Part IIIa

Intrinsic Realism

1 | Intrinsic Realism

A commonsense belief is that at any given moment any physical quantity must have a value even if we do not know what it is. The intrinsic realist interpretations are those in which probabilities of measurement outcomes are assumed to be determined by the intrinsic properties of the observed system. The notion of reality is closely linked with the values of observables in these interpretations.

- Intrinsic Realism
  - ψ-ontic
    - Many worlds, etc.
  - ψ-epistemic
    - Ballentine [3]
    - Spekkens [4]
    - Sabine Hossenfelder?

The intrinsic realist interpretation is ψ-ontic if they view the quantum state as an intrinsic property of the system. The interpretation is ψ-epistemic if they view the quantum state as having information about the intrinsic properties of the system. Here we will discuss stuff related to ψ-epistemic interpretations. ψ-ontic interpretations such as many worlds interpretation will not be discussed.

This intrinsic realist view comes from our belief in classical physics. This is not problematic in classical physics since the underlying structure for observables and states fits this view perfectly. However such a view comes into lots of problems with the quantum formalism.

1.1 | Hidden Variable Model

In statistical mechanics the states of the system are probability distribution over microstates. The microstates correspond to the dirac delta distribution over the phase space. These are similar to how states are in classical mechanics. The microstates represent the reality, and are the ontic states of the theory. These microstates are variables whose values we don't know accurately for whatever reason. This can be states as saying, 'In statisical mechanics we don't exactly know the microstate' i.e., we don't have complete knowledge about the variables. The macrostates represent this known information about the microstates. This is the idea behind hidden variables, that there exist some ontic 'real' state and the quantum states represent only the knowledge we know about them. For example, a coin flip has a hidden variable determining the probability of outcomes. If we know all the parameters such as force, torque, etc from the finger exerted on the coin and complete information about the system we can in theory know what the result of the experiment will be exactly. These parameters together represent the hidden variable for the coin toss experiment.

The natural question to ask now is if it is possible to find a hidden variable for the quantum case, i.e., does there exist a hidden variable knowing values of which will determine the values of the measurement. As a spoiler we would like to inform the reader that the problem comes from the non-commutativity of quantum theory.

Kochen-Specker Hidden Variable Model

One of the very well understood systems are spin 1/2 systems that we can perform Stern-Gerlach experiments on. In these experiments we can measure the 'spin' of a particle in some direction. To every direction in space we can measure if the particle is 'up' in that direction. The idea is that to every pure state of this particle there corresponds a unique direction in space such that the spin is 'up' in that direction. Therefore its space of pure states is isomorphic to the set of all directions in space. This space of pure states is called Bloch shere, antipodal points on the Bloch sphere correspond to states that have spin up in opposite directions. Bloch ball, whose boundary is the Bloch sphere corresponds to the space of density matrices and can be parametrized as,

$$\rho = \begin{bmatrix} \frac{1}{2}+z & x-iy \\ x+iy & \frac{1}{2}-z \end{bmatrix}$$

This can be treated as an expansion in terms of Pauli matrices, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, as,

$$\rho = \frac{1}{2}\mathbb{I} + \vec{\tau}\cdot\vec{\sigma}$$

$\vec{\tau}$ is called the Bloch vector such that,

$$x^2 + y^2 + z^2 \leq 1/4.$$

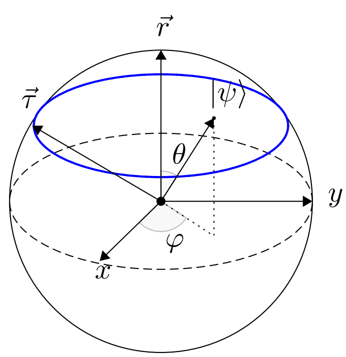
These systems are called qubits. A measurement correspond to projector projecting onto the eigenstate corresponding to the eigenvalue ± 1 of spin along some direction $\vec{r}$ is given by,

$$P_{\pm 1, \vec{r}} = \frac{1}{2}(\mathbb{I} + (\pm 1)\vec{r}\cdot\vec{\sigma}),$$

where $|\vec{r}| = 1$. The expectation value is given by,

$$Tr(\rho P_{\pm 1, \vec{r}}) = \vec{\tau}\cdot\vec{r}.$$

A qubit can be represented by the following diagram where the direction of measurement is represented by the z axis,



Kochen and Specker gave a hidden variable model for this system. The hidden variable state is given by the Bloch vector $\vec{r}$ of the quantum state and a unit vector $\vec{\lambda} = (\theta, \phi)$ uniformly distributed on the sphere given by,

$$\rho(\vec{\lambda})d\vec{\lambda} = \frac{1}{4\pi} \sin\theta d\theta d\phi$$

The outcome is deterministically computed as follows,

$$a(\lambda) = \text{sign}[(\vec{r} - \vec{\lambda}) \cdot \vec{m}].$$

This term is either $+1$ or -1. To every $\vec{\lambda}$ in the cup above blue circle corresponds to a outcome -1. We have,

$$p(-1, \vec{r}) = \frac{1}{4\pi} \int_0^{\alpha} d\theta \sin\theta \int_0^{2\pi} d\psi,$$

where $\alpha = \arccos(\vec{r} \cdot \vec{r})$. The complement area will yield $+1$. The mean value is given by,

$$\langle s \rangle_{\vec{r}} = \int \rho(\vec{\lambda}) a(\vec{\lambda}) d\vec{\lambda} = \vec{r} \cdot \vec{r}$$

So the Kochen-Specker hidden variable model yields same result as quantum theory. Such a hidden variable model is not possible for higher dimensions however. This is shown by the Kochen-Specker theorem.

1.1.1 | Kochen-Specker Theorem

In classical physics, the states are specified as points in phase space which correspond to the values of position and momentum, the observables of the system. Such states specify the values of every observable that the system can take. The observer can know with certainty the values of all observables. The states in classical mechanics represent a state of reality. If Ω is the state space in classical physics, an observable is a map,

$$X :\, \Omega \to \mathbb{R}.$$

Each state would fix a value for the observables. If we denote the valuation map associated with a state ρ by λ_ρ then,

$$\lambda_\rho :\, X \mapsto \lambda_\rho(X).$$

All physical quantities possess a value in any state. If $h : \mathbb{R} \to \mathbb{R}$ is a real-valued measurable function we can construct new observables from old ones, the values of which are $h(X) := h \circ X : \Omega \to \mathbb{R}$. In such cases, we should expect the valuation of the new observable to be,

$$\lambda_\rho(h(X)) = h(\lambda_\rho(X)).$$

The observable $h(X)$ is defined by saying that its value in any state is the result of applying the function h to the value of X.

Effects in the quantum case are projection operators. If a valuation as above exists then the valuation map associated with a state of reality should assign to each projection operator the values 1 or 0 based on whether the system was measured with the said property or not. Such maps are called valuation maps or valuations. If λ is such a valuation map then $\lambda(\mathbb{I}) = 1$. If A and B are self-adjoint operators such that for some real-valued function h, $B = h(A)$ then,

$$\lambda(B) = h(\lambda(A)).$$

Valuation maps represent non-contextual hidden variables i.e., the observables have predefined values and the values are independent of measurement context. A valuation map associated with a state ρ is a homomorphism from the algebra of projection operators to the set $\{0, 1\}$.

$$\lambda_\rho : \mathcal{P}(\mathcal{H}) \to \{0, 1\}.$$

Assuming such a valuation map exists, it must satisfy the valuation conditions $\lambda_\rho(\mathbb{I}) = 1$ and $\lambda_\rho(\sum_i E_i) = \sum_i \lambda_\rho(E_i)$. Such a map satisfies the conditions of Gleason's theorem, hence must take continuous values in $[0, 1]$. Since valuation maps can only take discrete values $\{0, 1\}$, such a map cannot exist.

Theorem 1.1. (Kochen-Specker) *If* $\dim(\mathcal{H}) \geq 3$ *then there exist no valuations.* □

We used Gleason's theorem for the Kochen-Specker theorem. Since the Gleason's theorem makes lots of people uncomfortable we also give below a more elementary 'pentagram' proof of the theorem.

Sketch of Proof

Consider a pentagram with each vertex representing an observable and any two vertices that have an edge between them commute. Denote the observables by A_i, $i \in 1, \cdots, 5$. If there exists a non-contextual hidden variable λ that describes the system then to each of the five observables we assign definite values that's independent of measurement context. So,

$$A_i(\lambda) = \pm 1, \quad i = 1, \ldots, 5.$$

Now some algebra shows us that,

$$-3 \leq A_1(\lambda)A_3(\lambda) + A_3(\lambda)A_5(\lambda) + A_5(\lambda)A_2(\lambda) + A_2(\lambda)A_1(\lambda) \leq 5.$$

So the average should also lie in the same interval, $-3 \leq \langle A_1(\lambda)A_3(\lambda)\rangle + \langle A_3(\lambda)A_5(\lambda)\rangle + \langle A_5(\lambda)A_2(\lambda)\rangle + \langle A_2(\lambda)A_1(\lambda)\rangle \leq 5$. Now to arrive at a contradiction the idea is to choose a system and a set of 5 observables as above and show the expectation value of above expression lies outside the interval required by non-contextual hidden variables. See [**?**] for the full proof. □

The theorem asserts that it is impossible to assign values to all physical observables while simultaneously preserving the functional relations between them. It should, however, be noted that when restricted to commutative subalgebras valuations do exist. Due to the non-commutativity of quantum theory, the values of all the observables can't be known at once and any such notion has to be contextual, value of the observable depends on the experimental context. The states in quantum theories cannot be interpreted completely ontically. Non-contextual hidden variable theories are also not viable. A 'state of reality' is meaningless in quantum theory. Any attempt to view the quantum state as ontic states would require serious mutilation of objects of quantum theory.

1.2 | Local Realism or Bell Locality

The starting assumptions of quantum theory are quite general. It should be possible to model all observed phenomenon using quantum theory. Now the aim is write down a quantum theory that satisfy the constraints put forth by the theory of relativity of Einstein. The constraint of interest to us is causality. A basic characteristic of physics in the context of relativity is that causal influences on spacetime propagate in timelike or lightlike directions but not spacelike. Communication should only possible at a speed less than that of light according to relativity.

Now the first step to formalize this statement is to formalize the concept of local realism. We start with Bell's definition of locality also called called Local causality or Bell locality. We want to define local realism for two experimenters say, Alice and Bob, in space-like separated regions performing two experiments A and B respectively. Let the values of the observables by A_i and B_j respectively. Bell assumed there existed some hidden variable λ and the probability of observables are distribution over the hidden variable. The probability that the combined system in the hidden variable λ has values A_i and B_j for the obseravbles A and B is given by,

$$\mu(A_i, B_j|\lambda) = \mu(A_i|\lambda)\mu(B_j|\lambda)$$

The product structure comes from the assumption that space-like separated events can't influence each other.

$$\mu(A_i, B_j) = \int_\lambda \mu(A_i, B_j|\lambda)\rho(\lambda)d\lambda = \int_\lambda \mu(A_i|\lambda)\mu(B_j|\lambda)\rho(\lambda)d\lambda,$$

where $\rho(\lambda)$ tells us how the probabilities are distributed over λ. We will call this condition Bell locality. The normalization implies that, $\sum_{A_i,B_j} \mu(A_i, B_j) = 1$. Note that we are already assuming non contextual hidden variables determining the values of observables. We will denote the set of all probabilistic vectors which obey the Bell locality $\mathcal{L}$, call this set local set. It's easy to verify that this is a convex set.

1.2.1 | Bell's Theorem

Now for the case of quantum theory the expectations are given by,

$$\mu_\rho(A_i, B_j) = Tr(\rho(P_{A_i} \otimes P_{B_j})),$$

where ρ is the state of the system and P_{A_i} and P_{B_j} are POVMs corresponding to the events associated with values A_i and B_j of observables A and B respectively with $\sum_i P_{A_i} = \mathbb{I}_A$ and $\sum_j P_{B_j} = \mathbb{I}_B$. Denote by,

$$\mathcal{Q} = \{\mu_\rho(A_i, B_j) \mid \rho \in \mathcal{S}(\mathcal{H}_A \otimes \mathcal{H}_B)\}$$

the set of all probability vectors. Since $Tr(\cdot(P_{A_i} \otimes P_{B_j}))$ is a continuous linear map, it maps convex bounded set to convex bounded set. Now basic topology tells us every continuous function $f : C \to \mathbb{R}$ where C is convex, compact will attain maxima/minima at extreme points of C. This is due to the fact that compact sets get mapped to compact sets, since C is a convex set $f(C)$ is a connected interval, interior of C is mapped to interior of $f(C)$.

Our aim is to show that there exist states that don't satisfy Bell locality, i.e., the local set is a strict subset of the quantum set.

Theorem 1.2. (Bell's Theorem) $\mathcal{L} \subsetneq \mathcal{Q}$ □

Proofs of Bell's theorem usually involve constructing an entangled state that will not satisfy the Bell locality condition. These proofs don't reveal anything about what mathematical structure of quantum theory is causing the problem. One strategy is to show that states satisfying Bell locality will have certain bounds and then construct a quantum state that disobeys such a bound. We will sketch below one such proof that uses the so called CHSH inequality.

Sketch of Proof

The starting point is the expectation value of the product of the outcomes of the experiment. Assume the observables A and B take two values say ± 1. The expectation of the product of outcomes is given by,

$$E(a,b) = \int A(a,\lambda)B(b,\lambda)\rho(\lambda)d\lambda$$

where $A(a,\lambda)$ is the value of the observable A given the system is in the hidden variable λ and measurement setting a for the instrument and similarly for $B(b,\lambda)$. This is where we have imposed the Bell locality condition. Since possible values are ± 1 we get that $|A| \leq 1$ and $|B| \leq 1$. By considering intruments with two settings a_1, a_{-1} and b_1, b_{-1} respectively, we get the following constraint

$$|S| = |E(a_1,b_1) - E(a_1,b_{-1}) + E(a_{-1},b_1) + E(a_{-1},b_{-1})| \leq 2.$$

for all probability vectors in $\mathcal{L}$. The wikipedia article on CHSH inequality has sufficient details and an interested reader should read it.

To arrive at a contradiction one considers a two qubit system. One starts with measurement basis, $|0\rangle_A, |1\rangle_A$ and $|0\rangle_B, |1\rangle_B$ and then considers Bell entangled states. The expectations will turn out to be $E(a_i,b_j) = \pm 1/\sqrt{2}$ and $|S| = 2\sqrt{2}$ hence violating the CHSH bound and proving the theorem. $\square$

For any self-adjoint operator A, the norm of A is the same as the spectral radius, $\|A\| = \sup_{\lambda\in\sigma(A)}\{|\lambda|\}$. So, for 'binary' observables A_i, B_j with eigenvalues ± 1, consider the operator,

$$S = A_1 \otimes B_1 + A_1 \otimes B_2 + A_2 \otimes B_1 - A_2 \otimes B_2$$

Each $A_i = P_{+1_{A_i}} - P_{-1_{A_i}}$, where $P_{\pm 1_{A_i}}$ is the projection corresponding to the value ± 1. We get, $A_i^2 = \mathbb{I}$, and similarly for B_j. Calculating S^2, we get, $S^2 = 4(\mathbb{I}\otimes\mathbb{I}) + [A_1, A_2] \otimes [B_2, B_1]$. This yields,

$$\|S^2\| \leq 8$$

or equivalently,

$$\|S\| \leq 2\sqrt{2}.$$

This is called the Tsirelson bound, and Bell states attain the maximal value.

A probability distribution is said to satisfy no-signalling condition if,

$$\mu(A_i|\lambda_k) = \textstyle\sum_{B_j} \mu(A_i, B_j \mid \lambda_k, \lambda_l)$$

and similarly for B_j. Distributions which satisfy this no-signalling condition are denoted by $\mathcal{NS}$. The quantum set satisfies the no-signalling condition.

Theorem 1.3.

$$\mathcal{L} \subsetneq \mathcal{Q} \subsetneq \mathcal{NS}.$$

The proof is a constructive proof like the Bell's theorem, the example construction is called the Popescu-Rorhlich box or PR box which we will describe in the sketch below.

Sketch of Proof

Consider the distribution, $\mu_{PR}(A_i, B_j|\lambda_k, \lambda_l) = \frac{1}{2}$, $\mu_{PR}(A_i, B_i|\lambda_k, \lambda_l) = 1$, $\mu_{PR}(A_i, B_{j\neq i}|1, 1) = 1$ where $i, j = \pm 1$, and $\lambda_{k,l} = \{0, 1\}$. This satisfies the no-signalling condition, and doesn't belong to $\mathcal{Q}$. Suppose $\mu \in \mathcal{Q}$, then we have,

$$\mu(A_i, B_j|\lambda_k, \lambda_l) = Tr(\rho(P^k_{A_i} \otimes P^l_{B_j}))$$

Then the probabilities will be, $\sum_i \mu(A_i, B_j|\lambda_k, \lambda_l) = \sum_i Tr(\rho(P^k_{A_i} \otimes P^l_{B_j})) = Tr(\rho(\mathbb{I} \otimes P^l_{B_j}))$. So, the probability doesn't depend on the choice of the setting λ_k. So, $\mathcal{Q}$ is indeed contained in $\mathcal{NS}$. □

On an operational level, the hidden variable models cannot be distinguished from quantum mechanics. The question then is about the plausibility of such models and if they are useful as physical theories in terms of predictability, etc. If a model requires an infinite number of hidden variables to describe stuff, it's not a very good model or as Hardy calls it, such models carry "ontological excess baggage".

Let Λ be the set of all hidden variables, let $\mu(\lambda|\varphi)$ denote the probability distribution of the hidden variable corresponding a given state φ. Then we should have,

$$\sum_{\lambda \in \Lambda} \mu(\lambda|\varphi) = 1$$

For normalized states we have, $|\langle\varphi|\varphi\rangle|^2 = 1$, so we have, $\sum_{\lambda\in\Lambda} \mu(\lambda|\varphi)\mu(\varphi|\lambda) = |\langle\varphi|\varphi\rangle|^2 = 1$. This can happen only if each $\mu(\varphi|\lambda) = 1$. Denote by Λ_φ all $\lambda \in \Lambda$ for which $\mu(\lambda|\varphi) > 0$.

Theorem 1.4. (Hardy's Theorem) *Any hidden variable theory that reproduces all measurements of a quantum system must have an infinite number of hidden variable states.*

Sketch of Proof

Consider a two level system, the idea is that if Λ_φ is a complete set of hidden variables then for all vectors $\varkappa$ we should have $\sum_{\lambda\in\Lambda} \mu(\lambda|\varkappa) = 1$. Now consider the set of M states (not orthogonal) given by,

$$|\varphi_i\rangle = \cos(\tfrac{\pi i}{2M})|0\rangle + \sin(\tfrac{\pi i}{2M})|1\rangle.$$

For this set we have, $|\langle\varphi_i|\varphi_j\rangle| < 1$ which implies $\sum_{\lambda\in\Lambda} \mu(\lambda|\varphi_i)\mu(\varphi_j|\lambda) = |\langle\varphi_i|\varphi_j\rangle|^2 < 1$ which can happen only if some $\mu(\varphi_i|\lambda) < 1$. So Λ_{φ_i} must have different elements not already in Λ_φ. This means there are atleast M distinct subsets of Λ. Now since M can be arbitrarily large we conclude that Λ must be infinite. □

1.3 | Ontological Model

A non-contextual ontological model of an operational theory is an attempt to provide a causal explanation of the operational statistics. It says that the response of the measurement is determined by the ontic state λ of the system, while preparation procedures determine the distribution over the space of ontic states, Λ, from which λ is sampled. An ontological model associates to each preparation ρ a probability distribution μ_ρ representing the agents' knowledge of the ontic state given the preparation ρ. If we denote the set of such distributions by $\mathcal{D}(\lambda)$, the ontological model specifies a map,

$$\mu : \mathcal{S}(\mathcal{H}) \to \mathcal{D}(\Lambda).$$

An ontological model associates to each operational effect a response function on Λ representing the probability assigned to the outcome R_i in a measurement of R if the ontic state of the system fed into the measurement device were known to be $\lambda \in \Lambda$. If we denote the set of response functions by $\mathcal{F}(\Lambda)$, the ontological model specifies a map,

$$\eta : \mathcal{P}(\mathcal{H}) \to \mathcal{F}(\Lambda).$$

These two maps must preserve the convex structure i.e, if ρ is a mixture of ρ_1 and ρ_2 with weights λ and $1-\lambda$ then $\mu_\rho = \lambda\mu_{\rho_1} + (1-\lambda)\mu_{\rho_2}$ and similarly for effects. Furthermore, an ontological model should produce the same probability rule as the operational theory. Assuming Λ is discrete for simplicity, we have,

$$\mu(\rho, R_i) = \textstyle\sum_\lambda \eta_{R_i}(\lambda)\mu_\rho(\lambda).$$

An ontological model of an operational theory is said to satisfy the generalized noncontextuality if every two operationally equivalent procedures have identical representations in the ontological model. That is to say, $\rho \sim \rho' \implies \mu_\rho = \mu_{\rho'}$ and similarly for effects. The same can be defined for GPTs and it's shown in [18] that they are equivalent. We will now go back to the quantum case, and the PBR theorem [31].

1.3.1 | Pusey-Barret-Rudolph Theorem

Suppose quantum state ρ is a state of knowledge, representing the uncertainty about the real underlying ontic state of the system λ. The quantum state ρ results in a physical state λ with a probability distribution $\mu_\rho(\lambda)$. If the distributions for distinct quantum states do not overlap then the quantum state can be uniquely inferred from the physical state. If the distributions overlap, then the quantum states can be said to only contain some knowledge about the physical state. Suppose we have two quantum states ρ_1 and ρ_2 with overlapping distributions, $\mu_{\rho_1}(\lambda)$ and $\mu_{\rho_2}(\lambda)$ then for any λ in the overlap Δ, there is a $q > 0$ probability that the physical state is compatible with both quantum states. Now consider two uncorrelated systems that are prepared with two copies of the same preparation device. If the physical states λ_1 and λ_2 lie in the overlap Δ, there must be some $q > 0$ such that with q^2 probability the quantum states $\rho_1 \otimes \rho_1$, $\rho_1 \otimes \rho_2$, $\rho_2 \otimes \rho_1$ and $\rho_2 \otimes \rho_2$ will be in this physical state. The measurement on this system can be cleverly chosen such that the first outcome is orthogonal to the first state, the second outcome orthogonal to the second state, and so on.

To arrive at a contradiction consider $\rho_1 = |0\rangle$ and $\rho_2 = |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, and choose the measurement which projects onto the following orthogonal vectors,

$$\begin{aligned}
&(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle)/\sqrt{2} \\
&(|0\rangle \otimes |-\rangle + |-\rangle \otimes |0\rangle)/\sqrt{2} \\
&(|+\rangle \otimes |1\rangle + |1\rangle \otimes |+\rangle)/\sqrt{2} \\
&(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle)/\sqrt{2},
\end{aligned}$$

where $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$. The measuring device should have been uncertain at least q^2 of the time about which quantum state was used in the measurement.

Theorem 1.5. (Pusey-Barret-Rudolph) *Quantum state interpreted as information about an objective physical state cannot reproduce the predictions of quantum theory.* □

This will imply it will give an outcome that is predicted to not happen quantum mechanically. Hence, interpreting quantum states as having information about an underlying objective physical state cannot reproduce the predictions of quantum theory. Kochen-Specker theorem rejects non-contextual hidden variable theories and the PBR theorem rejects the existence of a non-contextual ontological model or the statistical interpretation. Frequentist interpretation of probabilities occurring in quantum mechanics is problematic. Kochen-Spekker theorem can be evaded by arguments of the sort 'the values are never accurately known'. But we don't like to go down this path. We respect the idealization procedure of quantum theory.

The interpretation is ψ-epistemic if they view the quantum state as containing knowledge about an underlying reality similar to how we view states in classical statistical mechanics. The point of view given in [3] is the statistical interpretation, the quantum states represent partial knowledge about an underlying state of reality. In classical statistical mechanics, probability distributions are introduced on the phase space. These distributions represent the likelihood of the occurrence of the values. However, if the position and momenta of all the particles are known then we have complete knowledge of the system. These states of complete knowledge of the system correspond to delta distributions which are in a one-to-one correspondence with the points in phase space. The PBR theorem is a contradiction to interpreting quantum states statistically.

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