MAE 243 Written #1

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1 Problem 1

1. State whether the problem is convex or not. Justify your answer
2. State whether you should use a linear programming solver, a mixed integer linear programming solver, or a nonlinear solver
3. Assume that $c_1, c_2, d_1, d_2, d_3 > 0$

A

$$\begin{aligned}
&\min_x c^T x \\
&s.t. \quad Ax > b \\
&x_i \in [0,1], \forall i = 1, ..., n
\end{aligned}$$

This problem is convex because both the objective function and the feasible region are themselves convex. Since the values of the decision variable x are $[0,1]$, a mixed integer LP solver shoud be used.

B

$$\begin{aligned}
&\min_{x,y} c_1 x + c_2 y \\
&s.t. \quad d_1 x^2 + d_2 y^2 \leq d_3 \\
&x, y \geq 0
\end{aligned}$$

This problem is convex (as both the objective and constraints are convex functions), but since the constraints include a squared term, it's a quadratic programming problem, which you'd need a nonlinear solver for.

C

$$\begin{aligned}
&\min_{x,y} c_1 x + c_2 xy \\
&s.t. \quad d_1 x + d_2 y \leq d_3 \\
&x, y \geq 0
\end{aligned}$$

Due to the c_2xy term, the objective function is nonlinear and nonconvex as the Hessian of $c_1x + c_2xy$ is $\left(\begin{smallmatrix} 0 & c_2 \\ c_2 & 0 \end{smallmatrix}\right)$, which is **not** positive semi-definite. You would therefore have to use a nonlinear solver.

2 Problem 2

Consider the optimization problem:

$$\begin{aligned}
\max_{x,y} \quad & 10x + 5y \\
\text{s.t.} \quad & 2x + 5y \leq 50, \\
& 5x + y \leq 30 \\
& 8 \geq x \geq 2, \\
& y \geq 0
\end{aligned}$$

A

The model gives the output as following:

```
Objective: 84.78260869565219
Value of x: 4.347826086956522
Value of y: 8.26086956521739
```

B

We can convert this to standard form with slack variables s_i:

$$\begin{aligned} \min_{x,y} \; & -(10x + 5y) \\ \text{s.t.} \; & 2x + 5y + s_1 = 50 \\ & 5x + y + s_2 = 30 \\ & x + s_3 = 8 \\ & x - s_4 = 2 \\ & x, y, s_1, s_2, s_3, s_4 \geq 0 \end{aligned}$$

This can then be written in matrix notation:

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

Where:

$$\mathbf{A} = \begin{bmatrix} 2 & 5 & 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

and $b = [50, 30, 8, 2]^T$, $x = [x, y, s_1, s_2, s_3, s_4]^T$.

C

D

If we use the duals from JuMP, we find the first two constraints to be binding.

```
2-element Array{ConstraintRef,1}:
 c1 : 2 x + 5 y  50.0
 c2 : 5 x + y  30.0
```

Which is sensible