

ES/STT 7140: Homework 6

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Wednesday, April 4, 2018 (due Wednesday, April 11, 2018)

Question 1 This problem refers to data from a study of nesting horseshoe crabs (J. Brockmann, Ethology 1996); see also Agresti (1996) Sec. 4.3 and Agresti (2002) Sec. 4.3. Each female horseshoe crab in the study had a male crab attached to her in her nest. The study investigated factors that affect whether the female crab had any other males, called satellites, residing near her. Explanatory variables that are thought to affect this included the female crab's color (`color`), spine condition (`spine`), weight (`weight`), and carapace width (`width`). The response outcome for each female crab is her number of satellites (`satell`). There are 173 females in this study.

- a) Run the following code to load the data into R and plot the raw number of satellites versus the carapace width. Describe the trend, if any? Does it appear that the width of the female's back can explain the number of satellites attached?

```
# Load the data
url <- paste0("http://users.stat.ufl.edu/~presnell/Courses/",
              "sta4504-2000sp/R/Data/crabs.dat")
crabs <- read.table(url, header = TRUE)
head(crabs) # print first few observations

# Scatterplot of raw data
plot(
  formula = satell ~ weight,
  data = crabs,
  xlab = "Width (cm)",
  ylab = "Number of satellites"
)
```

Answer: While there certainly seems to be a relationship between `width` and `satell`, the relationship is not very strong (at least according to the plot). From the plot, the number of satellites tends to increase with carapace width.

- b) For a female crab, let μ be the expected number of satellites and $x = \text{width}$. Using the R code below, fit a Poisson loglinear model of the form

$$\log(\mu) = \beta_0 + \beta_1 x.$$

For a one-unit increase in width (e.g., one centimeter), what is the multiplicative effect on $\hat{\mu}$?

```
# Fit a Poisson loglinear model
fit <- glm(satell ~ width, data = crabs, family = poisson(link = "log"))
summary(fit) # print model summary
```

Answer: For a one-unit increase in width (e.g., one centimeter), the multiplicative effect on $\hat{\mu}$ (i.e., the mean number of satellites) is given by $e^{\hat{\beta}_1} = e^{0.16405} \approx 1.18$. The easiest way to obtain this in R is to type `exp(coef(fit)[2])`!

Question 2 (extra credit) The R data frame `ldeaths` contains monthly death rates from three lung diseases in the UK over a period of several years (see `?ldeaths` for details and a reference). One possible model for the data is that they can be treated as Poisson random variables with a seasonal component and a long term trend component, as follows:

$$E(\text{deaths}) = \beta_0 + \beta_1 t + \alpha \sin(2\pi \times \text{toy}/12 + \phi),$$

where β_0 , β_1 , α , and ϕ are parameters, t is time since the start of the data, and toy is time of year, in months (January being month 1). The data can be loaded into R using the following snippet of code:

```
death <- as.numeric(ldeaths)
month <- rep(1:12, 6)
time <- 1:72
```

By making use of properties of sines and cosines (i.e., basic trigonometry), we can get this model into a form suitable for fitting using `glm()`, and fit it using the following code:

```
fit <- glm(death ~ sin(month/12*2*pi) + cos(month/12*2*pi),
          family = poisson(link = "identity"))
summary(fit)
```

Use the following code to plot the raw data time series on a plot, with the predicted time series overlaid. Additionally, the code also constructs a scatterplot of the (deviance) residuals versus the fitted values and a Q-Q plot of the residuals. Based on these plots alone, does the model appear to provide an adequate fit to the data? Write your answer using a paragraph (i.e., 5 – 7 complete sentences). If possible, see if you can figure out the model that is being fit by the `glm()` function!

```
# Plot fitted mean response
plot(time, death, type = "l")
lines(time, fitted(fit), col = 2)

# Residual plots
par(mfrow = c(1, 2))
plot(fit, which = 1:2)
```