Student Manual
Supersonic Wind Tunnel Practical

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The purpose of the supersonic wind tunnel practical is to gain a physical insight on channel flow and high speed wind tunnel operation following the lectures given in the compressible aerodynamics course. The practical consists of three parts:

Water Channel Demonstration A demonstration is given to illustrate some aspects of supersonic flows using a water channel analogy. Visual examples are given to show the difference between supersonic and subsonic flow, the influence of back pressure, the behavior of oblique shock waves in a channel, and the starting conditions of a supersonic wind tunnel.

Wind Tunnel Experiment An experiment is conducted in a small-scale wind tunnel where modifications to the tunnel geometry are used to produce various flow conditions, ranging from subsonic to supersonic. Measurements will be performed using steady pressure ports and optical methods (Schlieren).

Report Compilation Directly following the experiment, a report is compiled in which the measurements from the experiment are analyzed and compared with theoretical predictions.

Before attending this practical, it is vital to read over this manual and the theory of compressible channel flow, discussed in Chapter 10 of Anderson's 'Fundamentals of Aerodynamics.' Additionally, ensure that one of your group members has access to Matlab or Python on their computer.

1 Theory

1.1 The Mach-Area Relation

For a quasi one-dimensional flow without the influence of viscosity, it is possible to derive simple algebraic relations linking the local flow conditions to the local geometry. This 1-D assumption is reasonable for channels with a gradual variation in cross-sectional area, and where the local radius of curvature is large compared to the channel height. The derivations can be found in Chapter 10 of Anderson. The most important result is the *Mach-Area relation*, which establishes a relation between the local area ratio (ratio between the local cross section and the throat cross section) and the Mach number,

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/(\gamma-1)} \tag{1}$$

where $A = A(x)$ is the local cross-sectional area at position x, A^* is the cross-sectional area at sonic conditions, M is the local Mach number, and γ is the ratio of specific heats. There are two critical points to note:

1. For a given area ratio A/A^*, two Mach numbers can be obtained from equation 1, corresponding to a subsonic and a supersonic solution.

2. Defining a contour with a variable area ratio is *not* enough to create a flow through the channel. Generating a flow requires a proper inlet and outlet boundary condition across the channel, which typically means that there is a pressure difference between the inlet and the outlet (i.e., a pressure ratio).

For an isentropic flow the pressure ratio can be related to the local Mach number by,

$$\frac{p}{p_t} = \left(1+\frac{\gamma-1}{2}M^2\right)^{-\left(\frac{\gamma}{\gamma-1}\right)} \tag{2}$$

where p is the local static pressure and p_t is the local total pressure. In general, the total pressure is measured in the settling chamber of the wind tunnel. The total pressure may be assumed constant and equal to the value in the settling chamber throughout the flow in *absence* of shock waves and other non-isentropic flow phenomena. By combining the geometry requirements outlined in equation 1 and the pressure requirements of equation 2, a unique flow condition is established in the channel.

In figure 1, the flow through a convergent-divergent channel is shown. The free stream flow direction is from left to right. Far upstream in the settling chamber, the cross section is considered to be infinitely large and thus the velocity and the Mach number are zero. In this location the pressure is equal to the total pressure (p_t). A pressure ratio across the channel is established by varying (decreasing) the pressure at the exit. The value of the pressure ratio governs the flow in the channel.

When the pressure at the end is decreased (going from curves 0 to 1 and 2 in figure 1), the local Mach number increases. As the exit pressure is further decreased to $(p/p_t)_3$, the flow becomes sonic ($M = 1$) in the throat but remains subsonic in all other locations. Therefore, for all $(p/p_t) > (p/p_t)_3$ the flow is subsonic throughout the channel.

As the exit pressure is further decreased, a supersonic flow will be established downstream of the throat. When the exit pressure ratio is greater than $(p/p_t)_5$, the boundary conditions at the channel exit cannot be satisfied. Therefore a shock wave is established within the channel in order to satisfy the pressure boundary condition at the exit (see for example curve 4 in figure 1).

As the exit pressure is further decreased, the shock wave moves toward the exit of the channel. In case the pressure ratio over the channel is equal to $(p/p_t)_5$ a normal shock wave is located at the exit. As the exit pressure is decreased below $(p/p_t)_5$, the flow is *over-expanded*, and the normal shock becomes an oblique shock outside of the channel. If the exit pressure is equal to $(p/p_t)_6$, no shock waves are formed leading to the *ideally-expanded* case. When the pressure ratio is smaller than $(p/p_t)_6$ the flow is *under-expanded* and expansion waves will be formed at the exit of the channel.

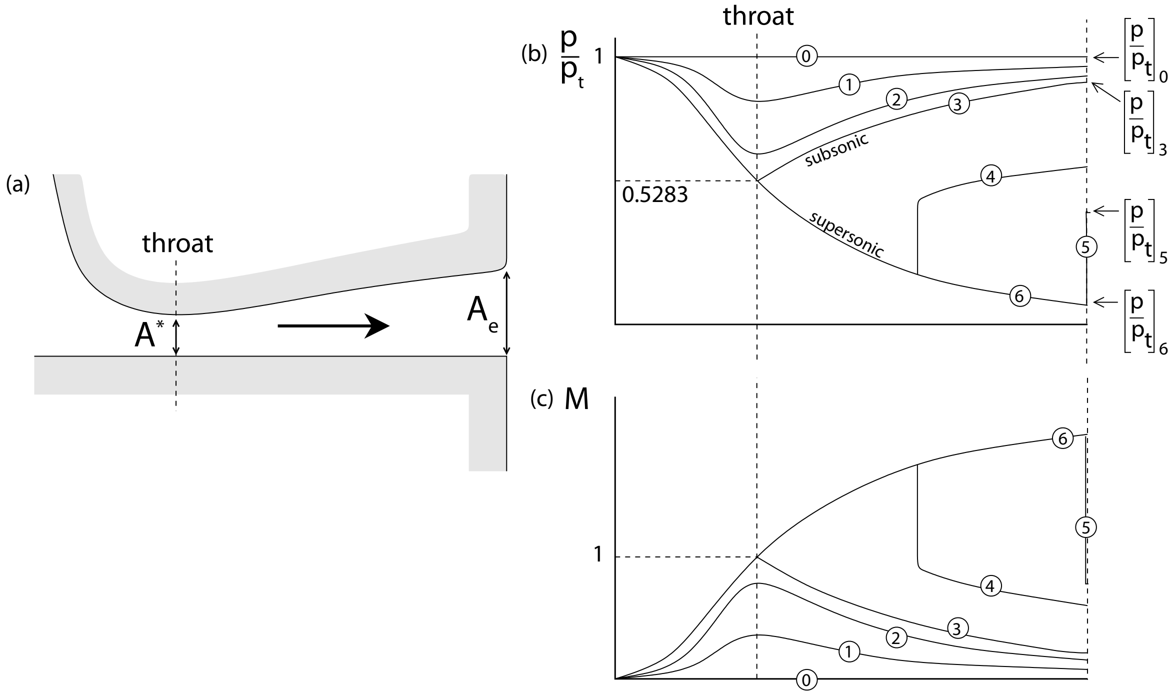


Figure 1: (a) Schematic nozzle flow, (b) Possible pressure distributions in the nozzle, (c) Possible Mach number distributions

1.2 Pressure Regulation Using a Second Throat

The previous section demonstrated how modifying the exit pressure influences the flow through the channel. In case of a vacuum tunnel (air is sucked through the tunnel by means of a vacuum pump) this can be achieved by changing the strength of the pump or by partially opening the channel to allow bleed air to enter.

Another possibility is the application of an extra throat (or choke) downstream of the first throat. If the cross-section of the second throat is larger than the first, both throats may have sonic conditions. Thus the flow situation will be as shown in figure 2): the flow upstream of the first throat is subsonic and the flow downstream is partially supersonic. A normal shock wave is present after which it causes the flow to become subsonic. Moving further downstream the flow is accelerated when going through the converging part that belongs to the second throat. Downstream of the second throat the flow is supersonic again.

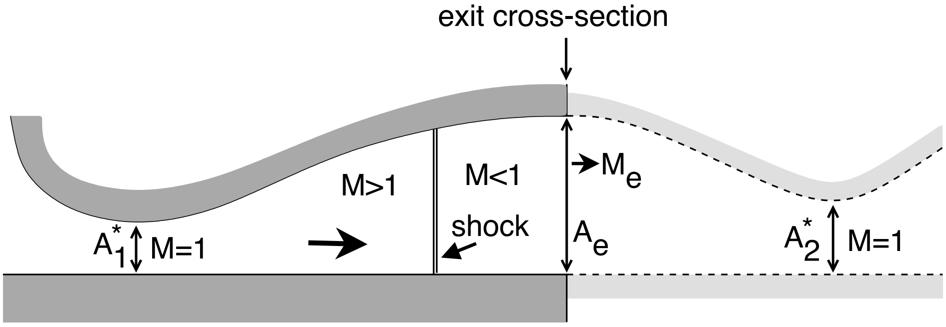


Figure 2: Flow through two choked throats, A_1^* and A_2^*, with a NSW in between.

In order to find the relationship between the occurence of the normal shock wave and the presence of a second throat, we consider the continuity equation and apply it in both throats having sonic conditions:

$$\rho_1^* u_1^* A_1^* = \rho_2^* u_2^* A_2^* \tag{3}$$

After some manipulation (see Anderson) the following relation results:

$$\frac{A_2^*}{A_1^*} = \frac{p_{t,1}}{p_{t,2}} \sqrt{\frac{T_{t,1}}{T_{t,2}}} \tag{4}$$

And for an adiabatic flow ($T_{t,1} = T_{t,2}$), this simplifies to:

$$\frac{A_2^*}{A_1^*} = \frac{p_{t_1}}{p_{t_2}} \tag{5}$$

Therefore when the throat cross-section area ratio $\frac{A_2^*}{A_1^*}$ decreases, but stays larger than 1 (A_2^* approaches A_1^*), the total pressure loss also decreases. This means that the normal shock wave must become weaker by occurring at a lower Mach number i.e. the shock moves upstream. Therefore, by varying the throat cross-section area ratio A_1^*/A_2^* allows the position of the normal shock wave to be precisely controlled.

1.3 The Mach-Area Relation for Subsonic Flow

For a subsonic flow through the throat ($M < 1$), the Mach-Area relation cannot be used to compute the theoretical Mach number and pressure-ratio distribution directly because the area ratio A/A^* is based on the area of the tunnel at sonic conditions, A^*. For supersonic flow, the sonic area is equal to the throat area: $A^* = A_t$. For subsonic flow through the throat there is no sonic area.

In order to still use the Mach-Area relation for a fully subsonic flow, a reference sonic area can be approximated by using a reference pressure measurement (i.e., first point, $x = 0$). Using this reference pressure measurement, the local Mach number can be obtained from the isentropic relation and also the area ratio at this particular location x_0 (using 1 and 2):

$$\left(\frac{p}{p_t}\right)_{x_0,\text{measured}} \rightarrow \frac{A(x_0)}{A^*}$$

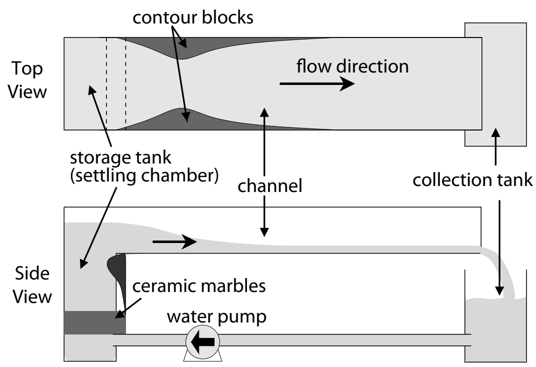
Since the first throat does not have sonic conditions for a fully subsonic flow we do not call it A^* but A_t. The geometry of the tunnel (given in table 1) allows the geometric area ratio $A(x_0)/A_t$ to be determined. Using the sonic area ratio obtained from the pressure measurement $A(x_0)/A^*$ the coefficient to convert from the geometric area ratio to the sonic area ratio is determined by $A_t/A^* = A(x_0)/A^* \times A_t/A(x_0)$. Once this coefficient is found, the procedure to determine the pressure ratio and Mach number throughout the flow becomes:

1. Determine local geometric area ratio at position x (from table): $A(x)/A_t$.
2. Convert the local geometric area ratio to the local sonic area ratio via the correction coefficient: $A(x)/A^* = A_t/A^* \times A(x)/A_t$.
3. Use equations 1 and 2 to determine the Mach number and pressure ratio.

2 Facilities

2.1 Water Channel

The water channel is used to give a simple visualization of flow phenomena that occur for compressible air flow through a nozzle. A schematic diagram of the water channel is shown on the right.



There is an analogy between flow in a shallow water channel and compressible air flows. This analogy only exists for 2D flows and can be explained by considering the governing equations for the two cases, which are a (nearly) identical set of partial differential equations,

$$
\begin{array}{lcc}
 & \text{air} & \text{water} \\
\text{continuity:} & \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 & \frac{\partial h}{\partial t} + \nabla \cdot (h \mathbf{u}) = 0 \\
\text{momentum:} & \frac{D\mathbf{u}}{Dt} + \frac{\gamma p_t}{\rho_t^\gamma} \rho^{\gamma - 2} \nabla \rho = 0 & \frac{D\mathbf{u}}{Dt} + g \nabla h = 0
\end{array}
\tag{6}
$$

where the operator $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$ is the material flux, $\mathbf{u}$ the velocity vector, g acceleration of gravity and h the water level. Comparing the continuity equations shows an analogy between the air density ρ and the water level h. The two momentum equations are analogous if the term ahead of $\nabla\rho$ is constant for air flow. This will only be the case for a ratio of the specific heats $\gamma = 2$, which is not true for a real gas. This means that the analogy is only approximate. However, in this approximate case, there is analogy between the following quantities:

$$
\begin{array}{ccc}
\rho & \leftrightarrow & h \\
p & \leftrightarrow & h^2 \\
T & \leftrightarrow & h
\end{array}
$$

For an air flow we have the Mach number, which is the ratio between the velocity and the speed of sound: $M = u/a$, as the governing similarty parameter for the occurence of shock and expansion waves. For a shallow water flow a similar non-dimensional number is the Froude number, which is the ratio between the flow velocity and the speed of the surface waves $(Fr = u\,/\,\sqrt{gh})$. For a shallow water flow with $Fr > 1$ we can have a jump in the water level, which is analogous to shock waves that occur in supersonic air flow.

2.2 Wind Tunnel

The ST-3 supersonic wind tunnel is a vacuum-type wind tunnel originally built for the NLR (Netherlands Aerospace Centre) in Amsterdam. In its initial form, the ST-3 was the very first supersonic wind tunnel in the Netherlands. In 1969, the tunnel was donated to TUDelft. The tunnel is driven by a 36 kW vacuum pump located in an adjacent room. This drives the tunnel by reducing the exit pressure. The inlet to the tunnel is dried air at atmospheric pressure. To vary the Mach number in the wind tunnel, the geometry is modified.

A schematic representation of the tunnel is given in figure 3. The tunnel section is a half-channel, where the bottom part of the channel is flat and the top part describes the channel contour. A fixed nozzle block is used to create first throat. This block has a design exit Mach number of 2.1. In the bottom half of the channel, 153 pressure ports are used to measure the static pressure. The flexible upper wall and adjustable diffuser downstream of the first throat are used to vary the geometrical cross section in this part of the channel.

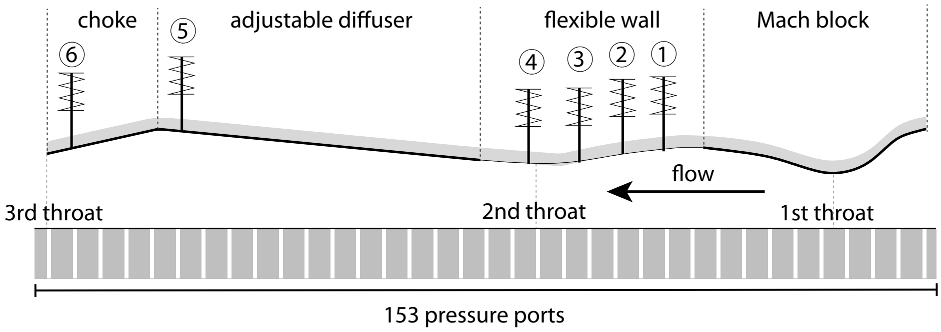


Figure 3: Cross view of the ST-3 tunnel section

2.3 Pressure Measurement

The static pressures are measured through the orifices in the lower flat channel wall and the pitot pressure is measured in the settling chamber using a pitot tube. The pressures are measured by means of a pressure transducer. This is an instrument that converts a pressure into an electrical signal (normally a voltage). The pressure transducer consists of a membrane instrumented with strain gauges that deforms because of the pressure. The deformation of the strain gauges causes a variation in the electrical resistance which is measured by means of a digital volt meter. Finally the measured voltage is converted into pressure using a calibration curve. The static pressure is not measured for all 153 pressure orifices since only 48 connections are available.

2.4 Schlieren Visualization

For visualization of the flow in the wind tunnel, a Schlieren system is used. This system makes use of the physical principle that a light-ray, when passing through a transparent medium with changing index of refraction, is deflected in the direction of increasing index of refraction. When the indices of refraction (n_1 and n_2) and the incoming light ray angle (α_1) are known, the deflection (α_2) can be calculated by Snell's law (see figure 4):

$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2) \tag{7}$$

For a gas, the index of refraction is proportional to the density expressed by the Gladstone-Dale relation:

$$n = 1 + K\rho \tag{8}$$

Thus a light ray will be deflected when it encounters density gradients. In a compressible flow these density gradients are present in shock waves, expansion fans and boundary layers.

The principle of a Schlieren system can be illustrated using figure 5. A light source is imaged onto a diaphragm (pinhole). The pinhole is located in the focal point of a lens, which thus creates a collimated light beam through the test section. In the schematic two (density) disturbances are modeled by means of prisms (the flow direction is into the paper). The light beam will remain collimated except for the part that encounters the disturbances. The non-deflected part of the beam is converged by a second lens and an image of the diaphragm is established in its focal point. In the focal point a Schlieren knife is positioned that cuts off approximately half of the light. The other half that is not stopped by the Schlieren knife is projected onto a camera or a screen that shows an image of the test section with uniform light intensity, except for locations where disturbances are projected. Assume that the light passing through the upper disturbance is deflected downward. This light ray is stopped by the Schlieren knife and the corresponding location in the image plane stays dark. The light passing through the lower disturbance is deflected upward. This light ray can pass the Schlieren knife unhindered and doubles the intensity in the image plane.

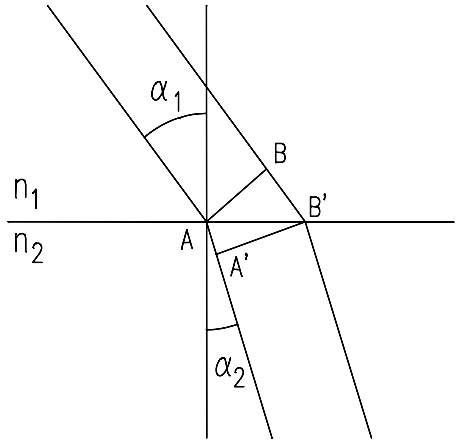


Figure 4: Illustration of Snell's Law.

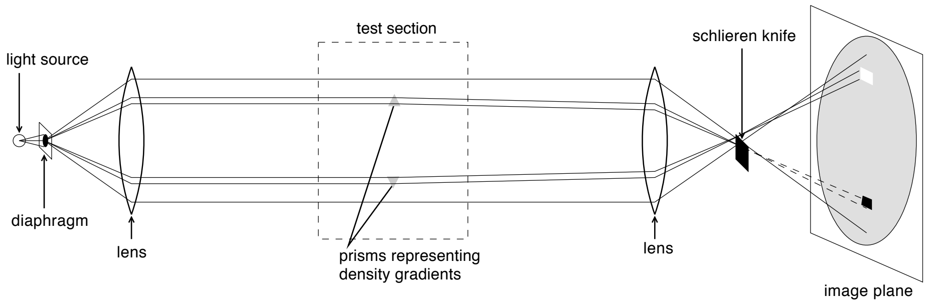


Figure 5: Schematic of a single pass Schlieren system

3 Measurements and Report

The practical is given for a group of 6 students at a time; however, this group of six will be broken up into two groups of three (group A and group B). Each group of three will then acquire a set of experimental data and write a report.

3.1 Measurements

The measurements are divided into two parts. In the first part, two measurements are performed that focus on the flow through the fixed nozzle. The geometrical cross section for this Mach block is given in table 1 which allows to compare the measurements with theory. The measurements are performed on a subsonic-supersonic flow (the throat is choked) and on a completely subsonic flow (throat is not choked). The measurement parameters are given in table 1.

Table 1: Measurements performed for part 1.

Measurement	Meter 4 [mm]	Meter 5 [mm]	Meter 6 [mm]	Shock Position [mm]
1A and 1B	0.0	0.0	-5.0	-
2A	15.0	0.0	-5.0	-
2B	18.0	0.0	-5.0	-

The second part focuses on the flow in the adjustable diffuser having a gradually increasing cross section. Measurements will be done on a completely subsonic flow and on a mixed subsonic/supersonic flow where the position of the shock wave will be varied. The measurement parameters are given in table 2.

Table 2: Measurements performed for part 2.

Measurement	Meter 4 [mm]	Meter 5 [mm]	Meter 6 [mm]	Shock Position [mm]
3A	15.0	10.0	-	630.0
4A	15.0	10.0	-	530.0
5A	15.0	10.0	-	Second Throat
3B	18.0	10.0	-	630.0
4B	18.0	10.0	-	530.0
5B	18.0	10.0	-	Second Throat

3.2 Report Overview

The report must be emailed to your practical supervisor no later than 2 hours after finishing the practical. You must include the following information at the top of the report:

1. Name and student number of each member of the group and the group number.
2. Date and time of the practical.
3. Name of your practical supervisor.

The content of the report is broken up into two parts. Both parts require a comparison to theoretical estimates based on the isentropic flow relations and the normal shock relations. To generate these estimates, it is *highly recommended* that you use the flowisentropic/flownormalshock command given in the Matlab Aerospace Toolbox. If your group is unfamiliar or does not have access to Matlab, a set of Python codes are provided on Brightspace which emulate the functionality of the Matlab code.

3.3 Report Part 1

This part analyzes the data from measurements 1A/B and 2A/B, which pertains to the flow through the fixed nozzle block from 44.8 $mm \leq x \leq 194.8$ mm. The geometry of the block at each pressure port is given in table 1. In your report, present the following:

1. A plot of the measured *and* theoretical pressure ratio p/p_t versus the position x for 44.8 $mm \leq x \leq 194.8$ mm.
2. A plot of the measured *and* theoretical Mach number M versus the position x for 44.8 $mm \leq x \leq 194.8$ mm.
3. A short discussion comparing the theoretical and the measured values. Provide an explanation for discrepancies seen in the figures (effect of the boundary layer and 1D assumption).

These plots and discussion should be able to fit *on a single page*. Note: for the subsonic portion of the plots, an area ratio correction is needed as discussed in section 1.3 of this guide.

3.4 Report Part 2

This part analyzes the data from measurements 3A/B, 4A/B, and 5A/B. In these measurements, the flow through the first throat in the fixed nozzle block is subsonic, while the flow through the second throat in the adjustable nozzle section is sonic. The geometry of the adjustable nozzle section is given as a function of the micrometer settings in table 2. In your report, present the following *within the same figure*:

1. A plot of the measured pressure ratio p/p_t for measurements 3, 4, and 5.
2. A set of vertical lines indicating the shock locations as seen in the Schlieren visualization.

3. For the two shock locations downstream of the second throat, you have to *compute* three points:

    i the maximum pressure ratio at that location for subsonic flow through the nozzle $(p_e/p_t)_3$;

    ii the minimum pressure ratio at that location for supersonic flow through the nozzle $(p_e/p_t)_6$);

    iii the pressure ratio after a normal shock positioned at that location $(p_e/p_t)_5$).

    Plot these values in the same graph as the measurements.

4. A short discussion comparing the theoretical and the measured values. Provide an explanation for discrepancies seen in the figure.

This plot and discussion should be able to fit *on a single page*. Note: the calculation of the pressure ratios and the normal shock properties requires the Mach number at the shock location. This can be computed by the geometrical information given in table 2. The height of the adjustable diffuser is a linear distribution: $h = (dh/dx)x + h_0$.

4 TABLES

Table 1: Distribution of channel height and area ratio at pressure port locations within the fixed nozzle block section. The fixed nozzle corresponds to $M_{nom} = 2.1$, $x_{throat} = 65.0$ mm, $h_{throat} = 16.34$ mm.

x [mm]	h [mm]	$A/A_{throat}[-]$
44.8	19.204	1.175
56.8	16.819	1.029
59.8	16.532	1.012
62.8	16.372	1.002
74.8	16.958	1.038
79.8	17.664	1.081
84.8	18.553	1.136
96.8	21.067	1.290
99.8	21.698	1.328
102.8	22.306	1.365
114.8	24.488	1.499
119.8	25.275	1.547
124.8	25.992	1.591
129.8	26.638	1.631
134.8	27.219	1.666
139.8	27.734	1.698
144.8	28.188	1.725
149.8	28.584	1.750
154.8	28.924	1.770
159.8	29.212	1.788
164.8	29.450	1.803
176.8	29.832	1.826
179.8	29.890	1.830
182.8	29.935	1.832
194.8	30.000	1.836

Table 2: Height of 2nd throat (h_{k_2}) and height of flexible diffuser $\left(h = \frac{dh}{dx}x + h_0 \text{ for } 410\text{mm} < x < 760\text{mm}\right)$ as a function of the micrometers 4 and 5.

Meter 4 [mm]	Meter 5 [mm]	h_{k_2} [mm]	$\frac{dh}{dx}$ [-]	h_0 [mm]
12	0	17.9	.03582	3.9
12	1	17.9	.03279	5.0
12	2	17.9	.02995	6.0
12	3	17.8	.02725	7.0
12	4	17.8	.02464	8.0
12	5	17.8	.02209	9.0
12	6	17.8	.01954	10.0
12	7	17.7	.01699	10.9
12	8	17.7	.01440	11.9
12	9	17.6	.01178	12.9
12	10	17.5	.00911	13.9
12	11	17.5	.00641	14.9
12	12	17.4	.00368	15.9
12	13	17.3	.00095	17.0
12	14	17.3	-.00175	18.0
15	0	15.1	.04397	-2.2
15	1	15.1	.04101	-1.0
15	2	15.1	.03822	.1
15	3	15.1	.03555	1.1
15	4	15.0	.03293	2.1
15	5	14.9	.03033	3.0
15	6	14.8	.02772	3.9
15	7	14.7	.02508	4.9
15	8	14.7	.02240	5.9
15	9	14.6	.01967	7.0
15	10	14.6	.01692	8.0
15	11	14.6	.01417	9.1
15	12	14.6	.01145	10.1
15	13	14.5	.00880	11.1
15	14	14.5	.00628	12.0
18	0	12.2	.05194	-8.2
18	1	12.2	.04912	-7.2
18	2	12.1	.04635	-6.2
18	3	12.1	.04362	-5.1
18	4	12.1	.04091	-4.0
18	5	12.0	.03821	-2.9
18	6	12.0	.03551	-1.8
18	7	11.9	.03281	-.8
18	8	11.9	.03011	.2
18	9	11.8	.02741	1.2
18	10	11.8	.02472	2.2
18	11	11.8	.02204	3.2
18	12	11.7	.01940	4.1
18	13	11.7	.01680	5.1
18	14	11.6	.01428	6.1