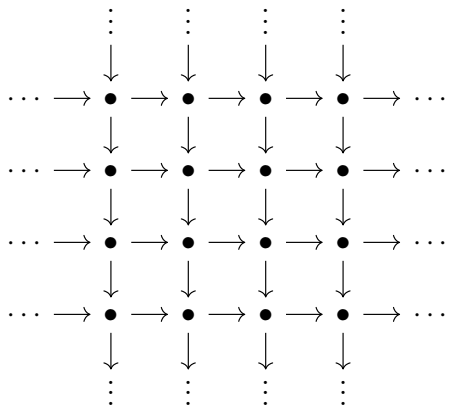
Part IV

Diagram Chasing

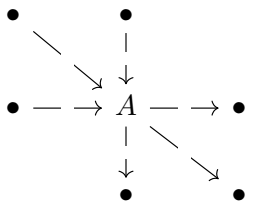
Diagram chases usually lead to simple connection between distant objects in a complex. In this chapter we discuss some of the diagram chase lemmas. The approach here is that of [1], which is more visual.

1 | Salamander Lemma

The starting point is a double complex in an abelian category $\mathcal{A}$,



where each $\bullet$ is an object in $\mathcal{A}$. Focusing on what happens around an object A in this double complex, consider the following morphisms to and from the object,



1.1 | Corollaries of Salamander Lemma

Lemma 1.1. (Five Lemma) *Consider two exact sequences $A^\bullet$ and $B^\bullet$,*

$$\begin{array}{ccccccccccc}
\cdots & \longrightarrow & A^i & \longrightarrow & A^{i+1} & \longrightarrow & A^{i+2} & \longrightarrow & A^{i+3} & \longrightarrow & \cdots \\
 & & \downarrow{\scriptstyle u_i} & & \downarrow{\scriptstyle u_{i+1}} & & \downarrow{\scriptstyle u_{i+2}} & & \downarrow{\scriptstyle u_{i+3}} & & \\
\cdots & \longrightarrow & B^i & \longrightarrow & B^{i+1} & \longrightarrow & B^{i+2} & \longrightarrow & B^{i+3} & \longrightarrow & \cdots
\end{array}$$

References

[1] G M Bergman, On Diagram-Chasing in Double Complexes http://arxiv.org/abs/1108.0958v2, 2011