </listOfReactants>
                t0fProducts>
                    <speciesReference species="S4" stoichiometry="1"/>
                </listOfProducts>
                <listOfModifiers>
                    <modifierSpeciesReference species="S3"/>
                </listofModifiers>
                <kineticLaw>
                    <math xmlns="http://www.w3.org/1998/Math/MathML">
                        <apply>
                            <times/>
                            <ci> k1 </ci>
                            <ci> S1 </ci>
                            <ci> S2 </ci>
                            <ci> S3 </ci>
                         </apply>
                    </kineticLaw>
            </reaction>
        </listOfReactions>
    </model>
</sbml>
```

5.6 Use of Function Definition Feature in a Model

This section contains a model which uses the function definition feature of SBML. Consider the following hypothetical system:

$$S_1 \xrightarrow{f(S_1)} S_2$$

where

$$f(x) = x * 2$$

The following is the XML document that encodes the model shown above:

```
<?xml version="1.0"?>
<sbml xmlns="http://www.sbml.org/sbml/level2" version="1" level="2">
    <model id="Branch">
        <listOfFunctionDefinitons>
            <functionDefinition id="f">
                <math xmlns="http://www.w3.org/1998/Math/MathML">
                        <br/>
<br/>
dvar><ci> x </ci></bvar>
                        <apply>
                            <times/>
                            <ci> x </ci>
                            <cn> 2 </cn>
                        </apply>
                    </lamdba>
                </functionDefinition>
        </listOfFunctionDefinitions>
        <listOfCompartments>
            <compartment id="compartmentOne" volume="1"/>
        </listOfCompartments>
        <listOfSpecies>
            <species id="S1" initialAmount="0" compartment="compartmentOne"/>
            <species id="S2" initialAmount="0" compartment="compartmentOne"/>
        </listOfSpecies>
        <listOfReactions>
            <reaction id="reaction_1" reversible="false">
                <listOfReactants>
                    <speciesReference species="S1" stoichiometry="1"/>
                </listOfReactants>
```

5.7 Use of the *delay* Function in a Model

The following is a simple model illustrating the use of *delay* to represent a gene that suppresses its own expression. The model can be expressed in a single rule:

$$\frac{dP}{dt} = \frac{\frac{1}{1 + m(P_{delayed})^q} - P}{\tau}$$

where

 $P_{delayed}$ is $delay(P, \Delta_t)$ or P at $t - \Delta_t$ P is protein concentration τ is the response time m is a multiplier or equilibrium constant q is the Hill coefficient

The SBML form of this model is as follows:

```
<?xml version="1.0"?>
<sbml xmlns="http://www.sbml.org/sbml/level2" version="1" level="2">
    <model>
        <listOfCompartments>
            <compartment id="cell"/>
        </listOfCompartments>
        <listOfSpecies>
            <species id="P" compartment="cell" initialAmount="0"/>
        </listOfSpecies>
        <listOfParameters>
            <parameter id="tau" value="1"/>
            <parameter id="m" value="0.5"/>
            <parameter id="q" value="1"/>
            <parameter id="delta_t" value="1"/>
        </listOfParameters>
        tOfRules>
            <assignmentRule variable="P" type="rate">
                <math xmlns="http://www.w3.org/1998/Math/MathML">
                 <apply>
                  <divide/>
                  <apply>
                   <minus/>
                   <apply>
                    <divide/>
                    <cn> 1 </cn>
                    <apply>
                     <plus/>
                     <cn> 1 </cn>
                     <apply>
                      <times/>
                      <ci> m </ci>
```

```
<apply>
                        <power/>
                        <apply>
                         <csymbol encoding="SBML"</pre>
                                  definitionURL="http://www.sbml.org/symbols/delay">
                             delay
                         </csymbol>
                         <ci>P </ci>
                         <ci> delta_t </ci>
                        </apply>
                        <ci> q </ci>
                       </apply>
                      </apply>
                     </apply>
                    </apply>
                    <ci> P </ci>
                   </apply>
                   <ci> tau </ci>
                  </apply>
                 </assignmentRule>
        </listOfRules>
    </model>
</sbml>
```

5.8 Use of Events Feature in a Model

This section contains a simple model system that demonstrates the use of an events. Consider a system with two genes: k_1 and k_2 . k_1 is initially on and k_2 is initially off. The genes when on produce products, P_1 and P_2 respectively, at a fixed rate when switched on. When P_1 reaches a given concentration k_2 switches. This can be represented mathematically as follows:

$$\frac{dP_1}{dt} = k_1 - P_1$$
$$\frac{dP_2}{dt} = k_2 - P_2$$

```
when P_1 > \tau then k_2 = 1
when P_1 \le \tau then k_2 = 0
```

initially

$$k_1 = 1$$
 $k_2 = 0$ $\tau = 0.25$ $P_1 = 0$ $P_2 = 0$

The SBML Level 2 representation of this as follows:

```
<?xml version="1.0"?>
<sbml xmlns="http://www.sbml.org/sbml/level2" version="1" level="2"</pre>
     xmlns:math="http://www.w3.org/1998/Math/MathML">
   <model>
       <listOfCompartments>
           <compartment id="cell"/>
       </listOfCompartments>
       <listOfSpecies>
           <species id="P1" compartment="cell" initialAmount="0"/>
           <species id="P2" compartment="cell" initialAmount="0"/>
       </listOfSpecies>
       <listOfParameters>
           <parameter id="k1" value="1" constant="false"/>
           <parameter id="tau" value="0.25"/>
       </listOfParameters>
       tOfRules>
```

```
<assignmentRule variable="P1" type="rate">
                <math:math>
                    <math:apply>
                        <math:minus/>
                        <math:ci> k1 </math:ci>
                        <math:ci> P1 </math:ci>
                    </math:apply>
                </math:math>
            </assignmentRule>
            <assignmentRule variable="P2" type="rate">
                <math:math>
                    <math:apply>
                        <math:minus/>
                        <math:ci> k2 </math:ci>
                        <math:ci> P2 </math:ci>
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        <listOfEvents>
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                <trigger>
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                            <math:gt/>
                            <math:ci> P1 </math:ci>
                            <math:ci> tau </math:ci>
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                            <math:leq/>
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                            <math:ci> tau </math:ci>
                        </math:apply>
                    </math:math>
                </trigger>
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        </listOfEvents>
    </model>
</sbml>
```

6 Discussion

The volume of data now emerging from molecular biotechnology leave little doubt that extensive computer-based modeling, simulation and analysis will be critical to understanding and interpreting the data (Abbott, 1999; Gilman, 2000; Popel and Winslow, 1998; Smaglik, 2000). This has lead to an explosion in the development of computer tools by many research groups across the world. The explosive rate of progress is exciting, but the rapid growth of the field is accompanied by problems and pressing needs.

One problem is that simulation models and results often cannot be directly compared, shared or re-used, because the tools developed by different groups often are not compatible with each other. As the field

of systems biology matures, researchers increasingly need to communicate their results as computational models rather than box-and-arrow diagrams. They also need to reuse published and curated models as library components in order to succeed with large-scale efforts (e.g., the Alliance for Cellular Signaling; Gilman, 2000; Smaglik, 2000). These needs require that models implemented in one software package be portable to other software packages, to maximize public understanding and to allow building up libraries of curated computational models.

We offer SBML to the systems biology community as a suggested format for exchanging models between simulation/analysis tools. SBML is an open model representation language oriented specifically towards representing biochemical network models.

Our vision for SBML is to create an open standard that will enable simulation software to exchange models. SBML is not static; we continue to develop and experiment with it, and we interact with other groups who seek to develop similar markup languages. We plan on continuing to evolve SBML with the help of the systems biology community to make SBML increasingly more powerful, flexible and useful.

6.1 Future Enhancements: SBML Level 3 and Beyond

As mentioned above, SBML Level 2 is intended to provide the foundations for modeling biochemical networks. A number of significant capabilities are lacking from Level 2 as described in the main part of this document. Features to implement these capabilities will be introduced in future levels of SBML. The following summarizes additional features that are under consideration to be included in SBML Level 3:

- Arrays. This will enable the creation of arrays of components (species, reactions, compartments and submodels).
- Connections. This will be a mechanism for describing the connections between items in an array. For example, it should be possible to create a 2-D array of compartments and then a 3-D array of reactions which transport species between the compartments, where the third dimension is the connections between the compartments. Two possible ways of describing a connection scheme are: (1) sparse/explicit, simply listing the relative co-coordinates of connected objects for patterns of points; (2) algebraic, where a conditional equation describes whether two objects are connected.
- Geometry. We will develop a scheme for representing the 3-D structure of compartments.
- *Model Composition*. This will enable a large model to be built up out of instances of other models. It will also allow the reuse of model components and the creation of several instances of the same model.
- Multi-state and Complex Species. This will allow the straight-forward construction of models involving species with a large number of states or species composed of subcomponents. The representation scheme would be designed to contain the combinatorial explosion of objects that often results from these types of models.
- Component Identification. This will enable components to be described using some stable universal identification scheme.
- Diagrams. This feature will allow components to be annotated with data to enable the display of the model in a diagram.
- Conditional rules. This will enable rules and reactions to have their effect conditional on the state of the model system. For example in SBML Level 2 it is possible to create a rule with the effect:

$$\frac{ds}{dt} = \begin{cases} 0 & \text{if } s > 0\\ y & \text{otherwise} \end{cases}$$

Conditional rules would enable the expression of the following example:

if
$$s > 0$$
 $\frac{ds}{dt} = y$

where s is not determined by the rule when $s \leq 0$.

6.2 Relationships to Other Efforts

There are a number of ongoing efforts with similar goals as those of SBML. Many of them are oriented more specifically toward describing protein sequences, genes and related entities for database storage and search. These are generally not intended to be computational models, in the sense that they do not describe entities and behavioral rules in such a way that a simulation package could "run" the models.

The effort perhaps closest in spirit to SBML is CellMLTM (Hedley et al., 2001). CellML is an XML-based markup language designed for storing and exchanging computer-based biological models. It includes facilities for representing model structure, mathematics and additional information for database storage and search. Models are described in terms of networks of connections between discrete components, where a component is a functional unit that may correspond to a physical compartment or simply a convenient modeling abstraction. Components contain variables and connections contain mappings between the variables of connected components. CellML provides facilities for grouping components and specifying the kinds of relationships that may exist between components. It also uses MathML (W3C, 2000b) for expressing mathematical relationships between components and provides the ability to use ECMAScript (formerly known as JavaScript) to define functions.

The constructs in CellML tend to be at a more abstract and general level than those in SBML Level 2, and describes the structure and underlying mathematics of cellular models in a very general way. By contrast, SBML is closer to the internal object model used in model analysis software. Because SBML Level 2 is being developed in the context of interacting with a number of existing simulation packages, it is a more concrete language than CellML and may be better suited to its purpose of enabling interoperability with existing simulation tools.

The development of SBML Level 2 has benefited from discussions with the developers of CellML. The developers of SBML and CellML are actively engaged in ensuring that the two representations can be translated between each other.

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SBML was first conceived at the JST/ERATO-sponsored First Workshop on Software Platforms for Systems Biology, held in April, 2000, at the California Institute of Technology in Pasadena, California, USA. The participants collectively decided to begin developing a common XML-based declarative language for representing models. A draft version of the Systems Biology Markup Language was developed by the Caltech ERATO team and delivered to all collaborators in August, 2000. This draft version underwent extensive discussion over mailing lists and then again during the Second Workshop on Software Platforms for Systems Biology held in Tokyo, Japan, November 2000. A revised version of SBML was issued by the Caltech ERATO team in December, 2000, and after further discussions over mailing lists and in meetings, we produced a description of SBML Level 1 (Hucka et al., 2001).

SBML Level 2 was conceived at the 5th Workshop on Software Platforms for Systems Biology, held in July 2002, at the University of Hertfordshire, UK. The participants collectively decided to revise the form of SBML in Level 2. The first draft of this document was released in August 2002. The final set of features in SBML Level 2 described in this document was finalized in December 2002 at the 6th Workshop on Software Platforms for Systems Biology at ICSB 2002 in Stockholm.

SBML Level 2 was developed with the help of many people, especially the authors of BioSketchPad, BioSpice, DBSolve, Cellerator, COPASI, E-Cell, Gepasi, Jarnac, JDesigner, JigCell, MCell, NetBuilder, Promot/DIVA, StochSim, and Virtual Cell, and members of the sysbio and sbml-discuss mailing lists. We are particularly grateful to the following people for discussions, advice and comments: Adam Arkin, Hamid Bolouri, Benjamin Bornstein, Dennis Bray, Roger Brent, Claudine Chaouiya, Kwang Cho, Athel Cornish-Bowden, Autumn Cuellar, Serge Dronov, Drew Endy, David Fell, Carl Firth, Ed Frank, Akira Funahashi, Warren Hedley,

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Appendix

A Summary of Notation

The definitive explanation for the notation used in this document can be found in the companion notation document (Hucka, 2000). Here we briefly summarize some of the main components of the notations used in describing SBML.

Within the definitions of the various object classes introduced in this document, the following types of expressions are used many times:

```
field1 : float
field2 : integer[0..*]
field3 : (XHTML)
field4 : float {use = "default" value = "0.0"}
```

The symbols field1, field2, etc., represents fields in a data structure. The colon immediately after the name separates the name of the attribute from the type of data that it stores.

More complex specifications use square brackets ([]) just after a type name. This is used to indicate that the field contains a list of elements. Specifically, the notation [0..*] signifies a list containing zero or more elements; the notation [1..*] signifies a list containing at least one element; and so on. The approach used here to translate from a list form into XML is, first, create a subelement named list0f____s, where the blank indicates the capitalized name of the field, and then put a list of elements named after the field as the content of the list0f____s element.

Expressions in curly braces ($\{\}$) shown after an attribute type indicate additional constraints placed on the field. We express constraints using XML Schema language. In the examples above, the expression $\{use="default" value="0.0"\}$ indicates that the field field4 is optional and that it has a default value of 0.0. A constraint of the form $\{namespace="X"\}$ indicates that the field is not in the SBML Level 2 XML namespace but resides in the given XML namespace X. If a field is in a different namespace then the type of the field will not be defined by the SBML UML.

A field definition of the form X:(A:B) defines an element X that contains a field A with type B. If A is the string ANY then the element X contains an arbitrary sequence of elements. A field definition of the form X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is constrained by constraint X:(A:B) is similar except that the field X and its content is X:(A:B) is similar except that the field X and its content is X:(A:B) is similar except that X:(A:B) is si

B XML Schema for SBML

The following is an XML Schema definition for the Systems Biology Markup Language, using the W3C Recommendation for XML Schema version 1.0 of 2 May 2001 (Biron and Malhotra, 2000; Fallside, 2000; Thompson et al., 2000).

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