1 Chapter 3

1. Since $P(A_1 \cap A_2) = 0.64 = (0.8)(0.8) = P(A_1) P(A_2)$, we can conclude that A_1 and A_2 are independent; hence, the answer is (c).

2. (a) $P(A \cap B) = P(B \cap A) = P(B|A) = (0.95)(0.05) = 0.0475$.
   (b) $P(B) = P(B \cap A) + P(B \cap A^c) = 0.0475 + (.03)(1 - 0.05) = 0.076$.
   (c) $P(A|B) = P(A \cap B) / P(B) = 0.0475/0.076 = 0.625$.

3. Let A be the event that an adult gets the flu and let B be the event that an adult gets the flu shot.

   (a) $P(A \cap B) = P(A|B) P(B) = (0.1)(0.42) = 0.042$.
   (b) $P(A) = P(A \cap B) + P(A \cap B^c) = 0.042 + (0.7)(1 - 0.42) = 0.448$.

4. Let X denote the number of people who have asthma. Then $X \sim \mathit{Binomial}(n = 50, p = 0.2)$. (Think about why!)

   (a) $P(X = 19) = \binom{50}{19}(0.2)^{19}(0.8)^{50-19} =\approx 0.001579$.
   (b) The standard deviation is $\sigma = \sqrt{np(1-p)} = \sqrt{(50)(0.2)(0.8)} = 2.828427$ and the mean/expected value is $\mu = np = (50)(0.2) = 10$. Hence, the z-score is $(19 - 10)/2.828427 \approx 3.181981$ which implies that 19 is a little over three standard deviations above the mean.
   (c) Using the empirical rule, we have that $P(X \geq 19) \approx P(X \geq \mu + 3\sigma) \approx (1 - 0.997)/2 = 0.003/2 = 0.0015$. (Draw a picture!). The exact answer is

   ```
   1 - pbinom(18, size = 50, prob = 0.2)

   ## [1] 0.002511203
   ```

5. $E[X] = \sum_{r=1}^{3} rP(X = r) = (1)(1/3) + (2)(1/3) + (3)(1/3) = 2$ envelopes.

6. (a) $E[X] = \mu = \sum_{r=0}^{4} rP(X = r) = (0)(0.2) + (1)(0.3) + (2)(0.3) + (3)(0.1) + (4)(0.1) = 1.6$ egg masses.
   (b) $Var[X] = \sum_{r=0}^{4} (r - \mu)^2 P(X = r) = (0 - 1.6)^2 (0.2) + (1 - 1.6)^2 (0.3) + (2 - 1.6)^2 (0.3) + (3 - 1.6)^2 (0.1) + (4 - 1.6)^2 (0.1) = 1.44$. Hence, the standard deviation is $\sqrt{1.44} = 1.2$ egg masses.

7. Let A be the event that a subject is taking the drug (then A^c is the event that the subject is taking the placebo) and let B be the event that a subject improves.

   (a) $P(B \cap A) = P(B|A) P(A) = (0.6)(0.5) = 0.3$.
   (b) $P(B) = P(B \cap A) + P(B \cap A^c) = 0.3 + (0.35)(0.5) = 0.475$.

8. Let Y be a random variable denoting the number of chickens out of 20 with the bird flu. It then follows that $Y \sim Binomial\,(n = 20, p = 0.1)$.

    (a) $P\,(Y = 5) = \binom{20}{5}(0.1)^5(0.9)^15 = (15504)\left(10^{-5}\right)(0.2058911) \approx 0.031921$.

    (b) $E\,[Y] = np = (20)(0.1) = 2$ chickens.

    (c) $\sqrt{Var\,[Y]} = \sqrt{np(1-p)} = \sqrt{(20)(0.1)(0.9)} \approx 1.3416$ chickens.

9. Let X be a random variable denoting the number of frog eggs that hatch out of 100. Then, $X \sim Binomial\,(n = 100, p = 0.87)$. (Since the frog eggs hatch independently of each other!)

    (a) $P\,(X = 80) = \binom{100}{80}(0.87)^{80}(0.13)^{20} \approx 0.01477606$. You should be able to do this with a calculator, but in R we would just use

    ```
    dbinom(80, size = 100, prob = 0.87)

    ## [1] 0.01477606
    ```

    (b) For a binomial random variable, $E\,[X] = np = (100)(0.87) = 87$ eggs.

    (c) To use the empirical rule, we must first calculate the standard deviation of X. The stahndard deviation of a binomial random variable is given by $\sqrt{Var\,[X]} = \sqrt{np(1-p)} = \sqrt{3.31} = 3.363034$. Computing the z-score yields

    $$Z = \frac{77 - 87}{3.363034} = -2.973505 \approx -3.$$

    In other words, 77 eggs is about three standard deviations below the mean. So, $P\,(X \leq 77) \approx$ the probability of being three or more standard deviations below the mean $\approx 0.003/2 = 0.0015$.

10. (a) $\binom{20}{5} = \frac{20!}{5!(20-5)!} = \frac{(20)(19)(18)(17)(16)(15!)}{5!(15!)} = 15504$.

    (b) $P\,(X = 0) = \binom{5}{0}(7/20)^0\,(1 - 7/20)^5 = 0.116$.

11. Let X be a random variable that represents the number of white croaker fish with high mercury levels out of $n = 100$. It follows that $X \sim Binomial\,(n = 100, p = 0.4)$.

    (a) $P\,(X = 100) = (0.4)^100 \approx 1.6069 \times 10^{-40}$.

    (b) $P\,(X = 45) = \binom{100}{45}(0.4)^{45}(1 - 0.4)^{55} = 0.0478$.

    (c) $E\,[X] = np = (100)(0.4) = 40$ fish and $\sqrt{Var\,[X]} = \sqrt{np(1-p)} = \sqrt{(100)(0.4)(0.6)} = 4.898979$ fish.

    (d) $P\,(X \geq 55) = 1 - P\,(X \leq 54) = 1 - [P\,(X = 0) + P\,(X = 1) + \ldots P\,(X = 54)]$. In R, we get

```r
1 - pbinom(54, size = 100, prob  = 0.4)
## [1] 0.001710927
```

2 Chapter 4

1. No (it looks bimodal).
2. The best answer is (d); bimodal.
3. The population might consist of both males and females, and each of these subpopulations probably has its own mean.
4. The best answer is (a); 31.
5. The best answer is (c); 0.22.
6. The best answer is (e); 0.94.
7. It will remain the same. Go back to the properties about correlation and linear transformations!
8. The best answer is (d); 0.27.
9. The best answer is (d); 0.58.
10. Use the fact that $X \sim N\left(\mu = 5.28, \sigma = 0.4\right)$.

    (a)

$$\begin{aligned} P\left(X > 5.4\right) &= 1 - P\left(X \leq 5.4\right) \\ &= 1 - P\left(\frac{X - 5.28}{0.4} < \frac{5.4 - 5.28}{0.4}\right) \\ &= 1 - P\left(Z < \frac{5.4 - 5.28}{0.4}\right) \\ &= 1 - P\left(Z < 0.3\right) \\ &= 1 - \Phi\left(0.3\right) \end{aligned}$$

In R, we get

```r
1 - pnorm(0.3)
## [1] 0.3820886
```

(b)

$$
\begin{aligned}
P(5 < X < 6) &= P(X < 6) - P(X < 5) \\
&= P\left(\frac{X - 5.28}{0.4} < \frac{6 - 5.28}{0.4}\right) - P\left(\frac{X - 5.28}{0.4} < \frac{5 - 5.28}{0.4}\right) \\
&= P\left(Z < \frac{6 - 5.28}{0.4}\right) - P\left(Z < \frac{5 - 5.28}{0.4}\right) \\
&= P(Z < 1.8) - P(Z < -0.7) \\
&= \Phi(1.8) - \Phi(-0.7)
\end{aligned}
$$

In R, we get

```
pnorm(1.8) - pnorm(-0.7)

## [1] 0.722106
```

(c) The general formula for the p-th percentile, denoted x_p, of a normal distribution with mean μ and standard deviation σ is

$$x_p = \mu + \sigma z_p,$$

where z_p is the p-th percentile of a standard normal distribution (which we can obtain using `qnorm(p)` in R). Hence, the 95-th percentile is $x_{0.95} = 5.28 + 0.4 z_{0.95}$. Using `qnorm(0.95)` in R, we obtain $x_{0.95} = 5.28 + 0.4\,(1.644854) = 5.937941$.

(d) Since the data are a random sample from a normal distribution, we know that the sample mean also has a normal distribution; in particular, $\bar{X} \sim N\left(\mu = 5.28, \sigma = 0.4/\sqrt{50}\right)$. Hence,

$$
\begin{aligned}
P\left(\bar{X} > 5.4\right) &= 1 - P\left(\bar{X} \leq 5.4\right) \\
&= 1 - P\left(\frac{\bar{X} - 5.28}{0.4/\sqrt{50}} < \frac{5.4 - 5.28}{0.4/\sqrt{50}}\right) \\
&= 1 - P\left(Z < \frac{5.4 - 5.28}{0.4/\sqrt{50}}\right) \\
&= 1 - P(Z < 2.12132) \\
&= 1 - \Phi(2.12132)
\end{aligned}
$$

In R, we get

```
1 - pnorm(2.12132)

## [1] 0.01694744
```

11. Use the fact that $X \sim N(\mu = 170, \sigma = 20)$.

(a)

$$
\begin{aligned}
P\left(X > 200\right) &= 1 - P\left(X \leq 200\right) \\
&= 1 - P\left(\frac{X - 170}{20} < \frac{200 - 170}{20}\right) \\
&= 1 - P\left(Z < \frac{200 - 170}{20}\right) \\
&= 1 - P\left(Z < 1.5\right) \\
&= 1 - \Phi\left(1.5\right)
\end{aligned}
$$

In R, we get

```
1 - pnorm(1.5)

## [1] 0.0668072
```

(b) Using the fact that $\bar{X} \sim N\left(\mu = 170, \sigma = 20/\sqrt{20}\right)$, we get

$$
\begin{aligned}
P\left(\bar{X} > 200\right) &= 1 - P\left(\bar{X} \leq 200\right) \\
&= 1 - P\left(\frac{\bar{X} - 170}{20/\sqrt{20}} < \frac{200 - 170}{20/\sqrt{20}}\right) \\
&= 1 - P\left(Z < \frac{200 - 170}{20/\sqrt{20}}\right) \\
&= 1 - P\left(Z < 6.708204\right) \\
&= 1 - \Phi\left(6.708204\right)
\end{aligned}
$$

In R, we get

```
1 - pnorm(6.708204)

## [1] 9.851675e-12
```

(c) $x_{0.95} = \mu + \sigma z_{0.95} = 170 + 20\left(1.644854\right) = 202.8971$ (mg/dL).