Part

Derived Categories of Sheaves

1 | Abelian Sheaves

2 | Triangulated Categories

2.1 | Adjoint Functor

Two functors $F : \mathcal{C} \to \mathcal{D}$ and $G : \mathcal{D} \to \mathcal{C}$ are called an adjoint pair if

$$\mathrm{Hom}_{\mathcal{D}}(F(X), Y) = \mathrm{Hom}_{\mathcal{C}}(X, G(Y))$$

for all $X \in \mathcal{C}$ and $Y \in \mathcal{D}$. F is a left adjoint to G and G is a right adjoint to F. This is denoted by, $F \dashv G$. Adjoints are unique upto isomorphism and is the representative of the functor,

$$X \mapsto \mathrm{Hom}_{\mathcal{D}}(F(X), Y)$$

The isomorphism gives us,

$$\mathrm{Hom}_{\mathcal{C}}(G(X), G(Y)) \cong \mathrm{Hom}_{\mathcal{D}}(F \circ G(X), Y)$$

and similarly, $\mathrm{Hom}_{\mathcal{D}}(F(X), F(Y)) \cong \mathrm{Hom}_{\mathcal{C}}(X, G \circ F(Y))$.

Suppose $\mathcal{A}$ is an abelian category, a complex in $\mathcal{A}$ is a diagram,

$$A^\bullet \equiv (\cdots \xrightarrow{d_{i-2}} A^{i-1} \xrightarrow{d_{i-1}} A^i \xrightarrow{d_i} A^{i+1} \xrightarrow{d_{i+1}} \cdots)$$

in $\mathcal{A}$, such that $d_i \circ d_{i-1} = 0$.

3 | Derived Categories

References