Structure of Physical Theories

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Aim: Take a closer look at the structure of physical theories

What's the point?
We can get a better understanding of the problem we are trying to solve.

Structure of this talk:

- Structure of a physical theory
- From physical ideas to physical theory
- Relation between different physical theories
- Problems of interest to us,
  1. Quantum Thermodynamics
  2. Quantum Gravity

TLDR History

Most of the reconstruction of physical theories, structure of physical theories type stuff can be traced back to Vienna circle, Carnap and all.

This then branched to mathematics side and physics side.
Mathematics side of structure and reconstruction is very rich.

Driven by influential people like Grothendiek

Physics side is quite dry,

(Probably due to Feynman's charm and rejection of these approaches as waste of time. and later on the cult following maybe by physicists)

A completely unrelated remark: Mathematics side has seen huge development compared to the physics side.

Whatever did happen on the physics side happened mostly in Germany and other European places. and some places in America, but very rarely.

Ludwig, Sheibe, etc. come to mind. Arguably, operational quantum theories, GPTs can be traced back to Ludwig's book.
and even AQFT can be included in this category.

Goal for this talk

How is mathematics done?

- Start with sets, put structure on it, make it into vector space, group, etc. then study them

  Or
- Assume you have some object and then study maps from and to the object, i.e., study the object by studying the maps to and from it.

This second approach is the 'categorical approach'.
The question:
I want to see if it's possible to study physics this way.

We will divide the structure of a physical theory into 3 different parts,

- Domain
- Mathematical Theory
- Correspondence

Physical theories can only be approximations of reality,

So, each physical theory PT_α will have an application domain,

$$A_\alpha \subset W,$$

where the principles and predictions work.

Goal is to maximize the application domain.

Domain

Domain of a physical theory PT_α itself consists of 3 types,

- Physical world/physical domain W.
- Application domain A_α,
  - Restriction of W, the stuff that PT_α considers.
- Fundamental domain G_α
  - Restriction of A_α that PT_α can describe

We can denote the relation as,

$$W \supset A_\alpha \supset G_\alpha.$$

The next step is to assign to the application a mathematical theory

$$A \longleftrightarrow MT$$

This is where the syntax-semantics, logic, etc. enter the discussion. I will skip this part. Because that will make the already dry subject drier.

Aim of a physical theory is to axiomatize the observed behavior of systems. So the first step is observing systems and recording properties.

- So in this sense reality consists stating basic properties of objects which we can measure and relation between these objects.
- The objects and their supposed properties are postulated.
- A collection of maps between objects.

For example consider quantum mechanics,

- Observables and states are objects.
- Schrödinger equation and collapse rules are the maps between objects.

So the mathematical theory MT_α of a physical theory PT_α consists of the following,

- Objects of the physical theory
- Morphisms between these objects.

A PT_α is then,

$$PT_\alpha \equiv A_\alpha \longleftrightarrow MT = (O, \text{morph}(O)).$$

A physical theory associates to reality a collection of objects, whose properties can be measured through actual measurements. The postulated relation between these objects gives rise to mathematics.

So, each physical theory is a category,
O of objects of the theory, and morph(O) of morphisms between objects.

Now how do we get this association?

$$A_\alpha \longleftrightarrow MT_\alpha$$

We observe some application domain A_α, get some ideas about the systems that belong to this domain $\mathcal{I}_\alpha$ and this set of ideas provides us with maps $A_\alpha \longleftrightarrow MT_\alpha$,

$$\begin{tikzcd} & \mathcal{I}_\alpha \arrow[d, dashed] & \\ A_\alpha \arrow[rr, leftrightarrow] & {} & MT_\alpha \end{tikzcd}$$

Each physical theory PT_α will have a collection of physical ideas $\mathcal{I}_\alpha$, and they provide us the maps,

$$A_\alpha \longleftrightarrow MT_\alpha$$

Let $\mathcal{I}$ be the collection of all physical ideas in physics. Now, the physical ideas of each physical theory will be a subset of this collection.

Now each $\mathcal{I}_\alpha \subset \mathcal{I}$ corresponds to some physical theory PT_α. We are interested in understanding how changing $\mathcal{I}_\alpha$ changes physical theories.

$\mathcal{I}$ is a partially ordered set with the order given by set inclusion.

$$(\mathcal{I}, \leq)$$

is a category.
So we have a functor,

$$(S(\mathcal{I}), \leq) \to \{PT_\alpha\}$$

$S(\mathcal{I})$ is some collection of subsets of $\mathcal{I}$.

Now we note some basic properties of the association $I_\alpha \mapsto PT_\alpha$.

- Bigger $\mathcal{I}_\alpha$ is more constrained the theory becomes.

$$I_\alpha \subset \mathcal{I}_\beta \implies PT_\alpha \supset PT_\beta.$$

  This means that $A_\alpha \supset A_\beta$. Application domain gets larger as we relax the assumptions.

Example, GPT vs Quantum theory vs Classical theory. (without evolution for simplicity)

Let $\mathcal{I}_{GPT}$ be the set of ideas needed for the formulation of GPTs i.e.,

- assumptions about measuring instruments gives rise to the mathematical notion of effects
- assumptions about preparation instruments gives rise to the mathematical notion of ensembles and hence states.
- Convexity assumptions
- Principle of tomography/effect-ensemble are mutually separating.
- ...

,

In quantum theory,

$$\mathcal{I}_{QM} = \mathcal{I}_{GPT} \cup \{effects = \mathcal{P}(\mathcal{H})\}$$

In classical theory,

$$\mathcal{I}_{CM} = \mathcal{I}_{QM} \cup \{effects = commutative\ \ algebra\}$$

The commutative algebra= functions on some geometric object. like functions on $\mathbb{R}^n$ or something like that.

Clearly we know that,

$$GPTs \supset QT \supset CT$$

Note that this sort of application domain expansion is useless. What we did was increase the application domain but the fundamental domain is actually reduced.

GPTs consider everything, but don't describe stuff that quantum theory can.
So we have to try to get to this sweet spot where the fundamental domain is everything, just having a large application domain is not enough.

Another example, String theory.

String theory we have too many assumptions, i.e., $\mathcal{I}_{string}$ is too big, might end up describing more than what actually exists.

Might end up not finding anything at all in experiments. So we have to be careful about adding random ideas to the physical theory.

How to reach the sweet spot? i.e., how to maximize the fundamental domain?

Reconstructions !
Example, reconstruction of quantum theory makes us realise which postulates are necessary and so on. We get better understanding about stuff.

Let PT_α and PT_β be two physical theories, they might have connections,

$$PT_\alpha \to PT_\beta$$

these map objects of one physical theory to objects of other physical theory and similarly map morphisms to morphisms.

These connections are 'functors'.

Example, “Approximation”

$$QT \to CM$$

is an approximation functor.

$$CM \to TD$$

is the “statistcal mechanics” functor.

These connections require additional ideas.
‘higher’ ideas maybe :)

What are we trying to do with quantum thermodynamics?

$$
\begin{tikzcd}
QT \arrow[d] \arrow[dr] & \\
CM \arrow[r] & TD
\end{tikzcd}
$$

CM $\rightarrow$ *TD* required the ergodic hypothesis, some assumption about microstates and all.

This is not allowed in *QM*, due to Kochen-Specker theorem and PBR theorem. Conflict between 'higher ideas'.
So the composition *QM* $\rightarrow$ *CM* $\rightarrow$ *TD* is forbidden. Or we will lose all the intricacies of quantum theory

Physics is a collection of physical theories along with connections between physical theories.

Quantum gravity?

Now we have two sets $\mathcal{I}_G$ and $\mathcal{I}_{QM}$ and their union $\mathcal{I}_G \cup \mathcal{I}_{QM}$.
Goal is to modify $\mathcal{I}_G \cup \mathcal{I}_{QM}$, the resulting physical theory is quantum gravity.

$$\mathcal{I}_{QG} \subseteq \mathcal{I}_G \cup \mathcal{I}_{QM}$$

We have to determine which ideas to throw and which to keep. This is where reconstruction becomes important again.

Reconstruction of quantum theory, reconstruction of relativity?...

Reconstruction of relativity is currently mostly inactive area. But it's also hard because it will involve foundations of geometry and all that.

Black hole thermodynamics and Quantum gravity?

People working in the area talk about lots of stuff which I find weird.

- Yes, thermodynamics has a very large application domain, it's applicable to blackholes.
- But it's important to understand what tools we are using and to which physical theory it belongs. and be careful when applying some tools to stuff it's not applicable to.

Then Verlinde, et al start making claims about particles of space-time, etc.

What did they mean by this?

Whenever thermodynamics enters the discussion, physicists usually start thinking in terms of some statistical mechanics stuff.

$$\begin{tikzcd} QT \arrow[d] \arrow[dr] & \\ CM \arrow[r] & TD \end{tikzcd}$$

Now we are trying to go back to CM which we trying not to do.

Blackhole entropy

We should be careful here, what's entropy?

We have different notions of entropy,

- Thermodynamics entropy
- Statistical mechanics entropy
- Information theory entropy

With respect to the statistical mechanics functor all these are equivalent. But is this really the case in quantum case? Is this 'entropy' von Neumann entropy? Why?

It's easy to sweep these issues under the rug but we should be careful.

Thank you!