# Estimating the number of COVID-19 victims by using a Monte Carlo algorithm on a generalized logistic equation

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- Idea: if it works, repeat now to predict June/July...
- Chosen Countries: Germany, France, Italy.

# The model in theory: ODE description

$X(t)$ : number of deaths in time. Modeled via a simple 1-d ODE with initial conditions $X_0$. Generalized logistic map (S-shaped, exp initially, then flat):

$$X'(t) = \frac{q}{v} X(t) \left(1 - \left(\frac{X(t)}{Q}\right)^{v}\right) \tag{1}$$

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- It has a close form solution (here not written);
- Depends on the parameter $P = \{q, Q, v\}$ (more next slide).
- In others words: assume $\exists P \in \mathbb{R}^3$, s.t. $X^P(t)$ correctly describe the (past and future) number of victims. Want to find $P$.

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- Keep in mind: $Q$ large, $> 10.000$, $v$, $q$ small, $\in [0, 1]$.

# The model in practice: observed vector

The available dataset are "numbers of total deaths until day $t_i$": discrete, limited, ODE trajectory of $X^P_{X_0}(t)$. Use them to infer $P$.

Fix $T+1$ times $\{t_i\}_{i=0,\dots,T}$, e.g. 21 days. The observed vector $\mathbf{y} \in \mathbb{R}^{T+1}$ is the random variable defined componentwise as:

$$y_i(\omega) = X^P_{X_0}(t_i) + \eta_i(\omega) \qquad (2)$$

where $\eta_i \sim \mathcal{N}(0, \sigma_i^2)$, $i \in \{0, \dots, T\}$.

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- In other words, $\mathbf{y}$ is what we measure: "noised truth".

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To assume an error of the form

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is a strong assumption (I know). Need to decide the standard variations $\sigma_i$ for each day. Abritrately.

Quantile formula: $\eta_i \sim N(0, \sigma_i^2) \implies \mathbb{P}[-2\sigma_i \leq \eta_i \leq 2\sigma_i] \geq 95\%$

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- Case 2: $\sigma_i$ s.t. we trust $y_i \pm y_i$, i.e. "100% of error";
- Warning: the inference algorithm [not yet described] is destroyed for higher errors, but not with our cases - checked with toy model data.

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- In brief: we infer $P$ by using a Monte Carlo algorithm.

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- **Infer $P$ = sample from the posterior**

# The pCN Monte Carlo method

Let $\mathbb{P}[P|\mathbf{y}]$ be the posterior on $\mathbb{R}^3$. Let $0 < \beta < 1$ the speed parameter, and the acceptance probability, for $v, u \in \mathbb{R}^3$:

$$a(u, v) \doteq \min\{1, \frac{\mathbb{P}(\mathbf{y}|v)}{\mathbb{P}(\mathbf{y}|u)}\} \tag{5}$$

To produce a **single sample**, construct a chain $\{x_i\}_{i\in\mathbb{N}}$ as follows:

1. set $x_0 \in \mathbb{R}^n$ **arbitrarily**. Then, for each $k > 0$:
2. sample a point $R \in \mathbb{R}^3$ from the **gaussian** prior $\mathbb{P}(P \in dx)$;
3. propose a candidate as $\hat{x}_k = \sqrt{(1-\beta^2)}x_{k-1} + \beta R$;
4. accept it (i.e. set $x_k = \hat{x}_k$) with probability $a(x_{k-1}, \hat{x}_k)$;
5. (accepted or not) repeat from 2;

Run 120.000 multiple chains in parallel, different starting point, each stopped after 250.000 iterations. Beta around 0.05, s.t. acceptance rate $\sim 25\%$. Last missing info: which prior?

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- Therefore: we only need to choose the $\sigma$, and then we can move on the simulations!

# Prior measure on $P$: part 2

Goal of this slide: explain how the values for $\sigma_q$, $\sigma_Q$ and $\sigma_v$ have been chosen.

- The idea is: the prior represents the "guess", the "default" choice. But if "nothing is done", the phenomenon propagates exponentially. Therefore we run an exponential interpolation, and register the number of victims in 1 Month: $K$. Then choose $\sigma_Q$ in order to have the Gaussian 95% quantile just below this quantity;

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- Dealing with $\sigma_q$ and $\sigma_v$, they have been chosen in order to have that quantile between 0 and 1.
- These idea formalize the intuition that we expect parameters in these range, without *forcing* any constraint.

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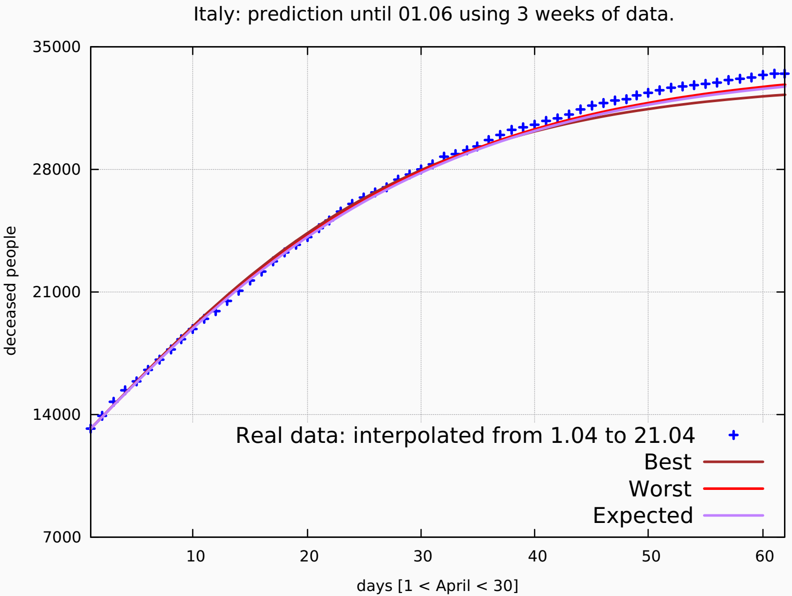
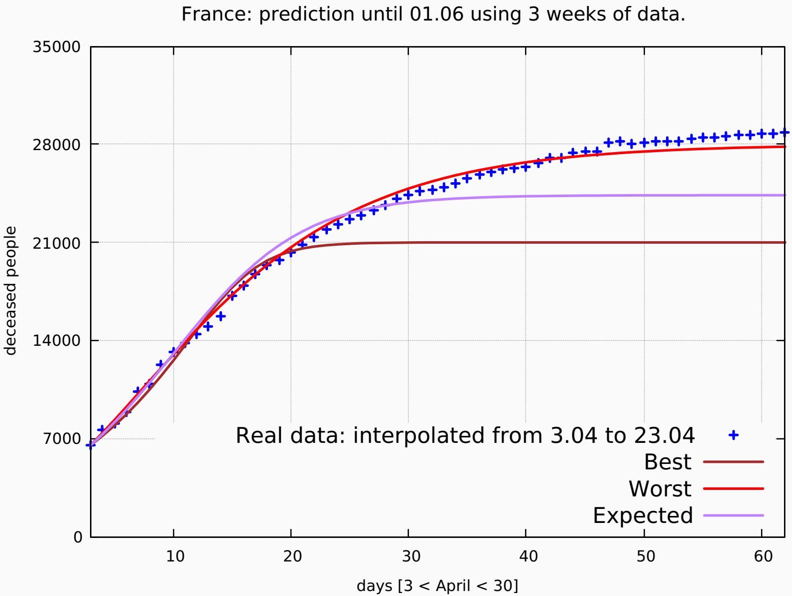
- the worst case, i.e.the triple with the highest value for $Q$; the best (converse) and the expectation.
- We have plots for the "10%" error scenario, as well as for the "100%" (*meaning explained before*)

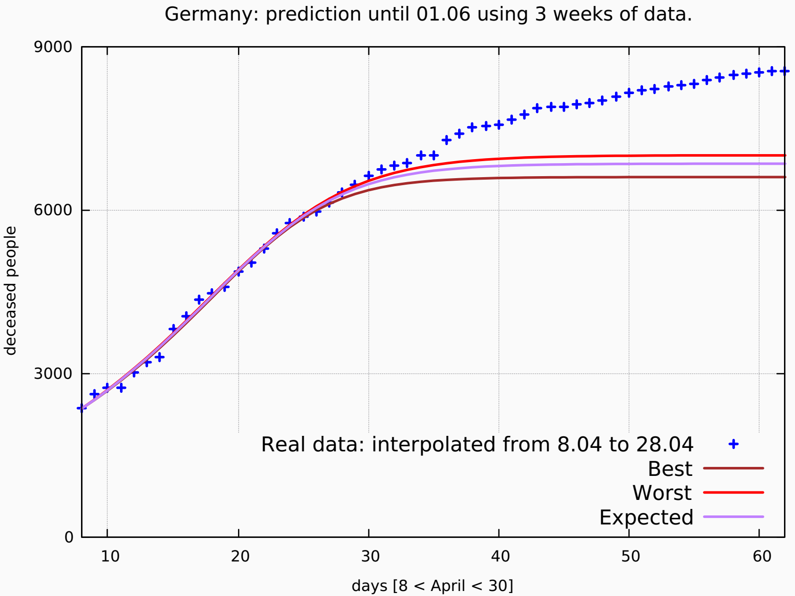
# Results: Italy, assuming $10\%$ of "error"



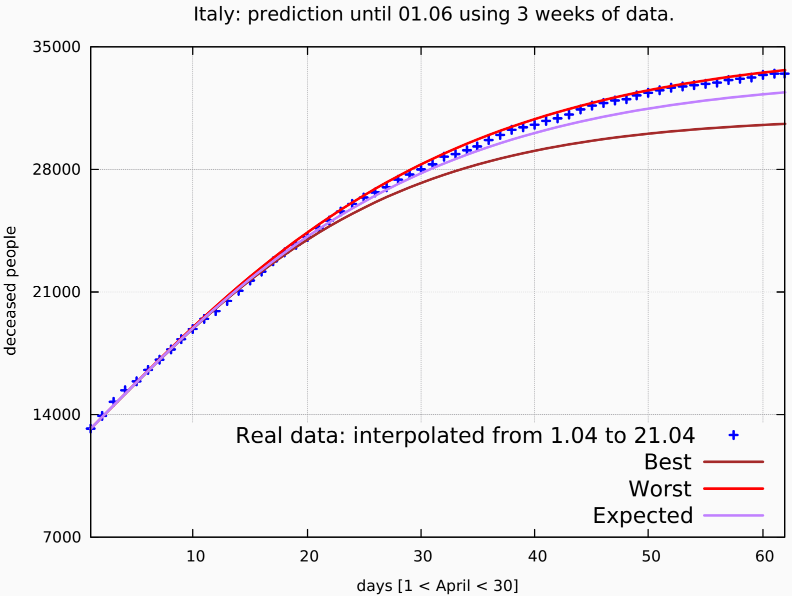
# Results: France, assuming $10\%$ of "error"



# Results: Germany, assuming 10% of "error"



# Results: Italy, assuming $100\%$ of "error"



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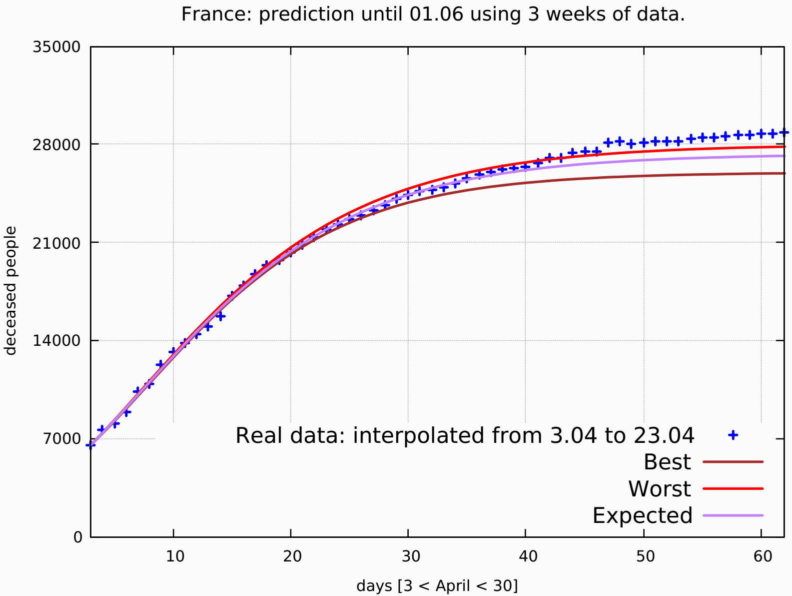

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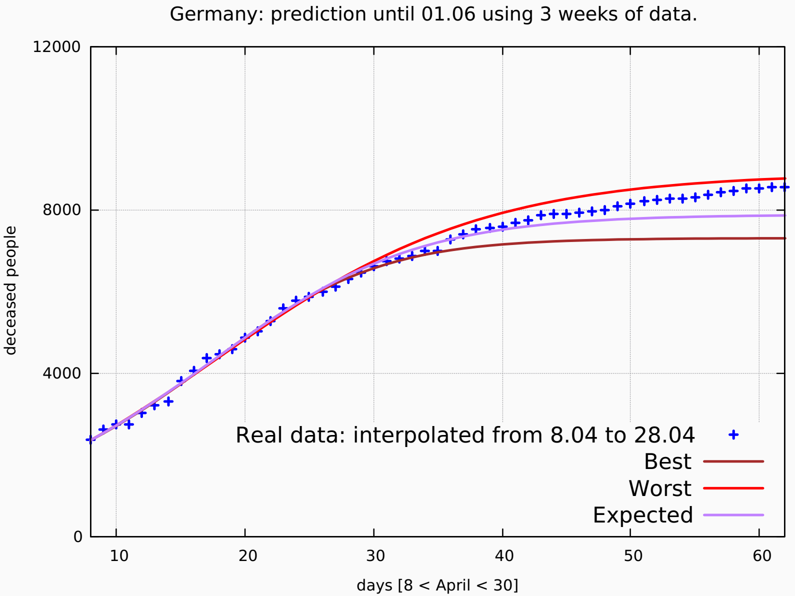

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- Or try with another more appropriate Monte Carlo methods with better prior measures;
- Or instead of the Bayesian approach, try maximizing the Likelihood (i.e. probabilistic least square);
- Or simply to consider this example as a nice application of Monte Carlo Bayesian techniques, before moving back to the main PhD topic.

Thanks for the attention!