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## 1 Introduction

In this assignment I experiment with three learning algorithms in a simple discrete maze environment: the 9x9 maze shown in Figure 1. In this figure, white squares represent walls and black squares represent free spaces. In all experiments, the goal square is in (7,7).

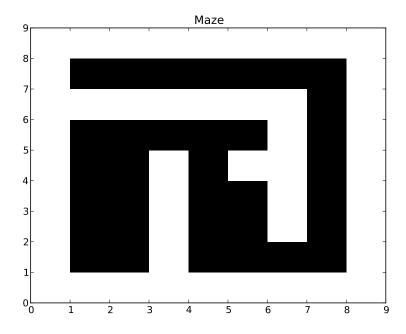


Figure 1: Maze

My learning agents use SARSA(0), Q(0), and Q( $\lambda$ ) with  $\epsilon$ -greedy exploration and the following parameters (where applicable):

$$\alpha = 0.5 \tag{1}$$

$$\gamma = 0.99 \tag{2}$$

$$\epsilon = 0.3 \tag{3}$$

$$\epsilon_{\text{decay}} = 0.9999 \tag{4}$$

$$\lambda = 0.9 \tag{5}$$

 $\epsilon$  is multiplied by  $\epsilon_{\text{decay}}$  after every learning step. I use the algorithm implementations and experimental framework from PyBrain[1]. By inspection, the Q( $\lambda$ ) variant implemented appears to be Peng's[2].

An agent receives a reward of 1.0 for reaching the goal state and a reward of 0.0 at every other step. Running into walls does not result in an explicit penalty, it is implicitly penalized through the discount factor  $\gamma$ . This task is modeled as a continuing, discounted task; after an agent reaches the goal state, it is initialized to a new random start state. Unless explicitly stated, I optimistically initialize all action-values to 1.0.

## 2 State-Value Tables

First, I inspect the action-value tables of the three algorithms. In Figures 2, 3, and 4, I show a representation of the maximal action-value for each state after one "episode"; in other words, I run the algorithm until the first reward is received and show the resulting update. In these figures I do not initialize optimistically to make it more clear what the algorithms are doing.

In Figures 2 and 3, only the state previous to the goal state receives an update, whereas in Figure 4, several states previous to the goal state are updated after only one episode. This is as expected, and indicates that  $Q(\lambda)$  should converge more quickly to the optimal policy.

In Figures 5, 6, and 7, I show the same representation after 5000 steps, or "interactions" with the maze. As expected,  $Q(\lambda)$  seems to have already converged to a reasonable representation of the maze, with less complete representations computed by Q(0) and then SARSA.

## 3 Average Cumulative Reward

In this section I present graphs of the cumulative reward of each algorithm, averaged over five agents.

In Figure 8 I show the cumulative reward up to 5000 interactions.  $Q(\lambda)$  takes an early lead, and SARSA starts off dominating Q(0). This is expected, as  $Q(\lambda)$  is especially suited to this task; "On tasks with many steps per episode, or many steps within the half-life of discounting, it appears significantly better to use eligibility traces than not to", ([2], Section 7.11). SARSA dominates Q in the beginning because it is an on-policy method, and it's strategy takes  $\epsilon$  into account.

This ordering changes in Figures 9, 10, and 11. As  $\epsilon$  decreases due to the  $\epsilon_{\rm decay}$  factor, Q(0) is able to make better decisions than SARSA, as it has already learned (close to) the true optimal policy. SARSA(0) and Q(0) seem to converge to the same slope (policy), though Q(0)'s early lead means a permanent, constant, gap between their cumulative performance.

I do not know why SARSA(0) and Q(0) dominate  $Q(\lambda)$  in the long term. As described in Sutton and Barto, Peng's  $Q(\lambda)$  should converge to the optimal policy if the policy is gradually made more greedy, which I am doing here using the  $\epsilon_{\rm decay}$  factor. Looking at its slope on the longest-term plot, Figure 11, it appears that  $Q(\lambda)$  has converged to a suboptimal policy. There may be an error in the PyBrain implementation of  $Q(\lambda)$ , my usage of it, or perhaps the  $\epsilon_{\rm decay}$  factor is not being used (though I do specify it). I have investigated all of these possibilities in the time I have had available, but I do not yet have an answer.

## References

- [1] Tom Schaul, Justin Bayer, Daan Wierstra, Yi Sun, Martin Felder, Frank Sehnke, Thomas Rückstieß, and Jürgen Schmidhuber. PyBrain. Journal of Machine Learning Research, 2010.
- [2] Richard S. Sutton and Andrew G. Barto. Reinforcement learning: An Introduction. MIT Press, 2005.

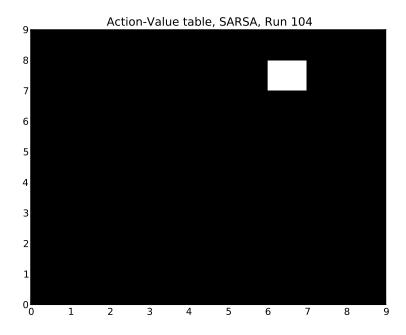


Figure 2: SARSA, Single Episode

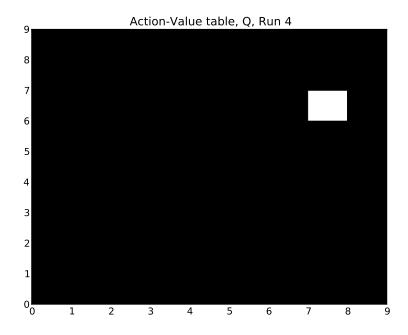


Figure 3: Q(0), Single Episode

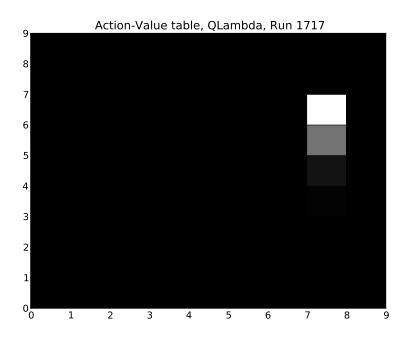


Figure 4:  $Q(\lambda)$ , Single Episode

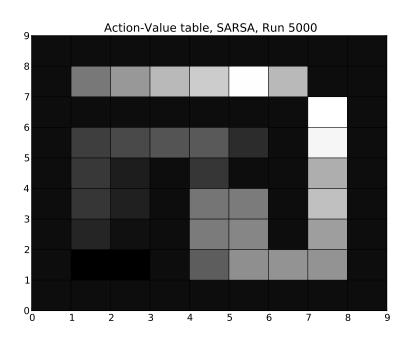


Figure 5: SARSA, 5000 Interactions

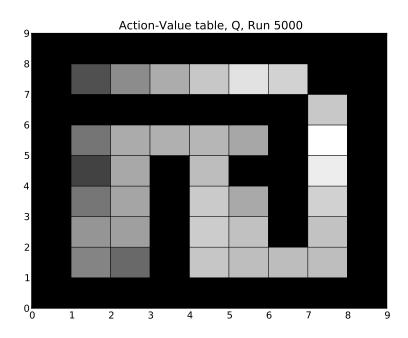


Figure 6: Q(0), 5000 Interactions

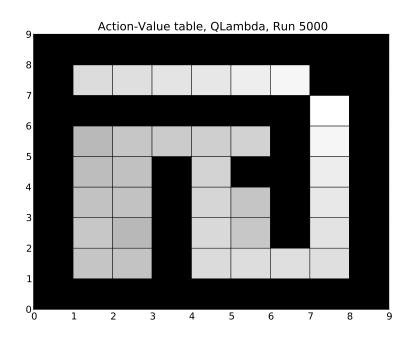


Figure 7:  $Q(\lambda)$ , 5000 Interactions

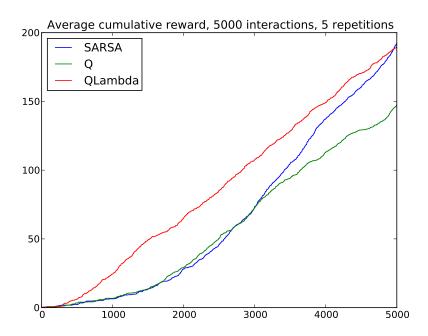


Figure 8:

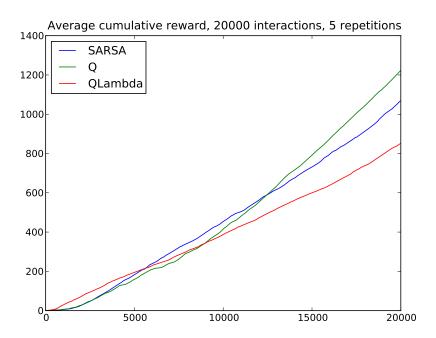


Figure 9:

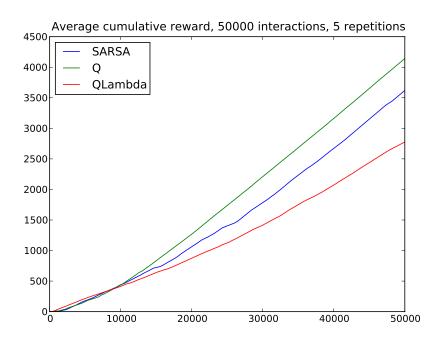


Figure 10:

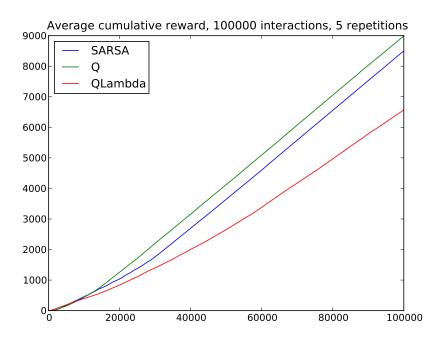


Figure 11: