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Math 4610

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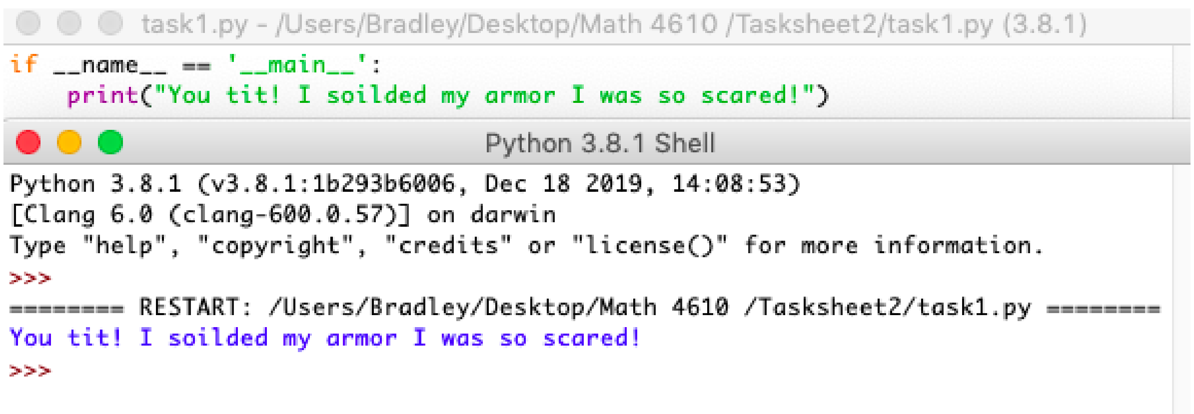
Dr. Koebbe

TaskSheet 2

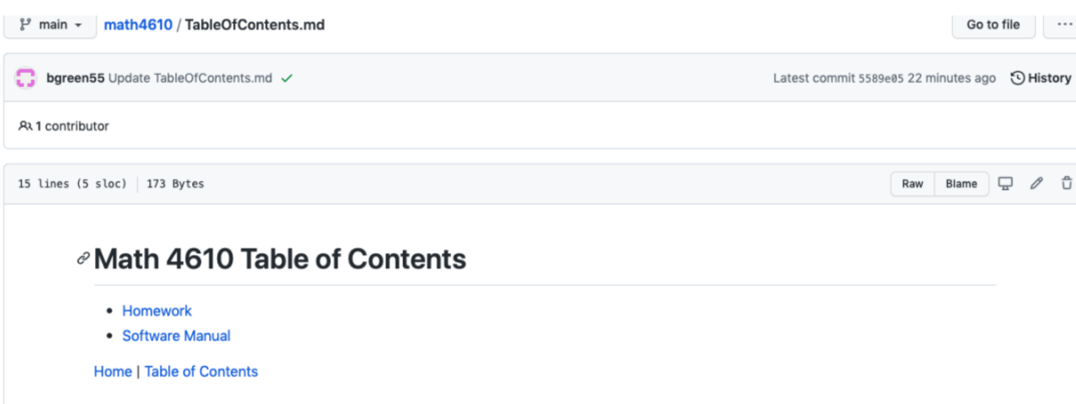
Task 1:

I am using the programming language python. The program prints to the console “You tit! I soiled my armor I was so scared!”



Task 2:

Added links between my home page and the table of contents and my software manual on my github page.



Task 3:

<u>Task 3:</u>

$$f'(a) = \frac{f(a+h)-f(a-h)}{2h}$$

$$|error| = \left| f'(a) - \frac{f(a+h)-f(a-h)}{2h} \right|$$

$$= \left|f'(a) - \frac{1}{2h}\big(f(a+h) - f(a-h)\big)\right|$$

$$f(a+h) = f(a) + f'(a)(a+h-a) + \tfrac{1}{2} f''(a) h^2 + \tfrac{1}{6} f'''(a) h^3 + \ldots$$

$$f(a-h) = f(a) + f'(a)(a-h-a) + \tfrac{1}{2} f''(a)(-h)^2 + \tfrac{1}{6} f'''(a)(-h)^3 + \ldots$$

$$|error| = \left| f'(a) - \tfrac{1}{2h}\Big(\big(\cancel{f(a)} + f'(a)h + \cancel{\tfrac{1}{2} f''(a)h^2} + \tfrac{1}{6} f'''(a)h^3 + \ldots\big) - \big(\cancel{f(a)} - f'(a)h + \cancel{\tfrac{1}{2} f''(a)h^2} - \tfrac{1}{6} f'''(a)h^3 + \ldots\big)\Big)\right|$$

$$= \left| f'(a) - \tfrac{1}{\cancel{2h}}\big(2 f'(a)\cancel{h} + \tfrac{2}{6} f'''(a)h^3 + \ldots\big)\right|$$

$$= \left| f'(a) - \big(f'(a) + \tfrac{1}{6} f'''(a)h^2 + \ldots\big)\right| \qquad = \left| -\tfrac{1}{6} f'''(a)h^2 + \ldots \right|$$

The approximation is second order b/c of (h^2)

Task 4:

<u>Task 4:</u>

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$|error| = \left| f''(x) - \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \right|$$

$$f(x+h) = f(x) + f'(x)(x+h-x) + \tfrac{1}{2} f''(x) h^2 + \tfrac{1}{6} f'''(x) h^3 + \tfrac{1}{24} f^{4}(x) h^4 + \ldots$$

$$f(x-h) = f(x) + f'(x)(x-h-x) + \tfrac{1}{2} f''(x)(-h)^2 + \tfrac{1}{6} f'''(x)(-h)^3 + \tfrac{1}{24} f^{4}(x) h^4 + \ldots$$

$$|error| = \Big| f''(x) - \tfrac{1}{h^2}\Big(\cancel{-2f(x)} + \big(\cancel{f(x)} + \cancel{f'(x)(x+h-x)} + \tfrac{1}{2} f''(x)h^2 + \cancel{\tfrac{1}{6} f'''(x)h^3} + \tfrac{1}{24} f^{4}(x)h^4 + \ldots\big)$$

$$+ \big(\cancel{f(x)} + \cancel{f'(x)(x-h-x)} + \tfrac{1}{2} f''(x)(-h)^2 + \cancel{\tfrac{1}{6} f'''(x)(-h)^3} + \tfrac{1}{24} f^{4}(x)h^4 + \ldots\big)\Big)\Big|$$

$$= \left| f''(x) - \tfrac{1}{h^2}\Big(\big(\tfrac{1}{2} f''(x)h^2 + \tfrac{1}{24} f^{4}(x)h^4 + \ldots\big) + \big(\tfrac{1}{2} f''(x)(-h)^2 + \tfrac{1}{24} f^{4}(x)h^4 + \ldots\big)\Big)\right|$$

$$= \left| f''(x) - \tfrac{1}{h^2}\big(f''(x)h^2 + \tfrac{1}{12} f^{4}(x)h^4 + \ldots\big)\right|$$

$$= \left| f''(x) - f''(x) - \tfrac{1}{12} f^{4}(x)h^2 + \ldots\right| \qquad = \underline{\left| -\tfrac{1}{12} f^{4}(x)h^2 + \ldots \right|}$$

The approximation is second order b/c of (h^2).

I used python code and got the following output.

Iter	h	Approx	Error
1	1	0.38260348236197916	0.03354335418516324
2	0.5	0.4075490368602161	0.00859779968692631
3	10e-1	0.41580016309240014	0.0003466734547422634
4	10e-2	0.416143368670574	3.4678765684081903e-06
5	10e-3	0.4161468019070469	3.4640095514237856e-08
6	10e-4	0.41614681700608	1.9541062379335727e-08
7	10e-5	0.4161471167662966	-2.802191542139454e-07
8	10e-6	0.41600056732704616	0.0001462692200962512
9	10e-7	0.4385380947269369	-0.022391258179794482
10	10e-8	1.1102230246251563	-0.694076188078014
11	10e-9	55.51115123125782	-55.09500439471068
12	10e-10	0.0	0.4161468365471424
13	10e-11	555111.5123125783	-555111.0961657417
14	10e-12	0.0	0.4161468365471424
15	10e-13	0.0	0.4161468365471424
16	10e-14	-1665334536937.7349	1665334536938.1511
17	10e-15	222044604925031.28	-222044604925030.88
18	10e-16	0.0	0.4161468365471424

Task 5:

Finite difference approximations are made to replace derivatives in differential equations with finite approximations. This simplifies the equations enough so a computer can compute them. This is because computers cannot deal with limits as x tends to infinity. In higher ordered approximations an important property of the centered formula is that the error only contains odd derivative terms.

https://archive.siam.org/books/ot98/sample/OT98Chapter1.pdf

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