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Appendix A
Math Handbook

I. Basic Math Calculations

Fractions, decimals, and percents

To express a fraction as a decimal, divide the numerator by the denominator. The resulting number, the quotient, will be the decimal equivalent of the fraction. To express a fraction as a percent, multiply the quotient by 100%. Round the result to the correct number of significant digits.

Example: Express $\frac{33}{59}$ as a decimal and as a percent.

Strategy:

- Divide 33 by 59, expressing the answer in two significant digits.
- Multiply the decimal by 100 to determine the percent.

Solution: $\frac{33}{59} = 0.5593 = 0.56$

$$0.56 \times 100 = 56\%$$

To express a percent as a decimal, write the percent in the form of a fraction, $\frac{x}{100}$, and then find the quotient. A shortcut is to simply move the decimal point two places left and remove the percent symbol.

Example: Express 91.6% as a decimal.

Strategy:

- Move the decimal point two places to the left.

Solution: $91.6\% \Rightarrow 0.916$

Calculating relative uncertainty and relative error

Whenever you measure a physical quantity, there is some degree of uncertainty in the measurement. The type of measuring device chosen, as shown by the two metersticks in **Figure 1,** and how carefully the measuring device was used affect precision and accuracy.

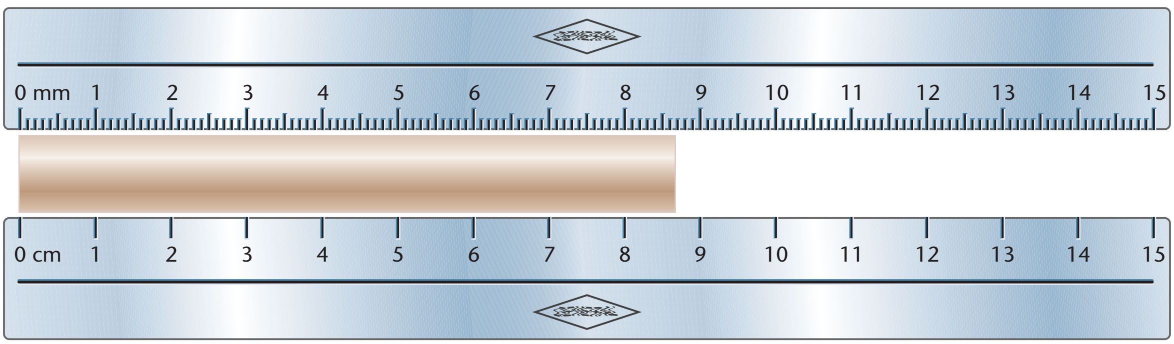


FIGURE 1

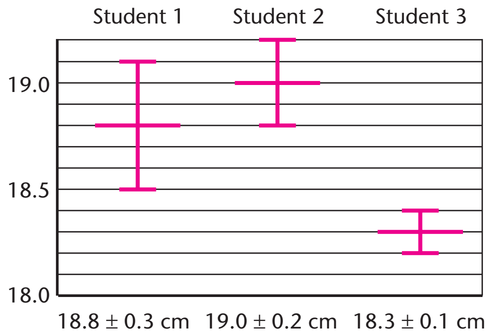


FIGURE 2

The precision of an experimental result can be expressed as estimated uncertainty. Examine the experimental results in **Figure 2** reported by the three students in Chapter 2. Each student measured the length of a block of wood. Student 1's result was reported as (18.8 ± 0.3) cm. The estimated uncertainty in that measurement is represented by $\pm$ 0.3. Notice that each student reported a different estimated uncertainty.

The students also could have reported each result using relative uncertainty.

$$\text{relative uncertainty (\%)} = \frac{\text{estimated uncertainty}}{\text{actual measurement}} \times 100$$

Often, experimental data are compared to accepted values. Relative error is the percent deviation from an accepted value, that is, the uncertainty of a measurement in terms of accuracy. The relative error is calculated according to the following formula.

$$\text{relative error (\%)} = \frac{|\text{accepted value} - \text{experimental value}|}{\text{accepted value}} \times 100$$

Example: Compare the relative error and relative uncertainty of each student's measurement shown in **Figure 2.** The actual length of the block of wood is 19.0 cm.

Strategy:

- Identify the experimental value and estimated uncertainty measured by each student.
- Use the formulas above to calculate both unknown quantities.
- Round the answers to the correct number of significant digits.

Solution:

$$\text{relative error \#1} = \frac{|19.0\text{ cm} - 18.8\text{ cm}|}{19.0\text{ cm}} \times 100 = 1.05\%$$

$$\text{relative error \#2} = \frac{|19.0\text{ cm} - 19.0\text{ cm}|}{19.0\text{ cm}} \times 100 = 0.00\%$$

$$\text{relative error \#3} = \frac{|19.0\text{ cm} - 18.3\text{ cm}|}{19.0\text{ cm}} \times 100 = 3.68\%$$

$$\text{relative uncertainty \#1} = \frac{0.3\text{ cm}}{18.8\text{ cm}} \times 100 = 1.59\% = 2\%$$

$$\text{relative uncertainty \#2} = \frac{0.2\text{ cm}}{19.0\text{ cm}} \times 100 = 1.05\% = 1\%$$

$$\text{relative uncertainty \#3} = \frac{0.1\text{ cm}}{18.3\text{ cm}} \times 100 = 0.54\% = 0.5\%$$

Student 3 reported the smallest relative uncertainty. His measurement was the most precise. Student 2's measurement was the most accurate. She had the smallest relative error.

Ratios, rates, and proportions

A ratio is a comparison between two numbers by division. Ratios are often expressed as fractions. A rate is a ratio between two measurements with different units. For example, the ratio, $\frac{\text{meters}}{\text{second}}$, compares the distance traveled to a period of time. In physics, you will need to solve problems that relate ratios. A proportion is a statement of equality of two or more ratios. To solve for the unknown quantity in a proportion, cross multiply the terms in the ratios and solve for the unknown. Notice that the cross products *ad* and *cb* are equal.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } ad = cb.$$

Example: $\frac{1.0\text{ in.}}{2.54\text{ cm}} = \frac{3.5\text{ in.}}{x}$

Strategy:

- Cross multiply.
- Solve for x.
- Round the answer to two significant digits.

Solution: $\frac{1.0\text{ in.}}{2.54\text{ cm}} = \frac{3.5\text{ in.}}{x}$

$$(1.0\text{ in.})x = (2.54\text{ cm})(3.5\text{ in.})$$

$$x = \frac{(2.54\text{ cm})(3.5\cancel{\text{ in.}})}{1.0\cancel{\text{ in.}}}$$

$$x = 8.89\text{ cm}$$

$$x = 8.9\text{ cm}$$

II. Algebra

Solving equations

To solve for one unknown, perform arithmetic operations on both sides of the equal sign until the unknown is by itself on one side of the equation.

Example: Solve the following equation for x.

$$\frac{ay}{x} = cb + 5$$

Strategy:

- Multiply both sides by x.
- Divide both sides by the term $cb + 5$.

Solution:

$$\frac{ay}{x} = cb + 5$$

$$x\left(\frac{ay}{x}\right) = x(cb + 5)$$

$$ay = x(cb + 5)$$

$$x = \frac{ay}{cb + 5}$$

Unit operations/dimensional analysis

Most physical quantities have units as well as numerical values. When you substitute a value into an equation, you must write both the value and the unit. You have learned in the factor-label method of unit conversion that, when a term has several units, you can operate on the units like any other mathematical quantity. You will often be able to tell when you have set up the equation incorrectly by inspecting the units. This procedure is often called dimensional analysis. If your answer has the wrong units, you have made an error in the calculation of your answer.

Example: Find d when v = 67 meters/second and t = 5.0 minutes.

Strategy:

- v, t, and d are related by the equation $d = vt$.
- Set up the equation and operate on the units.
- Be sure the resulting unit is correct for d.

Solution: $d = vt$

$$d = \frac{67 \text{ meters}}{\text{second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times 5.0 \text{ minutes}$$

$$d = \frac{67 \text{ meters}}{\cancel{\text{second}}} \times \frac{60 \cancel{\text{ seconds}}}{1 \cancel{\text{ minute}}} \times 5.0 \cancel{\text{ minutes}}$$

$$d = 2.0 \times 10^4 \text{ meters}$$

d is measured in units of length. The solution is correct.

Properties of exponents

An exponent tells how many times a number, called a base, is used as a factor. In the example, $a \times a \times a = a^3$, a is raised to the third power.

For any nonzero number a and any integer n, the following properties apply.

- Exponent of Zero: $a^0 = 1$
- Exponent of One: $a^1 = \text{a}$
- Negative Exponents: $a^{-n} = \dfrac{1}{a^n}$

For all integers a and b and all integers m, n, and p, the following properties apply.

- Product of Powers: $a^m \times a^n = a^{m+n}$
- Power of Powers: $(a^m)^n = a^{mn}$
- Quotient of Powers: $\dfrac{a^m}{a^n} = a^{m-n}$
- The n-Root of Powers: $\sqrt[n]{a^m} = a^{m/n}$
- Power of a Product: $(ab)^m = a^m b^m$
- Power of a Monomial: $(a^m b^n)^p = a^{mp} b^{np}$

Example: Simplify $(2a^4b)^3[(-2b)^3]^2$

Strategy:
- Use the power of powers property.
- Use the power of a monomial property.
- Use the product of powers property.

Solution:

$$\begin{aligned}(2a^4b)^3[(-2b)^3]^2 &= (2a^4b)^3(-2b)^6\\ &= 2^3(a^4)^3b^3(-2)^6b^6\\ &= 8a^{12}b^3(64)b^6\\ &= 512a^{12}b^9\end{aligned}$$

Example: Simplify $\sqrt[4]{\dfrac{1}{a^2}}$

Strategy:
- Use the negative exponents property.
- Use the n-root of powers property.

Solution:

$$\begin{aligned}\sqrt[4]{\frac{1}{a^2}} &= \sqrt[4]{a^{-2}}\\ &= a^{-2/4}\\ &= a^{-1/2}\end{aligned}$$

The quadratic formula

Any equation in one variable, where the highest power is two, is a quadratic equation. The graph of a quadratic equation is a parabola. The roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quantities a, b, and c typically are given.

The expression $b^2 - 4ac$ is called the discriminant. The discriminant tells us the nature of the roots of the quadratic equation.

Discriminant	Nature of the Roots
$b^2 - 4ac > 0$	two distinct real roots
$b^2 - 4ac = 0$	exactly one real root
$b^2 - 4ac < 0$	two distinct imaginary roots

Example: Solve $x^2 - 6x - 40 = 0$.

Strategy:

- Substitute the values in the quadratic formula.
- $a = 1$, $b = -6$, and $c = -40$.

Solution:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-40)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 + 160}}{2}$$

$$x = \frac{6 \pm \sqrt{196}}{2}$$

$$x = \frac{6 \pm 14}{2}$$

$$x = \frac{6 + 14}{2} \quad \text{or} \quad x = \frac{6 - 14}{2}$$

$$= 10 \qquad\qquad = -4$$

Notice in the example that there are two solutions, $x = 10$ and $x = -4$. Sometimes in physics problems, only one solution corresponds to a real-life situation. In that case, one of the solutions would be discarded.

III. Geometry and Trigonometry

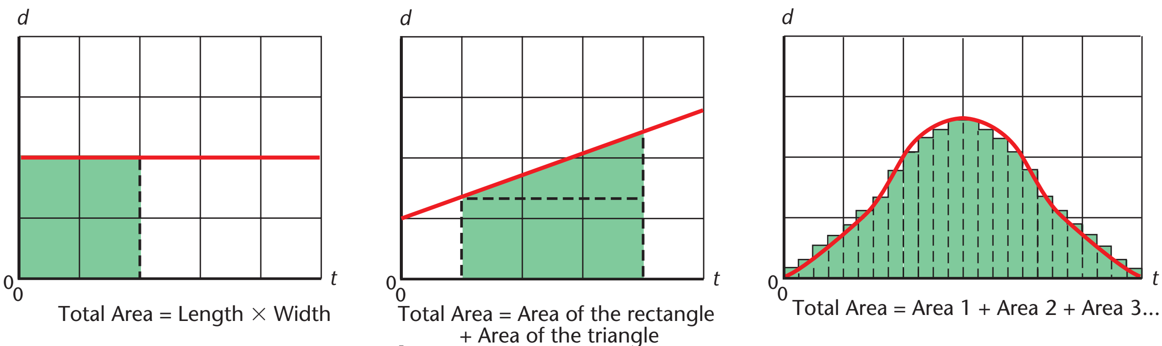
Perimeter, area, and volume

Use the following table to solve problems involving perimeter, circumference, area, and volume.

	Perimeter/ Circumference	Area	Surface Area	Volume
Circle radius r	$C = 2\pi r$	$A = \pi r^2$		
Square side a	$P = 4a$	$A = a^2$		
Rectangle length l width w	$P = 2l + 2w$	$A = lw$		
Triangle base b height h		$A = \frac{1}{2}bh$		
Cylinder radius r height h			$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
Sphere radius r			$SA = 4\pi r^2$	$V = \frac{4}{3}(\pi r^3)$
Cube side a			$SA = 6a^2$	$V = a^3$

Calculating the area under a graph

The calculation of the area under a graph, as shown in **Figure 3-a** and **b,** can often yield useful information. When you do not know the formula for the area of a figure with a curved edge, you can approximate the area by drawing rectangles at small intervals, as shown in **Figure 3-b.** The smaller the intervals, the closer the sum of the areas of the rectangles will be to the actual area under the curve.



a **b** **c**

FIGURE 3

Pythagorean Theorem

If a and b represent the measures of the legs of a right triangle and c represents the measure of the hypotenuse, then $c^2 = a^2 + b^2$, or $c = \sqrt{a^2 + b^2}$.

Example: Find the distance c from A to B in **Figure 4.**

Strategy:

- Use the graph to determine a and b.
- Use the Pythagorean Theorem to find c.

Solution: Distance between B and $C = a = |4 - 1| = 3$

Distance between A and $C = b = |1 - 5| = 4$

$$c = \sqrt{4^2 + 3^2}$$
$$= \sqrt{6 + 9}$$
$$= 5$$

The distance from A to B is 5.

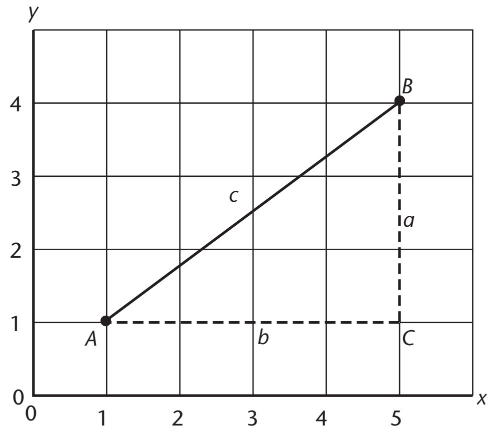
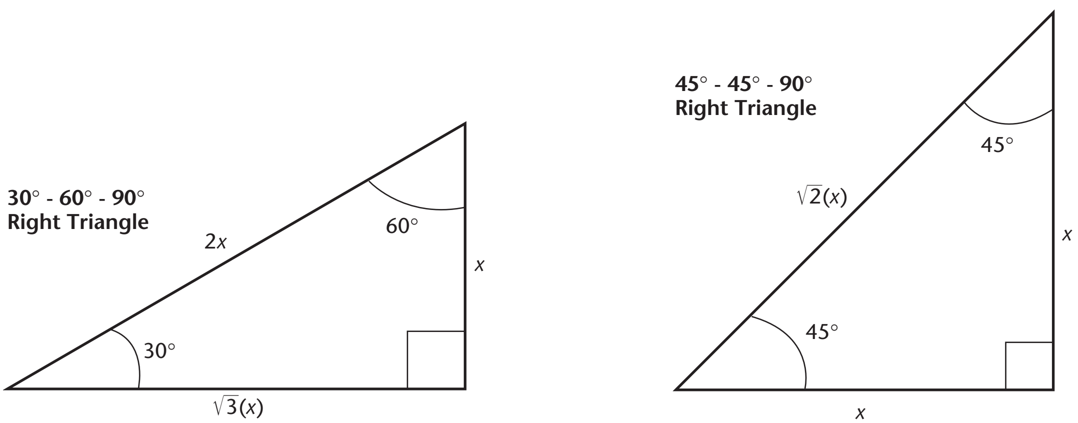


FIGURE 4

Special triangles

In physics, it is useful to know the relationships between the sides of a 30°-60°-90° right triangle and the sides of a 45°-45°-90° right triangle, shown in **Figure 5.** If the length of one side of the triangle is known, the unknown sides can be easily calculated.



a **b**

FIGURE 5

The trigonometric ratios

The ratios of the lengths of the sides of a right triangle can be used to define the basic trigonometric functions, sine (sin), cosine (cos), and tangent (tan). The sides a and b form the right angle, $\angle C$. The angle θ is formed by sides b and c. Side a is opposite angle θ. Side c, opposite the right angle, is called the hypotenuse.

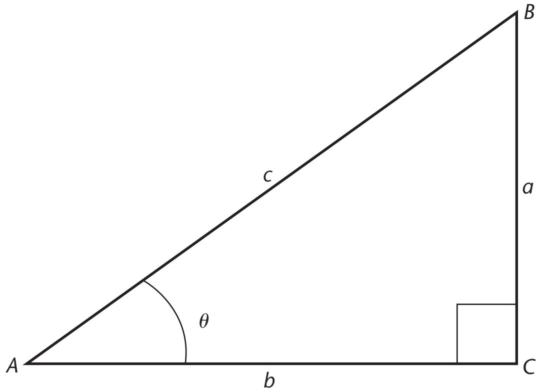


FIGURE 6

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

Use the first letter from the terms in each relationship to form the acronym SOH-CAH-TOA, an easy way to remember the trigonometric ratios.

Example: For the triangle ABC in **Figure 6,** find $\sin\theta$, $\cos\theta$, and $\tan\theta$, if $a = 48$ cm, $b = 55$ cm, and $c = 73$ cm.

Strategy:

- Use the trigonometric ratios, SOH-CAH-TOA.

Solution: SOH: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{48\text{ cm}}{73\text{ cm}} = 0.66$

CAH: $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{55\text{ cm}}{73\text{ cm}} = 0.75$

TOA: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{48\text{ cm}}{55\text{ cm}} = 0.87$

If the value of the sine, cosine, or tangent can be determined from the lengths of two sides of a triangle, the corresponding angle can be found by using a trigonometric functions table or by using the inverse function ($\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$) on a calculator.

The Law of Cosines and Law of Sines

Sometimes, you will need to work with a triangle that is not a right triangle. The Law of Cosines and Law of Sines apply to all triangles.

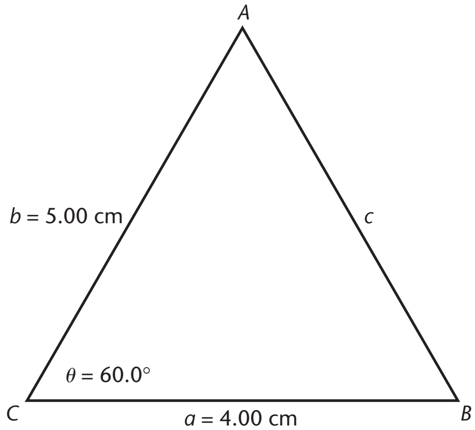


FIGURE 7

The Law of Cosines is useful when you know the measure of two sides and the angle formed by them or the measures of all three sides of a triangle.

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Example: For triangle *ABC* in **Figure 7,** find the length of side *c*.

Strategy:

- Substitute the known values into the Law of Cosines.
- $a = 4.00$ cm, $b = 5.00$ cm, and $\theta = 60.0°$.

Solution:

$$c = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

$$c = \sqrt{(4.00\text{ cm})^2 + (5.00\text{ cm})^2 - 2(4.00\text{ cm})(5.00\text{ cm})\cos 60.0°}$$

$$c = \sqrt{16.0\text{ cm}^2 + 25.0\text{ cm}^2 - 40.0\text{ cm}^2(0.500)}$$

$$c = \sqrt{21.0\text{ cm}^2}$$

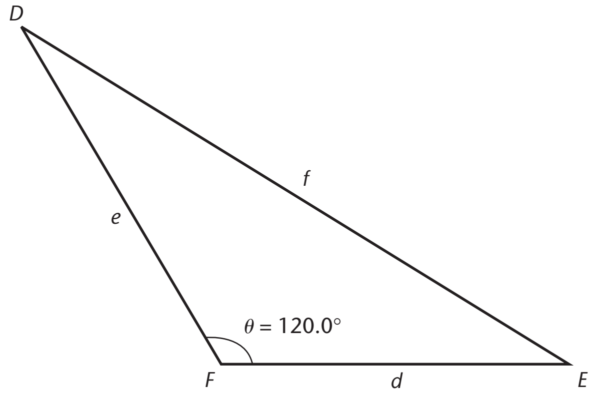
$$c = 4.58\text{ cm}$$

Similarly, it is true for any triangle such as *ABC* in **Figure 7** that

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B.$$

If an angle is larger than 90°, its cosine is negative and is numerically equal to the cosine of its supplement. In the triangle *DEF* below, angle *F* is 120.0°. Therefore, its cosine is the negative of the cosine of (180.0° − 120.0°) or 60.0°. The cosine of 60.0° is 0.500. Thus, the cosine of 120.0° is −0.500.



The Law of Sines is useful when you know the measures of two angles and any side of a triangle or the measures of two sides of a triangle and an angle opposite one of these sides.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example: For the triangle *ABC* in **Figure 7,** find the measure of angle *A*.

Strategy:

* Substitute the known values into the Law of Sines.
* Use a calculator or a trig table to go from sin *A* to *A*.

Solution:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\sin A = \frac{a \sin C}{c}$$

$$\sin A = \frac{4.00\text{ cm}(\sin 60.0°)}{4.58\text{ cm}}$$

$$\sin A = \frac{(4.00\text{ cm})(0.867)}{4.58\text{ cm}}$$

$$\sin A = 0.757$$

$$A = 49.2°$$