# Emergence of Time in a Participatory Universe

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**Abstract**

After stating the measurement problem, physicists usually assume the problem to be coming from the measurement part. Since classical probabilities also collapse when updating information, there is nothing special about quantum state collapse. I believe the problem comes from the unitary evolution part of quantum theory. The question we should be asking is not 'what happens during measurement?', but 'what is time?'. After discussing the problems with time evolution in quantum theory, we propose a new approach to interpret time and argue how it would emerge from the non-commutativity of quantum theory, assuming participatory realism. Its relation to the familiar mechanical or unitary notion of time is discussed.

**Keywords:** quantum foundations, interpretation, participatory universe, quantum first, QBism, foundations of thermodynamics, resource theory, problem of time

# Introduction

The notion of time is closely linked with the foundations of thermodynamics and quantum gravity. As we discuss in this paper it's also closely related to the measurement problem. This makes studying the origins of time important to develop a clear understanding of the foundations of quantum physics as well. We study the measurement problem and problem of time in the foundations of quantum physics and discuss the emergence of time from a purely quantum viewpoint. The idea that the notion of time is closely related to change has been noted by many philosophers [71]-[72]. Alain Connes, [80],[81], has been saying for a long time that non-commutativity of the algebra of observables is closely related to the flow of time. I believe both of these play an important role in the notion of time.

Physicists aim to understand the physical world with a minimum number of 'laws'. In quantum physics, we have two laws for the evolution of quantum systems. The measurement collapse law, given by Lüders-von Neumann rule, and the unitary evolution law, given by the Schrödinger's equation. Many consider this a problem. It is called the measurement problem. What we expect from a solution to the measurement problem is a single law of evolution for quantum theories that explains both the measurement process and unitary evolution. There are different ways of attacking the problem. One common attempt to solve the problem is to try to explain the measurement process through unitary evolution, i.e., take the law of unitary evolution seriously and try to deduce the measurement process from it as some sort of partial tracing operation on a larger system, and so on. Approaches such as decoherence fall under this category. The other less popular approach is to explain the unitary evolution through the measurement process, i.e., the measurement process is taken seriously and unitary evolution is 'derived'. Stochastic approaches fall under this category. It's however important to provide a clear explanation as to why we prefer one law over the other. None of these approaches question the validity of the unitary evolution.

We obtain the law of Schrödinger equation from classical physics. In classical physics, the laws of evolution came from some symmetry groups such as the Galilean group or the Lorentz group. When quantum mechanics was being developed in the early twentieth century this law was implemented on the new objects of quantum theory consisting of observables and states. Quantum mechanics is an irreducibly probabilistic theory, the measurement law tells us the rule for updating the state of the system based on the new information obtained through measurement. Since classical probabilities also collapse when updating information, there is nothing special about quantum state collapse. I believe the problem comes from the unitary evolution part of quantum theory. Our belief in unitary evolution lies in the success of evolution in classical physics. This notion of evolution is already in deep trouble within classical physics itself. General relativity for example doesn't obey such a law of evolution. So we are in a situation where we trust the unitary evolution in quantum theory based on its success in *some* classical theories. This is an extremely weird situation to be in not only because we are relying on a faulty pre-theory for evolution in quantum theory but this notion of evolution wasn't even sufficient for classical theories. There is no reason why we should be trusting the laws from classical physics that's not even general enough to be taken as standard evolution for all classical theories. Our trust in unitary evolution is misguided. Since our law of evolution and notion of time in quantum theory comes from this unitary evolution, it's this notion of evolution that needs a replacement. The question we should have been asking is not 'what happens during measurement?', but 'what is time?'. We propose a new approach to interpret time and argue how it would emerge from the non-commutativity of quantum theory, assuming participatory realism. We will define this notion of time using only a weak version of the equilibrium principle of thermodynamics. Its relation to the familiar notion of time coming from unitary evolution is discussed. There needs to be a unifying 'law' for both the measurement process and whatever replaces the Schrödinger equation. Our approach is to start over, take the measurement process seriously. The unifying law is Bayesian inference i.e., updating the state based on the newly available information. Since inference applies to every probabilistic theory it is a very general law.

Time is treated as an emergent concept arising from quantum processes. It is automatically compatible with the inference rules. We use the weak equilibrium principle to get a number for time (not unique). Since the equilibrium principle doesn't cause any conflict with the measurement postulate the notion of time is compatible with the measurement postulate. Our aim is not to define a notion of time compatible with general relativity or some other theory but to understand how time would emerge using purely quantum reasoning. This notion of time will exist on a framework level and not on a model level. A connection between general relativity and our notion of time would be a bit indirect and through thermodynamics. We don't try to make any such premature connection at the current stage.

## Organization of the Paper

We aim to develop a path towards a resolution of the measurement problem through the emergence of time. In §2.4, we describe how the non-commutativity of quantum theory is responsible for the emergence of time and later define time using methods of resource theory of asymmetry.

In §1.1, we discuss the different notions of evolution in quantum mechanics leading to the statement of the measurement problem in §1.2.

In §2.1, we discuss the laws of thermodynamics relevant to our discussion, how the equilibrium principle is more important than the second law for us.

In §2.4.2, we build up the tools needed to assign a number to each event that should correspond to the notion of time. In §2.2.0.1, we discuss the relation to the thermal time hypothesis. In §2.2.1.2, we define 'time' using a weak version of the equilibrium principle.

In §2.3, we discuss some of the consequences of our definition of time and in §2.3.2 we relate it with the standard unitary evolution.

Since we can't evade questions regarding the treatment of observers and instruments in quantum theory we have discussed these issues very briefly. We have also indicated how the choice of treatment of observers and instruments will not affect the message of the paper.

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# 1 | Statement of the Problem

Operational quantum theory is a minimalist approach to formulating quantum theory, initiated by Günther Ludwig [3]-[14]. Usually when one tries to formulate quantum theory one starts with a pre-theory such as classical mechanics then 'quantizes' the theory. This makes the theory very messy and the underlying physical ideas hidden and unclear. In operational quantum theory or instrumentalist approach, one starts with instruments. The construction and behavior of instruments are not of interest to us. Any changes occurring in the instruments during 'measurements' are accepted as objective events. Quantum mechanics is then interpreted entirely in terms of such instruments and events. These instruments and events are our links to 'objective reality'. The notion of 'state' is defined in terms of the preparation procedure. A preparation procedure is characterized by the kind of system it prepares. The other important thing is the existence of a measuring instrument that is capable of undergoing changes upon their interaction. The observable change is called an 'effect'. Any measurement can be interpreted as a combination of yes-no measurements. These yes-no instruments can be used to build any general instrument. Suppose we have such an instrument, label its registration procedure by $R$. If the experiment is conducted a lot of times, we get a relative frequency of occurrence of 'yes'. To every preparation procedure $\rho$ and registration procedure $R_i$ there exists a probability $\mu(\rho, R_i)$ of occurrence of 'yes' associated with the pair.

$$(\rho, R_i) \longrightarrow \mu(\rho|R_i).$$

The numbers $\mu(\rho|R_i)$ are called operational statistics. Two completely different preparation procedures may give the same probabilities for all experiments $R$. Such preparation procedures must be considered equivalent. Such preparation procedures are called operationally equivalent preparations. A precursor to the notion of a state of the system is an equivalence class of preparations procedures yielding the same result. They are called ensembles.

Denote the class of ensembles by $S$ and the class of effects by $E$. The maps of interest to us are the following,

$$S \times E \xrightarrow{\mu} [0, 1].$$

There may be two experiments that give the same probabilities for every ensemble. Such apparatuses must be considered equivalent. They are called operationally equivalent effects. An effect is the equivalence class of apparatuses yielding the same result. In general, a registration procedure $R$ for an experiment will have outcomes $\{R_i\}$. For an outcome, $R_i$ of the registration procedure $R$, denotes the corresponding equivalence class of measurement procedures by $E_{R_i}$. Each outcome $R_i$ of the registration procedure corresponds to a functional $E_{R_i}$ called the effect of $R_i$ that acts on the ensemble of the system to yield the corresponding probability.

$$E_{R_i} : \rho \mapsto E_{R_i}(\rho) = \mu(\rho|R_i).$$

Maps of interest to us will be those that assign to each of its outcomes $R_i$ its associated effect $E_{R_i}$. Since each ensemble fixes a probability distribution we have,

$$\mu_\rho : R_i \mapsto \mu_\rho(R_i) = \mu(\rho|R_i).$$

The above-given map $\mu_\rho$ is determined by the instrument and the registration procedure. If we can form mixtures and the set of ensembles is convex we call the theory convex operational theory. Since a mixture of ensembles corresponds to a convex combination of probabilities each functional $E_{R_i}$ preserves the convex structure. Since two preparations giving the same result on every effect represent the same ensemble and two measurement procedures that can't distinguish ensemble represent the same effect, ensembles and effects are mutually separating. A generalized probabilistic theory or a GPT for short is an association of a convex state space and effect vectors to a given system, such that the states and effects are uniquely determined by the probabilities they produce. This is known as the principle of tomography.

One takes an operational theory and 'quotients' with operational equivalences to obtain a GPT. Denote by $\mathcal{S}$ the set of maps, $f : E \longrightarrow \mathbb{R}$ such that $f(X) = \sum_i \alpha_i \mu(\rho_i|X)$ and denote by $\mathcal{E}$ the set of maps, $g : S \longrightarrow \mathbb{R}$ such that $g(\rho) = \sum_i \beta_i \mu(\rho|R_i)$ where $\rho_i$ and $R_i$ are ensembles and effects respectively and $\alpha_i, \beta_i \in \mathbb{R}$. Clearly $\mathcal{S}$ and $\mathcal{E}$ are real vector spaces. We can embed ensembles inside $\mathcal{S}$ with the map,

$$\rho \longmapsto \mu_\rho,$$

and similarly embed effects inside $\mathcal{E}$ with the map,

$$R_i \longmapsto E_{R_i}.$$

The bilinear map $\langle \cdot|\cdot \rangle : \mathcal{S} \times \mathcal{E} \to \mathbb{R}$ which coincides with $\mu$ is then uniquely determined.

It's important to note here that the notion of effect is an abstract concept. A measuring instrument is only a representative of the equivalence class corresponding to the effect. Such a measurement apparatus need not exist. Similarly for ensembles. These abstract notions apply to any generalized probabilistic theories or GPTs for short.

Let $(\mathcal{H}, \langle \cdot|\cdot \rangle)$ be a complex Hilbert space. $\mathcal{P}(\mathcal{H})$ denote the set of all closed subspaces. Denote $\mathcal{H}_i \leq \mathcal{H}_j$ if and only if $\mathcal{H}_i \subseteq \mathcal{H}_j$. The relation $\leq$ is a partial ordering in $\mathcal{P}(\mathcal{H})$. Join $\vee$ of a family $\{\mathcal{H}_i\}_{i \in I}$ is the linear span of the family denoted $\vee_i \mathcal{H}_i$. Meet $\wedge$ of a family $\{\mathcal{H}_i\}_{i \in I}$ is the intersection of the family, denoted $\wedge_i \mathcal{H}_i$. The orthocomplement of $\mathcal{H}_i$ in $\mathcal{P}(\mathcal{H})$ denoted by $\mathcal{H}_i^\perp$ is the closed subspace of vectors $\varphi \in \mathcal{H}$ such that $\langle \varphi|\mathcal{H}_i \rangle = 0$. Since there is a bijection between closed subspaces of a Hilbert space and projection operators acting on the Hilbert space, the set of all projection operators on the Hilbert space inherits a lattice structure from the lattice of closed subspaces. Abusing notation, we will denote the projection operators on $\mathcal{H}$ by $\mathcal{P}(\mathcal{H})$. The orthocomplement of the projection $E$ is the projection onto the orthogonal complement of the subspace corresponding to the projection operator $E$ and is denoted by $E^\perp$. The lattice structure of $\mathcal{P}(\mathcal{H})$ coming from the above relations gives us the necessary structure to get the mathematical representatives of physical observables. The non-Boolean lattice $\mathcal{P}(\mathcal{H})$ of projections should act as the space of effects in quantum theories. Or more generally they correspond to projection operators in a von Neumann algebra.

A quantum mechanical observable is a map of the form,

$$E_A : \Sigma_A \to \mathcal{P}(\mathcal{H}),$$

a projection valued function and $\Sigma_A$ is a Boolean lattice. Usually in physical experiments, the statements that can be made are of the type 'the value of the observable lies in some set $\epsilon_i$ of real numbers'. To accommodate the fact that the measurement scale is composed of real numbers, we identify $\Sigma_A$ with the Borel sets of $\mathbb{R}$. The quantum observables are analogous to classical random variables, namely, that of a projection valued measure,

$$E_A : \mathcal{B}(\mathbb{R}) \to \mathcal{P}(\mathcal{H}).$$

This generalizes the classical case, for which mathematical representatives were the measure space $(\Omega, \Sigma(\Omega), \mu)$, where the $\sigma$-algebra, $\Sigma(\Omega)$ is a class of subsets of the set $\Omega$ which correspond to events and $\mu$ is a probability measure. A classical random variable is defined as a map $X : \Omega \to \mathbb{R}$. The map doing the work in assigning necessary probabilities is its inverse, considered as a set map,

$$X^{-1} : \mathcal{B}(\mathbb{R}) \to \Sigma(\Omega).$$

A spectral measure is a projection operator-valued function $E$ defined on the sets of $\mathbb{R}$ such that, $E(\mathbb{R}) = I$ and $E(\sqcup_i \epsilon_i) = \sum_i E(\epsilon_i)$, where $\epsilon_i$s are disjoint Borel sets of $\mathbb{R}$. The spectral theorem says that every self-adjoint operator $A$ corresponds to a spectral measure $E_A$ such that,

$$A = \int \lambda \, dE_A(\lambda),$$

and conversely, every spectral measure corresponds to a self-adjoint operator. In the finite-dimensional case this reduces to $A = \sum_i \lambda_i E_i$ where $E_i$s are projections onto eigenspaces of $\lambda_i$s. Observables in quantum theories are represented by self-adjoint operators on some complex Hilbert space and the orthogonal projections of the self-adjoint operator correspond to the events. The values of the observable are the spectrum of the operator. The characteristic feature of quantum theory is that the space of effects is a non-commutative entity.

The mathematical representatives of the physical states for the quantum case are the maps, $\omega : \mathcal{P}(\mathcal{H}) \to [0, 1]$, such that $\omega(0) = 0$, $\omega(E^{\perp}) = 1 - \omega(E)$ and $\omega(\vee_i E_i) = \sum_i \omega(E_i)$ for mutually orthogonal $E_i$. For an observable with the associated self-adjoint operator $A$, the map

$$\mu^A = \omega \circ E_A : \Sigma_A \to [0, 1],$$

By Gleason's theorem, these maps correspond to density matrices on the Hilbert space $\mathcal{H}$. If the complex separable Hilbert spaces $\mathcal{H}$ of dimension greater than 2, then every $\omega$ is of the form

$$\omega(E) = Tr(\rho E).$$

where $\rho$ is a positive semidefinite self-adjoint operator of unit trace or density matrix. Conversely, every density matrix determines a state as defined in the above formula. In general, physical states correspond to normal states on von Neumann algebras.

## 1.1 | Evolution in Quantum Theories

Physical laws axiomatize the regularities found in the evolution of the system. These physical laws act as constraints on the evolution of physical systems. There was a time when one tried to base a physical theory on experimental facts. The physical laws were guessed. When it comes to verification of the laws it can only be stated that all the experiments conducted until then were, within the scope of inaccuracies, not in contradiction with the law. But how do we come up with these laws? Is it possible to develop a method of guessing physical laws? and why do we trust physical laws? It is sometimes not difficult to guess from experimental data a mathematical representation of this relation. Everything is permitted in the guessing of physical laws. One commonly employed way of finding the physical laws is to take the laws from physical theories with a common application domain and reformulate it in terms of the objects of the new physical theory. Symmetries of the system for example act as common laws for many physical theories. It can also be that we trust a new law even if we have made no critical experiments. Before Einstein's new gravitational theory there were two other well-tried theories: relativity and Newton's gravitational theory for different domains of application. But it was not possible to unify these two theories. Einstein succeeded in achieving this by introducing a new law with an enlarged application domain.

Physics has always been about developing a theoretical framework that explains experimental observations. The experimental observations roughly speaking are then 'interpreted' in terms of this theory. Quantum mechanics is a theoretical framework using which we intend to interpret experiments. Each physical theory consists of a set of physical ideas which allow us to assign to the physical world a set of physical objects, and laws. The laws of the physical theory provide morphisms between the physical objects. A physical theory $PT$ is composed of a mathematical theory $MT$, an application domain $A$, and the correspondence rules $\leftrightarrow$. The correspondence rules $\leftrightarrow$ are prescriptions of how to translate facts detected in the world into mathematical theory. A physical theory is then,

$$\mathcal{I} \to PT \equiv MT \longleftrightarrow A.$$

The facts about the physical domain are then 'interpreted' using the mathematical theory. For the description of the facts in the physical domain, one uses 'pre-theories' which already come with their own interpretation. Ludwig's instrumentalist approach greatly simplifies this part. The construction and behavior of instruments will not be of interest to us.

Suppose a system prepared in a state $\rho$ undergoes a process. Assign to this system a quantum state. The original preparation procedure along with the process can be considered a new preparation procedure. The equivalence class of the new preparation procedure will define the new quantum state after the process. This state depends on the original preparation procedure and the process. Each process corresponds to a linear map,

$$\alpha : \mathcal{B} \to \mathcal{A}.$$

The algebras of observables $\mathcal{A}$ and $\mathcal{B}$ represent the input and output systems respectively. To an initial state $\rho$ of $\mathcal{A}$ the channel associates the output state $\rho \circ \alpha$ of $\mathcal{B}$.

If density matrices are used to describe quantum states in quantum mechanics, then a process must be some operation that sends density matrices to density matrices. So for finite-dimensional state spaces, a process should be a linear map of vector spaces of matrices. It preserves the trace of matrices and takes hermitian matrices with non-negative eigenvalues to hermitian matrices with non-negative eigenvalues. It must take positive operators to positive operators. A map is called positive. Suppose the process acts only on some part of the system then it must still be a process on the total system. The map corresponding to a process should be positive for the bigger system as well. A general quantum process corresponds to a completely positive unital mapping.

### 1.1.1 | Quantum Measurement

In quantum theory, the description of the system requires two physical objects. The first being the state of the system which contains the information known about the system. Second, the observables, which are objects the information is about. Bayes' theorem says that additional information about a system will alter the probabilities of possible outcomes. The notion of information is closely related to the notion of probability. Probability gives one way to describe information about the events. We are interested in quantifying the amount of information contained in a state relative to another state.

The relative entropy of two states $\rho$ and $\sigma$ is the informational divergence of $\rho$ from $\sigma$. Suppose the state $\sigma$ contains information only about a subsystem $\mathcal{B}$ of $\mathcal{A}$ and $E$ is a projection of norm one of $\mathcal{A}$ onto $\mathcal{B}$ then the state $\sigma$ should satisfy $\sigma \circ E = \sigma$. In such a case the informational divergence should have two components. First component is the divergence of $\rho$ from $\sigma$ on the subalgebra $\mathcal{B}$ which is the divergence between the states $\rho|_{\mathcal{B}}$ and $\sigma|_{\mathcal{B}}$. The other component is the remaining information $\rho$ has and this will be the divergence between the states $\rho$ and $\rho \circ E$. If $R(\cdot\,,\cdot)$ is such a function then,

$$R(\rho, \sigma) = R(\rho|_{\mathcal{B}}, \sigma|_{\mathcal{B}}) + R(\rho, \rho \circ E).$$

Any automorphism $\alpha$ of the algebra $\mathcal{A}$ should change the information contained in the two states similarly hence the information divergence should be invariant under automorphisms of the algebra,

$$R(\rho, \sigma) = R(\rho \circ \alpha, \sigma \circ \alpha).$$

The informational divergence of a state with respect to itself should be zero $R(\rho, \rho) = 0$. If $R(\cdot\,,\cdot)$ is a real-valued functional satisfying the above conditions then there exists a constant $c \in \mathbb{R}$ such that,

$$R(\rho, \sigma) = c\,\mathrm{Tr}\left(\rho\left(\log\rho - \log\sigma\right)\right).$$

The relative entropy of the state $\rho$ with respect to $\sigma$ is defined as,

$$J(\rho, \sigma) = \mathrm{Tr}\left(\rho\left(\log\rho - \log\sigma\right)\right).$$

In the classical case, the Bayes' rule has been shown to be a special case of the constrained maximization of relative entropy [40]. The quantum version of this result is obtained in [48]. We will state the result here.

Suppose an observable $A$ has been subjected to measurement. For simplicity we consider the observable to be a discrete observable. Let $A$ be a discrete observable with effects given by the set $\{A_i\}_{i\in I}$ and the corresponding projection operators $\{E_{A_i}\}_{i\in I}$. If the quantum state of the system after the measurement is $\sigma$, it carries information that has to be compatible with the possibility of measuring all eigenvalues of $A$ precisely. Such a situation is given by the condition $[\sigma, A] = 0$. Suppose the result of the measurement is $A_k$ then the probability of measuring $A_k$ again should be $Tr(E_{A_k}\sigma) = 1$. Repeated measurements add no new information. The set of all such states such that $Tr(E_{A_k}\sigma) = 1$ is a convex set. Let $p = \{p_i\}_{i\in I}$ such that $\sum_i p_i = 1$. The set,

$$\mathcal{S}_p = \{\sigma \in \mathcal{S}(\mathcal{H}) \mid [E_{A_i}, \sigma] = 0,\ Tr(\sigma E_{A_i}) = p_i\},$$

encodes the data that the measurement outcome $A_i$ corresponding to the projection $E_{A_i}$ occurs with probability $p_i$. The commutation condition says that they posses a common eigenbasis and also means that $[\sigma, A] = 0$.

**Theorem 1.1.1. (Hellmann-Kamiński-Kostecki)**

$$\arg\inf_{\sigma\in\mathcal{S}_p}\{J(\rho,\sigma)\} = \sum_i p_i E_{A_i}\rho E_{A_i}/Tr(E_{A_i}\rho E_{A_i}). \quad \square$$

The strong collapse or the Lüders-von Neumann rule of collapse is a limiting case of the above projection with all $p_i$ going to zero except one. By taking the limit $p_i \to 0$ for $i \neq j$ we get the Lüders-von Neumann's rule of collapse,

$$\rho \to E_{A_j}\rho E_{A_j}/Tr(E_{A_j}\rho E_{A_j}).$$

This amounts to selecting the quantum state that is least distinguishable from the original state among all the states that satisfy the constraint. For proof and generalization of the result to the algebraic case, the interested reader should read the original papers [48],[49],[58] and the references therein. The origins of principles of maximum entropy and minimum relative entropy go back to the works by Jaynes, Bayer & Ochs, Ingarden & Urbanik, Shore & Johnson, and others [44],[41],[46],[18]. These maps are called measurement channels. In general measurement channels are given by positive operator-valued measures, where for a measure space, $(\Omega, \Sigma(\Omega))$, and $\epsilon \in \Sigma(\Omega)$, $E(\epsilon)$ is a positive operator, $E(\Omega) = 1$ and for pairwise disjoint $\epsilon_i$, $\sum_i E(\epsilon_i) = E(\vee_i \epsilon_i)$. It should, however, be noted that the Lüders-von Neumann rule is about calibrating with the experimental result and has no predictivity. We will abuse the notation and denote an event characterized by the effect $E_{A_i}$ by $E_{A_i}$ only.

**Postulate. (Lüders-von Neumann Collapse)** *If an observable $A$, with values $A_i$ with corresponding projections $E_{A_i}$, is measured on the system in a state $\rho$, then the state transforms to,*

$$E_{A_i}\rho E_{A_i}/Tr(E_{A_i}\rho E_{A_i}),$$

*on the condition that the result $A_i$ was obtained.*

Since classical probabilities also collapse when updating information, there is nothing special about quantum state collapse. We will denote the physical objects of quantum theory along with this notion of state update by $\mathcal{QM}_M$.

### 1.1.2 | Why Unitary?

Here we give a brief review of the unitary evolution. The purpose of this subsection is to remind ourselves why unitary evolution is used in quantum theory. When it comes to time evolution, the quantum theory continues on with the received view. The symmetries of classical theories are implemented on objects of quantum theory.

The simplest structure a symmetric map should preserve is the convexity of the space of states, physically corresponding to the fact that a state arises from mixing states with certain statistical weights. Symmetry operations may modify the constituent states but do not change the weights. A bijection $\alpha : \mathcal{S}(\mathcal{H}) \to \mathcal{S}(\mathcal{H})$ is a symmetry if it preserves the convex structure of $\mathcal{S}(\mathcal{H})$. For $p_i \in [0,1]$ and $\sum_i p_i = 1$,

$$\alpha(\sum_i p_i\rho_i) = \sum_i p_i \alpha(\rho_i).$$

Such a map is called a Kadison automorphism. If a map $\alpha$ is a Kadison automorphism, then Kadison-Wigner theorem says $\alpha$ is of the form,

$$\alpha(\rho) = U\rho\, U^{-1},$$

where $U$ is unitary or antiunitary and is determined up to phase. A unitary operator is a map $U$ such that $\langle Ux, Uy\rangle = \langle x, y\rangle$ and an antiunitary operator is a map $U$ such that $\langle Ux, Uy\rangle = \overline{\langle x, y\rangle}$ where $\langle \cdot, \cdot\rangle$ is the inner product on the Hilbert space.

To implement the symmetries of the system, the symmetries must be represented in terms of Kadison automorphisms. We seek maps from some group to the set of Kadison automorphisms. Whether a specific transformation is unitary or antiunitary depends on its physical nature. Transformations that belong to a continuous group, such as translations and rotations, can only be unitary because in that case any finite transformation can be generated by a sequence of infinitesimal steps. Let $\mathcal{G}$ be the group of symmetries of the system, then to each $g \in \mathcal{G}$ there should correspond a Kadison automorphism,

$$\alpha : g \mapsto \alpha_g.$$

By Kadison-Wigner theorem we get a unitary or antiunitary representative $U(g)$ to each element $g \in \mathcal{G}$. For now we will assume $U(g)$ to be unitary. Given $g, h \in \mathcal{G}$ we know that,

$$\alpha_g \alpha_h = \alpha_{gh}.$$

For compatible representative $U$ we have,

$$U(g)U(h) = \lambda(g, h)U(gh),$$

where $\lambda(g, h)$ is a phase factor. A map $U : g \mapsto U(g)$ satisfying the above relation is called a projective unitary representation. $\lambda(g, h)$s are called multipliers. For $g = e$ we get,

$$U(e) = \lambda(e, e)I.$$

We get some conditions on the multipliers $\lambda(g, h)$. Applying several times to $f, g, h$ we get,

$$\lambda(f, g)\lambda(fg, h) = \lambda(g, h)\lambda(f, gh).$$

We also get,

$$\lambda(e, g) = \lambda(g, e) = \lambda(e, e).$$

A projective unitary representation with $\lambda(e, g) = \lambda(g, e) = \lambda(e, e) = 1$ for every $g \in \mathcal{G}$ is said to be normalized. A map $g \mapsto U(g)$ is called a unitary representation of $\mathcal{G}$ on $\mathcal{H}$ if $U(e) = I$ and satisfies,

$$U(g)U(h) = U(gh).$$

Unitary representations are usually much easier to work with. A theorem of Bargmann says for some groups with nicer properties (connected and simply connected) it's possible to get a unitary representation. One can always consider the universal covering group and get a unitary representation of that anyway.

Given a self-adjoint operator $A$, one can construct a family of unitary operators, $U(t) = e^{-itA}$. Stone's theorem says the opposite is also true. If $t \mapsto U(t)$ is a strongly continuous one-parameter unitary group in the complex Hilbert space $\mathcal{H}$, there exists a unique self-adjoint operator $A$ called the generator of the group such that,

$$U(t) = e^{-itA}.$$

We can, therefore, by Stone's theorem, associate with every one-parameter subgroup of $\mathcal{G}$ a unique self-adjoint operator $A_i$. The Lie algebra of the group $\mathcal{G}$ is represented by the self-adjoint operators $A_i$. From the Lie algebra of the group of symmetries, we can obtain the unitary representatives with a factor of $-i$. If the Lie algebra has the basic structure equation, $[a_i, a_j] = \sum c_{ij}^m a_m$, then the self-adjoint operators $A_i$ corresponding to $a_i$ satisfy the commutator relations, $i[A_i, A_j] = \sum c_{ij}^m A_m$.

We get the Schrödinger equation by implementing Galilean symmetries. When the symmetries are taken to be the Galilean group, the time evolution is generated by the Hamiltonian of the system and corresponds to the time translation symmetry of the system.

$$\rho \mapsto e^{-itH} \rho \, e^{itH}.$$

This is the Schrödinger equation.

**Postulate. (Schrödinger)** *Time translation is given by,* $\rho \mapsto e^{-itH} \rho e^{itH}$.

This notion of evolution is a direct copy-paste of the classical laws formulated for quantum objects. We will denote the physical objects of quantum theory along with this notion of state update by $\mathcal{QM}_U$.

## 1.2 | The Contradiction

A physical theory is said to be universal if its domain of application is everything. Classical physics was supposed to be such a universal theory but the experiments of the twentieth century showed it to be not the case. Quantum theory was created as a replacement. Quantum mechanics is supposed to be universal. It is supposed to explain all the observed phenomenon and all non-quantum theories should be approximation theories relative to quantum mechanics. Although the successes of quantum theory may make the idea of the universality of quantum mechanics more compelling, it is important to note that these successes are not proof of the validity of the fundamental principles of the theory. Even if the adherents of the universality of quantum mechanics avoid the problem of elaborating the limits of quantum mechanics, they necessarily introduce a new difficulty, namely, How do we obtain a 'determination' of measurement results? This is known as the measurement problem. Here again, we find a variety of different approaches, which range from attempts to show that probability theory itself gives the valid determination to the introduction of consciousness of the observer or the so-called 'many-worlds' interpretation of quantum mechanics.

What we have are two theories for the same domain of facts, one given by $\mathcal{QM}_M$ and another by $\mathcal{QM}_U$. From the point of view of $\mathcal{QM}_U$, the representation of the Galileo group (or some other group depending on the situation) determines the laws. We shall only examine the condition for the physically important time translation. From this point of view, a description means that for every system there is a corresponding trajectory $\rho(t)$ of the state. What we expect is a reasonable relation between $\mathcal{QM}_M$ and $\mathcal{QM}_U$. There must exist in some sense, an equivalence $N$ between $\mathcal{QM}_M$ and $\mathcal{QM}_U$.

$$\mathcal{QM}_M \xleftrightarrow{N?} \mathcal{QM}_U.$$

According to $\mathcal{QM}_U$, there exists, a measurement of the state 'at time' $t$. In $\mathcal{QM}_M$ such a measurement 'at time' $t$ is not defined. One can then ask if there exists a theory whose laws reduce to these in special situations. But then there are complications like which law should apply when and why? The decoherence approaches try to 'explain' measurement using unitary evolution[1] and the stochastic interpretation tries to 'explain' unitary evolution through the measurement process. Others feel there is no need for unification and that quantum theory in its current form is perfectly fine. People belonging to this group (mostly belonging to epistemic interpretation or other 'Copenhagenish' interpretation category) view both unitary evolution and measurement process to be perfectly fine and that one should decide whether the unitary evolution applies or measurement process applies depending on the situation. In this case, we have two rules for updating the states of the system and it rests on the observer to decide which to apply when. We are however not convinced by this. We seek a universal theory with a unifying physical idea for the evolution of systems. Such a general theory that contains both doesn't yet exist in our opinion. The question we should

[1] Interested reader should check out [50] and [51] for a discussion as to why decoherence doesn't say anything about the measurement problem.

be asking now is if it's possible to find some workaround for this contradiction. What we are dealing with are different physical theories. A false view of inter-theory relations has been the source of many false opinions concerning the truth of a physical theory. Thus the misconception has arisen that no theory is really 'true' but that during the development of physics a later theory becoming valid makes an older theory untrue. Each physical theory will have its own application domain. What we expect from a better theory is a larger application domain. We can think of physical theories as a category with objects being physical theories. The inter-theory relations are the morphisms. What we expect is a physical theory from which we can obtain other theories either by approximation or some domain change. This would be a 'universal theory'. Quantum theory in its current stage is not a universal theory. Its application domain is not good enough. For example, it cannot explain a phenomenon like gravity. What we seek is a modification of the laws of quantum mechanics that would make it universal. Based on the needs, physical theories are revised to suit us. Once in a while, the change required will be so radical that the objects of the theory themselves need to be replaced. In this case, we are fine with the objects of quantum theory. What we are interested in is a theory consisting of objects of quantum theory and a new law of evolution compatible with measurement such that the law of evolution in $\mathcal{QM}_U$ would approximate to the law of evolution in the new theory under the suitable domain of application conditions. We will denote this theory by $\mathcal{QM}$.

Physical theories do not begin their development based on some well-defined foundations. The methods of the new physical theory are initially intuitively conceived and applied. The theories usually encounter contradictions on the way, and the discovery of the cause of these contradictions lets us rectify or clarify them. The clarifications for the contradictions help us avoid the contradictions in the future. The contradictions are crucial to the development of any physical theory. It is these clarifications that develop the conceptual foundation of the said physical theory. Einstein's theory of gravity used the old classical physical objects and introduced a new physical law. In quantum mechanics, the physical objects are newly introduced but the physical laws are old received view. We believe this is where the problem comes from. The question we have to ask is not 'what happens during measurement?' but 'what is time?'. The notion of time in general relativity and quantum mechanics are different. The evolution in quantum mechanics is adopted from a faulty notion of evolution in a pre-theory and it's this notion of evolution that needs to be replaced. We feel the problem is coming from the unitary evolution part. Synthesis of quantum theory and gravity would require revision of the physical laws in quantum theory. Now the problem is how to guess the new law? As we stated before, one commonly employed way of finding the physical laws is to take the laws from existing physical theories with common application domain and reformulate it in terms of the objects of the new physical theory or try to guess a law that fits with the law of the existing physical theory. One physical theory that has a large application domain similar to what we expect from quantum theory is thermodynamics and we could seek guidance from thermodynamics.

Physics does not consist of one theory. It's however a common belief that there should be one theory behind all these physical theories. We will say two theories are compatible if it's possible to go from the objects and laws of one theory to the other. We expect two theories with the same application domain to be compatible. If the application domain of one theory contains the other then it should be possible to approximate the other using the first theory. Suppose we have a class of physical theories $\{PT_\alpha\}_{\alpha\in I}$,

$$\mathcal{I}_\alpha \to PT_\alpha \equiv MT_\alpha \longleftrightarrow A_\alpha,$$

where $A_\alpha$ is the application domain and $MT_\alpha$ is the corresponding mathematical theory and $\mathcal{I}_\alpha$ is the set of physical ideas of the physical theory $PT_\alpha$. The physical ideas in $\mathcal{I}_\alpha$ provide us the correspondence rules associate with facts in $A_\alpha$ physical objects in $MT_\alpha$ and the laws of the theory provide morphisms between these objects. The physical ideas for different physical theories don't have to be disjoint. We call these ideas the common ideas of the physical theories. If $PT_\alpha$ and $PT_\beta$ are two physical theories then denote by, $\mathcal{I}_{\alpha\beta}$, the common physical ideas.

$$\mathcal{I}_{\alpha\beta} = \mathcal{I}_\alpha \cap \mathcal{I}_\beta.$$

For example all classical theories assume that effects correspond to a commutative C*-algebra. Other example would be that lot of physical theories share the idea that energy conserved quantity.

The ideas for evolution in quantum mechanics are same as those in classical mechanics i.e., in classical physics such as Newtonian mechanics the evolution was given by a symmetry group such as the Galilean group or in case of special relativity the Lorentz group. When it comes to evolution in quantum theory we have the same physical idea applied to the new objects of quantum theory. By Wigner's theorem the symmetries must be implemented via a unitary representation of the group, and this is how we obtain evolution in quantum theory. If we denote this idea by $\Xi$ then,

$$\Xi \in \mathcal{I}_{\mathcal{QM}} \cap \mathcal{I}_{\mathcal{CM}}$$

We believe this is where the problem lies. This notion of evolution, $\Xi$, is not even sufficient for all classical theories such as general relativity. By classical theories we mean theories where the effects are given by a commutative algebra. So we believe it's $\Xi$ that needs to be removed from $\mathcal{I}_{\mathcal{QM}}$ and replaced with something else, say $\Xi'$. To do this we use the idea of evolution in thermodynamics as starting point along with ideas of probability and inference. Our starting assumption is that,

$$\Xi' \in \mathcal{I}_{\mathcal{QM}} \cap \mathcal{I}_{\mathcal{TD}}$$

By idea of evolution in thermodynamics we mean the equilibrium principle or the assumption that systems trend to equilibrium. As we will show, though this assumption is much weaker than $\Xi$ it does lead to a notion of time with expected properties of time.

We are interested in comparing physical theories. Suppose $PT_\alpha$ and $PT_\beta$ are physical theories with the same application domain $A_\alpha \equiv A_\beta$. Now each theory will have its own physical objects, idealization process, and correspondence rules. The two theories are the same if all these are the same. These will be different forms of the same theory. We are interested in a relation of the form, $PT_\alpha$ is better than $PT_\beta$ for $\alpha, \beta \in I$,

$$PT_\alpha \succ PT_\beta,$$

that is, $PT_\alpha$ says more about the structure of $A_\alpha \equiv A_\beta$ than $PT_\beta$. All these can be made precise, see [2]. In such a case there must exist some kind of forgetful map,

$$PT_\alpha \xrightarrow{F} PT_\beta.$$

We expect these maps to map objects of $PT_\alpha$ to objects of $PT_\beta$ and laws of $PT_\alpha$ to laws of $PT_\beta$. The maps between physical theories need not necessarily be forgetful maps, they can be approximations like in the case of quantum to classical maps or some other general map. But they should take objects of the physical theory to objects and laws to laws. The theories in question are quantum mechanics $\mathcal{QM}$ and thermodynamics $\mathcal{TD}$. We expect both thermodynamics and quantum mechanics to be large domain of application theories and we expect quantum mechanics to be better than thermodynamics.

Before quantum physics, classical mechanics was considered universal. Statistical mechanics provides a connection between classical mechanics and thermodynamics. Denote classical mechanics by $\mathcal{CM}$. Statistical mechanics is the map,

$$\mathcal{CM} \xrightarrow{F_C} \mathcal{TD}.$$

Since quantum mechanics should replace classical mechanics as an attempt at a universal theory we should expect a statistical mechanics type connection between quantum mechanics and thermodynamics.

$$\mathcal{QM} \xrightarrow{F_Q} \mathcal{TD}.$$

What we want to do is guess the laws of $\mathcal{QM}$ intelligently. We know $F_Q$ should exist. We know what $\mathcal{TD}$ is, at least vaguely. The aim is to find a law for the quantum theory that would satisfy the above relation. The thing we *cannot* do is a statistical mechanics-like procedure for the quantum case because probabilities in quantum mechanics cannot be viewed statistically, because of the PBR theorem.

- Interpretations
  - Intrinsic Realism
    - $\psi$-ontic
      - Many worlds, etc.
    - $\psi$-epistemic
      - Ballentine [20]
      - Spekkens [33]
      - Sabine Hossenfelder and others?
  - Participatory Realism
    - About Belief ($\psi$-doxastic)
      - QBism [22],[23],[79]
      - ConSol [32]
    - About Knowledge
      - Neo-Copenhagen [14],[74]
      - Wheeler [26],[27]
      - Relational [28]
      - Fuchs & Peres [21]

The PBR theorem rejects $\psi$-epistemic interpretations, i.e., quantum states can't be interpreted as having information about an underlying ontic state. Its conclusions however don't have any effect on the participatory realist interpretations. Instead of trying to find some workaround that would allow us to evade this constraint, we would like to take the PBR theorem seriously. In the next section will be participatory realist perspective.

# 2 | Guidance from Thermodynamics

This chapter aims to seek guidance from thermodynamics to guess a law of evolution for quantum theory. In particular, we are interested in the notion of time and the constraints on evolution. For that purpose, it's important to study the relation between quantum theory and thermodynamics.

## 2.1 | Laws of Thermodynamics

It's important to note what the laws of thermodynamics say and what they don't. Thermodynamics is the study of thermal equilibrium. Its state space is the space of equilibrium states of the system. These equilibrium states, by definition, do not evolve if left to themselves. The states of thermodynamical systems change because we do things to them. The transitions between the equilibrium states are called thermodynamic processes. These processes are a result of an outside intervention under a set of control variables. Thermodynamic laws are different from other physical theories. Thermodynamic laws don't say how systems evolve from one state to another. The laws of thermodynamics describe the properties these equilibrium states have, their relation with control variables, and thermodynamic processes. The zeroth law expresses the transitivity of thermal equilibrium. It also implies the existence of temperature as a parametrization of equilibrium states. The first law introduces the notion of energy. It relates mechanical work and the notion of heat using the notion of energy for systems at equilibrium. The existence of the special variable called energy and the fact that the amount of energy in an equilibrium state is independent of how the state was arrived at is the first law of thermodynamics. It provides a connection between mechanics and thermodynamics. It allows the notion of energy to be used as one of the parameters describing the equilibrium states. The different formulations of the second law tell us what kind of transitions of the system are not allowed to happen. The second law leads us to the existence of an entropy function that tells us which processes can occur and which cannot. The second law from a purely classical thermodynamics sense does not talk about the trend of non-equilibrium states towards equilibrium states. There are various axiomatizations of thermodynamics. Depending on what they choose as fundamental each author has their own way of axiomatization. A system satisfies the second law of thermodynamics if there exists a function, entropy, that tells us which processes are allowed. Lieb & Yngvason highlight some basic intuitive assumptions classical systems satisfy which guarantee that the system will satisfy the second law. Starting from a few basic assumptions they explicitly construct an entropy function.

The equilibrium states correspond to points in the space of states $\Gamma$ of the thermodynamic system. In the next step, the space $\Gamma$ is given the structure such that one variable is the energy $U$ and the remaining variables come from some pre-theory. It's at this stage the notion of energy enters physics. In classical thermodynamics, the variables coming from pre-theory were volume, pressure, etc. The space of states $\Gamma$ is taken to have the structure of a convex set in $\mathbb{R}^{2n+1}$, where one variable is energy $U$ and the remaining are mechanical variables. The scaling of the system $\Gamma$ by $\lambda \in \mathbb{R}^+$ will be denoted by $\Gamma^\lambda$.[1] For $\lambda, \mu \in \mathbb{R}^+$, we should expect $(\Gamma^\lambda)^\mu = \Gamma^{\lambda\mu}$. Two systems $\Gamma_1$ and $\Gamma_2$ can be combined and the states are points from the cartesian product, $\Gamma_1 \times \Gamma_2$.

A state $\sigma$ is called adiabatically accessible from $\rho$ if there exists a thermodynamic process that takes $\sigma$ to $\rho$. We denote it as,

$$\rho \prec \sigma.$$

This relation $\prec$ is intended to introduce some ordering on the space of states $\Gamma$. A process is irreversible adiabatic if $\rho \prec \sigma$ and not $\sigma \prec \rho$ we denote this by $\rho \prec\prec \sigma$. Two states $\rho$ and $\sigma$ are adiabatically equivalent if $\rho \prec \sigma$ and $\sigma \prec \rho$ and we denote it by $\rho \sim \sigma$. If $\prec$ satisfies,

$$\rho \sim \rho.$$

$$\rho \sim (\lambda\rho, (1-\lambda)\rho), \qquad \lambda \in [0,1].$$

$$\rho \prec \sigma, \lambda > 0 \implies \lambda\rho \prec \lambda\sigma.$$

$$\rho \prec \sigma \text{ , } \sigma \prec \gamma \implies \rho \prec \gamma.$$

$$\rho \prec \rho', \sigma \prec \sigma' \implies (\rho, \sigma) \prec (\rho', \sigma').$$

$$(\rho, \epsilon\gamma) \prec (\sigma, \epsilon\gamma'), \epsilon \to 0 \implies \rho \prec \sigma.$$

All of these have very simple physical meaning. A state-space $\Gamma$ is said to satisfy the comparison hypothesis if all pairs of states in $\Gamma$ are comparable i.e., either $\rho \prec \sigma$ or $\sigma \prec \rho$. For a state-space $\Gamma$ satisfying the above order relations and the comparison hypothesis one can construct an entropy function as follows. For systems satisfying these conditions, Lieb and Yngvason, [61],[62],[63], define a function,

$$J(\rho) := \sup\{\lambda \mid ((1-\lambda)\rho_0, \lambda\rho_1) \prec \rho\}.$$

This function is the entropy function and hence the function serves as our entropy function. The function is unique upto affine transformation. A system with the $\prec$ satisfying the above conditions will have the second law. The existence of such a function $J$ is called the entropy principle.

The second law, in this view, refers to processes of a system that begin and end in equilibrium states and says that the entropy of the final state is never less than that of the initial state. The second law does not say anything about systems reaching an equilibrium state. The laws of thermodynamics don't say anything about the evolution of isolated systems. Summarizing, the zeroth law says about transitivity of equilibrium and gives rise to the concept of temperature, first law introduces the variable, energy, as a special variable that characterizes equilibrium states, and the second law introduces the concept of entropy that says what transitions between equilibrium states are allowed.

### 2.1.1 | The Equilibrium Principle

It's a common belief among physicists that thermodynamics says something about the flow of time. These discussions often assume that the second law is where time asymmetry enters thermodynamics. Is it really the second law where time asymmetry enters? The starting point of most formulations of thermodynamics is equilibrium states which by definition don't change in time. The second law, after all, provides a relation between variables at equilibrium.

[1] This scaling will not be possible in the quantum case.

The question we should be asking about the second law is regarding its scope. What's the scope of the second law of thermodynamics? If we take the definition of equilibrium seriously, and if the second law is concerned with transitions from one equilibrium state to another, is it applicable to the universe as a whole? so that we can say the universe's entropy is increasing or does it only apply to sub-systems of the universe? If it does apply to the whole universe, how does the universe go from one equilibrium state to another and why? Equilibrium states are by definition supposed to stay unchanged and the universe has nothing influencing it from 'outside'. Isn't it supposed to remain in the same state once attained? It is usually assumed that the approach to equilibrium is accompanied by an increase in entropy and that this is a consequence of the second law. Perhaps the second law could be extended to include non-equilibrium systems, but thermodynamics in its traditional sense doesn't say anything about systems trending towards equilibrium. The idea that the second law says something about the flow of time lacks a good theoretical foundation. The origins of the flow of time might be something deeper as pointed out by Brown and Uffink in [65].

There is however no question that thermodynamics, if not its second law, makes time-asymmetric claims. The spontaneous movement from non-equilibrium to equilibrium happens and is assumed throughout the field. This most certainly is a time asymmetric notion. This fact comes logically before the laws of thermodynamics. The existence of this tendency should not be confused with the zeroth law of thermodynamics which says about the transitivity of interbody thermal equilibrium. The tendency towards equilibrium is a more basic principle than that of transitivity. Brown, Uffink [65] call this the Equilibrium principle,

**Postulate. (Equilibrium Principle)** *An isolated system in an initial state will spontaneously attain a unique state of equilibrium.*

The equilibrium principle guarantees the existence of equilibrium states for isolated systems. These states by definition remain the same once they are attained. The equilibrium principle tells us about the uniqueness of the equilibrium state; i.e. for any initial state of an isolated system, there is exactly one state of equilibrium to which the system will reach. This is where the time asymmetry arises at the most basic level. The existence of such states itself is a time-asymmetric notion. The time asymmetry is built into thermodynamics by way of the notion of equilibrium. Time asymmetry may not be needed for a statement of the second law but is definitely required in the interpretation of the notion of equilibrium. Instead of getting into a discussion about the existence or the non-existence of a connection between the second law of thermodynamics and the notion of time[2], we will focus on the equilibrium principle which we know for sure has a connection with the notion of time. The equilibrium principle is more relevant to our discussion than the second law of thermodynamics.

## 2.2 | Evolution in Quantum Theories, Revisited

In this section, we try to guess the time evolution in the quantum case. The equilibrium principle is the only law of thermodynamics that's similar to the laws of other physical theories. It tells us about states going to other states like in other physical theories. It doesn't however say anything about how states are going to evolve. Our aim here is limited.

[2]It's important to be able to identify patterns, but it's more important to differentiate between real patterns and false patterns. I believe the naive assumption of the second law's importance in the arrow of time, commonly held by most physicists, might have been detrimental to progress in our understanding of the notion of time.

It's to construct a quantum counterpart that is compatible with the equilibrium principle of thermodynamics. That is to say, this new construction would allow us to arrive at the notion of equilibrium from the quantum side.

This task can be further divided into the following questions; how does a system evolve given the structure of observables and states? why that might be the case? what mathematical structure of the physical theory plays a crucial role in the said behavior of the system? We will start with the second and the third question first.

### 2.2.1 | The First Law

The purpose of the laws of physics is to recognize some regularity in naturally occurring processes that are in general complicated and unpredictable. These regularities are then formulated as laws of physics. The class of objective states should form the building blocks for everything objective we see in quantum theory including the laws of evolution. The objective states are 'limit states' that don't evolve once reached. In thermodynamics, the equilibrium states are the states that don't change by definition. Objective states should correspond to equilibrium states in thermodynamics. This allows us to make use of the laws of thermodynamics. Equilibrium states are closely related to the Hamiltonian of the system. The first law, in the sense of [61], says that energy is a coordinate in the description of equilibrium states.

**Postulate. (First law of Thermdynamics)** *Energy is a coordinate for the description of equilibrium states.*

Let the operator corresponding to energy observable be $H$. This postulate relates the class of objective states and the Hamiltonian of the system. For a finite system $\mathcal{A}$ with the Hamiltonian $H$, the time translation symmetry is given by, $\alpha_t = e^{-iHt}(\cdot)e^{iHt}$. The equilibrium state at temperature $1/\beta$ is characterized by the Gibbs' condition given by,

$$\rho^\beta(A) = Tr(e^{-\beta H}A)/Tr(e^{-\beta H}).$$

The set,

$$\mathcal{S}_E = \{\rho \in \mathcal{S}(\mathcal{H}) \mid \; Tr(\rho H) = E\}.$$

represents the states that have the expectation value $E$ for the operator $H$. It was shown in [41], [44], that equilibrium states maximize entropy under the constraint of the expectation value of energy. For a fixed expectation value of $H$ the Gibbs states are maximum entropy states.

$$\arg\sup_{\sigma\in\mathcal{S}_E} J(\rho) = \rho^\beta.$$

The state $\rho^\beta = e^{-\beta H}/Tr(e^{-\beta H})$ commutes with every $P$ such that $[P, H] = 0$. The symmetries of the system are generated by self-adjoint operators that commute with $H$. So, $\rho^\beta \in \mathcal{S}_{\mathfrak{g}}$, what we expect in an intersubjective situation. Though this hints some links between objective states and symmetries through QRF transformations, we will not study it further as it's not important for the rest of the paper.

For the case of any general system, the KMS condition given by,

$$\rho^\beta(A\alpha_{i\beta}(B)) = \rho^\beta(BA).$$

characterizes the equilibrium states.

Given an equilibrium state, the thermal time hypothesis allows us to recover the Hamiltonian. The notion of thermal time was put forward by Connes and Rovelli [80]. The thermal time of a system is the natural flow induced by its state on its algebra of observables. So far, however, the thermal time has remained a rather abstract notion with a few concrete applications [82]. For a von Neumann algebra $\mathcal{A}$ acting on a Hilbert space $\mathcal{H}$, with a faithful normal state $\rho(\cdot) = \langle \Omega_\rho | \cdot \Omega_\rho \rangle$, where $|\Omega_\rho\rangle$ is a cyclic and separating vector for $\mathcal{A}$, we have the embedding of $\mathcal{A}$ in $\mathcal{H}$ via the map, $A \mapsto A|\Omega_\rho\rangle$, the $^*$-operation on $\mathcal{A}$ gives an anti-linear operator $S$ on $\mathcal{H}$,

$$\begin{tikzcd}
\mathcal{A} \arrow[r, "A \mapsto A^*"] \arrow[d, "A \mapsto A|\Omega_\rho\rangle"'] & \mathcal{A} \arrow[d, "A \mapsto A|\Omega_\rho\rangle"] \\
\mathcal{H} \arrow[r, "S\colon A|\Omega_\rho\rangle \mapsto A^*|\Omega_\rho\rangle"] & \mathcal{H}
\end{tikzcd}$$

Then, $\Delta = S^* S$ is a positive self-adjoint operator. The Tomita modular operator $\log \Delta$ implements a one-parameter automorphism group $t \mapsto \alpha_t^\rho$ of $\mathcal{A}$ where,

$$\alpha_t^\rho(A) = e^{it \log \Delta} A e^{-it \log \Delta}.$$

called modular automorphism. The parameter $t$ is called modular time. Tomita-Takesaki theorem says that the state $\rho$ satisfies the KMS condition with respect to the flow $\alpha_t^\rho$ with inverse temperature $\beta = -1$. Given the time translation, one can determine the equilibrium states by the KMS condition, and assuming a state is in equilibrium one can construct the one-parameter group of automorphisms with respect to which the state satisfies the KMS condition.

### 2.2.2 | Why not Unitary?

Before discussing why we should get rid of the unitary evolution we should discuss why the unitary evolution was adapted into quantum physics in the first place. By unitary we mean the Schrödinger equation, we have no hostility with any other unitaries.

The first step is getting the unitaries through Kadison-Wigner's theorem. Each process in quantum theory is described by a unique completely positive unitary map $\alpha$ on the algebra of observables on a Hilbert space $\mathcal{H}$. If the transformation is such that it preserves the convex structure of state space, Kadison-Wigner theorem says that the transformation should be of the form,

$$\alpha(\rho) = U \rho \, U^*,$$

for some unitary or anti-unitary map $U$ on the Hilbert space $\mathcal{H}$. The next step is to assume that the system has certain symmetries, i.e., there exists a group of transformations that preserve the convex structure of state-space. Denote the group of transformations by $\mathcal{G}$. Now by Kadison-Wigner theorem, this requirement means that we have for each element $g \in \mathcal{G}$ a Kadison-Wigner automorphism,

$$g \mapsto \alpha_g.$$

For each element $g \in \mathcal{G}$ we have a unitary or anti-unitary map $U(g)$. So in the case of a continuous group we end up requiring a unitary representation of the group $\mathcal{G}$ on the Hilbert space.

It's very important to understand how we reached this stage i.e., how did we assume the group $\mathcal{G}$ should be the symmetries of the system. This assumption comes from a pre-theory, i.e., classical mechanics. In classical mechanics, our laws of evolution came from certain symmetry groups such as the Galilean group or Lorentz group. When quantum mechanics was developing in the early twentieth century, Heisenberg changed the objects of the theory. Observables were changed from real-valued functions to self-adjoint operators on a Hilbert space. However, the evolution part was taken from the classical theories and implemented on the new objects of quantum theory. Our belief in unitary evolution lies in the success of evolution in classical physics. However, this notion of evolution is already in deep trouble within classical physics itself. General relativity for example doesn't obey such a law of evolution. So we are in a situation where we trust the unitary evolution in quantum theory based on its success in *some* classical theories. This is an extremely weird situation to be in not only because we are relying on a faulty pre-theory for evolution in quantum theory but this notion of evolution wasn't even sufficient for classical theories. There is no reason why we should be trusting the laws from classical physics that's not even general enough to be taken as standard evolution for all classical theories. Our trust in unitary evolution is misguided. No offense to Erwin Schrödinger, but I believe it's the Schrödinger equation that needs to go. Recent work of Höhn, Smith and Lock [56] show that a unitary notion of evolution cannot be covariant. Our aim is different, we expect quantum theory to stand on its own feet. It should be noted that a replacement for Schrödinger equation will not be compatible with the measurement process just by being non-unitary. There needs to be a unifying 'law' for both the measurement process and the replacement for Schrödinger equation. Our approach is to start over, take seriously the measurement process and the unifying law is Bayesian inference i.e., updating the state based on the newly available information. Inference rules apply to every probabilistic theory and hence a very general law. For us, time is an emergent concept arising from quantum processes. So it is compatible with the inference rules.

#### 2.2.2.1 | The Role of Thermal Time Hypothesis

Given a state $\rho$ of the system $\mathcal{A}$, the thermal time flow $\alpha^{\rho}_{-t}$ is the modular flow associated with the system $\mathcal{A}$. The problem however is that the thermal time hypothesis associates to every state a flow. Not all these flows have physical meaning. Random states don't correspond to physically meaningful flows. The modular group cannot be directly related to the flow of time. The thermal time hypothesis does however tell us something about the notion of time and its relation to time. There is a need to explain why certain states are special compared to others. So the question then is what is the connection between the modular theory of operator algebras and thermodynamics? What it allows us to do is construct the Hamiltonian. This is similar to what the first law says, that energy is a variable for the description of equilibrium states. What modular theory and the thermal time hypothesis allow us to do is construct this variable from the equilibrium states. The equilibrium states contain data regarding the Hamiltonian of the system and by Tomita-Takesaki theory this data can be recovered. Starting from the Hamiltonian of the system one obtains the equilibrium states by the KMS condition and conversely assuming a state is an equilibrium state one can construct the Hamiltonian. One can then ask what is the difference between these two? Does using equilibrium states as starting point add any new methods to physics? If the two were the same then this approach wouldn't add anything new to physics. It's important to clarify the difference between the approach where Schrödinger equation is the starting point and the approach where the equilibrium states are the starting point. Schrödinger equation is a constraint on the evolution of the system. It tells us that systems must follow the unitary evolution, coming from the symmetries of the system. If our starting point is equilibrium states, the constraint is that the system tends to the equilibrium states. It is not as strict a constraint as Schrödinger equation. One of the biggest disadvantages of the Schrödinger equation is the following; if the constraints coming from Schrödinger equation were true, i.e, the evolution of a system was given by unitary one-parameter group generated by the Hamiltonian, it would mean an isolated system, out of equilibrium, would never approach equilibrium. Equilibrium states are stationary points for this one-parameter group of transformations. We expect this to be not the case. We expect systems out of equilibrium to trend to equilibrium. Then to work around this issue, more weird assumptions are made in quantum thermodynamics literature adding to more confusion. If the constraints are relaxed a bit and we postulate only that the systems trend to equilibrium then we have a lot more flexibility and many more physically observed phenomenon can be explained while also recovering the information regarding Hamiltonian through modular theory. The only difference between the two is the starting point, in the standard case, the symmetries of the system are the starting point, in our case the equilibrium states are the starting point. The standard approach is more constrained. The equilibrium principle only says things about the system at an asymptotic behavior level. It's doesn't have anything to say at an individual quantum process level. So it doesn't cause any conflict with the individual quantum processes. Since the equilibrium principle doesn't cause any conflict with the measurement postulate the notion of time is compatible with the measurement postulate.

To explain why equilibrium states are special out of all the states, we, unfortunately (fortunately?) have to invoke the maximum entropy-minimum relative entropy[3] methods for now and hope some better explanation appears in the future.

#### 2.2.2.2 | A Number for Time

The processes of interest to us are those that take the state closer to an equilibrium state. Our next step should be to define a notion of 'time' for systems tending to equilibrium. From a pragmatic point of view, the dynamics of the system should be about studying how the system tends to an equilibrium state. Since the equilibrium states are invariant under the time-translation symmetry of the system, this boils down to studying the loss of asymmetry. To make quantitative statements regarding this, we rely on the resource theory of asymmetry.

#### Resource Theory of Asymmetry

The origins of resource theory approaches can be traced back to the ideas in Lieb-Yngvason's approach to thermodynamics. A resource theory consists of a subset $\mathcal{F}$ of all quantum channels called free operations, closed under composition, contains the identity and a set of free states $\mathcal{S}$. Any state that cannot be created from free states by performing the free operations is called a resource state. If $\rho$ is a resource state and some other state $\sigma$ can be reached by performing a free operation on $\rho$ then the state $\rho$ is a better resource than $\sigma$.

$$\alpha(\rho) = \sigma \iff \rho \succ \sigma,$$

where $\alpha \in \mathcal{F}$. If the two states can be interconverted i.e, $\rho \succ \sigma$ and $\sigma \succ \rho$ then they lie in the same equivalence class $[\rho]$. The above-defined relation gives a partial order on the equivalence classes. Since it's not possible to convert a resource state into a free state using free operations alone we are interested in is what kind of quantum operations are needed to convert a resource state into a free state. The constraints on such operations will come from physical requirements.

Resource theory of asymmetry is a framework for quantifying the asymmetry of states and operations. Suppose we have a unitary representation of a group $\mathcal{G}$, $g \mapsto U(g)$ on the

[3] Jaynes'-Ochs-Bayer maximum entropy principle states that equilibrium states are maximum entropy states subjected to the constraint that the expectation value of energy is fixed. [41], [44].

Hilbert space $\mathcal{H}$, free states of the resource theory of asymmetry of the group $\mathcal{G}$ are states with no asymmetry,

$$U(g)\,\rho\,U^{-1}(g) = \rho, \quad \forall\, g \in \mathcal{G}.$$

We denote the totality of such states by $\mathcal{S}_{\mathcal{G}}$. A quantum channel $\alpha$ is a free operation if,

$$U(g)\,\alpha(\rho)\,U^{-1}(g) = \alpha(U(g)\rho\,U^{-1}(g)), \quad \forall\, g \in \mathcal{G}.$$

We denote the set of all free operations by $\mathcal{F}_{\mathcal{G}}$. In order to study asymmetry of states, we first need a measure of asymmetry. A function $f : \mathcal{S}(\mathcal{H}) \to \mathbb{R}$ is a measure of asymmetry with respect to the symmetry group $\mathcal{G}$ if it satisfies the following conditions,

$$f(\alpha(\rho)) \leq f(\rho), \quad \forall\, \alpha \in \mathcal{F}_{\mathcal{G}}.$$

$$f(\rho) = 0, \quad \forall\, \rho \in \mathcal{S}_{\mathcal{G}}.$$

The first condition says that allowed operations can only make the system more symmetric. The second convention fixes the value for symmetric states and makes the function non-negative. For the existence of asymmetry measures check out [94]. If the system is acted upon by an operation that takes it closer to an equilibrium state, then the state must lose asymmetry in the process. The measures of asymmetry can be used to measure this change in asymmetry. Any changes in the asymmetry measure would indicate some change in the system. The measures of asymmetry can be used to parametrize this notion of time.

#### 2.2.2.3 | A Description of Time

The equilibrium principle is one of the very basic postulates of thermodynamics. We can't say for sure whether it's applicable for quantum theory i.e., a quantum system in an initial state reaches a unique equilibrium state. However, we still expect systems to reach an equilibrium state, not necessarily uniquely determined by the initial state. Instead of the full equilibrium principle, we will use the following weaker equilibrium principle,

**Postulate. (Weak Equilibrium Principle)** *Every isolated system in an initial state reaches an equilibrium state.*

The weak equilibrium principle removes the uniqueness requirement in the equilibrium principle. The operations of interest to us take the system closer to equilibrium. The allowed operations or processes are those which reduce the asymmetry of the state of the system concerning 'time-translation' symmetry denote this collection of allowed operations by $\mathcal{F}_{\mathcal{T}}$. Every event changes the state through some allowed operation and hence the state before the event and the state after the event will have different associated asymmetry. For a system tending to an equilibrium state, asymmetry measures can be used to distinguish different instants of time. So, the weak version of the equilibrium principle should be enough to define 'time'. Asymmetry measures map states onto real numbers, so to each state, one associates a real number.

A tree $\mathcal{T}$ consists of an initial state and a collection of quantum processes $\mathcal{F}_{\mathcal{T}}$ taking the initial state to a final equilibrium state. We have an order on the tree $\mathcal{T}$ as follows, we say $\rho$ precedes $\sigma$ if there is an operation $\alpha \in \mathcal{F}_{\mathcal{T}}$ such that $\alpha(\rho) = \sigma$.

$$\rho \prec_t \sigma, \quad \text{if} \;\; \exists\, \alpha \in \mathcal{F}_{\mathcal{T}},\; \alpha(\rho) = \sigma.$$

As stated in our postulate for time earlier in §2.4, time changes for the system because of these processes, and we want to associate numbers to each state $\rho \in \mathcal{T}$. Suppose the system has followed a tree of events, $\mathcal{T}$, we can associate to each state $\rho \in \mathcal{T}$, its associated asymmetry $f(\rho)$. Define the associated 'time' for the state $\rho \in \mathcal{T}$ to be,

$$t(\rho) = 1/f(\rho).$$

We expect the notion of time to have a few properties. Given two states $\rho$ and $\sigma$ in $\mathcal{T}$ with $\rho \prec_t \sigma$, by definition we have, $f(\rho) > f(\sigma)$. This means that,

$$t(\rho) < t(\sigma) \quad \text{whenever} \quad \rho \prec_t \sigma.$$

The notion of time inherits an order and hence a topological structure from these maps. It should be noted that if we remove the requirement that allowed operations decrease asymmetry then we will be able to 'go back' in time. It doesn't mean the system physically went back in time, just the number we associated with events went back. It should also be noted that the number time we assigned is not unique. Depending on how the asymmetry measure changes, we can make claims about discreteness or continuity of the flow of time. We would like to stay agnostic about this.

As stated before, we expect a few features from whatever number we associate with the notion of time. These features should include, the distinction between various instants of time, the existence of a notion of time instances close to each other, the possibility of arranging the instants of time on a one-dimensional manifold, an order, a notion of 'now', and a metric structure. As discussed before every observation brings with it a notion of before and after. This is closely related to a preferred instance of time representing the 'now'. We cannot however recover any metric structure from this notion of time. It's an a priori assumption that time is comparable from the start of the universe to its end. The question one could ask then is the following, Is the flow of time in the early universe really comparable to the flow of time now? Is such an assumption necessary?

#### From Synthetic to Analytic?

The description of time we have given so far is a synthetic one. It's described in terms of some inequalities and such. The physics community is usually adjusted to more analytic stuff, we usually work with objects that have nicer mathematical structures that allow us to do calculus for example. Reaching this stage however is complicated, one has to understand the properties of the object of interest well enough and those additional properties give us the necessary structure to develop analytic methods. We don't have anything to say as to how to reach this stage. Perhaps there needs to be a lot of experimental data on how very-large-scale quantum systems trend to equilibrium and from this data extract property that would tell us about some underlying structure. Information-theoretic postulates such as Landauer's principle can also act as useful methods in this regard. While it's tempting to use Dilation theorems, we feel it's a faulty shortcut path to take. We expect quantum mechanics to have the capacity to explain closed systems as well and we don't feel it's right to think of the completely positive map of the process as a unitary operator on some bigger Hilbert space, such a unitary map need not have any physical meaning. We would hence refrain from making any premature claims about an equation of state.

## 2.3 | Consequences

The notion of time we just defined is purely quantum and independent of any quantization procedures. The only structure needed was the non-commutativity of observables to explain why time emerges and a weak version of the equilibrium principle to assign 'time' numbers. Time as we have defined applies to any quantum theory and should even apply to generalized probabilistic theories with non-commutative structure. In this section, we will discuss some of the consequences and explain how it's closely related to the notion of time we experience in real life.

### 2.3.1 | Basic Properties

Time as we defined in the previous section satisfies the basic properties expected from 'time'. Since the asymmetry decreases as the system evolves we have that,

$$t(\rho) < t(\alpha(\rho)),$$

for every allowed operation $\alpha$. So the notion of time carries a natural order, arrow of time is built in the definition. Every quantum process carries with it a notion of before and after. The notion of before and after is closely related to the notion of 'now'. The map,

$$t : \rho \mapsto t(\rho)$$

arranges states $\rho \in \mathcal{T}$ on a one-dimensional manifold and hence we have the notion of 'close to each other' or 'distant from each other' for different events.

#### 2.3.1.1 | Relational Nature of Time

What we have discussed so far is the notion of time in isolated systems tending to equilibrium. What's important and revealing is how this applies to open systems. To demonstrate this we will consider a modified Wigner's friend type experiment.

Let Wigner be capable of observing a system $\mathcal{A}_W$ acting on a Hilbert space $\mathcal{H}_W$ and his friend is capable of observing a system $\mathcal{A}_F$ acting on a Hilbert space $\mathcal{H}_F$. The combined system is $\mathcal{A}_W \otimes \mathcal{A}_F$ acting on $\mathcal{H}_W \otimes \mathcal{H}_F$. They are allowed to come together and discuss the information they have i.e., Wigner's friend acts as instruments that allow Wigner to acquire information about the observables in $\mathcal{A}_F$. We say they are in contact when Wigner and his friend come together and update their information about the other's system. We say the two systems are in equilibrium with each other if the systems remain in the same state after contact. Wigner and his friend have reached equilibrium means their states are not changing after contact. Denote by $\mathfrak{O}_{WF}$ the equilibrium states for the system $\mathcal{A}_W \otimes \mathcal{A}_F$, and $\mathfrak{O}_W = \mathfrak{O}_{WF}|_{\mathcal{A}_W \otimes I}$. We can assign two different notions of time, one for the combined system and one for the individual systems.

Wigner starts with state $\rho_1$ on $\mathcal{A}_W \otimes \mathcal{A}_F$ and his friend starts with state $\rho_2$. Wigner was two notions of time, one for the system $\mathcal{A}_W$ and one for the combined system $\mathcal{A}_W \otimes \mathcal{A}_F$ denote them by $t_W = 1/f_W$ and $t_{WF} = 1/f_{FW}$ respectively. When not in contact, Wigner can only know about what's happening to the system $\mathcal{A}_W$ and update $\rho_1|_{\mathcal{A}_W}$ and hence only the time $t_W$ is known, he can only speculate about the combined system $\mathcal{A}_W \otimes \mathcal{A}_F$.[4] On the other hand, his friend has her own time $t_F$ for the system $\mathcal{A}_F$. After contact, they update their information about the total system and adjust their clock accordingly.

The interpretation of this thought experiment is pretty intuitive. Suppose Wigner is locked in a room without any contact with the outside world, Wigner's clock updates based on the stuff that influences it. The outside world might be evolving at a different rate. But once Wigner opens the door and goes outside he might notice that his clock was running faster or slower based on what he observes and will have to recalibrate the clock.

[4] If he assumes the universe evolves at the same rate then he can say $t_{FW} = t_W$.

### 2.3.2 | Relation to Unitary Time

What we have tried to do so far is to develop a replacement for unitary evolution. The unitary evolution, however, is a very good approximation for most purposes. So, it is important to explain how the unitary evolution can approximate to the notion of time we just described.

What we are interested in is the relation between dynamics and symmetries. That is, a relation between the notion of time we have described so far and the standard textbook version of evolution. For $\epsilon > 0$ define,

$$\tau_\epsilon(\rho) = \inf\{s \mid J(\rho,\ e^{-isH}\rho\, e^{isH}) \geq \epsilon\},$$

and is infinity if for all $s$, $J(\rho,\ e^{-isH}\rho\, e^{isH})$ is less than $\epsilon$. Here $J(\cdot\,,\cdot)$ is the relative entropy. As shown in [94], for a 'time translation' invariant quantum channel $\alpha$, using the monotonicity of the measure of distinguishability and the commutativity of $\alpha$ with 'time translation' we get,

$$\begin{aligned}
\tau_\epsilon(\alpha(\rho)) &= \inf\{s \mid J(\alpha(\rho), (e^{-isH}\alpha(\rho)e^{isH})) \geq \epsilon\} \\
&= \inf\{s \mid J(\alpha(\rho), \alpha(e^{-isH}\rho\, e^{isH})) \geq \epsilon\} \\
&\geq \inf\{s \mid J(\rho, \rho(s)) \geq \epsilon\} \\
&= \tau_\epsilon(\rho).
\end{aligned}$$

So we have,

$$1/\tau_\epsilon(\alpha(\rho)) \leq 1/\tau_\epsilon(\rho).$$

Since symmetric states are invariant under the transformation $\rho \mapsto\ e^{-isH}\rho\, e^{isH}$, $1/\tau_\epsilon(\rho)$ will be zero in that case for all $\epsilon > 0$. The function $f = 1/\tau_\epsilon$ is a measure of asymmetry. So the standard unitary evolution is indeed related to our definition of time. This is applicable to any measure of distinguishability, $D : \mathcal{S}(\mathcal{H}) \times \mathcal{S}(\mathcal{H}) \to \mathbb{R}$ that only decreases the distinguishability between pairs of states for any quantum operation $\alpha$,

$$D(\alpha(\rho), \alpha(\sigma)) \leq D(\rho, \sigma).$$

If the two states are identical then the measure of distinguishability should vanish.

$$D(\rho, \rho) = 0.$$

Define $\tau_\epsilon^D(\rho) = \inf\{s \mid D(\rho,\ e^{-isH}\rho\ e^{isH}) \geq \epsilon\}$, the function $1/\tau_\epsilon^D$ will be a measure of asymmetry. The one-parameter group generated by the Hamiltonian of the system though not the same as the flow of time is closely related to it.

It's important to note that the above quantification of the approach to equilibrium is unlikely the only way to study it. This approach reveals not much about the approach to equilibrium. The important point of the discussion is that we can define the notion of time, based on the system's approach to equilibrium. What methods we use to quantify this approach to equilibrium is not important to us. Here we have used the invariance of the equilibrium states under the action of time-translation symmetry for the purpose. The definition of time depends on the choice of asymmetry measure. Misner, Thorne, and Wheeler [95], have remarked 'Time is defined so that motion looks simple!'. So, the choice should be such that the dynamics becomes simplified. For most practical purposes the asymmetry measure coming from unitary translation is sufficient. However, such an evolution is not sufficient for the covariant case as discussed in [56]. It's also not compatible with the equilibrium principle of thermodynamics if one wishes to treat the universe as a closed system. Discussing what asymmetry measure we should choose for such theories is not within the scope of this article.

## 2.4 | Emergence of Time in Quantum Theory

Time is a very basic physical quantity. Pre-relativity physics had a very clear notion of time, the absolute time. It was believed that time flows without relations to anything intrinsic to the system. Initiated by Lorentz, Einstein, and others, the twentieth century questioned this notion of time for the first time. Quantum physics which was under development at the time however continued with the old notion of time. Time is absolute in quantum physics and not so in relativity. This conceptual difference is referred to as the problem of time. In classical physics, the notion of time is associated with the time translation symmetry of the system. The symmetries of the system are assumed apriori in quantum theory. The notion of time and other important physical objects in quantum theory comes from these pre-theories. This places quantum theory in an awkward situation of being a more fundamental theory that relies on an outdated theory. The distinction is between the variable 'time' that appears in physical theories for instance in Newtonian physics and the 'time' that flows in our experience. The first is well defined and is unproblematic by itself and is related to the symmetries of the system (as suggested by Rovelli, we can just forget about this notion of time [86]). This notion of time is reversible, in the sense that the dynamics of the physical processes remain well defined when the direction of time is reversed. This is in strong contradiction with our experience of time as we perceive it and also as it exists in other physical theories such as thermodynamics. It does not justify the sense of passage and flowing that we associate with time. So, it is the second that needs explanation. Any notion of 'physical time' should come equipped with a few peculiar features. These features should include, the distinction between various instants of time, the existence of a notion of time instances close to each other, the possibility of arranging the instants of time on a one-dimensional manifold, a metric structure, an order, and a notion of 'now'. One can say time is whatever clocks measure, in such a case the question would be what *do* the clocks measure? What we will be describing in this section is the emergence of such a notion of time in quantum theory.

Consider two observables $A$ and $B$ that are being measured consecutively. If the system is in the state $\rho$ and the outcome of the measurement for the observable $B$ is $B_j$ then the state after the measurement will be given by, $\rho_{B_j} = E_{B_j}\rho E_{B_j}/Tr[E_{B_j}\rho E_{B_j}]$. Suppose the observable $A$ is measured after the above measurement then the probability that the observable $A$ takes the value $A_i$ will be given by $p(A_i|B_j) = Tr[\rho_{B_j}E_{A_i}]$. If the measurement gives the value $A_i$ then the state of the system will be, $\rho_{A_iB_j} = E_{A_i}E_{B_j}\rho E_{B_j}E_{A_i}/Tr[E_{A_i}E_{B_j}\rho E_{B_j}E_{A_i}]$. If the two observables $A$ and $B$ are compatible then $\rho_{A_iB_j} = \rho_{B_jA_i}$. Suppose $A$ and $B$ are compatible then the measurement sequence $A \to B \to A$ is equivalent to the measurement sequence $A \to A \to B$. Hence in the first sequence of measurements, the value of the observable $A$ obtained in the last measurement will be the same as the value in the first measurement. Commuting observables can be measured so that the values of all the observables can be known at once, in the sense that if you conduct a measurement of either observable $A$ or $B$, then it will be possible to say with certainty what the values are going to be. Since the algebra of observables is non-commutative, having complete knowledge about the observables is not possible. The system can always be measured to get new information. Non-commutativity forces the system to always have some information about it and the observer[5] can keep obtaining new information from it.[6] This is the reason for irreversibility in quantum theory. We will call a process by which an observer updates their knowledge about the system an observation. An observation need not necessarily change the state, for example, measuring

[5]By observer we don't mean a conscious being.

[6]They would also lose some of the already known information about the system

the same observable again won't give any new information.

Perception of time is an elementary experience. Time changes for an observer if the observer notices some change in the system. If the observer notices no change at all then it's equivalent to no change in time. So perhaps it's the other way round? When a system is measured for an observable, before the measurement the value of the observable is unknown, and after the measurement, we know the value of the observable. Every quantum process brings with it a notion of before and after. So it might not be that if the observer notices no changes in the system means time is not changing, but the other way around. It's **because** of the observations time changes for the system. Every quantum process changing the state of the system should correspond to some time change and conversely, to each change in time, there is some change in the state.

**Postulate. (Time)** *Time changes for a system* ***because*** *of quantum processes.*

The idea that the notion of time is closely related to change has been noted by many philosophers [71]-[72]. Alain Connes, [80],[81], has been saying for a long time that non-commutativity of the algebra of observables is closely related to the flow of time. Connes has indicated that time might arise from the lack of knowledge of observables. In our case both of these play an important role in the notion of time.

I am not so well read on the philosophy side of the subject, I apologize for not citing important works on the subject which I am sure there are many. In our case, the non-commutativity naturally forces this relation between change and time. In the commutative case, it's possible to have complete information about all observables. This is however not the case in non-commutative cases. Complete knowledge about an observable $A$ means the observer can't have complete knowledge about an observable $B$ that doesn't commute with $A$. An observer can keep obtaining new information from the system. Constant change exists by the mere act of quantum processes and this is exclusive to the non-commutative case. We believe, if we want to take the measurement postulate seriously and think of time as an emergent concept then this relation between change and time plays a crucial role. Time as emerging due to changes occurring in systems is artificial in the commutative case.

This can be stated in a more clickbaity way as 'observation of the system changes time for the observer.' The purpose of using the term observation instead of terms like process or event is that it might provide some intuition to the reader. This tells us why the notion of time even exists. The mathematical structure responsible for this is the non-commutativity of observables in quantum theory. Any generalized probabilistic theory that has some similar mathematical structure will possess this notion of time. This notion of time **cannot** exist in the commutative case, for example, say classical physics. In quantum theory, the non-commutative structure of observables naturally forces such a notion of time and this structure is absent in commutative cases. This is closely related to the notion of 'events' Haag talks about in [90]. Jürg Fröhlich and collaborators at ETH have an interpretation which they call Events-Trees-Histories interpretation or cleverly abbreviated as the 'ETH interpretation' [83], which seem to be closely related to Haag's ideas and hence to the ideas presented here. However, in their approach time is an irreducible concept and events/observations monitor time. In our case time is an emergent concept and events/observations are the reason for the existence of time. The notion of time exists just because of the structure of observables and no additional structure is required. The notion of time as emerging out of random observations or events doesn't give any structure to it. One gets stuck after saying time emerges due to quantum processes changing the system. Philosophers usually stop here.

How does one obtain the number 'time' from such a description of time? We don't have any additional structure that gives us the 'flow' of time yet. We expect time to have this additional structure. To completely describe the emergence of time we have to associate a number to time and show how it has the properties we expect from time. Our next step is to understand how this structure could come into quantum theory when our collection of events have some additional constraints. We will then relate it with the already existing notions of time in quantum mechanics, and show how the traditional unitary evolution is an approximation of this notion of time.

### 2.4.1 | Observers and Instruments

The notion of time we have described so far is defined for the observer. One common feature among all the interpretations belonging to the participatory realism category is their acceptance of objects of quantum theory along with the collapse rule. Participatory realist interpretations such as QBism and relational interpretation have some important roles for observers or agents. These epistemic interpretations of quantum states treat quantum states as carriers of information the observer has about the system. There is however a lot of difference in how observers are treated in different approaches. There is less clarity about what is meant by an observer, or agent in different interpretations. We will do a quick discussion to clarify what we mean by an observer in this paper.

Physics has always been about describing the world without getting into discussions about observers. To describe observer or agent we have to rely on some pre-theory that's not the quantum theory itself and this can lead us into a never-ending, unproductive philosophical discussion. By using the instrumentalist approach, also known as the device independent approach among quantum foundations groups, Ludwig tried to remove this dependency of quantum theory on pre-theories. We will try to describe the notion of an observer in the instrumentalist lingo. One of Ludwig's main motivations was to show that quantum mechanics, with all its subtleties, can be formulated and interpreted consistently without radical changes in our concept of physical reality and to reject the decisive role of human consciousness in the creation of observable events. Now we are in an awkward situation where we intend to describe the notion of an observer using Ludwig's instrumentalist lingo.

#### Observers in Structural Approach

Observables make sense only in the presence of observers. Quantum states are the mathematical representatives of knowledge or belief an observer has about the observables. To each observer, we have an associated quantum state corresponding to their knowledge or belief about the system and conversely, each quantum state represents the knowledge a hypothetical observer has about the system. The mathematical structure of quantum states already contains all the structures needed for the description of an observer. The observer can (should?) be identified with the quantum states of the system.

$$\text{Observer} \hookrightarrow \text{Quantum States}$$

Note that this is a transition from the physical world to abstract mathematical world.

$$A_{QM} \longleftrightarrow \mathcal{QM}.$$

However just like every other physical theory, the objects are fixed by the starting physical ideas of the theory. In case of quantum mechanics the objects we assigned to the physical world are effects and ensembles, identified with projection operators on some Hilbert space and states. The notion of observer is 'emergent' or the structure needed for its description is already contained in effects and ensembles.

The motivation for this treatment is to take the idea seriously that there are no observers external to the universe. When one talks about the value of an observable, there is no need to specify a special state of the observer on the 'observer's Hilbert space'. The state of the system is sufficient and all that matters is how this state changes. Similarly, in this sense facts are 'relative to observers'. This is close to how observers are treated in QBism as well.

There is however a huge part of the quantum foundations community that treats observers as quantum systems. Treating observers as quantum systems give contradictions such as the Frauchinger-Renner no-go theorem [75]. Jeffrey Bub, [77] & [78], and Matthew Pusey, [76], have discussed how the origin of this contradiction is coming from treating observers as quantum systems. An observer is implied when one says a system is in a quantum state $\rho$. The approach where the observers or agents are treated as quantum systems themselves is called the quantum information approach. The approach we have adopted in this paper is called the structural approach, see [53] for details. The structural approach disregards the external relatum altogether and treats reference frames as internal to the system. Though observers and reference frames are different objects, here we think of the reference frame as representing the observer's perspective of the system. We will use the terms reference frame and observer interchangably. The distinction between quantum systems and their reference frames is not fundamental. We will be using the structural approach for the description of observers or reference frames. Treating observers as quantum systems can lead to certain absurdities. If the quantum states of observers' system represent observer's knowledge, what does the quantum state of the system represent?

We would like to extend the structural approach to the treatment of instruments. We would like to take the physical idea seriously that there are no instruments external to the universe. The kinematics of the theory are described using a set of effects. In classical theories, the kinematics of the theory was described using a set of coordinates. These coordinates are measured using instruments such as rods and clocks. These rods and clocks are representatives of the equivalence class for their respective effects. Effects are equivalence classes of observable changes in the system as discussed in §1. The notion of effects is an abstract concept. A measuring instrument can act as a representative of the equivalence class of an effect. Such an instrument might not exist. We did not assign a separate state space for instruments in classical physics, this situation doesn't change for quantum theory. The difference between classical and quantum theories lies in the mathematical structure of effects and states not how we treat observers and instruments. When we talked about the values of an observable in classical theory, we did not discuss how 'instruments are themselves classical systems'. The structure needed is already contained in effects.

$$\text{Instruments} \hookrightarrow \text{Effects}$$

A measuring instrument can act as a representative for the equivalence class of an effect. The behavior of the idealized instruments is already contained in the equivalence class of the corresponding effect. There is no need to introduce another quantum system representing the measuring instrument. Operational theories call this device independent approach. These notions are inherent in the formulation of quantum theory in the instrumentalist approach to quantum theory.

QBists usually view measuring instruments as essentially part of the observer or agent [31]. This is also where QBism departs from other Copenhagenish interpretations. Though our reasoning for treating instruments as intrinsic to the system was from an instrumentalist point of view, it's still closer to how QBist view measuring instruments. In relational quantum mechanics or RQM, a measurement is treated as an interaction event between the observer system and the system being observed, and measurement is treated as a correlation between these systems. In QBism, a measurement is interpreted as an agent's subjective experience of something that happens to them. Though our notion of time can be formulated in the relational language, we are not comfortable with treating observers as quantum systems. We prefer the QBist route for the treatment of observers even though we feel the interpretation itself is too subjective. Fuchs, et. al have suggested in [37], that the concept of 'measurement outcomes' can be extended beyond agents' experiences to encompass a more general notion of 'little moments of creation' that can occur whether or not any agents are present. So maybe it's not so subjective. Their treatment of observers is closer to our instrumentalist or structuralist approach to observers. There is also a difference between QBism and relational quantum mechanics in their treatment of measurement or general quantum process. QBism they are 'moments of creation' and for RQM they are 'interaction events'. For this reason, QBism is closer to our view than RQM. See [30] for a comparison between QBism and RQM. Both seem to remain flexible about where to draw the boundary between observer and observed system. It's important to note that by observer we don't mean a human or a conscious being. We would like to stay agnostic regarding these issues. We feel these issues are not of concern to physicists. The universe will exist even if there are no living beings. It's not important to discuss which of the many participatory realist interpretations is better, most of them will be compatible with what we wish to do in the following sections. My preferred interpretation is the instrumentalist approach with some input from QBism and relational interpretation.

### Observers in QI Approach

Though it is important how one treats observers or reference frames in quantum theory, the treatment will not affect our description of time. In the quantum information approach, observers or reference frames are thought of as separate quantum systems, what is of interest to us is the observer's quantum system. Time flows in relation to the observers, how the observer's state changes is what is important. So the mathematical objects necessary for the description of time's flow for the observer are quantum states of the observer's quantum system and quantum processes that change the observer's state. What is important to us is the changes happening to the observer's state, what effect the outside environment has on the observer is not of interest. The observer is treated 'in isolation' in the sense that other physical systems to which the observer could be related are disregarded. There might be measuring instruments or other instruments responsible for processes through which the observer updates their information. However, the relevant part is the corresponding quantum channel acting on the observer's Hilbert space and everything else is irrelevant. Time's flow is in relation to observer.

The bottom line is that the notion of time we are about to define can be defined also when observers are treated in the quantum information approach. We will however define it for the structural approach. The structural approach to observers is also minimalist in the sense that no additional description of observers is needed. Usually when one makes additional assumptions about how things are one usually ends up with paradoxical situations needing more explanations. Through the use of a minimalist approach like this, our aim is to avoid any mathematical issues that might come up.

### 2.4.2 | Intersubjectivity & Approach to Equilibrium

The question we could ask is if it's possible to arrive at the equilibrium principle from more general ideas of inference and state updating. We will only partially answer this using thought experiments regarding intersubjectivity and provide how one could arrive at equilibrium principle of thermodynamics from purely quantum side.

Everything we have done so far on the quantum side is subjective to the observer. 'How come one world out of many observer participants?' was one of Wheeler's three questions in [26]. QBism's ontology seeks to explain not how 'physical phenomena' arise, but rather, 'how physical phenomena co-arise together with agents' experiences of them'. What we will try to do in this subsection is try to motivate the equilibrium principle for the quantum case. The equilibrium principle can also be taken as a postulate coming from a pre-theory and can be assumed to be true for quantum mechanics but for quantum purists this might be unacceptable. This is similar to how in the unitary evolution case the postulate for evolution in classical, i.e., coming from symmetries of the system, was assumed to be true for quantum case, hence we could assume the equilibrium principle to be true in the quantum case.

If the reader's preferred approach to observers is the quantum information approach then this subsection is likely not going to provide any motivation for the equilibrium principle. Such a reader should assume the equilibrium principle. We prefer the structural approach to observers because philosophically there are no observers outside the universe and hence the observers should be explained from within. In this sense, the structural approach can also be thought of as an observer independent approach.

#### Wigner's Friend Type Thought Experiment

One way to bring objectivity into quantum theory is through intersubjectivity. To describe how objectivity might come into the quantum picture consider the following Wigner's friend-type situation. From here on we will use the structuralist approach to observers and instruments. In the structuralist approach, when one says the system is in a certain quantum state, the observer is assumed a priori. The quantum state represents the knowledge or belief of this observer.

Suppose Wigner's friend performs a measurement in her laboratory and obtains an outcome that Wigner who is not in the laboratory doesn't know. The quantum state for Wigner gets updated once his friend tells him the result of the experiment. Here Wigner's measuring apparatus includes his friend. Hervé Zwirn in [32], has similar viewpoint on Wigner's friend-type situation. In particular, there is no retrocausality issues, see [32] §5.2, 'past events', for a discussion on this. Zwirn in the paper, also does a careful analysis and a discussion of the delayed choice thought experiment which is also closely related to Wigner's friend thought experiment. According to these QBistic approaches, It is simply updating probability with new info. Bayesian inference is the most straightforward description of quantum eraser snd delayed choice, observer has to update their information once they learn about what happened in the experiment. In case of multiple observers, they update their state after communication. This issue has been treated in the QBist literature, for example [79]. If the two performed the experiments together, they have to agree with the result of the experiment.

'Objective fact' only makes sense if the observers are observing the same observables. Let the first observer be represented by $(\mathcal{A}_1, \rho_1)$ and a second observer by $(\mathcal{A}_2, \rho_2)$, where the first term is the algebra of observables and the second term is the state which corresponds to the observer's knowledge or belief (if you are a QBist) about the observables. To make sense of objectivity the algebra of observables for both observers should have some intersection and the observable being measured should be in this intersection. After measuring an observable $A \in \mathcal{A}_1 \cap \mathcal{A}_2$, the result is the same for both the observers if the restriction of their respective states to the subalgebra generated by the operator $A$ coincide. Denote the algebra generated by $A$, by $[A]$. The two observers having the same information about the observable $A$ means,

$$\rho_1|_{[A]} = \rho_2|_{[A]}.$$

If this happens we can say that the information the two observers have about the observable $A$ is the same. Similarly, we say two observers with states $\rho_1$ and $\rho_2$ have the same information about a system $\mathcal{B}$ if $\mathcal{B} \subset \mathcal{A}_1$ and $\mathcal{B} \subset \mathcal{A}_2$ and

$$\rho_1|_{\mathcal{B}} = \rho_2|_{\mathcal{B}}.$$

By performing a measurement an observer obtains information about the observable. We can think of quantum processes as information updating processes for the observers i.e., maps corresponding to quantum processes are to be viewed as inductive inference, change of state is due to change of information.[7] What we should expect in an intersubjective situation is that the quantum processes will be such that the information the observers have about the common observables converges. Such states should have maximum possible information constrained by observer relations. What we mean by observer relations will become clear soon. This could be thought of as the quantum formulation of the equilibrium principle.

Once all observers agree on the information about the system no additional processes are 'needed' to reach an agreement about the observables. Any additional process would take the system 'out of equilibrium'. Since in our view time is an emergent phenomenon due to quantum processes, time would not 'flow' after reaching such a state for the system as there won't be any changes happening. Call such states objective states, i.e., the information is objective to all observers of the system. By objective state, we don't mean an underlying ontic state or a state of reality. Such an ontic state cannot exist because of the Kochen-Spekker theorem and PBR theorem. For each of the observers, their state updating is different. Though they will eventually end up agreeing about the results of the experiments, the way the state updating happens for each of them is different. That is, the way time changes for the two observers is different. Time is relational and is defined for the observer, in particular there is no retrocausality. This view is very close to Zwirn's approach and hence to the QBist approach to the issue, see [32]. However, the information they have about the experiments will be the same in the end. If multiple agents are observing the system, they should eventually come to terms with their knowledge of the observables. There is no way to know this class of objective states of the system if only the algebra of observables is known. The existence of such states is an additional physical hypothesis just like how the existence of equilibrium states is an additional postulate in thermodynamics.

**Postulate. (Objectivity)** *Every system $\mathcal{A}$ there exists a class of objective states $\mathfrak{O}_{\mathcal{A}}$.*

This, in some way, answers the question, why does a system evolve? The answer would be because it has to reach one of the objective states. What we have now is a collection of events $\mathcal{T}$ which we will call a tree, that takes the system towards some objective state in $\mathfrak{O}_{\mathcal{A}}$. Within each tree, the processes provide a natural order on states. Which of these trees is realized can only be determined through Bayesian inference i.e., updating the state based on

[7] Ryszard Kostecki seems to have been working on viewing quantum theory as a theory of inductive inference for a long time. Interested reader should read his insightful articles on the subject see for example, [17].

the information available to us. What we want to be able to do is assign to each state in a tree its associated 'time'. In order to do that we must first define some properties the class of objective states should have, we will then use these properties to define to each event a 'time' that coincides with our intuition regarding the notion of time.

By endorsing relative facts, both QBism and RQM reject measurement outcomes as absolute facts. Measurement outcomes can sometimes correspond to entirely private reality relevant only to the observer. For example, in the Wigner's friend experiment, Wigner's friend measures something while sealed inside an isolated laboratory, the event corresponding to the measurement result only exists relative to her as long as the laboratory remains sealed. Neither QBism nor RQM says all facts are completely private. When the sealed laboratory is opened, the friend can communicate with Wigner. In QBism, Wigner updates his information after communication and his friend to him is part of the measurement instrument. In QBism, it's a normative activity. In RQM, communication is a physical interaction between two physical systems. This is an important distinction mainly coming from the different treatment of observers or agents in the two approaches. What we are interested in is understanding this shared reality where observers or agents agree when they communicate. This is an important assumption because communication is treated differently in QBism and RQM. In QBism, communication doesn't necessarily mean agreement. In RQM, it does. We are assuming the observers agreed on the method of the measurement process and what the results mean and hence they will always agree on the result after communication.

### QRF Transformations and Symmetries

For a fixed system $\mathcal{A}$ we are interested in understanding how the states of two observers given by $\rho_1$ and $\rho_2$ are related. Different observers or reference frames are related to each other through quantum reference frame transformations or QRF transformation for short. The area of reference frames and QRF transformations has seen increased activity recently, see [52]-[57]. We will collect the relevant parts here and formulate them in the way we will use them. How are observer transformations, also called QRF transformations, different from any other automorphism of the system?. The role of symmetries is important to answer this question. The idea is that moving from one observer to another should leave the 'physics invariant'. [53],[55] show the QRF transformations as the physical symmetries of the system i.e.,

$$\rho_1 = U(g)\rho_2 U^*(g),$$

where $g \in \mathcal{G}$, the group of coordinate transformations of the system. To achieve this they start with a classical configuration space and note that observers at different positions should be related through some symmetry group. If $\rho \mapsto \alpha(\rho)$ is a QRF transformation between two observers or reference frames then there exists some $g \in \mathcal{G}$ such that $\alpha(\rho) = U(g)\rho U(g)^*$. Note that this is not the same as saying evolution is governed by a representation of the group $\mathcal{G}$.

In an intersubjective situation, the quantum processes should be such that the information the observers have about the common observables converges. The objective states are such limit states. An objective state, if it exists, represents the common 'final' information all the observers agree on. The final states $\rho_1^F$ and $\rho_2^F$ for the two observers satisfy,

$$\rho_1^F = \rho_2^F.$$

Denote this final state by $\rho$. From the previous relation between two different observers we have,

$$\rho = U(g)\rho\, U^*(g).$$

If $\mathfrak{O}_{\mathcal{A}}$ represents states with such information then first of all we expect these states to be invariant under coordinate transformation of the system, i.e., if the affiliated generator of the symmetry is $P$, we should have $\rho = e^{itP}\rho e^{-itP}$ for every objective state $\rho$. This equivalently means that,

$$\rho P = P\rho.$$

In the finite-dimensional setting, this means, $\rho$ and $P$ can be simultaneously diagonalized. Let $\mathcal{G}$ be the group of symmetries of the system and $\mathfrak{g}$ be the set of the corresponding generators. Then from the previous relation,

$$\rho P = P\rho,$$

for every $P \in \mathfrak{g}$. The set,

$$\mathcal{S}_{\mathfrak{g}} = \{\rho \in \mathcal{S}(\mathcal{H}) \mid [P, \rho] = 0, \, \forall P \in \mathfrak{g}\},$$

encodes the constraint that the the state $\rho$ is invariant under the action of $\mathcal{G}$.

## 2.5 | Conclusion

In this paper, we have tried to address the measurement problem in the foundations of quantum mechanics. We discussed how the problem is coming not from the measurement part but from our (lack of) understanding of the notion of time. To fix the issue we described how time could be defined in quantum theory and discussed how it was emergent from the non-commutative structure of quantum theories. We used a weak version of the equilibrium principle to define a notion of time that has the necessary properties we expect from 'time'.

Our aim was to understand how time was emergent in a purely quantum setting, not use general relativity as a pre-theory and use the notion of evolution in general relativity to arrive at a covariant notion of time. We use the weak equilibrium principle to get a number for time (not unique). The flow of time is then related to the progressive loss of asymmetry between past and future that the system suffers as it evolves. What I have tried to do is describe a purely quantum notion of time applicable to any quantum theory. So it's not restricted to relativistic quantum theory or any other model. The notion of time is something more fundamental than general relativity and should apply to all quantum theories, including a yet-to-be-discovered quantum theory of gravity or quantum gravity. General relativity will eventually be an approximation of something more fundamental. Using general relativity to arrive at a notion of time, which is conceptually deeper than general relativity itself is a faulty path to take. Yes, in a 'limit' situation they should coincide but using the notion of time in general relativity and 'quantizing' is the wrong path to take I feel. Our aim is different, we expect quantum theory to stand on its own feet. It should be noted that a replacement for Schrödinger equation will not be compatible with the measurement process just by being non-unitary. There needs to be a unifying 'law' for both the measurement process and the replacement for Schrödinger equation. Our approach is to start over, take seriously the measurement process and the unifying law is Bayesian inference i.e., updating the state based on the newly available information. This applies to every probabilistic theory and is hence a very general law. For us, time is an emergent concept arising from quantum processes. It is automatically compatible with the inference rules. Since the equilibrium principle doesn't cause any conflict with the measurement postulate the notion of time is compatible with the measurement postulate. The problem however is to get a number 'time' for such an emergent notion of time. The notion of time we described works on a quantum framework level and not model level. It might work perhaps even on a GPT level if we can define symmetries and asymmetry measures and properties of equilibrium states in GPT setting. If one assumes the weak equilibrium principle in addition to the non-commutative structure of effects one can then assign numbers to each event that corresponds to our intuitive notion of time. The standard notion of time in quantum mechanics is also an approximate notion of time as we showed in last section of the paper. So it's much more general. Since the equilibrium principle doesn't cause any conflict with the measurement postulate the notion of time is compatible with the measurement postulate. While it's tempting to use Dilation theorems, we feel it's a faulty shortcut path to take (we expect quantum mechanics to have the capacity to explain closed systems as well and we don't feel it's right to think of the completely positive map of the process as a unitary operator on some bigger Hilbert space, the unitary map need not have any physical meaning). We would hence refrain from making any such premature claims about an equation of state.

A connection between our notion of time and general relativity would be a bit indirect and through thermodynamics. We wouldn't try to make any such premature connection at the current stage.

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