Research Statement

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Overview

My main scientific interests are quantum foundations (QF) and algebraic quantum field theory (AQFT) with a view towards quantum gravity, mainly motivated by the works of Bisognano-Wichmann, Unruh, Borchers, Wiesbrock, Shroer, Buchholz, Summers, Longo, Brunetti, Guido, and others.

In quantum foundations, I am mainly interested is in the measurement problem, interpretaion of quantum states, and the problem of time in particular. In my personal opinion, QBism provides an interpretation that has so far evaded all the problems thrown at it, with the downside of extreme subjectivity it associates with quantum states. I worked on the problem of time and measurement problem with a QBist line of thinking, although it is equally applicable to other participatory realist interpretations.

Algebraic quantum field theory setting has a strong physical foundations as well as rigorous formulation. AQFT has been seeing a renewed interest, especially among high energy groups. AQFT fits my interests pretty well in that it's foundations are carefully thought through. AQFT also provides the correct framework for describing quantum theory on curved space-time. In the past year, I have been reading AQFT and operator algebras literature, in hopes of contributing to the field, and using it in quantum gravity research. Works of Borchers, Wiesbrock, Buchholz and Summers indicate that we could be able to reconstruct the Poincaré group from the vacuum state using operator algebraic tools. Unruh effect tells us that uniformly accelerating observers correspond to equilibrium states, and hence relates the notion of curvature, which is related to acceleration in relativity, to equilibrium states on operator algebras. So, it appears that much of the geometric information we need about the 'underlying' space-time is contained in the operator algebraic structures. My aim is to work along these lines to develop 'quantum geometry' that would be needed in a quantum theory of gravity. My goals and plans in this regard will be briefly summarized in the following sections.

I. Quantum Foundations

Quantum foundations is very important as quantum theory underlies all of modern physics. Many problems in quantum foundations such as the problem of time are also very closely related to the problem of quantum gravity.

So far, my main focus of research has been in the foundations of physics, mainly studying structure of physical theories, quantum foundations, studying the interpretation of quantum states and related problems such as the measurement problem, and the problem of time.

Emergence of Time

Quantum measurements, from the perspective of epistemic interpretations, is a Bayesian updating of the state information based on the newly available information. Since classical probabilities also collapse when updating information there is nothing special about quantum collapse. But the unitary evolution is obtained by implementing symmetries on quantum objects via Wigner's theorem. So, even though the objects of quantum theory are new, the notion evolution is inherited from classical physics. I believe this is the problematic part.

Every quantum process brings with it a notion of before and after. In quantum theory, it's not possible to have complete information about the observables, due to non-commutativity. Non-commutativity forces the system to always have some information about it and the observer can keep obtaining new information from it. So, it might be **because** of the quantum processes time changes[1], and not the otherway round.

I spent a lot of time to turn this idea into a paper, [1], it still needs work.

Much of my remaining interests in quantum foundations revolve around subjects I needed to make the previous idea precise. I have studied the foundations of physics, in particular trying to approach it a bit more category theoretically, by treating each physical theory as an object, and studying the physical theory via its relations to other physical theories, instead of studying the theory from within. This was mainly to make precise and implement the ideas for the emergence of time project. I have also carefully studied the foundations of thermodynamics, mainly following the works of Lieb & Yngvason, and the works of Brown & Uffink relating equilibrium principle and the notion of time.

Future plans in QF

I have been interested in resource theory approaches to thermodynamics mainly due to their similarities to Lieb-Yngvason's thermodynamics, and clean formulation. I am interested in resource theory of asymmetry and motivation of trend to equilibrium via information theoretic arguments like Jaynes, etc. Statistical mechanics starts with ideas about microstates, and macro states, the

Non-Commutative Entropy

Every probability distribution carries with it the information about the possible events. In classical case, the quantification of this information about events, corresponds to the Shannon entropy. In the quantum case, the set of events corresponds to the projection operators on some Hilbert space. The von Neumann entropy measures the amount of mixing a state has, i.e., if we think of states as mixtures of pure states, the von Neumann entropy measures the amount of mixing, or the information about which pure state the system is in. This is however not the operational meaning we expect from entropy. We want to quantify the information about the possible events, and not the amount of mixing.

Since it's not possible to know the values of all possible events, due to non-commutativity, this entropy cannot be zero and should carry some non-commutative factor. I feel there is

[1] Connes and Rovelli have for long time been saying that non-commutativity has an important role to play in the emergence of time, and many philosophers have for a long time indicated that the notion of change and passing of time are closely related. I felt these two are closely related.

a need for a new, operationally meaningful, non-commutative generalization of Shannon's entropy.

II. AQFT/Quantum Geometry

The foundations of gravitation, as modelled by Einstein's general relativity is geometric. In quantum theory however, the starting point is effects and ensembles which correspond to projection operators and normal states on a von Neumann algebra. The path towards a quantum theory of gravity inevitably involves a description of geometry with operator algebras as the starting point. Riemannian geometry, which is the mathematical theory needed in relativity was already available for use for Einstein and his collaborators, our situation however a bit more complicated. We don't have the mathematical theory necessary to formulate the ideas of quantum theory and gravity together. So, the first step towards quantum gravity is developing a mathematical theory of geometry with operator algebras as the starting point.

Haag and Kastler's motivation for starting out with the net of algebras of observables was the idea that measuring instruments have particular associated regions of space-time. The possible events in quantum theory correspond to equivalence classes of measurement effects. So each observable must have an associated region in space-time. We expect the universe to be a closed system, the measurement instruments, and hence effects should remain the same even as the space-time evolves. So the corresponding representatives for the equivalence classes should remain the same as the system evolves. So, the algebra of observables for the system describing the universe should remain the same as the universe evolves. If we start with an instrument in a region, and the space-time evolves, the same equipment should exist in this evolved region as long as it's a closed system.

Assuming these regions of space-time are open subsets of the space-time X, we have a map that assigns to each open set $\mathcal{O}$ of the space-time the algebra of observables $\mathcal{A}(\mathcal{O})$ that are measured in the region.

$$\mathcal{O} \to \mathcal{A}(\mathcal{O}).$$

This is called a net of observable algebras. This has property similar to restriction property of presheaves. In case of presheaves, we had a restriction map which mapped functions on larger domains to give functions in smaller domains contained in the larger domain. The situation here is opposite to the presheaf situation.

If an observable can be measured in a space-time region $\mathcal{O}_1$, then it can be measured in any space-time region $\mathcal{O}_2$ containing $\mathcal{O}_1$. If $\mathcal{O}_1 \subset \mathcal{O}_2$ we have an associated co-restriction map, such that, for every observable $A \in \mathcal{A}(\mathcal{O}_1)$ we have the co-restriction $A|^{\mathcal{O}_2} \in \mathcal{A}(\mathcal{O}_2)$. The notation $|^{\mathcal{O}_2}$ is the co-restriction to $\mathcal{O}_2$. The map, $A \mapsto A|^{\mathcal{O}_2}$ is a function $\mathcal{A}(\mathcal{O}_1) \to \mathcal{A}(\mathcal{O}_2)$.

If $\mathcal{O}_1 \subset \mathcal{O}_2 \subset \mathcal{O}_3$ are nested open sets then the co-restriction is transitive.

$$(A|^{\mathcal{O}_2})|^{\mathcal{O}_3} = A|^{\mathcal{O}_3}.$$

This can be summarised by saying the assignment $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ is a functor,

$$\mathcal{A} : \mathcal{O}(X) \to \mathbf{C}^*,$$

where $\mathcal{O}(X)$ are open sets of X and the morphisms $\mathcal{O}_1 \to \mathcal{O}_2$ are inclusions $\mathcal{O}_1 \subset \mathcal{O}_2$. To each such inclusion morphism in $\mathcal{O}(X)$ we get restriction morphism in **Sets**, $\{\mathcal{O}_1 \subset \mathcal{O}_2\} \mapsto \{\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)\}$, given by

$$A \mapsto A|^{\mathcal{O}_2}$$

This is a co-presheaf. We will call for now, the subalgebras $\mathcal{A}(\mathcal{O})$ the local subalgebras.

Statement of the Problem

By quantum geometry we would like to do the opposite of what we do in quantum field theory. The goal is now to reverse this procedure, given a quantum system with an algebra of observables $\mathcal{A}$ and a state ω, we would like to associate to it a physically meaningful manifold. But it's important to explain and make precise what we mean by physically meaningful.

A quantum field theory is a functor, from the category of open sets of a Lorentzian manifolds to C^*-algebras. What we want to do is reverse this, i.e., a functor that assigns to each C^*-algebra a geometric object. For this functor to be physically meaningful, the geometric object associated to a quantum field theory should be the underlying space-time. This is categorically speaking an adjoint situation.

This requirement acts as a very strong constraint as to what we can do. AQFT results such as Unruh effect, Borchers-Wiesbrock reconstructions, should be respected. That's to say, the modular hamiltonian associated with the vacuum state should correspond to the orthogonal group on the tangent space of space-time. If the system is in an equilibrium states then the temperature should be related to the curvature via Unruh effect.

My Beliefs/Speculation

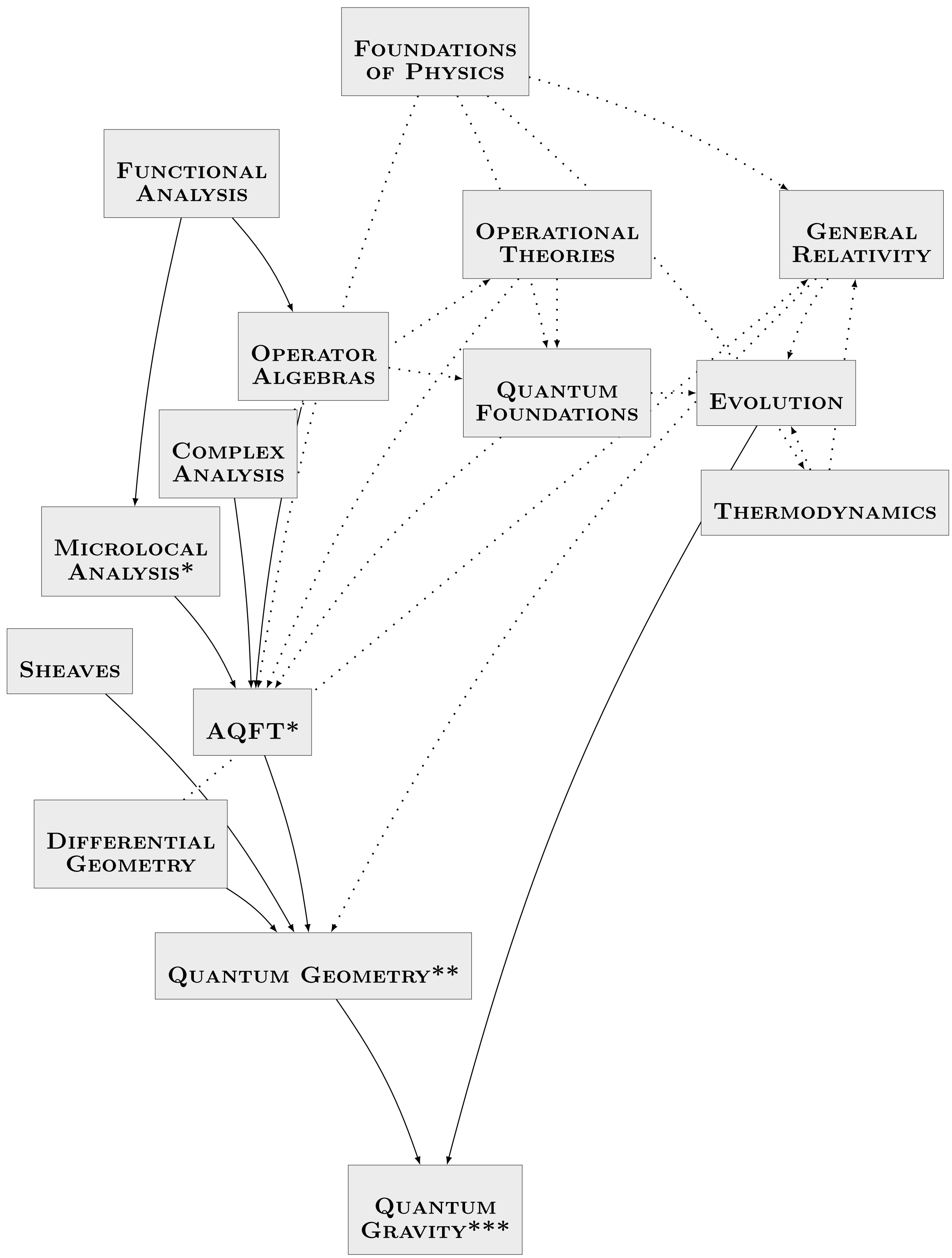
Suppose we have such a functor, then it must respect certain theorems and properties of AQFTs, such as the Unruh effect, Borchers-Wiesbrock works relating symmetries of space-time and modular operators associated with the vacuum state. So, we could start with these theorems as the starting point.

The goal is then to associate to a spanning class of subalgebras of $\mathcal{A}$ geometric objects, more precisely Lorentzian manifolds. This is a pre-sheaf like functor from the site of some special class of subalgebra of $\mathcal{A}$ to the category of Lorentzian manifolds of fixed dimension. This association might not correspond to a global lorentzian manifold, but only an pre-sheaf like association. The Lorentzian manifolds on subalgebras might not patch up to give a global Lorentzian manifold. Once we have such a pre-sheaf like functor, the 'quantum geometry' would be to patch it up. This would be analogous to 'sheafification' of pre-sheaves.

Now that the rough goal is set, we now have construct such a 'pre-sheaf' like functor. The AQFT theorems such as Bisognano-Wichmann, Borchers, and Unruh effect say that equilibrium states carry geometric information about the underlying geometry. This seem to indicate a path towards associating physically meaningful geometric objects to operator algebras by exploiting subalgebra relations and the vacuum state. To do this with a general state, we can approximate a given state to an equilibrium state (say, closest wrt relative entropy), and using this equilibrium state to associate the geometric information of the equilibrium state to this given state. This would correspond to how the system would look geometrically if the system is restricted to the subalgebra.

This, to me seems to be the path I should take. My studies have so far been trying to learn the tools from analysis and geometry that would be relevant to this plan. I would be very happy if I could make any progress in a small part of this path.

My Academic Interests



References

[1] https://bgr95.github.io/PDF/Research/Time.pdf