1 Chapter 3

1. Since $P\left(A_1 \cap A_2\right) = 0.64 = (0.8)(0.8) = P\left(A_1\right)P\left(A_2\right)$, we can conclude that A_1 and A_2 are independent; hence, the answer is (c).

2. (a) $P\left(A \cap B\right) = P\left(B \cap A\right) = P\left(B|A\right) = (0.95)(0.05) = 0.0475$.

   (b) $P\left(B\right) = P\left(B \cap A\right) + P\left(B \cap A^c\right) = 0.0475 + (.03)(1-0.05) = 0.076$.

   (c) $P\left(A|B\right) = P\left(A \cap B\right)/P\left(B\right) = 0.0475/0.076 = 0.625$.

3. Let A be the event that an adult gets the flu and let B be the event that an adult gets the flu shot.

   (a) $P\left(A \cap B\right) = P\left(A|B\right)P\left(B\right) = (0.1)(0.42) = 0.042$.

   (b) $P\left(A\right) = P\left(A \cap B\right) + P\left(A \cap B^c\right) = 0.042 + (0.7)(1-0.42) = 0.448$.

4. Let X denote the number of people who have asthma. Then $X \sim \mathit{Binomial}\left(n=50, p=0.2\right)$. (Think about why!)

   (a) $P\left(X=19\right) = \binom{50}{19}(0.2)^{19}(0.8)^{50-19} =\approx 0.001579$.

   (b) The standard deviation is $\sigma = \sqrt{np\left(1-p\right)} = \sqrt{(50)(0.2)(0.8)} = 2.828427$ and the mean/expected value is $\mu = np = (50)(0.2) = 10$. Hence, the z-score is $\left(19-10\right)/2.828427 \approx 3.181981$ which implies that 19 is a little over three standard deviations above the mean.

   (c) Using the empirical rule, we have that $P\left(X \geq 19\right) \approx P\left(X \geq \mu + 3\sigma\right) \approx \left(1-0.997\right)/2 = 0.003/2 = 0.0015$. (Draw a picture!). The exact answer is

   ```
   1 - pbinom(18, size = 50, prob = 0.2)

   ## [1] 0.002511203
   ```

5. $E\left[X\right] = \sum_{r=1}^{3} rP\left(X=r\right) = (1)\left(1/3\right) + (2)\left(1/3\right) + (3)\left(1/3\right) = 2$ envelopes.

6. (a) $E\left[X\right] = \mu = \sum_{r=0}^{4} rP\left(X=r\right) = (0)(0.2) + (1)(0.3) + (2)(0.3) + (3)(0.1) + (4)(0.1) = 1.6$ egg masses.

   (b) $Var\left[X\right] = \sum_{r=0}^{4}\left(r-\mu\right)^2 P\left(X=r\right) = \left(0-1.6\right)^2(0.2) + \left(1-1.6\right)^2(0.3) + \left(2-1.6\right)^2(0.3) + \left(3-1.6\right)^2(0.1) + \left(4-1.6\right)^2(0.1) = 1.44$. Hence, the standard deviation is $\sqrt{1.44} = 1.2$ egg masses.

7. Let A be the event that a subject is taking the drug (then A^c is the event that the subject is taking the placebo) and let B be the event that a subject improves.

   (a) $P\left(B \cap A\right) = P\left(B|A\right)P\left(A\right) = (0.6)(0.5) = 0.3$.

   (b) $P\left(B\right) = P\left(B \cap A\right) + P\left(B \cap A^c\right) = 0.3 + (0.35)(0.5) = 0.475$.

8. Let Y be a random variable denoting the number of chickens out of 20 with the bird flu. It then follows that $Y \sim Binomial\,(n = 20, p = 0.1)$.

   (a) $P\,(Y = 5) = \binom{20}{5}(0.1)^5(0.9)^15 = (15504)\left(10^{-5}\right)(0.2058911) \approx 0.031921.$

   (b) $E\,[Y] = np = (20)(0.1) = 2$ chickens.

   (c) $\sqrt{Var\,[Y]} = \sqrt{np(1-p)} = \sqrt{(20)(0.1)(0.9)} \approx 1.3416$ chickens.

9. Let X be a random variable denoting the number of frog eggs that hatch out of 100. Then, $X \sim Binomial\,(n = 100, p = 0.87)$. (Since the frog eggs hatch independently of each other!)

   (a) $P\,(X = 80) = \binom{100}{80}(0.87)^{80}(0.13)^{20} \approx 0.01477606$. You should be able to do this with a calculator, but in R we would just use

   ```
   dbinom(80, size = 100, prob = 0.87)

   ## [1] 0.01477606
   ```

   (b) For a binomial random variable, $E\,[X] = np = (100)(0.87) = 87$ eggs.

   (c) To use the empirical rule, we must first calculate the standard deviation of X. The stahndard deviation of a binomial random variable is given by $\sqrt{Var\,[X]} = \sqrt{np(1-p)} = \sqrt{3.31} = 3.363034$. Computing the z-score yields

   $$Z = \frac{77 - 87}{3.363034} = -2.973505 \approx -3.$$

   In other words, 77 eggs is about three standard deviations below the mean. So, $P\,(X \leq 77) \approx$ the probability of being three or more standard deviations below the mean $\approx 0.003/2 = 0.0015$.

10. (a) $\binom{20}{5} = \frac{20!}{5!(20-5)!} = \frac{(20)(19)(18)(17)(16)(15!)}{5!(15!)} = 15504.$

    (b) $P\,(X = 0) = \binom{5}{0}\,(7/20)^0\,(1 - 7/20)^5 = 0.116.$

11. Let X be a random variable that represents the number of white croaker fish with high mercury levels out of $n = 100$. It follows that $X \sim Binomial\,(n = 100, p = 0.4)$.

    (a) $P\,(X = 100) = (0.4)^100 \approx 1.6069 \times 10^{-40}.$

    (b) $P\,(X = 45) = \binom{100}{45}(0.4)^{45}(1 - 0.4)^{55} = 0.0478.$

    (c) $E\,[X] = np = (100)(0.4) = 40$ fish and $\sqrt{Var\,[X]} = \sqrt{np(1-p)} = \sqrt{(100)(0.4)(0.6)} = 4.898979$ fish.

    (d) $P\,(X \geq 55) = 1 - P\,(X \leq 54) = 1 - [P\,(X = 0) + P\,(X = 1) + \ldots P\,(X = 54)].$ In R, we get

```r
1 - pbinom(54, size = 100, prob  = 0.4)

## [1] 0.001710927
```

2 Chapter 4

1. No (it looks bimodal).
2. The best answer is (d); bimodal.
3. The population might consist of both males and females, and each of these subpopulations probably has its own mean.
4. The best answer is (a); 31.
5. The best answer is (c); 0.22.
6. The best answer is (e); 0.94.
7. It will remain the same. Go back to the properties about correlation and linear transformations!
8. The best answer is (d); 0.27.
9. The best answer is (d); 0.58.
10. Use the fact that $X \sim N\left(\mu = 5.28, \sigma = 0.4\right)$.

    (a)

$$\begin{aligned} P\left(X > 5.4\right) &= 1 - P\left(X \leq 5.4\right) \\ &= 1 - P\left(\frac{X - 5.28}{0.4} < \frac{5.4 - 5.28}{0.4}\right) \\ &= 1 - P\left(Z < \frac{5.4 - 5.28}{0.4}\right) \\ &= 1 - P\left(Z < 0.3\right) \\ &= 1 - \Phi\left(0.3\right) \end{aligned}$$

In R, we get

```r
1 - pnorm(0.3)

## [1] 0.3820886
```

(b)

$$
\begin{aligned}
P\left(5 < X < 6\right) &= P\left(X < 6\right) - P\left(X < 5\right) \\
&= P\left(\frac{X - 5.28}{0.4} < \frac{6 - 5.28}{0.4}\right) - P\left(\frac{X - 5.28}{0.4} < \frac{5 - 5.28}{0.4}\right) \\
&= P\left(Z < \frac{6 - 5.28}{0.4}\right) - P\left(Z < \frac{5 - 5.28}{0.4}\right) \\
&= P\left(Z < 1.8\right) - P\left(Z < -0.7\right) \\
&= \Phi\left(1.8\right) - \Phi\left(-0.7\right)
\end{aligned}
$$

In R, we get

```
pnorm(1.8) - pnorm(-0.7)

## [1] 0.722106
```

(c) The general formula for the p-th percentile, denoted x_p, of a normal distribution with mean μ and standard deviation σ is

$$x_p = \mu + \sigma z_p,$$

where z_p is the p-th percentile of a standard normal distribution (which we can obtain using `qnorm(p)` in R). Hence, the 95-th percentile is $x_{0.95} = 5.28 + 0.4 z_{0.95}$. Using `qnorm(0.95)` in R, we obtain $x_{0.95} = 5.28 + 0.4\left(1.644854\right) = 5.937941$.

(d) Since the data are a random sample from a normal distribution, we know that the sample mean also has a normal distribution; in particular, $\bar{X} \sim N\left(\mu = 5.28, \sigma = 0.4/\sqrt{50}\right)$. Hence,

$$
\begin{aligned}
P\left(\bar{X} > 5.4\right) &= 1 - P\left(\bar{X} \leq 5.4\right) \\
&= 1 - P\left(\frac{\bar{X} - 5.28}{0.4/\sqrt{50}} < \frac{5.4 - 5.28}{0.4/\sqrt{50}}\right) \\
&= 1 - P\left(Z < \frac{5.4 - 5.28}{0.4/\sqrt{50}}\right) \\
&= 1 - P\left(Z < 2.12132\right) \\
&= 1 - \Phi\left(2.12132\right)
\end{aligned}
$$

In R, we get

```
1 - pnorm(2.12132)

## [1] 0.01694744
```

11. Use the fact that $X \sim N\left(\mu = 170, \sigma = 20\right)$.

(a)

$$\begin{aligned}
P\left(X > 200\right) &= 1 - P\left(X \le 200\right) \\
&= 1 - P\left(\frac{X - 170}{20} < \frac{200 - 170}{20}\right) \\
&= 1 - P\left(Z < \frac{200 - 170}{20}\right) \\
&= 1 - P\left(Z < 1.5\right) \\
&= 1 - \Phi\left(1.5\right)
\end{aligned}$$

In R, we get

```
1 - pnorm(1.5)

## [1] 0.0668072
```

(b) Using the fact that $\bar{X} \sim N\left(\mu = 170, \sigma = 20/\sqrt{20}\right)$, we get

$$\begin{aligned}
P\left(\bar{X} > 200\right) &= 1 - P\left(\bar{X} \le 200\right) \\
&= 1 - P\left(\frac{\bar{X} - 170}{20/\sqrt{20}} < \frac{200 - 170}{20/\sqrt{20}}\right) \\
&= 1 - P\left(Z < \frac{200 - 170}{20/\sqrt{20}}\right) \\
&= 1 - P\left(Z < 6.708204\right) \\
&= 1 - \Phi\left(6.708204\right)
\end{aligned}$$

In R, we get

```
1 - pnorm(6.708204)

## [1] 9.851675e-12
```

(c) $x_{0.95} = \mu + \sigma z_{0.95} = 170 + 20\left(1.644854\right) = 202.8971$ (mg/dL).

3 Chapter 5

1. Using the central limit theorem, $\bar{X} \sim N\left(\mu = 19, \sigma = 7.8/\sqrt{30}\right)$. So,

$$\begin{aligned}
P\left(\bar{X} > 21.3\right) &= 1 - P\left(\bar{X} \le 21.3\right) \\
&= 1 - P\left(\frac{\bar{X} - 19}{7.8/\sqrt{30}} < \frac{21.3 - 19}{7.8/\sqrt{30}}\right) \\
&= 1 - P\left(Z < \frac{21.3 - 19}{7.8/\sqrt{30}}\right) \\
&= 1 - P\left(Z < 1.615079\right) \\
&= 1 - \Phi\left(1.615079\right)
\end{aligned}$$

In R, we get

```
1 - pnorm(1.615079)

## [1] 0.05314679
```

2. (a) Running the R script `maleturtle.R`, a 99% confidence interval for the mean carapace length is $(106.6246, 120.1254)$.

   (b) Running the R script `maleturtle.R`, a 99% confidence interval for the mean carapace width is $(84.23794, 92.34539)$.

   (c) Based on the plot, it seems reasonable that this sample belongs to the Painted Turtle species (the true mean happens to be captured in the 99% confidence intervals).

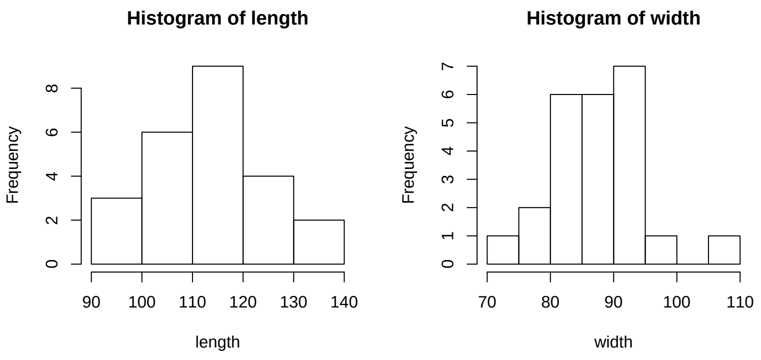
   (d) Narrower, since we are decreaing our confidence.

   (e) The normal approximation does not seem unreasonable here, but more data is needed to get a clearer picture. In R, try

```
# Carapace length
length <- c(93, 94, 96, 101, 102, 103, 104, 106, 107,
  112, 113, 114, 116, 117, 117, 119, 120, 120, 121, 125,
  127, 128, 131, 135)

# Carapce width
width <- c(74, 78, 80, 84, 85, 81, 83, 83, 82, 89, 88,
  86, 90, 90, 91, 93, 89, 93, 95, 93, 96, 95, 95, 106)

# Histograms
par(mfrow = c(1, 2))
hist(length, br = 5)
hist(width, br = 5)
```



3. In R, you could use

```
# Path to data set on my laptop
path <- "C:\\Users\\greenweb\\Desktop\\Filing cabinet\\STT 6300\\Data sets\\bodytemp.csv

# If you don't know how to find this, then just use: path <- file.choose()

# Load the data
bodytemp <- read.csv(path, header = TRUE)

# Temperature variable
temp <- bodytemp$temp

# Pulse rate
pulse <- bodytemp$pulse

# Part a)
t.test(temp, conf.level = 0.95)$conf.int

## [1] 98.12200 98.37646
## attr(,"conf.level")
## [1] 0.95

# Part b)
t.test(temp, conf.level = 0.99)$conf.int

## [1] 98.08111 98.41735
## attr(,"conf.level")
## [1] 0.99

# Part c)
#
# The 99% confidence interval for mean temperature is wider
# than the corresponding 95% confidence interval.

# Part d)
#
# No, since 98.6 is outside the range of both confidence
# intervals.

# Part e)
t.test(pulse, conf.level = 0.90)$conf.int

## [1] 72.73537 74.78771
## attr(,"conf.level")
## [1] 0.9
```

```
# Part f)
#
# False! Go back and read how we interpret confidence
# intervals!
```

4. The correct answer is c). Take a hard look at b) and try to determine why it is not the correct answer.

5. Skip. Extra credit.

6. The correct answer is c). For a given sample, the more confident you want to be, the wider your interval will be and vice versa.

7. The correct answer is f).

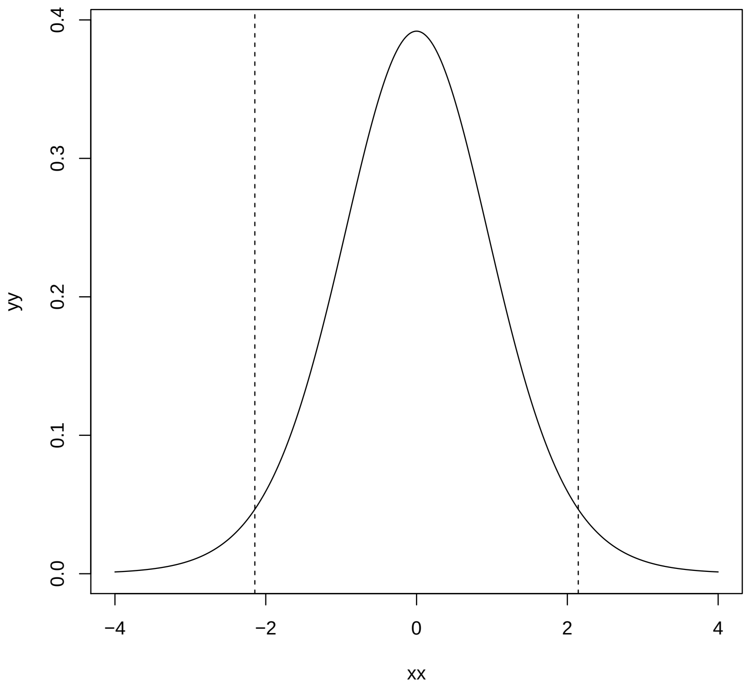
4 Chapter 6

Problem 1

1. $H_0 : \mu = 1.2$ vs. $H_1 : \mu \neq 1.2$
2. The type I error refers to the decision to reject the null hypothesis when it is true. In this case, concluding that the mean response time for rats injected with a unit dose of the experimental drug differes from 1.2 seconds, when in fact it does not.
3. The type II error refers to the decision to not reject the null hypothesis when it is false. In this case, concluding that the mean response time for rats injected with a unit dose of the experimental drug does not differ from 1.2 seconds, when in fact it does.
4. The test statistic is

$$\begin{aligned} t &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{1.05 - 1.2}{0.5/sqrt15} \\ &= -1.161895 \end{aligned}$$

5.
```
xx <- seq(from = -4, to = 4, length = 500)
yy <- dt(xx, df = 14)
plot(xx, yy, type = "l")
abline(v = c(-1, 1) * qt(0.975, df = 14), lty = 2)
```



6. Fail to reject H_0 since $|t| = 1.161895$ is not in the rejection region.

Problem 2 The correct answer is b) (since $0.01 < p \leq 0.05$).

Problem 4

1. The correct answer is that n will go UP!
2. The correct answer is that n will go DOWN!
3. The correct answer is that n will go UP!

Problem 6 Here, μ is the mean level of serum acid phosphatase in prostate cancer patients where the cancer has spread to surrounding lymph nodes. We wish to test the hypothesis

$$H_0 : \mu = 0.645 \quad vs. \quad H_1 : \mu > 0.645$$

The observed t-statistic is 2.5616 and the observed significance level (i.e., the p-value) is 0.009542. Hence, we would reject the null hypothesis at the 0.05 level of significance and conclude that the mean level of serum acid phosphatase in prostate cancer patients where the cancer has spread to surrounding lymph nodes is elevated.

5 Chapter 7

Question 1.

a) Yes, since $p_1 = 0.0076 < 0.01$ indicates strong statistical evidence against the null hypothesis (equality of average sperm concentration).

b) Yes, since $p2 = 0.13 > 0.1$ indicates weak evidence against the null hypothesis (equality of average sperm mortality).

c) Assuming equal variances, the degrees of freedom associated with the t-tests is $df = n_1 + n_2 = 32 + 43 - 2 = 73$.

Question 2.

a) Let μ_l and μ_p be the mean decrease in cholesterol level after taking lipitor and pravachol for a year, respectively. We are interested in testing

$$H_0 : \mu_l - \mu_p = 0 \quad vs. \quad H_1 : \mu_l - \mu_p > 0$$

b) The pooled estimate of the variance is given by

$$s_p^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2} = 628.2557.$$

Hence, the pooled estimate of the stadard deviation is $s_p = \sqrt{628.2557} = 25.06503$.

c) Assuming equal variances, the degrees of freedom associated with the t-tests is $df = n_1 + n_2 = 15 + 15 - 2 = 28$.

d) Since this is an upper one-sided test, the critical region is all values of the test statistic t such that $t > t_{1-\alpha,df} = 1.701131$.

e) The value of the test statistic is

$$t = \frac{\bar{x}_l - \bar{x}_p}{s_p\sqrt{1/n_1 + 1/n_2}} = 0.754.$$

f) Since $t < 1.701131$, we fail to reject the null hypothesis at the 0.05 significance level and conclude that the average decrease in cholesterol level is the same for each drug after taking for a year.

Question 6.

a) Let μ_m and μ_f be the average body temperatures for males and females, respectively. We are interested in testing

$$H_0 : \mu_m - \mu_f = 0 \quad vs. \quad H_1 : \mu_m - \mu_f \neq 0$$

b) Since this is a two-sided test, the rejection region consists of all values of the test statistic t such that $|t| > t_{1-\alpha/2,df} = 1.978671$.

c) The test statistic is $t = -2.285435$.

d) Running the R script, we can verify the result in part c).

e) Yes, hence, we will reject the null hypothesis at the 0.05 significance level and conclude that the average body temperature between males and females is different.

f) Same conclusion (just change `var.equal` to `FALSE` in the associated R code).

g) Running the associated R code, we obtain a 95% confidence interval of $(-0.5396, -0.0388)$. Hence, we can be 95% confident that the true difference $\mu_m - \mu_f$ lies in this interval. This interval also indicates that the average body temperature in males is statisticaly significantly less than that of females using a 0.05 significance level.

h) A 99% confidence interval for the same difference is $(-0.6201, 0.0417)$. This interval is wider (since we want to be more confident) and contains zero, hence, we would fail to reject the null hypothesis at the 0.01 level of signficance and conclude that the average body tempertures are the same.

Question 10.

a) Let X and Y be the before and after burn measurements, respectively. Further, let $D = Y - X$ be the difference and μ_D be the average difference. We are interested in testing

$$H_0 : \mu_D = 0 \quad vs. \quad H_1 : \mu_D \neq 0$$

b) Since the data are paired (i.e., before and after measurements), we should perform a paired t-test (or, equivalently, a one-samplt t-test on the paired differences).

c) The sample mean of the paired differences is $\bar{x}_D = -40.58333$ (kg) and the sample standard deviation is $s_D = 15.79101$ (kg). Since the sample size is 12, the test statistic is $t = \bar{x}_D / \left(s_D/\sqrt{n}\right) = -8.902836$. The associated p-value is $2.331e-06 < 0.001$, hence, we have strong statistical evidence against the null hypothesis and can reasonably conclude that fire does change calcium level in the soil.

d) R reports a 95% confidence interval of $(-50.61646, -30.55020)$. Thus, we are 95% confident that the average difference in calcium level in the soil after fire is between -50.62 and -30.55 kg.