Topic 1

Falling Raindrops

You're caught out in a rainstorm. As the rain pelts your exposed head, you might wonder how fast those drops are falling. If you check a book on weather, or do a search on the Internet, you'll find information like that shown in **Table 1.** Raindrops fall at a constant velocity, called the terminal velocity, that depends only on their size.

Can you explain those results using what you learned about objects moving with constant acceleration in Chapter 5? Let's assume that the drop starts at rest ($v_0 = 0$) at height d_0. It falls with constant acceleration $a = -g$. What is its velocity v when $d = 0$? According to **Table 5–2,** you can use the equation $v^2 = v_0^2 + 2a(d - d_0)$ and solve it for v. You'll find that $v = \sqrt{2gd_0}$.

TABLE 1

Diameter (mm)	Terminal Velocity (m/s)	Type of drop
0.2	0.70	Drizzle
1.0	4.0	Small raindrop
2.0	6.5	Average raindrop
5.0	9.0	Large raindrop

This equation predicts that the velocity depends on the height of the cloud. Another check on the Internet will tell you that rain falls from clouds 300 to 2000 m above Earth's surface. If we choose $d_0 = 1000$ m, then we find that $v = \sqrt{2(9.8\ \text{m/s}^2)(1000\ \text{m})}$ or $v = 140$ m/s.

Does a raindrop behave like a freely-falling object? The answer seems to be no for three reasons. First, raindrops fall at much lower, constant speeds. Second, the speed doesn't depend on the initial height. Third, the speed depends on the size of the drop. Because the raindrops' acceleration is small, and when they reach a constant speed, zero acceleration, there must be an upward force on them balancing the downward force. In Chapter 6, you learned that air exerts such a force, which is known as air drag or resistance. The drag force depends on the velocity, but is it directly proportional to the velocity? You can do a simple experiment to find out.

The Nature of Air Drag

You'll need a number of coffee filters, the kind that are used in drip coffee makers. Hold one above your head and gently release it. You'll notice that it falls slowly, at what looks like a constant velocity.

Draw a free-body diagram, like the one in **Figure 1,** and write Newton's second law of motion for the filter: If you choose up as the positive direction, then $F_{net} = -F_g + F_{drag}$. Because the force

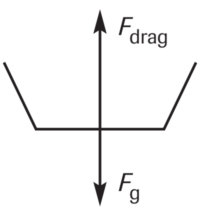


FIGURE 1 The two forces exerted on the falling coffee filter are the force due to gravity and the force due to air drag.

of air depends on velocity, when the filter is at rest, $F_{drag} = 0$. As the velocity increases, F_{drag} gets larger and larger. At some velocity $F_{drag} = F_g$ so the net force is zero. The filter is now moving at a constant velocity, called its terminal velocity.

How does the drag force depend on the velocity? Let's first assume that the drag force is proportional to the velocity. We can write that proportionality as $F_{drag} = -kv$, where k is a constant. The negative sign shows that the force is in the direction opposite the velocity. When the velocity is constant, the net force is zero, and $mg = -kv_t$, or $v_t = -mg/k$. That is, the terminal velocity of the falling filter should be proportional to its mass.

FIGURE 2 The two stacked filters are initially positioned at twice the height of one filter.

Is this prediction correct? Test it. You can double the mass by stacking two filters. The stacked filters should drop with twice the velocity of the single filter. How can you compare two velocities? You know that, for motion at a constant velocity, the time required to move a distance d is given by $t = d/v$. If you release the stacked filters from twice the height of the single filter, as shown in **Figure 2,** then, if they drop with twice the velocity, they should strike the ground at the same time. Try it.

They didn't hit the ground at the same time, did they? Let's try a different assumption. Assume $F_{drag} = kv^2$ (The sign is positive now because v^2 is positive for either direction of the velocity). When the net force is zero, $mg = kv_t^2$ or $v_t^2 = mg/k$. In order to double the velocity, you have to increase the mass by a factor of 2^2 or 4. Test this hypothesis by comparing the time it takes a stack of four filters to fall twice the distance as a single filter. Try it!

Now the two objects hit the ground at nearly the same time. You have shown that air drag on coffee filters is proportional to the square of the velocity. This dependence holds for most macroscopic objects moving at moderate speeds through air. Assuming that it is also true for raindrops, how does the speed of a drop depend on the time and distance it falls?

Modeling the Fall of Raindrops

To develop a model of the fall of raindrops, we need to know the size of the drag force at all velocities. To find that, we need to know the value of the constant k. Because you can measure the terminal velocity, it is useful to write $k = mg/v_t^2$ or $F_{drag} = mgv^2/v_t^2$. Thus

$$F_{net} = -mg + mgv^2/v_t^2 \text{ or}$$
$$a = -g(1 - v^2/v_t^2).$$

You cannot use algebra to find the velocity and position of an object moving with an acceleration that depends on velocity in this way. But, you can use Excel or any other spreadsheet to find numerical values of the velocity and position.

Using a Spreadsheet to Model Free Fall

A spreadsheet is used to break the motion into short time intervals during which the acceleration is approximately constant. The velocity and position are found at the end of that interval, and the calculation is repeated. The computer can do this hundreds or thousands of times very quickly. Let's try this method first for a freely-falling body so we can compare the result with what we found using the equations of motion under constant acceleration.

Call the short time interval Δt. If the initial velocity is v_0, then at the end $v_1 = v_0 + a_0\Delta t$. If the initial position is x_0 then $x_1 = x_0 + v_0\Delta t$. At the end of the next time interval, $v_2 = v_1 + a_1\Delta t$ and $x_2 = x_1 + v_1\Delta t$. This process can then be repeated and repeated.

Figure 3 shows part of an Excel spreadsheet that will calculate the velocity and position of a freely-falling object.

	A	B	C	D
1	**Δt (s)**	**a (m/s/s)**	**vo (m/s)**	**do (m)**
2	1.0	–9.8	0.0	1000.0
3				
4				
5				
6				
7				
8				
9	**t (s)**	**a (m/s/s)**	**v (m/s)**	**do (m)**
10	0.0	–9.8	0.0	1000.0
11	=A10+A2	=B10	=C10+B11*A$2	=D10+C11*A$2

FIGURE 3

The cells in a spreadsheet can contain text, as they do in rows 1 and 9, values, as they do in row 2, or equations, as they do in row 11. The time interval is placed in cell A2 so it can be changed easily. The values of the acceleration, initial velocity, and position are placed in cells B2, C2, and D2. The initial time is placed in cell A10. Cells B10–D10 contain copies of the same cells in row 2.

The formula in cell A11 sets the time to be $t_1 = t_0 + \Delta t$. The formula in cell B11 copies the acceleration into this cell. The formula in cell C11 calculates the new velocity, $v_2 = v_1 + a_1\Delta t$. Finally, the formula in cell D11 calculates the new position, $x_2 = x_1 + v_1\Delta t$.

Select the four cells A11:D11. Copy them and paste them into rows 12–25. Column A will now contain the time data, column B the constant acceleration, column C the velocity, and column D the position. Examine the formulas in row 12. You will find that cell A12 has the formula =A11+A$2. Cell A11 contains time t_1. The dollar sign in A$2 tells Excel to keep this reference constant, so that the same Δt will appear in each copied cell.

Construct the position-time graph by selecting the data in the range D10:D25. Then, from the Toolbar, choose Insert and select Chart. Follow the prompts of the ChartWizard. Use an XY (scatter) plot, with connecting lines, and use A10:A25 as the *x*-values. Next construct a velocity-time graph by selecting the data in the range C10:C25. Your results should look like those in **Figure 4.**

Note that the velocity-time graph shows the expected straight line with a constant -9.8 m/s^2 slope. At $t = 14$ s the speed is a little less than -140 m/s, but the object has fallen slightly more than 1000 m. Why is the velocity smaller than given by the exact results? Remember that this method assumes that the acceleration and velocity are constant over each time interval. The acceleration is, but the velocity is clearly not constant. Try reducing the length of the time interval and see if the results are more accurate. To make this test, reduce the size of Δt and copy cells A11:D11 into many more rows. Finally, change the series used in the graphs to graph the entire fall.

	A	B	C	D
1	*Δt (s)*	*a (m/s/s)*	*vo (m/s)*	*do (m)*
2	1.0	−9.8	0.0	1000.0
3				
4				
5				
6				
7				
8				
9	*t* (s)	*a (m/s/s)*	*v* (m/s)	*do (m)*
10	0.0	−9.8	0.0	1000.0
11	1.0	−9.8	−9.8	990.2
12	2.0	−9.8	−19.6	970.6
13	3.0	−9.8	−29.4	941.2
14	4.0	−9.8	−39.2	902.0
15	5.0	−9.8	−49.0	853.0
16	6.0	−9.8	−58.8	794.2
17	7.0	−9.8	−68.6	725.6
18	8.0	−9.8	−78.4	647.2
19	9.0	−9.8	−88.2	559.0
20	10.0	−9.8	−98.0	461.0
21	11.0	−9.8	−107.8	353.2
22	12.0	−9.8	−117.6	235.6
23	13.0	−9.8	−127.4	108.2
24	14.0	−9.8	−137.2	−29.0

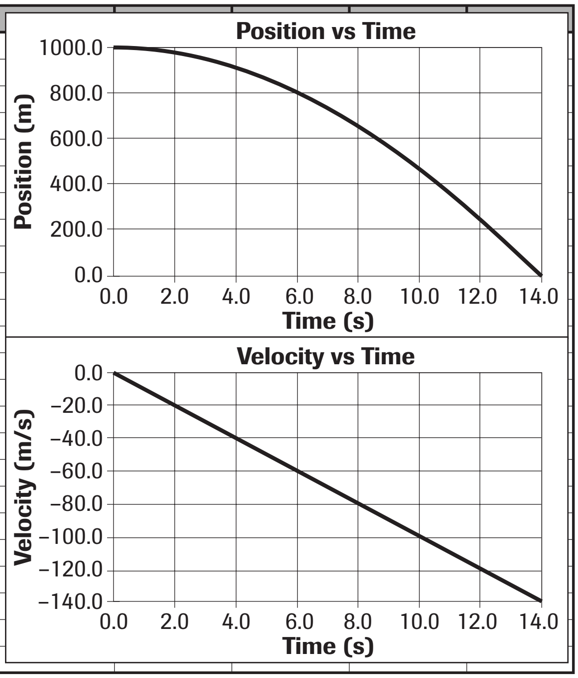


FIGURE 4

A Spreadsheet Model of Objects Falling Through Air

How can a spreadsheet be used to model the fall of an object when the acceleration is not constant? The only modification that is needed is to change column B. As you learned, if the air drag is proportional to the square of the velocity, the acceleration is given by $a = -g(1 - v^2/v_t^2)$. Note that when $v = 0$ then $a = -g$. That is, for very small velocities, the object behaves like one that is falling freely. When $v = v_t$ then $a = 0$. That is, when the velocity equals the terminal velocity, the acceleration is zero; the velocity is constant.

Use cell E2 to store the value of the terminal velocity. Enter the value 9.0, the terminal velocity of a large raindrop. Set the time interval to be 0.2 s. Next modify the formula in cell B11 to be =−B$2*(1 − (C11/E$2)^2). This equation says that the object's instantaneous acceleration (B11) depends on *g* (B$2), the terminal velocity (C$2), and its instantaneous velocity (C11). Copy this equation into the other cells in column B. Note that the values of the velocity and position immediately change, as shown in **Figure 5.**

Examine the acceleration and velocity columns. How long did the drop fall before it reached its terminal velocity? How far had it fallen? Note that the velocity-time graph is a horizontal line when the drop is moving with a constant velocity and that the position-time graph is a straight line with a slope of −9.0 m/s. How long would it take the drop to reach the ground?

Explore the model. As described above, results are accurate only if the changes in acceleration and velocity are small over the time interval. This will not be true if the interval is too large. Try changing the time interval, Δ*t*. Make it smaller and determine whether the time and distance at which the terminal velocity is reached change. Try a larger or a smaller terminal velocity.

	A	B	C	D	E	F	G	H	I
1	*Δt* (s)	*a* (m/s/s)	*vo* (m/s)	*do* (m)	*V*t (m/s)				
2	0.2	−9.80	0.0	1000.00	9.0				
3									
4									
5									
6									
7									
8									
9	*t* (s)	*a* (m/s/s)	*v* (m/s)	*do* (m)					
10	0.0	−9.80	0.0	1000.0					
11	0.2	−9.80	−1.96	999.6					
12	0.4	−9.34	−3.83	998.8					
13	0.6	−8.03	−5.43	997.8					
14	0.8	−6.23	−6.68	996.4					
15	1.0	−4.40	−7.56	994.9					
16	1.2	−2.89	−8.14	993.3					
17	1.4	−1.79	−8.49	991.6					
18	1.6	−1.07	−8.71	989.8					
19	1.8	−0.62	−8.83	988.1					
20	2.0	−0.36	−8.91	986.3					
21	2.2	−0.21	−8.95	984.5					
22	2.4	−0.12	−8.97	982.7					
23	2.6	−0.07	−8.98	980.9					
24	2.8	−0.04	−8.99	979.1					
25	3.0	−0.02	−9.00	977.3					
26	3.2	−0.01	−9.00	975.5					
27	3.4	−0.01	−9.00	973.7					
28	3.6	0.00	−9.00	971.9					
29	3.8	0.00	−9.00	970.1					
30	4.0	0.00	−9.00	968.3					
31	4.2	0.00	−9.00	966.5					
32	4.4	0.00	−9.00	964.7					
33	4.6	0.00	−9.00	962.9					
34	4.8	0.00	−9.00	961.1					

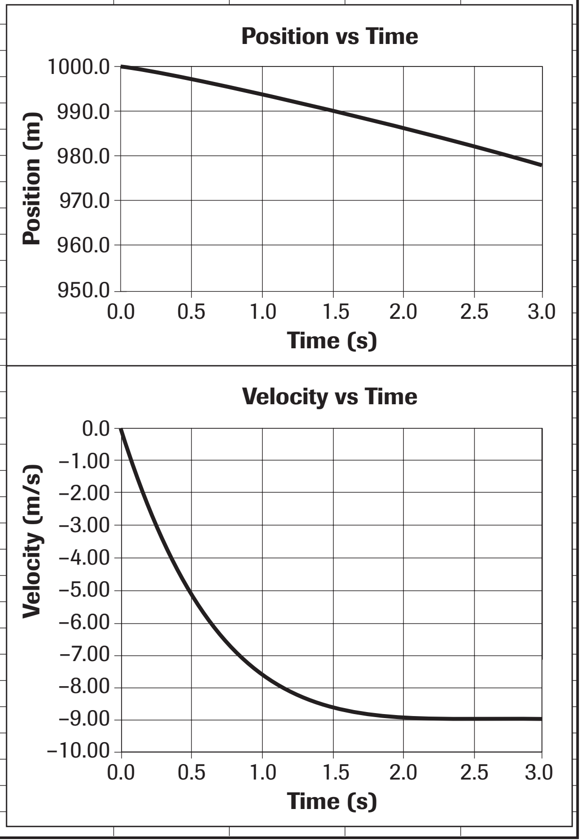


FIGURE 5

How the Terminal Velocity Depends on Drop Size

The proportionality constant k, in the equation for the drag force, $F_{drag} = kv^2$ can be computed by using an equation that is approximately true for objects moving at moderate speed through air, $k = (1/2)\rho_a cA$, where ρ_a is the density of the air ($\rho_a = 1.29\ \text{kg/m}^3$), A is the cross-sectional area of the falling object (in m^2), and c is a number that depends on the shape of the object. For spheres $c = 1/2$, for flat plates $c = 1$. You can find the relationship between k and v_t by noting that the net force is zero when $mg = kv_t^2$. That is, $v_t^2 = mg/k$, or $v_t^2 = 2mg/\rho_a cA$. For a raindrop of radius r, $A = \pi r^2$, and $m = \rho_w V = \rho_w(4/3)\pi r^3$. Thus $v_t^2 = \left(\frac{8}{3}\right)\left(\frac{\rho_w}{\rho_a}\right)\left(\frac{rg}{c}\right)$. That is, the terminal velocity is proportional to the square root of the radius of the drop.

Let's apply this equation to a raindrop of radius 1.0 mm (diameter 2.0 mm). Such a drop should fall with $v_t^2 = \left(\frac{8}{3}\right)\left(\frac{1000\ \text{kg/m}^3}{1.29\ \text{kg/m}^3}\right)\left(\frac{0.001\ \text{m} \times 9.8\ \text{m/s}^2}{0.5}\right) = 40.5\ \text{m}^2/\text{s}^2$ or $v_t = 6.4$ m/s, in good agreement with the experimental results quoted at the beginning of this lesson.

Additional Topics

Practice Problems

1. Use the spreadsheet model for objects with different terminal velocities. Would a larger raindrop fall a smaller or larger distance before reaching terminal velocity?
2. A coffee filter falls with a terminal velocity of 1.2 m/s. Use the spreadsheet model to find how far it fell before it reached terminal velocity.

Applying Concepts

1. Use the data given at the beginning of this lesson on the size and terminal velocities of raindrops. Does the drag force on raindrops of all sizes depend on the square of the velocity? Because such a drag force leads to a terminal velocity that is proportional to the square root of the radius, you can test this assumption by plotting the terminal velocity versus the square root of the radius, shown in **Figure 6.**

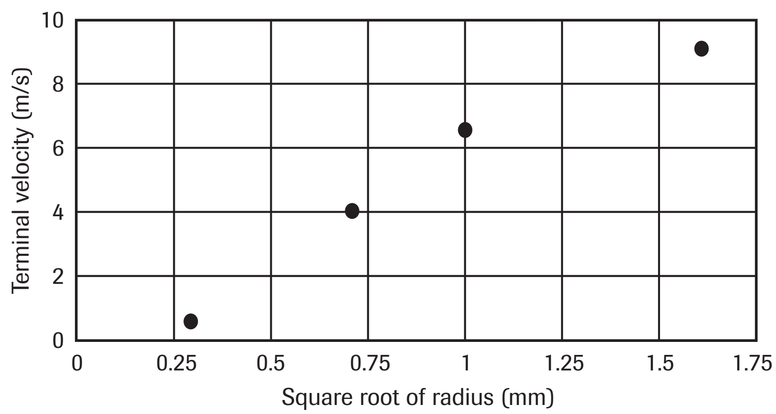


FIGURE 6

2. When you tried the demonstration that shows coffee filters experience a drag force proportional to the square of the velocity, you found that four filters fell 2 m in the same time one filter fell 1 m. You estimated that it took 1.5 s for the filters to fall. As you know, it takes some time for all objects to reach their terminal velocity, so the filters are not falling at constant speed during their entire fall. Use the spreadsheet model to estimate the actual time it would take the two sets of filters to fall. Does this result make the demonstration faulty? Explain.

Appendix D

Topic 2

Fundamentals of Rotation

Describing Rotary Motion

In this lesson, you will learn to apply the laws of motion to rotating bodies. So far we have described the motion of only a point object. In this model, a single value of x, y, and z gives the location of the object. For example, when the football in **Figure 1** is far away, it behaves as a point object. When it is close enough for you to throw or catch it, however, you have to consider the location of the whole ball. All bodies, in fact, take up space and have parts at different locations. That is, they are not point objects, but **extended objects.**

Earth's Motions

You have learned that all movement of point objects is a combination of straight-line motion and circular motion. Earth, for example, orbits around the sun in an almost circular path. In one year, Earth makes a complete revolution around the sun. Thus, we say the period of revolution of Earth is 365 days. We can also say that its frequency of revolution is one revolution per 365 days.

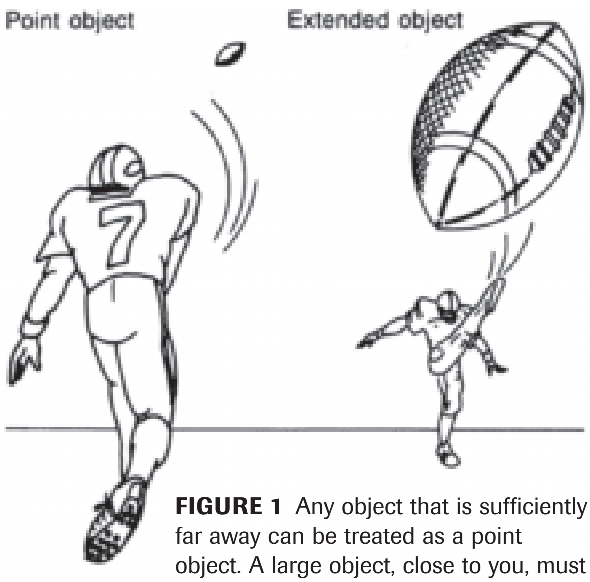


FIGURE 1 Any object that is sufficiently far away can be treated as a point object. A large object, close to you, must be treated as an extended object.

How far does Earth move in one year? It travels the circumference of a circle, given by the equation $C = 2\pi r$. Here r is the radius of the circle, the average distance from the sun to Earth, which is 1.49×10^8 km. The distance that Earth moves in one revolution is then $2\pi r = 2(3.14)(1.49 \times 10^8 \text{ km}) = 9.36 \times 10^8$ km. In this calculation, we treated Earth like a point object rather than a large sphere. This is a good approximation, because the size of Earth is small in comparison to its distance to the sun. Thus, every point on Earth travels almost exactly the same distance each year.

Earth not only revolves around the sun, it also rotates on its axis. It completes one rotation in one day, so its period of rotation is one day. Its frequency of rotation is one rotation per day. How far does an object rotating on Earth's surface move in one day? In this situation, we cannot approximate Earth as a point. As shown in **Figure 2,** the distance an object moves depends on how far it is from the axis of rotation. The axis of rotation of Earth is an imaginary line through its north and south poles. If you stand 1 m from the north pole, each day you will move in a circle with a 1.00 m radius. Thus, you will move $2\pi(1.00 \text{ m})$ or 6.28 m each day. If you are on the equator, however, you are 6.38×10^3 km from the axis. Thus, you will travel a much larger distance each day, namely 4.01×10^4 km.

Additional Topics

Angle of Rotation

The angle of rotation θ measures the amount of rotary motion. Angles are measured using the radian (rad). One radian equals approximately 57.3°. One complete rotation is defined as 2π rad. Thus, in 24 hours, Earth rotates through 2π rad. In 12 hours, the rotation is through π radians. Through what angle does Earth rotate in 6 hours?

Earth is an example of a rigid rotating object. Even though different points on Earth rotate different distances in each rotation, all points rotate through the same angle. All parts of a rigid body rotate with the same angular frequency. The sun, on the other hand, is not a rigid body. Different parts of the sun rotate with different frequencies. Most objects we consider in this lesson are rigid bodies.

Angular Velocity

How fast does an object rotate? The rate of rotation, or **angular velocity,** ω (omega), is the angle rotated per unit time, usually measured in radians/second. If the rotation rate is constant, $\omega = \theta/t$. For Earth $\omega_E = 2\pi$ rad/(24 h)(3600 s/h)$= 7.27 \times 10^{-5}$ rad/s.

We found the distance an object on a rotating body moves in one revolution by multiplying its distance from the axis by 2π. Recall that 2π rad is the angle of one complete rotation. Thus, if an object is a distance r from the axis of rotation and rotates through angle θ, the distance it moves is given by $d = r\theta$. In the same way, if the object's angular velocity is ω, then the linear velocity of a point a distance r from the axis of rotation is given by $v = r\omega$.

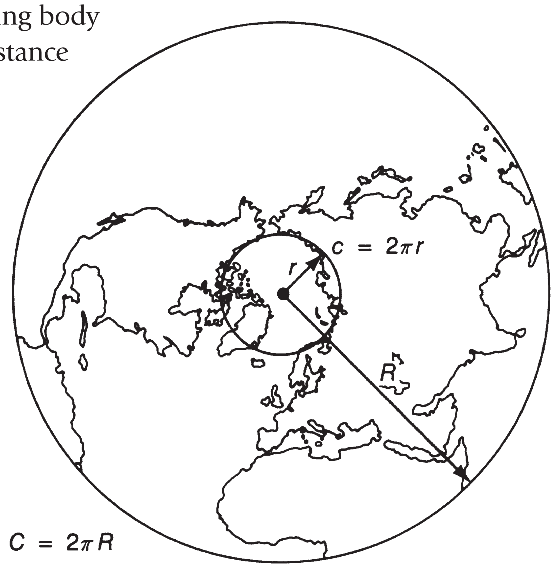


FIGURE 2 The distance traveled in one rotation by an object a distance *r* from the axis of rotation is the circumference of a circle of radius *r*.

Angular Acceleration

Angular acceleration α (alpha) is the rate of change of angular velocity. Just as $a = v/t$, $\alpha = \omega/t$. The linear acceleration of a point a distance r from the axis of a body with angular acceleration α is given by $a = r\alpha$. These relationships are summarized in **Table 1.**

TABLE 1

Linear and Angular Motion

Quantity	Linear Symbol	Angular Symbol	Relationships
Displacement	d	θ	$d = r\theta$
Velocity	v	ω	$v = r\omega$
Acceleration	a	α	$a = r\alpha$

Practice Problems

1. Describe the frequency of rotation of the hour, minute, and second hands of a clock.

2. **a.** What is the period of Earth's rotation in seconds?

b. What is the frequency of Earth's rotation in rotations per second?

3. What is the linear velocity of a person standing on Earth's surface at the equator, due only to the rotation of Earth?

4. What is the angular velocity in rad/s of the tip of the second hand of a watch?

5. The tip of a second hand of a watch is 11 mm from the axis. What is the velocity of the tip?

Torque: the Force of Rotational Motion

How do you start an object rotating? For example, how do you open a door? First, you push or pull—you exert a force. Second, to exert the least amount of force, you exert it as far from the axis of rotation as possible, as shown in **Figure 3a.** The axis of rotation is an imaginary vertical line through the hinges. The door knob is near the outer edge, not near the hinges. Third, you push at right angles to the door. You do not push toward or away from the hinges. Thus, force, distance from the axis, and direction of push determine the change in rate of rotation.

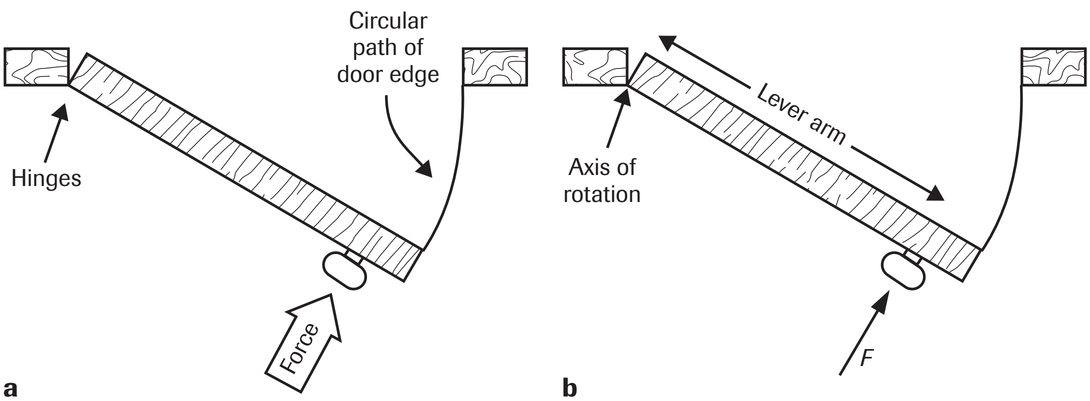


FIGURE 3 Overhead view of door **(a).** Lever arm is along the width of the door, from the hinge to the point where the force is exerted **(b).**

The Lever Arm

For a given applied force, the change in rotation rate depends on the lever arm, the perpendicular distance from the axis of rotation to the point where the force is exerted. If the force is perpendicular to the radius of rotation, then the lever arm is the distance from the axis, r. For the door, it is the distance from the hinges to the point you push, as illustrated in **Figure 3b.**

If a force is not exerted perpendicular to the radius, the lever arm is reduced, as shown in **Figure 4.** To find the lever arm, extend the line of the force until it forms a right angle with a line from the center of rotation. The distance between the intersection and the axis is the lever arm. This distance can also be found from trigonometry as $r \sin \theta$ where θ is the angle between the force and the radius.

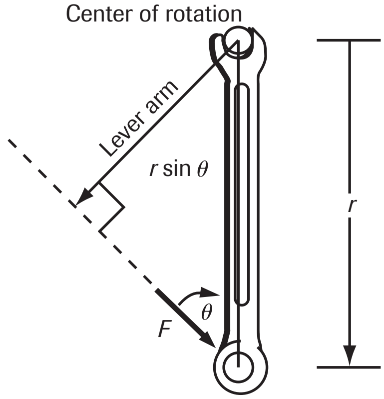


FIGURE 4 The lever arm

Torque

Torque is the product of the force and the lever arm. Its dimensions are force times distance. In the SI system, torque is measured in newton-meters (N·m). Torque is represented by the Greek letter tau, τ (rhymes with now). The equation for torque is

$$\tau = rF \sin \theta.$$

The direction of torque is important. It can be found from the direction of the θ rotation it causes. For a revolving door, the direction the door rotates depends on the direction you push and whether you push to the right or left of the axis. Whether you push on the right or pull on the left, the direction of the rotation, and thus of the torque, will be the same. Note that torque has both direction and size, like a vector.

Torques add, so they can be balanced or unbalanced. In **Figure 5,** two people, Mo and Joe, on opposite ends of a seesaw, exert torques in opposite directions. If the torques they exert are equal and opposite, the board does not rotate: $r_M F_M - r_J F_J = 0$. If Mo and Joe have equal weights, then the board balances when they sit equal distances from the axis: $F_M = F_J$, so $r_M = r_J$. Now suppose Mo is heavier. Mo must sit closer to the axis to balance Joe and have $r_M F_M = r_J F_J$. The board rotates if the two torques do not balance. All changes in rotational motion are the result of **net torque.**

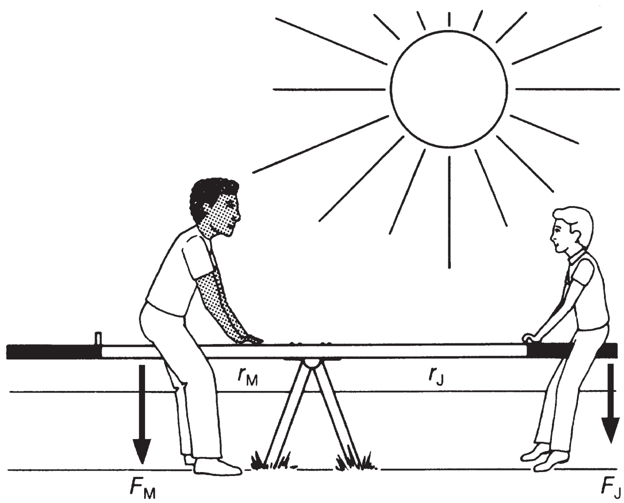


FIGURE 5 Addition of torques having opposite directions

Practice Problems

6. Your car has a flat tire. You get out your tools and find a lug wrench to take the nuts off the bolt studs. You find it impossible to turn the nuts. A friend suggests ways you might produce enough torque to turn them. What three ways might your friend suggest?

7. A bolt on a car engine is to be tightened with a torque of 35 N·m. If you have a 25 cm long wrench, what force should you exert?

8. What mass would have a weight equal to the force needed in the above problem?

9. Mo, whose mass is 43 kg, sits 1.8 m from the center of a seesaw. Joe, whose mass is 52 kg, wants to balance Mo. Where should Joe sit?

10. Jane (56 kg) and Joan (43 kg) want to balance on a 1.75 m long seesaw. Where should they place the pivot point?

Second Law of Rotational Motion

Newton's second law for linear motion is $F = ma$. We know that, for rotational motion, acceleration is replaced by angular acceleration α. Force is replaced by torque τ. A quantity that acts as rotational inertia takes the place of mass. What is rotational inertia? Its dimensions can be found by replacing F/a by τ/α. Thus, it has dimensions kg·m^2. Note that the radian disappears when doing dimensional analysis or unit cancellation. The radian is a unit without a dimension, a place holder.

Rotational inertia depends both on the mass of the object and the square of the distance of the mass from the axis of rotation. Rotational inertia is called the **moment of inertia.** It is represented by the symbol I. Thus, **Newton's second law of rotational motion** is

$$\alpha = \frac{\tau}{I} \text{ or } \tau = I\alpha.$$

That is, angular acceleration is directly proportional to the applied torque and inversely proportional to the moment of inertia. To get a feel for the moment of inertia, use a long pencil, two quarters, and some transparent tape. First tape the two coins to the pencil near its center, separated by about 2 cm, as shown in **Figure 6.** Hold the pencil with your thumb and first finger placed between the two coins. Rapidly rotate the pencil back and forth in a seesaw motion and feel the torque

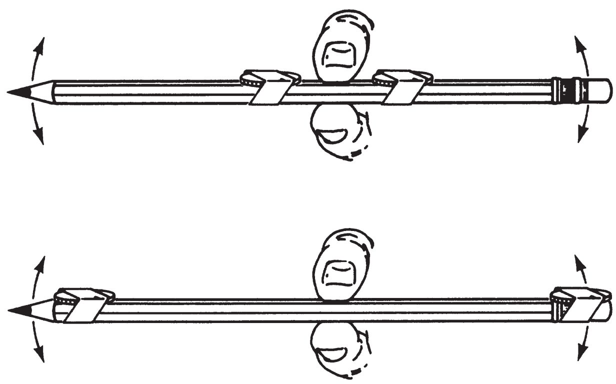


FIGURE 6 Feeling the moment of inertia.

your finger and thumb exert. Now tape the coins to the ends of the pencil and rotate it again. To rotate it at the same rate, that is, to give it the same angular acceleration, you have to exert much more torque. Even though the mass of the pencil and coins has not changed, its moment of inertia has because the distance of the coins to the axis of rotation is greater. The greater the moment of inertia I, the larger the torque τ needed to produce the same angular acceleration α.

Practice Problems

11. Two disks have the same mass, but one has twice the diameter of the other. Which would be harder to start rotating? Why?

12. When a bowling ball leaves the bowler's hand, it is not spinning, but after it has gone about half the length of the lane, it spins. Explain how its rotation rate is increased and why it does not continue to increase.

The Moment of Inertia

The moment of inertia has units of mass times the square of the distance. For a point mass located a distance r from the axis of rotation, the moment of inertia is

$$I = mr^2.$$

For example, a bicycle wheel has almost all its mass in the rim and tire. Its moment of inertia is almost exactly mr^2, where r is the radius of the wheel.

For most objects, however, the moment of inertia is smaller than mr^2. For example, for a solid disk of radius r, $I = \frac{1}{2}mr^2$, while for a sphere, $I = \frac{2}{5}mr^2$. The moment of inertia of an object depends on how the mass is distributed around its axis of rotation.

The moment of inertia also depends on the location of the rotational axis, as illustrated in **Figure 7.** To see this, grasp a book by placing your hands at the bottom of the book. Feel the torque needed to rock the book toward you, then away from you. Now put your hands in the middle of the book. Rock it again. Note that much less torque is needed. The average distance of the book's mass from the rotational axis is much less in the second case.

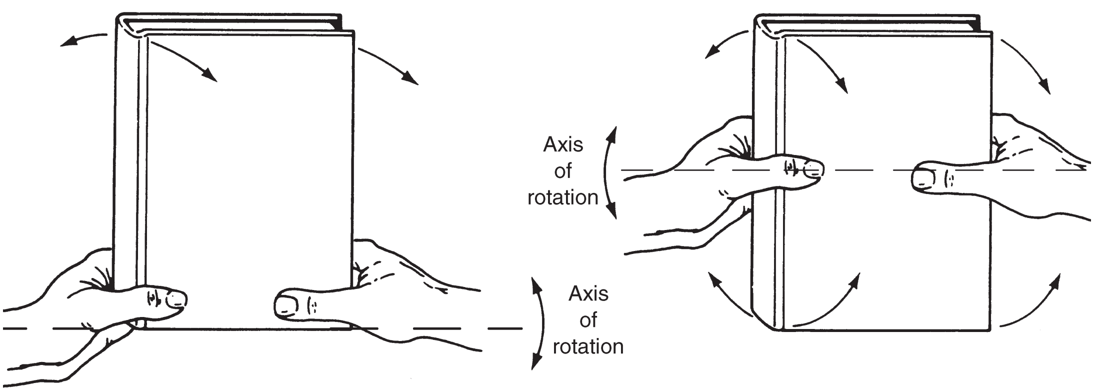


FIGURE 7 Moment of inertia of a book depends on the axis of rotation. The moment of inertia of the book on the left is larger than the moment of inertia of the book on the right.

Practice Problems

13. Two children first sit 0.3 m from the center of a seesaw. Assuming that their masses are much greater than that of the seesaw, by how much is the moment of inertia increased when they sit 0.6 m from the center?

14. Suppose there are two balls with equal diameters and masses. One is solid; the other is hollow, with all its mass near its surface.

a. Are their moments of inertia equal? If not, which is larger?

b. Describe an experiment you could do to see if the moments of inertia are equal.

15. You buy a piece of 10-cm-square lumber, 2.44 m long. Your friend buys the same size piece, but has it cut into two lengths, each 1.22 m long. You each carry your lumber on your shoulders.

a. Which would be easier to lift? Why?

b. You apply a torque with your hand to keep the lumber from rotating. Which would be easier to keep from rotating? Why?

The Center of Mass

Suppose you want to balance a block on the end of your finger. You try your finger at different spots on the block until you find where the block balances. Your finger must now be directly below the **center of mass** of the block. The force of gravity on all parts of an object can be replaced with the weight of a point mass located at the object's center of mass. When you balance a block on your finger, the force of gravity on the block is in the direction of your finger, so the lever arm for rotation of the block is zero. There is no torque and no angular acceleration.

Figure 8 shows how you can find the center of mass for any object. First suspend the object from any point. When it stops swinging, the center of mass must be at some height directly below the suspension point. Draw a vertical line from this point. Then suspend the object from another point. Again, the center of mass must be below this point. Draw a second vertical line. The center of mass is at the point where the two lines cross.

Find the center of mass of a block. Mark the point. Now toss the block, spinning, into the air. If you watch carefully, you will see that the

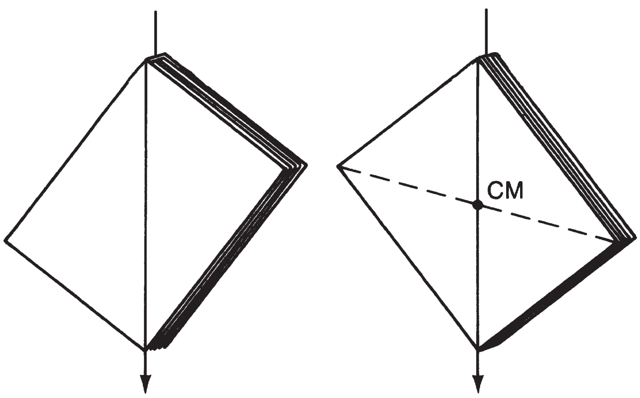


FIGURE 8 Finding the center of mass

Additional Topics

block spins about its center of mass. All freely rotating bodies rotate about an axis that goes through their center of mass.

The center of mass of a person who is standing with arms at sides is a few centimeters below the navel, midway between front and back. It is slightly higher in young children (because of their relatively larger heads). If you raise your hands above your head, your center of mass rises 6 to 10 cm. For example, a ballet dancer changes his center of mass in a leap so that he appears to be floating on air. By raising his arms and legs while in air, he moves his center of mass closer to his head. The path of the center of mass is a parabola, so the dancer's head stays at almost the same height for a surprisingly long time. A high jumper arches her back and rotates her body to pass over the bar while her center of mass remains below the bar. Thus, she can clear a bar higher than she could if her center of mass were inside her body.

Practice Problems

16. Where is the center of mass of a roll of masking tape?

17. When you walk in a strong wind, why do you have to lean into the wind to avoid falling down? Hint: consider torque produced by wind.

Topic 3

Applications of Rotation

Stability

Have you tried to tip over a box? A tall, narrow box, standing on end, tips much easier than a low, broad box. Why? To tip a box, you must rotate it. You apply a torque. The force of gravity acting on the center of mass applies an opposing torque. When the center of mass is directly above the point of support, this opposing torque becomes zero. The only torque is applied by you. As the box in **Figure 1** rotates farther, its center of mass is no longer above its base of support, and both torques act in the same direction. The box tips over rapidly.

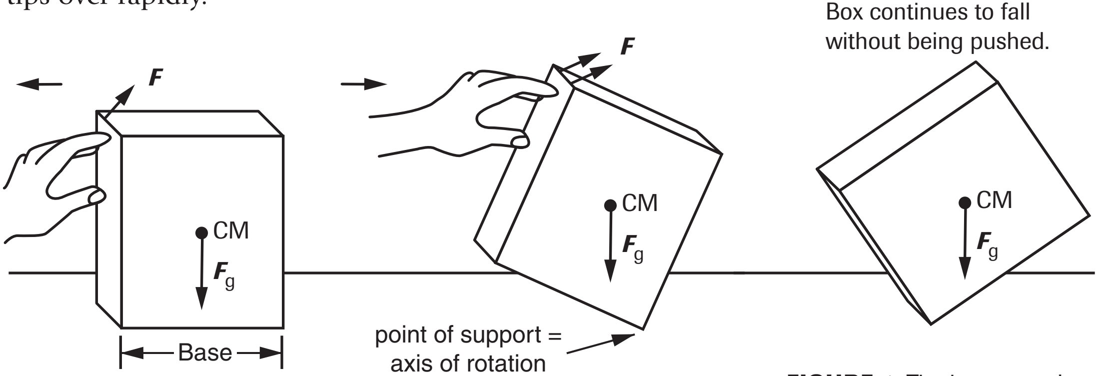


FIGURE 1 Tipping over a box

Center of Mass

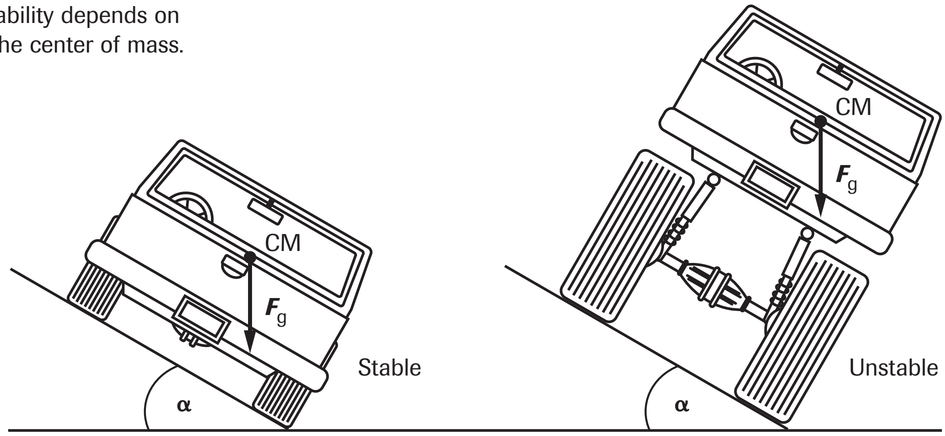
We see that an object is stable if its center of mass lies above its base. The broader the base, the more stable the object is. For that reason, if you are standing on a bus that is weaving through traffic and want to avoid falling down, you spread your feet apart.

To tip the box over, you must rotate its center of mass around the axis of rotation until it is no longer over the base of the box. Because the base of the box is flat, rotating it raises the center of mass. Thus, you must do work. The more the center of mass must be lifted during the rotation, the more work must be done. The more the center of mass must be lifted to move it beyond the edge of the base, the more stable the body is. Therefore, the lower the center of mass is in an object, the greater its stability. **Figure 2** shows how this principle applies to vehicles on a sloped roadway.

You are stable when you stand flat on your feet. When you stand on tiptoes, however, your center of mass moves forward directly above the balls of your feet, and you have very little stability. Try it! Stand with your toes against a wall. Now try to stand on tiptoes. You have to bring your center of mass above your toes, but you cannot because of the wall. Now try kneeling on the floor. Place your elbows against your knees and your forearms along the floor. Have someone place a small box at your fingertips. Now put your hands behind your back and try to touch the box with your nose. Generally, a male is unstable because he must move his higher center of mass in front of his knees. Females can usually do this trick. (Both demonstrations are taken from R.D. Edge, *String and Sticky Tape Experiments,* American Association of Physics Teachers.)

Additional Topics

FIGURE 2 Stability depends on the height of the center of mass.



A small person can use torque rather than force to defend himself or herself against a stronger person. In judo, aikido, and other martial arts, the fighter uses torque to rotate the opponent into an unstable position where his center of mass does not lie above his feet.

Practice Problems

1. Why is a vehicle in storage with its body raised high on blocks less stable than a similar vehicle with its body at normal height?

2. Circus tightrope walkers often carry long bars that sag so that the ends are lower than the center. Sometimes weights are attached to the ends. How does the pole increase the walker's stability? Hint: consider both center of mass and moment of inertia.

Angular Momentum

The concept of momentum is useful in understanding the motion of point bodies. The momentum of a body moving in a straight line (linear motion) is called linear momentum. For rotational motion, a similar quantity, angular momentum, L, is useful. The linear momentum of a body is mv. Replacing m and v by their rotational equivalents, moment of inertia I and angular velocity ω, gives the definition for angular momentum,

$$L = I\omega.$$

Conservation of Angular Momentum

Like linear momentum, angular momentum can be conserved. That is, if no net external torque acts on an object, then its angular momentum does not change. For example, Earth spins on its axis with no external torques. Thus, its angular momentum is constant. The length of the day does not change. A spinning ice skater also demonstrates conservation of angular momentum. When she pulls in her arms, she begins spinning faster. Without an external torque, her angular momentum does not change. That is, $L = I\omega$ is constant. Thus, her increased angular velocity must be accompanied by a decreased moment of inertia. By pulling her arms close to her body, the skater brings more mass closer to the axis of rotation, decreasing the radius and decreasing her moment of inertia.

If a torque-free object starts with no angular momentum, it must continue to have none. Thus, if part of a body rotates in one direction, another part must rotate in the opposite direction. For example, if you switch on a loosely-held electric drill, the drill body will rotate in the direction opposite to the rotation of the motor and bit.

We can calculate changes in angular velocity using conservation of angular momentum.

$$L_1 = L_2$$
$$I_1\omega_1 = I_2\omega_2$$
$$\frac{\omega_2}{\omega_1} = \frac{I_1}{I_2}$$

Since frequency $f = \omega/2\pi$, we also have

$$\frac{f_2}{f_1} = \frac{I_1}{I_2}.$$

Notice that since both f (or ω) and I appear as ratios in this equation, any units may be used, as long as the same unit is used for both values of the quantity.

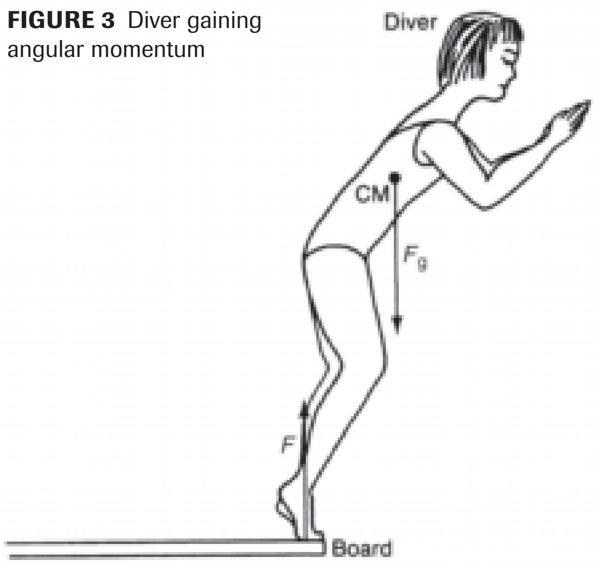
In many sporting events, the spin rate is increased or decreased by changing the moment of inertia. A diver may go into a tuck position, grabbing her knees with her hands. Her moment of inertia is decreased, and her spin rate is increased. When she nears the water, she stretches her body straight, increasing the moment of inertia, slowing the spin rate, and, she hopes, going straight into the water. A gymnast makes similar moves. A ballet dancer controls rotation in the same way an ice skater does.

Angular momentum is a vector quantity. Its direction is along the axis of rotation. A torque in the direction of the axis changes the magnitude of the angular momentum. This torque can either increase or decrease the angular velocity of the spinning object.

How does the diver in **Figure 3** start her body rotating? She uses the diving board to apply an external torque to her body. She moves her center of mass in front of her feet and then uses the board to give a final upward push to her feet.

Have you ever thrown a ball at a weather vane? The ball, moving in a straight line, can start the vane rotating. We can consider the ball and vane to be a system. With no external torques, angular momentum is conserved. The vane spins faster if the ball has large mass, large velocity, and hits at right angles as far as possible from the pivot of the vane. That is, the angular momentum of a moving object like the ball is given by $L = mvr$, where r is the perpendicular distance from the axis of rotation.

FIGURE 3 Diver gaining angular momentum



Additional Topics

Gyroscope Effect

Conservation of angular momentum also means that the direction of the rotation of a spinning object can be changed only by applying a torque. You probably played with a top or gyroscope as a child. You usually spin the top by pulling a string wrapped around its axle. When the top is vertical, there is no torque on it, and the direction of its rotation does not change. If the top is tipped as in **Figure 4,** a torque tries to rotate it downward. Rather than tipping, however, the upper end moves slowly **(precesses)** in a circle.

A gyroscope is a wheel or disk that spins rapidly around one axis while being free to rotate around one or two other axes. The direction of its large angular momentum can be changed only by applying an appropriate torque. Thus, it resists changes in the direction of the axis of rotation. Gyroscopes are used in airplanes, submarines, and spacecraft to keep an unchanging reference direction. Giant gyroscopes are used in cruise ships to reduce their motion in rough water.

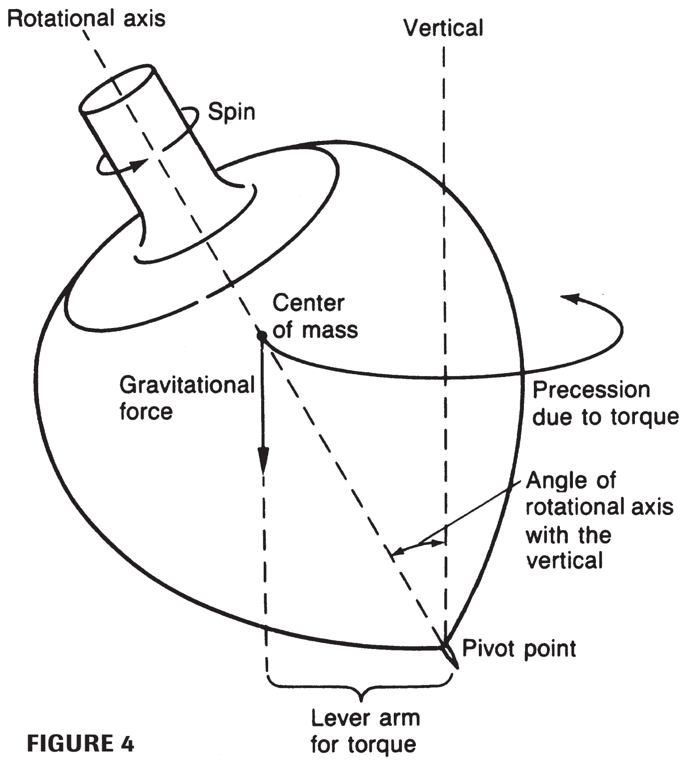


FIGURE 4
Precession of a top

A football quarterback uses the gyroscope effect to make an accurate forward pass. As he throws, he spins, or "spirals" the ball. If he throws it in the direction of its spin axis of rotation, the ball keeps its pointed end forward, reducing air resistance. The ball can be thrown far and accurately. If its spin direction is slightly off, it wobbles. If the ball is not spun, it tumbles end over end.

The flight of a plastic disk, such as a Frisbee, is also stabilized by spin. A well-spun disk can fly many meters without wobbling. You can do tricks with a yo-yo because its fast rotation speed keeps it rotating in one plane.

Practice Problems

3. The outer rim of a Frisbee is thick and heavy. Besides making it easier to catch, how would this affect its rotational properties?
4. A gymnast first does giant swings on the high bar, holding her body straight and pivoting around her hands. She then lets go and grabs her knees with her hands in the tuck position. Finally, she straightens up and lands on her feet.

a. In the second and final parts of the gymnast's routine, around what axis does she spin?

b. Rank in order, from largest to smallest, her moments of inertia for the three positions.

c. Rank in order her angular velocities in the three positions.

5. A student, holding a bicycle wheel with its axis vertical, sits on a stool that can rotate without friction. He uses his hand to get the wheel spinning. Would you expect the student and stool to turn? Which direction? Explain.

Rotational Kinetic Energy

The kinetic energy of a point object is given by the equation $K = \frac{1}{2}mv^2$. When an extended body moves, every part of it has kinetic energy. We can find the kinetic energy of such a body by separating its motion into the motion of the center of mass and its rotation about the center of mass. Then the total kinetic energy is the kinetic energy of the center of mass, $\frac{1}{2}mv^2$, plus the **kinetic energy of rotation,** $\frac{1}{2}I\omega^2$.

Suppose you turn your bicycle over to clean and oil it. You rotate the pedals, and the back wheel begins to spin. You have done work on the pedals, increasing the rotational kinetic energy of the pedals, sprocket, and back wheel. If you then rub your hand against the tire, it will stop, and your hand will get hot. Work will be done on your hand by the friction. The rotational energy will be turned into the thermal energy of your hand and the tire. Thus, rotational kinetic energy is another form of energy.

If an object slides down a frictionless ramp, all its potential energy is converted into the kinetic energy of its center of mass. But if it rolls down a ramp, some of the energy goes into rotation. Thus, the energy that can go into accelerating the center of mass is reduced. A rolling object thus accelerates more slowly than a sliding one.

We know that the speed a sliding object has at the bottom of a frictionless ramp depends only on the ramp height, not on the object's mass. The speed of a rolling object, however, depends on the ratio of its moment of inertia to its mass. If this ratio is small, then little of the energy goes into rotational energy. Thus, a sphere with $I/m = \frac{2}{5}mr^2/m = \frac{2}{5}r^2$ will beat a solid disk with $I/m = \frac{1}{2}r^2$. The disk will beat a hoop with $I/m = r^2$.

Practice Problems

6. The diameter of a bicycle wheel is 71.5 cm. The mass of the rim, tire, and inner tube is 0.925 kg.

a. Find the moment of inertia of the wheel (ignoring the spokes and hub).

b. When the bike is moving at 9.0 m/s, what is the angular velocity of the wheel?

c. What is the rotational kinetic energy of the wheel?

7. When a spinning ice skater pulls in her arms, by conservation of angular momentum, her angular velocity increases. What happens to kinetic energy? We write $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$. Thus, if L is constant and ω increases, K increases. But if kinetic energy increases, work must have been done. What did work on the skater?

Rotating Frames of Reference

Newton's laws apply to reference frames that are not accelerating. In such a frame, an object with no net force on it moves at constant speed in a straight line. But what happens on a rapidly-spinning merry-go-round? You have a sensation of a force pushing you to the outside. A pebble on the floor accelerates without any horizontal force on it, and it does not move in a straight line. A rotating frame of reference is an accelerated frame. Newton's laws are not valid under these conditions.

Motion in a rotating reference frame is important to us because Earth rotates, even though the rate is small. The effects of the rotation of Earth are too small to be noticed in the classroom or lab but are significant influences on the motion of the atmosphere and therefore on climate and weather.

Centrifugal "Force"

Suppose you fasten one end of a spring to the center of a rotating platform, as shown in **Figure 5.** An object lies on the platform and is attached to the other end of the spring. An observer on the platform sees the object stretch the spring. He might think that there is some force toward the outside of the platform that pulls on the object. This apparent force is called the **centrifugal "force."** It is not a real force, because there is no physical outward push on the object. Still, this "force" seems real, as any person on a carnival ride knows.

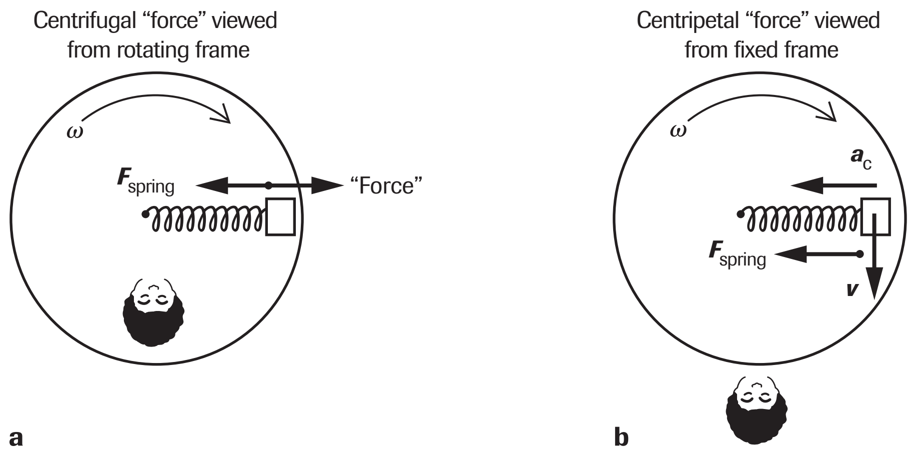


FIGURE 5 Centrifugal "force" is observed only in a rotating reference frame.

Appendix D

An observer on the ground sees things differently. This observer sees the object moving in a circle. The object accelerates toward the center because of the force of the spring. The acceleration is called "centripetal acceleration" because it is directed toward the center. It is given by $a = v^2/r$, or, in terms of angular velocity, $a = \omega^2 r$. Thus, the centripetal acceleration increases as the object moves away from the rotation axis. Note that it depends on the *square* of the angular velocity. If you double the rotation frequency, the acceleration increases by a factor of four.

Coriolis "Force"

A second effect of rotation is shown in **Figure 6.** Suppose a person standing at the center of a rotating disk throws a ball toward the edge of the disk. We study only the horizontal motion of the ball as seen by two observers. Thus, we ignore the vertical motion of the ball as it falls.

An observer standing outside the disk, as shown in **Figure 6a,** sees the ball travel in a straight line at a constant speed toward the edge of the disk. However, the other observer who is stationed on the disk and rotating with it, as shown in **Figure 6b,** sees the ball follow a curved path at a constant speed. A force seems to be acting to deflect the ball. This apparent force is called the Coriolis "force." Like centrifugal "force," the Coriolis "force" is not a real force. It seems to exist because we observe a deflection in horizontal motion when we are in a rotating frame of reference.

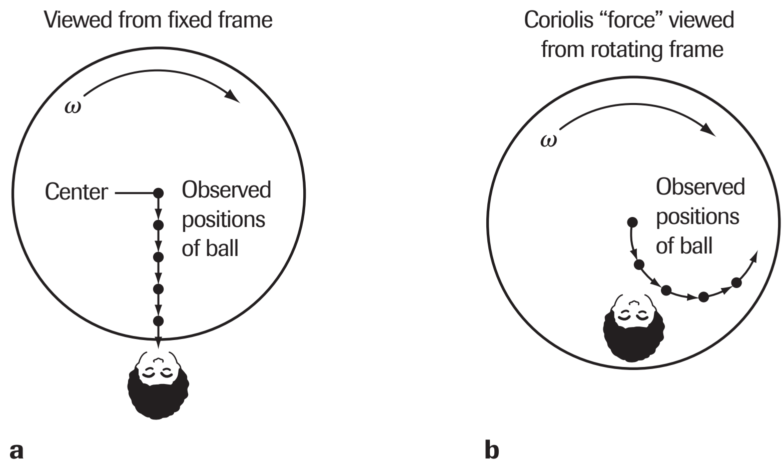


FIGURE 6 The Coriolis "force" exists only in rotating reference frames.

Suppose a rocket is launched from the European Space Agency base near the equator toward a target due north of it. If the rocket is launched directly north, it would also have an eastward velocity component due to the rotation of Earth. This eastward speed is greater at the equator than at any other latitude. Thus, as the rocket moves north, it also moves east faster than points on Earth below it do. The result is that the rocket lands east of the target. An observer on Earth could claim the miss was due to the Coriolis "force" on the rocket. The direction of the apparent force is westward for objects moving southward.

The direction of winds around high- and low-pressure areas also result from Coriolis "forces." Winds flow from areas of high to low pressure. Because of the Coriolis "force," in the northern hemisphere winds coming from the south go to the east of low-pressure areas. Winds from the north end up west of low-pressure areas. Therefore, winds rotate counterclockwise around low-pressure areas in the northern hemisphere.

Most amusement park rides thrill riders by putting them into accelerated reference frames. The "forces" felt by roller coaster riders at the tops and bottoms of hills, and when moving almost vertically downward, are mostly related to linear acceleration. On roller coaster curves, Ferris wheels, rotors, and other circular rides, centrifugal "forces" provide most of the excitement.

Practice Problems

8. While riding a merry-go-round, you toss a key to a friend standing on the ground. For your friend to catch the key, should you toss it a second or two before you reach your friend's position or wait until your friend is directly behind you? Explain.

9. People sometimes say that the moon stays in its orbit because the "centrifugal force just balances the centripetal force, giving no net force." Explain why this idea is wrong.

Topic 4

Statics

Equilibrium Conditions

If you prop a ladder against a wall, you want it to stay there. A falling ladder, you may have observed, accelerates downward and rotates. An object like a properly propped ladder is said to be in static equilibrium. It cannot be accelerating nor can its angular velocity be changing. Usually, it must be at rest and not rotating.

For an object to be in static equilibrium, it must meet two conditions.

- It must be in linear equilibrium—the net force on it is zero.
- It must be in rotational equilibrium—the net torque on it is zero.

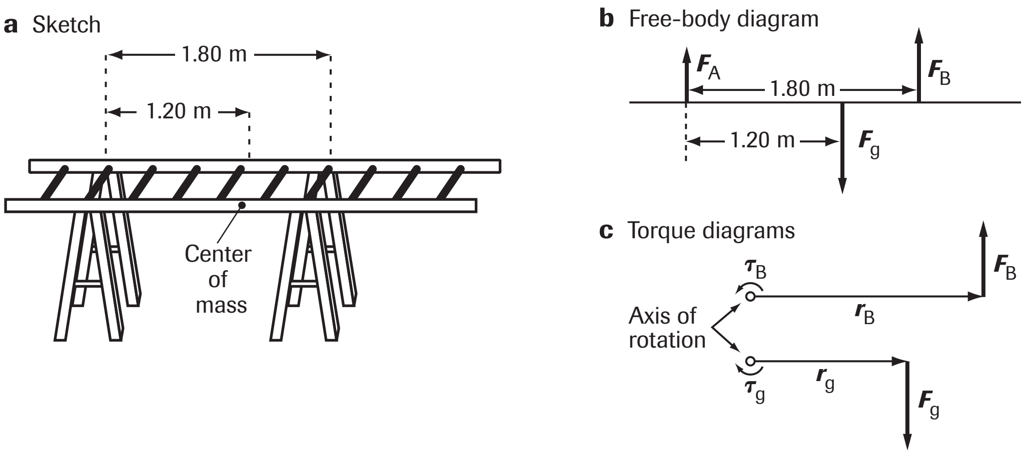
The following Example Problem shows you how to apply the two conditions of static equilibrium to a situation simpler than a propped ladder—a ladder resting on two sawhorses.

Example Problem

Parallel Forces

A ladder is resting on two sawhorses, as shown in **Figure 1a.** What force does each sawhorse exert on the ladder?

FIGURE 1 Forces on a ladder resting on two sawhorses



A free-body diagram of the ladder is shown in **Figure 1b.** The total weight of any object can be modeled as being exerted at the object's center of mass. For a ladder, which has a constant density, the center of mass is at the center rung. The force exerted by the left sawhorse is F_A, and the force exerted by the right sawhorse is F_B. Note that these forces are parallel to each other and act in the upward direction.

The net force acting on the ladder is $F_A + F_B - F_g$. For the ladder to be in linear equilibrium, the net force is zero. Therefore,

$$F_A + F_B - F_g = 0 \quad \text{or} \quad F_A + F_B = F_g.$$

As you can see, this equation is not sufficient to determine F_A or F_B. You must use the second condition of static equilibrium, that the ladder be in rotational equilibrium. This requires that the net torque on it is zero.

Torque τ is the product of the force and the lever arm, the perpendicular distance from the axis of rotation to a line that contains the force vector. If F is the force and r is the length of the lever arm, the equation for torque is

$$\tau = rF.$$

In **Figure 1c,** the axis of rotation was chosen to be at the point where F_A is acting on the ladder. While you can locate the axis of rotation anywhere you wish, placing it at the point where a force is exerted makes one torque zero. You can see that if F_g were the only force acting on the ladder, it would cause a clockwise rotation (negative direction), about this axis. Its torque is designated as τ_g. If F_B were the only force, it would cause a counter-clockwise rotation (positive). Its torque is designated as τ_B. Note that F_A does not exert a torque because the length of its lever arm is zero.

The net torque acting on the ladder is $\tau_B - \tau_g$. Because the net torque equals zero at equilibrium,

$$\tau_B - \tau_g = 0 \quad \text{or} \quad \tau_B = \tau_g.$$

In **Figure 1c,** length r_g, the lever arm of F_g, is 1.20 m, and length r_B, the lever arm of F_B, is 1.80 m. The weight F_g of the ladder is 228 N.

You can now solve the torque equation for F_B.

$$\tau_B = \tau_g$$

$$r_B F_B = r_g F_g \quad \text{or}$$

$$F_B = \frac{r_g F_g}{r_B}$$

Now that you know F_B, go back to the force equation to find F_A.

$$F_A + F_B = F_g \quad \text{or} \quad F_A = F_g - F_B$$

$$F_A = F_g - \frac{r_g}{r_B} F_g = (1 - r_g/r_B)F_g$$

Numerically, $F_B = \frac{(1.20\text{ m})}{(1.80\text{ m})}(228\text{ N}) = 1.80 \times 10^2\text{ N}$, $F_A = (1 - \frac{1.20\text{ m}}{1.80\text{ m}})(228\text{ N}) = 152\text{ N}.$

Example Problem

Forces at Angles

A banner is suspended from a horizontal pivoted pole, as shown in **Figure 2a.** The pole is 2.10 m long and weighs 175 N. The banner, which weighs 105 N, is suspended 1.80 m from the pivot point or axis of rotation. What is the tension in the cable supporting the pole?

A free-body diagram of the forces on the pole is shown in **Figure 2b.** The pole's weight is exerted at the center of mass, the middle of the pole. The tension F_T in the cable pulls the pole up and to the left. The force F_A at the pivot point pushes the pole up and to the right. By choosing the axis of rotation at the pivot point, F_A exerts no torque.

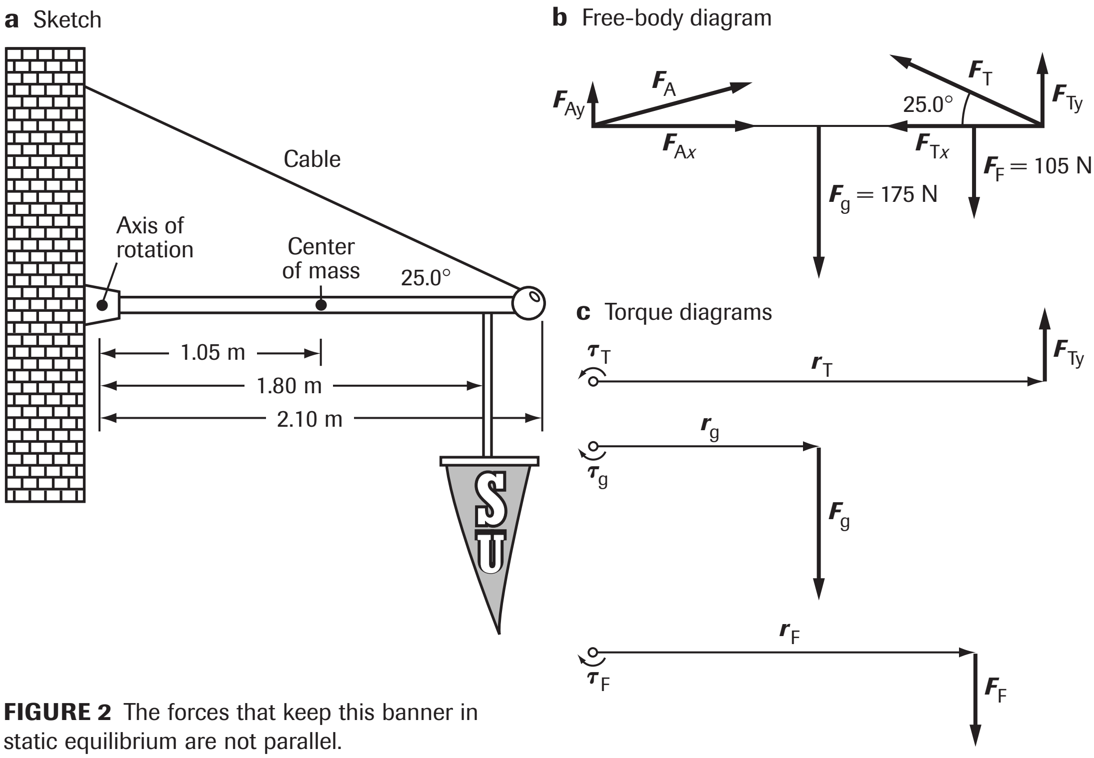


FIGURE 2 The forces that keep this banner in static equilibrium are not parallel.

Figure 2b shows the component vectors of F_A and F_T. Because the pole is in static equilibrium, it is in linear equilibrium. So the net horizontal force acting on it is

$$F_{x\text{net}} = F_{Ax} - F_{Tx} = 0.$$

The net vertical force acting on it is

$$F_{y\text{net}} = F_{Ay} + F_{Ty} - F_g - F_F$$
$$F_{y\text{net}} = 0,$$
$$\text{so } F_{Ay} + F_{Ty} = F_g + F_F.$$

Because the pole is in rotational equilibrium, the net torque on it is zero.

$$\tau_{cw} = \tau_{ccw}$$

Table 1 shows the torque caused by each force:

TABLE 1					
Clockwise Torque			**Counterclockwise Torque**		
Lever arm	**Force**	**Torque**	**Lever arm**	**Force**	**Torque**
			$r_T = 2.10$ m	F_{Ty}	$(2.10\text{ m})F_{Ty}$
$r_g = 1.05$ m	$F_g = 175$ N	184 N · m			
$r_F = 1.05$ m	$F_F = 105$ N	189 N · m			
		Total $\tau_{cw} = 373$ N · m			Total $\tau_{ccw} = (2.10\text{ m})F_{Ty}$

F_{Tx} does not produce torque because it is parallel to the pole.

Now set $\tau_{ccw} = \tau_{cw}$.

$$(2.10\ \text{m})\ F_{Ty} = 373\ \text{N·m}$$

$$F_{Ty} = \frac{373\ \text{N·m}}{2.10\ \text{m}} = 178\ \text{N}.$$

Knowing F_{Ty}, you can determine F_T by using the following equation:

$$F_T = \frac{F_{Ty}}{\sin 25.0°}$$

$$= \frac{178\ \text{N}}{\sin 25.0°} = 421\ \text{N}.$$

Practice Problem

1. Determine the tension in the rope supporting the pivoted lamp pole shown in **Figure 3** below. The pole weighs 27 N, and the lamp, 64 N. **Table 2** shows the torque caused by each force.

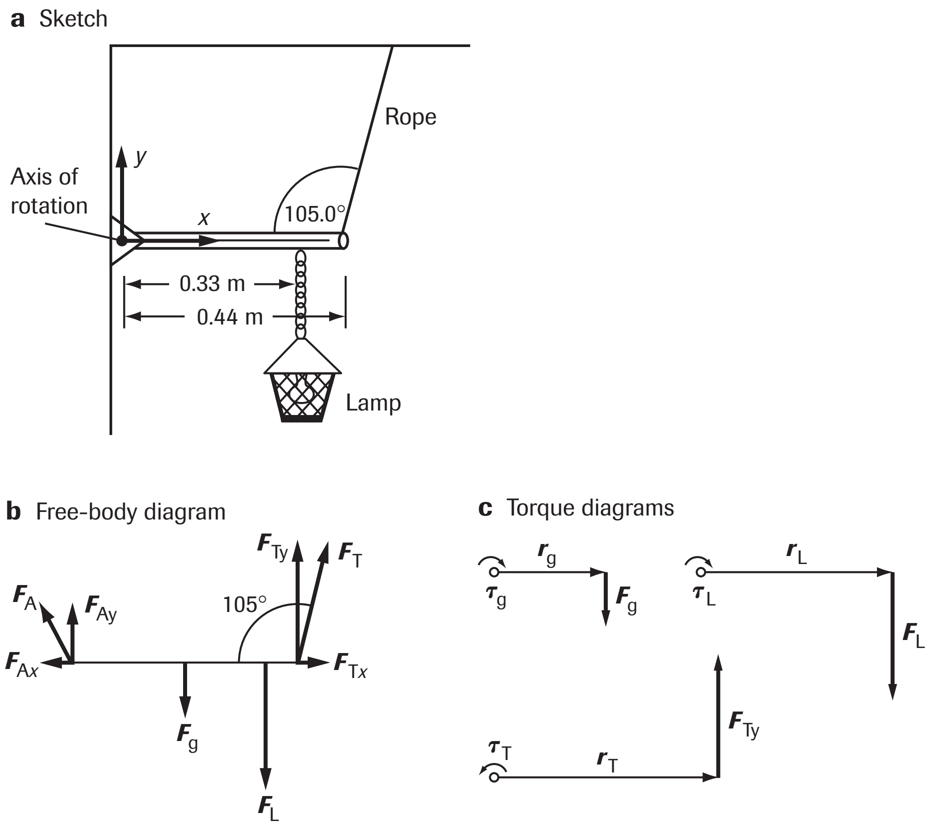


FIGURE 3 The lamp is suspended from a horizontal pole that is supported by a rope acting at an angle of 105° from the pole.

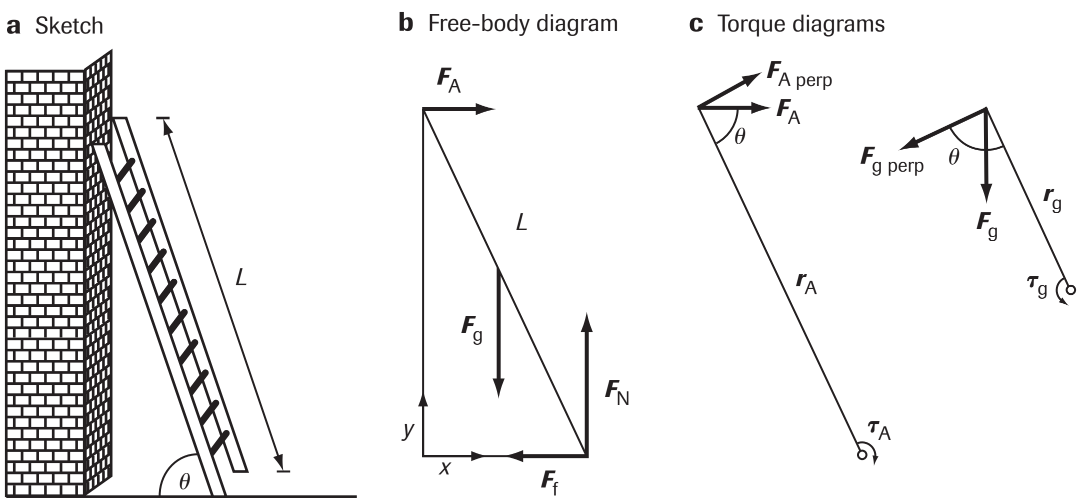
Appendix D

TABLE 2					
Clockwise Torque			Counterclockwise Torque		
Lever arm	Force	Torque	Lever arm	Force	Torque
			0.44 m	F_{Ty}	$\tau_T = (0.44\ m)F_{Ty}$
$r_g = 0.22$ m	$F_g = 27$ N	$\tau_g = 5.9$ N · m			
$r_L = 0.33$ m	$F_L = 64$ N	$\tau_L = 21$ N · m			
		Total $\tau_{cw} = 27$ N · m			Total $\tau_{ccw} = (0.44\ m)F_{Ty}$

Propped Objects

A ladder propped against a wall is in static equilibrium. What is needed for the propped ladder in **Figure 4a** to stay propped? That is, how can it remain in static equilibrium? Assume that there is friction between the ladder and the ground, but none between the ladder and the wall.

FIGURE 4 The forces on a ladder of length *L* propped against a frictionless wall



Using the free-body diagram in **Figure 4b,** you can find the vertical and horizontal forces.

The net horizontal force exerted on the ladder is

$$F_A - F_f = 0$$

where F_A is the normal force exerted on the ladder by the wall and F_f is the force of friction the ground exerts on the ladder.

The net vertical force exerted on the ladder is

$$F_N - F_g = 0$$

where F_N is the normal force exerted on the ladder by the ground and F_g is the force of gravity on the ladder.

Therefore, $F_A = F_f$ and $F_N = F_g$.

You can determine the torques acting on the ladder by placing the axis of rotation at the point where the ladder is in contact with the ground, as shown in **Figure 4c.** The calculations of the torques is shown in **Table 3.**

TABLE 3

Clockwise Torque			Counterclockwise Torque		
Lever arm	**Force**	**Torque**	**Lever arm**	**Force**	**Torque**
$r_A = L$	$F_A \sin\theta$	$LF_A \sin\theta$	$r_g = \frac{L}{2}$	$F_g \cos\theta$	$\frac{L}{2} F_g \cos\theta$
Total $\tau_{cw} = LF_A \sin\theta$			Total $\tau_{ccw} = \frac{L}{2} F_g \cos\theta$		

Because $F_A = F_f$, $\tau_{cw} = LF_f \sin\theta$ and $\tau_{ccw} = (L/2)F_g \cos\theta$.

$$\tau_{cw} = \tau_{ccw}$$

$$LF_f \sin\theta = (L/2)F_g \cos\theta$$

$$F_f = \frac{(L/2)F_g \cos\theta}{L \sin\theta} = \frac{1}{2} F_g \frac{\cos\theta}{\sin\theta}$$

Example Problem

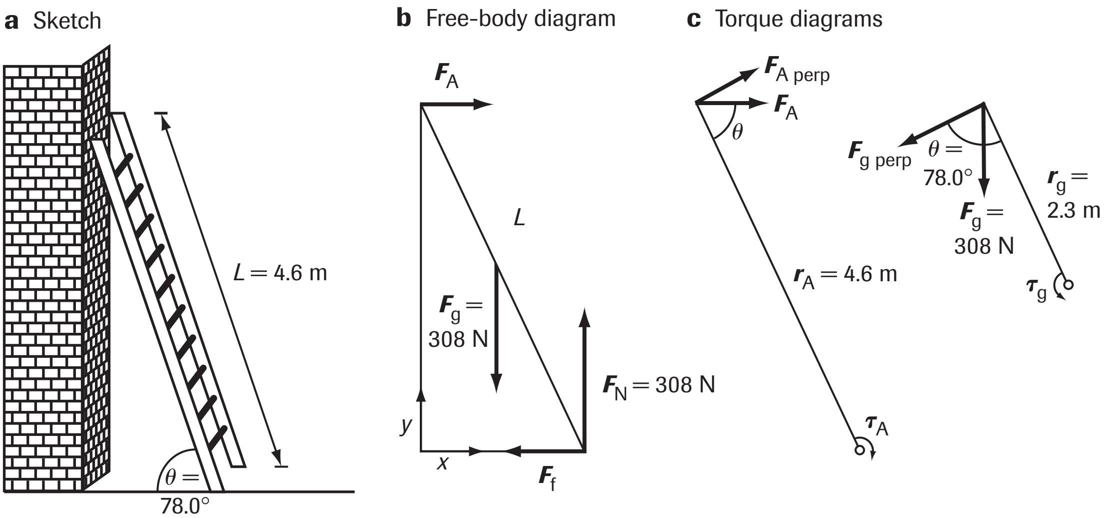
A Propped Ladder

A 4.6-m ladder weighing 308 N is propped against a wall at an angle of 78.0° from the horizontal. What is the frictional force acting on the ladder? What is the minimum coefficient of friction between the ladder and ground that would keep the ladder in equilibrium?

$$F_f = \frac{1}{2}(308\ \text{N})\frac{\cos 78.0°}{\sin 78.0°} = 32.7\ \text{N}$$

The frictional force on the ladder is 32.7 N. Since $F_f \leqslant \mu_s F_N$ and $F_N = F_g$, $\mu_s \geqslant F_F/F_g$ or $\mu_s \geqslant \frac{32.7\ \text{N}}{328\ \text{N}} = 0.100$

FIGURE 5



Practice Problems

2. What is the force of friction acting on a 1.7-m wooden plank weighing 145 N propped at an angle of 65° from the horizontal? There is friction only between the plank and the ground.

3. What coefficient of friction is needed to keep the plank propped up?

Reviewing Concepts

1. Why can you ignore forces that act on the axis of rotation of an object in static equilibrium when determining the net torque?

2. In solving problems about static equilibrium, why is the axis of rotation often placed at a point where one or more forces are acting on the object?

Topic 5

Gas Laws

Your first experience with the properties of gases may have come from blowing up a balloon. The air filled the entire balloon, not just one part. If you let go of the balloon, the air came rushing out with great force. You might have noticed that if you took a balloon outside on a very cold day, it shrank but then grew again when brought back inside. You may have had a helium-filled balloon that floated up out of reach when you let go. A balloon you blew up yourself never did that.

The kinetic-molecular model of gases that you studied in Chapter 12 was developed in the nineteenth century to explain the properties of gases such as those that fill balloons. Since that time, the model has been used to explain the properties of liquids and solids as well as gases. It is now called the kinetic-molecular theory of matter.

The kinetic theory explains the properties of an ideal gas, which is a model of a real gas. Under most conditions, real gases behave almost exactly as the ideal gas described by the kinetic theory. Later in this lesson we will discuss the conditions under which the model does not apply.

Pressure in the Kinetic Theory

The kinetic theory explains how gases exert pressure on the walls of a closed container. The randomly moving gas particles strike the sides of the container. **Figure 1** shows a particle with velocity $\boldsymbol{v}_1$ striking the wall of the container. The collision is elastic, so the particle rebounds with velocity $\boldsymbol{v}_2$. The change in momentum of the particle, $m\Delta\boldsymbol{v}$, exerts an outward impulse to the wall of the container, $\boldsymbol{F}\Delta t$. Thus during the brief time Δt that the particle is in contact with the wall, it exerts an outward force $\boldsymbol{F}$.

A sample of gas contains a huge number of particles, so many collisions are occurring every instant. There are so many collisions that the total force exerted by the gas is constant over any reasonable time interval. The larger the area of the wall, the larger the number of collisions, and so the greater is the total force. The number of collisions per unit area, however, is independent of the size of the wall. Thus the pressure, defined as the force per unit area, is a constant that does not depend on the size of the wall. Because the particles are moving randomly in all directions, the pressure is the same on all walls.

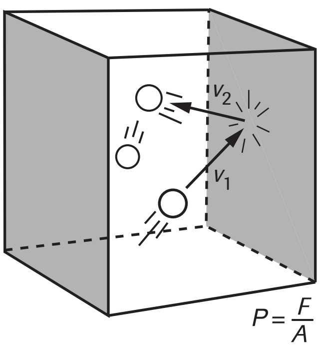


FIGURE 1 The force exerted per unit area by many particles of a gas randomly striking the walls of the container is called pressure.

Pressure of a Gas

You learned in Chapter 13 that pressure is defined as the force per unit surface area. That is, $P = F/A$. The pressure of gases, especially that of the atmosphere, is measured with a barometer. There are two types of barometers, aneroid and mercury. An aneroid barometer, shown in **Figure 2a,** contains a metal can, called an aneroid, from which most of the air has been removed. The atmosphere

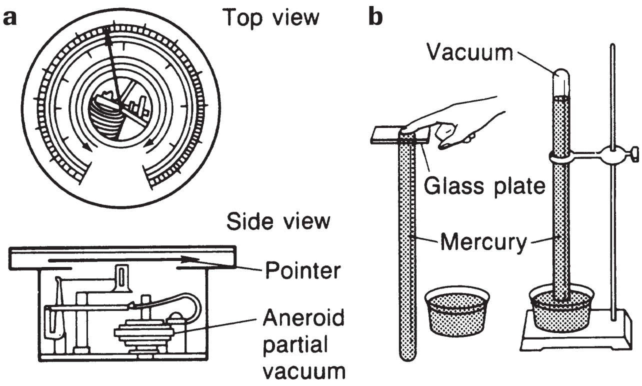


FIGURE 2 An aneroid barometer measures air pressure by means of changes in the size of an evacuated chamber **(a).** A mercury barometer measures air pressure by using the height of a column of mercury supported by the atmosphere **(b).**

exerts pressure on the can. When the atmosphere pressure changes, the shape of the aneroid changes slightly. A needle attached to the top of the can by means of a lever and gear system moves along a scale calibrated to read in air-pressure units.

Mercury barometers, illustrated in **Figure 2b,** are frequently used in laboratories. A glass tube, about 80 cm long and closed at one end, is completely filled with mercury. The filled tube is inverted and placed in a dish of mercury. Rather than emptying into the dish, the mercury in the tube falls only a little. That the column of mercury does not fall farther suggests that the downward force of the weight of mercury in the tube must be balanced by an upward force. There is nothing above the liquid mercury in the top of the tube but a few molecules of mercury vapor. There is certainly nothing that can pull mercury upward. Thus, the force that balances the weight of the column is the weight of the atmosphere pressing down on the mercury in the dish. When the air pressure changes, the force on the mercury in the dish changes, changing the height of the column of mercury in the tube. For this reason, the height of the column of mercury is a direct measure of the atmospheric pressure.

Standard Atmospheric Pressure

The average pressure of the atmosphere, measured at sea level, is called the **standard atmospheric pressure.** Standard atmospheric pressure is 101.3 kPa. This pressure will support a column of mercury 760 millimeters high. For this reason, standard atmospheric pressure is sometimes reported as 760 millimeters of mercury. The "millimeter of mercury" as a unit of pressure has been replaced by the Torr, named in honor of the Italian scientist Evangelista Torricelli (1608–1647). Torricelli studied atmospheric pressure and invented the barometer. In meteorology, atmospheric pressure is most commonly expressed in millibars (mb). One millibar equals 0.10 kPa, so standard atmospheric pressure is 1013 mb. Thus, one atmosphere of pressure (1 atm) is equivalent to 101.3 kPa, 760 Torr, or 1013 mb. Meteorologists watch barometric readings carefully because atmospheric pressure changes accompany weather changes.

Pressure gauges that measure air pressure in tires actually measure the amount by which the tire pressure exceeds the atmospheric pressure. Pressure defined in this way is called gauge pressure. For example, if the pressure gauge reads 225 kPa, the total pressure in the tire is 225 kPa + 101.3 kPa = 326.3 kPa.

Boyle's Law

Suppose a fixed amount of gas is put into a container of adjustable size such as a balloon or a cylinder with a movable piston. How does the pressure of the gas depend on the volume of the container? The relationship between pressure and volume of a gas is called Boyle's law, named for Robert Boyle (1627–1691), a British chemist and physicist. **Boyle's law** states that for a fixed sample of gas at a constant temperature, the volume of the gas V varies inversely with the pressure P. When two variables are inversely related, their product is a constant. That is, according to Boyle's law,

$$PV = \text{constant}.$$

If P_1 and V_1 are the original pressure and volume of a given mass of gas, and P_2 and V_2 are the final pressure and volume, then

$$P_1V_1 = P_2V_2 = \text{constant}.$$

That is, the pressure and volume of a fixed amount of gas at a constant temperature are related by the equation

$$P_1V_1 = P_2V_2.$$

Real gases follow Boyle's law closely except at very high pressures or low temperatures.

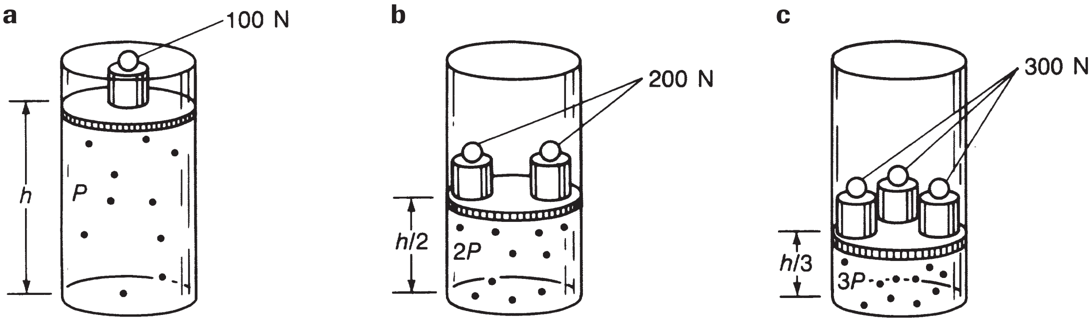


FIGURE 3 The volume of a gas decreases as the pressure applied to it increases.

Kinetic Theory and Boyle's Law

The kinetic theory can explain Boyle's law. Consider a gas-filled cylinder sealed with a weightless piston that is free to move up and down, as illustrated in **Figure 3.** The area of the piston is 0.010 m^2. A 100-N weight is placed on the piston. The piston moves down the cylinder until it comes to rest. The piston is a distance h = 10 cm or 0.10 m above the bottom of the cylinder. The piston is in equilibrium. That is, the downward force of the weight above the piston is balanced by the upward force of the gas below the piston. The volume of the gas is (0.010 m^2)(0.10 m) = 0.0010 m^3. The pressure of the gas is 100 N/0.010 m^2, equivalent to 10 kPa. The point that represents these values is plotted on the graph in **Figure 4.**

If the weight is increased to 200 N, the piston moves downward until the gas pressure is twice the original pressure. The gas temperature is constant, so the velocity of the gas particles has not changed. The gas pressure doubles only if the number of collisions with the

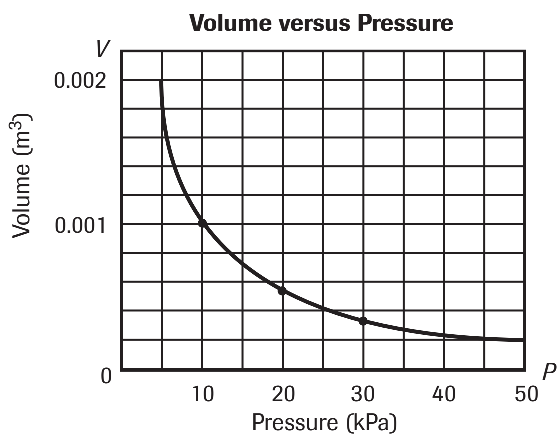


FIGURE 4 The curve is a hyperbola because pressure and volume are inversely related.

Additional Topics

piston each second doubles. This occurs if the vertical distance traveled by each particle is halved. Thus the piston moves down until it is a distance $h/2 = 5$ cm above the bottom. The volume of the gas is now half the original volume. This point, with $P = 20$ kPa and $V = 0.0005\ \text{m}^3$, is plotted on the graph.

If the weight is increased to 300 N, the pressure is three times the original pressure. The piston moves to a position where the number of collisions per second is three times the original number. At this location, $h/3$, the volume of the gas is one-third the original. **Figure 4** shows the three data points discussed and a smooth curve drawn through the points. The curve is a hyperbola, which shows that pressure and volume are inversely related.

Example Problem

Boyle's Law

A sample of gas is held in a 2.6 m^3 volume at 226 kPa. The temperature is kept constant while the volume is decreased until the pressure is 565 kPa. What is the new volume of the gas?

$P_1 = 226$ kPa
$V_1 = 2.6\ \text{m}^3$
$P_2 = 565$ kPa
Unknown: V_2
Boyle's Law: $P_1V_1 = P_2V_2$

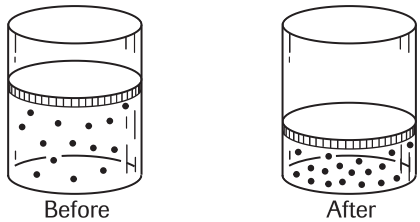


FIGURE 5

Solution: $P_1V_1 = P_2V_2$ or $V_2 = \dfrac{P_1V_1}{P_2}$

$$= \frac{(226\ \text{kPa})(2.6\ \text{m}^3)}{565\ \text{kPa}} = 1.0\ \text{m}^3$$

One way of exerting pressure on a gas is to put it under water. Just as standard atmospheric pressure supports a column of mercury 760 mm high, it will support a column of water 10.4 m high. This means that the pressure on an object under 10.4 m of water is 2.0 atm (203 kPa). One atmosphere of pressure is exerted by the water; and one atmosphere of pressure is exerted by Earth's atmosphere pushing downward on the water. Every additional 10.4 m of water exerts another atmosphere of pressure.

Example Problem

Boyle's Law Under Water

The volume of a balloon is 2.0 liters when the pressure on it is 1.0 atmosphere. It is tied with a string, weighted with a heavy stone, and tossed into a pond so that the balloon is 20.8 m deep. What is the balloon's volume when it reaches the bottom? A column of water 10.4 m deep exerts 1 atm of pressure.

$P_1 = 1.0$ atm
$V_1 = 2.0$ L
depth = 20.8 m
water pressure = 1 atm/10.4 m
Unknown: V_2
Boyle's Law: $P_1V_1 = P_2V_2$

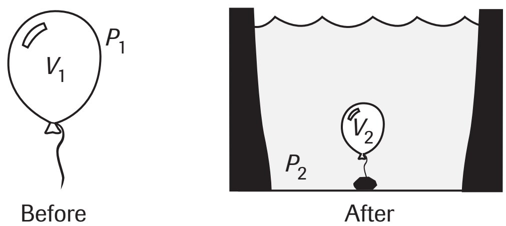


FIGURE 6

Solutions: $\dfrac{20.8\ \text{m}}{10.4\ \text{m/atm}} = 2.0$ atm of water pressure,

so $P_2 = 2.0\ \text{atm} + 1.0\ \text{atm} = 3.0\ \text{atm}$

$$P_1V_1 = P_2V_2 \text{ or } V_2 = \frac{P_1V_1}{P_2}$$

$$= \frac{(1.0\ \text{atm})(2.0\ \text{L})}{3.0\ \text{atm}} = 0.67\ \text{L}$$

Practice Problems

1. The volume of a cylinder with a movable piston is 0.063 m^3. It exerts 236 kPa on a certain amount of air. While the temperature is held constant, the pressure is increased to 354 kPa. What is the new volume of the air?
2. A pressure of 235 kPa holds neon gas in a cylinder whose volume is 0.0500 m^3. The volume increases to 0.125 m^3. What pressure is now exerted on the gas?
3. The volume of a helium-filled balloon is 2.0 m^3 at sea level. The balloon rises until its volume is 6.0 m^3. What is the pressure in kPa at this height?
4. A diver works at a depth of 52 m in fresh water. A bubble of air with a volume of 2.0 cm^3 escapes from her mouthpiece. What is the volume of the bubble just as it reaches the surface of the water?

Charles's Law

About one hundred years after Boyle, French scientist and balloonist Jacques Charles (1746–1823) studied how the volume of a gas depends on temperature. Charles discovered that the volume of a fixed amount of gas at constant pressure depends linearly on the temperature. That is, when he increased the temperature from 0°C to 1°C, the volume increased by 1/273 of the original volume. When he increased the temperature 2 C°, the volume increased by 2/273 of the original volume. If Charles increased the temperature by 273 C°, the volume would have doubled. He found that many gases behave this way.

When Charles cooled a gas, the volume shrank by 1/273 of its original volume for every degree the gas was cooled. Charles could not cool gases below −20°C, so, to see what lower limits might be possible, he extended the line of the graph to even lower temperatures. Extending a graph beyond measured points is called extrapolation. If the graph is based on precise data and extrapolation used with care, it can provide useful information. Charles's extrapolation had a startling implication. It suggested that if the temperature were reduced to −273°C, the gas would have zero volume. The temperature at which a gas would have zero volume is now called absolute zero. The zero of the Kelvin temperature scale is absolute zero.

The result of these experiments is called **Charles's law:** under constant pressure, the volume of a sample amount of gas varies directly as its Kelvin temperature.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{constant}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Real gases follow Charles's law except at low temperatures. At temperatures below about 2 K, no substances exist as gases. Thus, even though scientists can now obtain conditions of much lower temperatures than Charles could, they must still extrapolate their measurements of gas temperature and volume to find the temperature at which the volume of an ideal gas would be zero.

Example Problem

Charles's Law

A container of 0.22 m^3 of nitrogen gas at 20.0°C is heated under constant pressure to 167°C. What is its new volume?

$V_1 = 0.22\ \text{m}^3$
$T_1 = 20.0°\text{C}$
$T_2 = 167°\text{C}$

Unknown: V_2

Charles's Law: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

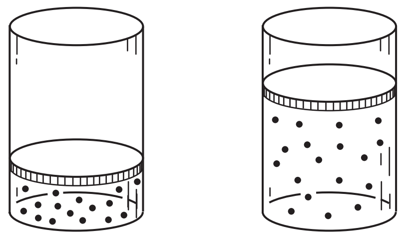


FIGURE 7

Solution: $T_1 = 20.0°\text{C} + 273°\text{C} = 293\ \text{K}$

$T_2 = 167°\text{C} + 273°\text{C} = 440\ \text{K}$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$V_2 = \frac{V_1 T_2}{T_1} = \frac{(0.22\ \text{m}^3)(440\ \text{K})}{(293\ \text{K})} = 0.33\ \text{m}^3$$

Practice Problems

5. A 30.0-m^3 volume of argon gas is heated from 20.0°C to 293°C under constant pressure. What is the new volume of the gas?

6. Thirty liters of oxygen gas are cooled from 20.0°C to −146°C under constant pressure. What is the new volume?

7. The volume of a sample of krypton gas at 60.0°C is 0.21 liters. Under constant pressure, it is heated to twice its original volume. To what temperature (in Celsius degrees) is it heated?

8. The volume of a balloon of helium is 63 liters at 20.0°C. At what temperature would its volume be only 19 liters?

The Ideal Gas Law

Boyle's law and Charles's law can be combined to obtain an equation that relates the pressure, temperature, and volume of a fixed amount of ideal gas.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3} = \ldots = \text{constant}$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

This equation is called the **combined gas law.** It reduces to Boyle's law if the temperature is constant. If the pressure is kept constant, it reduces to Charles's law. The law also shows that at constant volume the pressure varies directly with the temperature.

The Number of Particles

We can use the kinetic theory to discover how the constant in the combined gas law depends on the number N of particles in the sample. Suppose the volume and temperature are held constant. If we increase the number N of particles, the number of collisions the particles make with the container will increase, increasing the pressure. Removing particles will decrease the number of collisions and thus decrease the pressure. We can conclude that the constant in the combined gas law equation is proportional to N. That is,

$$\frac{PV}{T} = kN.$$

We now check this result by holding other variables constant. Suppose pressure and temperature are held constant. Increasing the number of particles increases the number of collisions. To keep the pressure constant, the volume is increased. The number of particles can be increased while maintaining constant pressure and temperature only if the volume is increased. This is consistent with the equation above. Similarly, removing particles requires a decreased volume.

Suppose the volume and pressure are held constant. Increasing the number of particles increases the number of collisions. Therefore, the pressure can be held constant only if the impulse given by each collision is reduced. This can be done by reducing the velocity, and therefore the average kinetic energy of the particles. Remember that temperature is proportional to the average kinetic energy of the particles. Reducing the temperature decreases the average kinetic energy. Thus the number of particles can be increased at constant pressure and volume only if the temperature is reduced. Similarly, removing particles requires a higher temperature. This is also consistent with the equation above.

The **ideal gas law** can be written

$$PV = NkT.$$

The constant $k = 1.38 \times 10^{-23}$ Pa·m^3/K is called Boltzmann's constant. Of course N, the number of particles, is an extremely large number. For that reason, we often use a unit called the **mole.** One mole is equal to 6.02×10^{23} particles. The number of particles in a mole is called **Avogadro's number.** It is numerically equal to the number of particles in a sample of matter whose mass equals the gram formula mass of the substance. You might think of a mole as a "chemist's dozen." Thus, the ideal gas law is written using n, the number of moles of gas in a sample, and the gas constant R.

$$PV = nRT$$

The value of R is 8.31 Pa·m^3/mol·K.

Example Problem

Combined Gas Law

A 20.0-L sample of argon gas at 273 K is at atmospheric pressure, 101.3 kPa. The temperature is lowered to the boiling point of nitrogen, 77 K, and the pressure is increased to 145 kPa. What is the new volume of the argon sample?

$V_1 = 20.0 \text{ L}$
$P_1 = 101.3 \text{ kPa}$
$T_1 = 273 \text{ K}$
$P_2 = 145 \text{ kPa}$
$T_2 = 77 \text{ K}$
Unknown: V_2

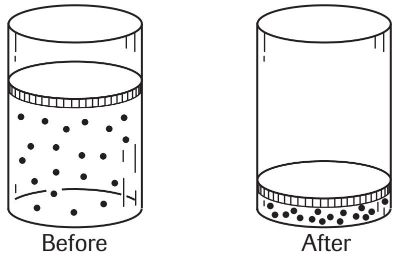


FIGURE 8

Combined Gas Law: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Solution: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$, so

$$V_2 = \frac{P_1V_1T_2}{P_2T_1}$$

$$= \frac{(101.3 \text{ kPa})(20.0 \text{ L})(77 \text{ K})}{(145 \text{ kPa})(273 \text{ K})} = 3.9 \text{ L}$$

Example Problem

Ideal Gas Law

Find the number of moles and the mass of the argon gas sample in the example above. The mean molecular mass of argon is 39.9 g/mol.

$V = 20.0 \text{ L}$
$P = 101.3 \text{ kPa}$
$T = 273 \text{ K}$
$R = 8.31 \text{ Pa·m}^3\text{/mol·K}$
$M = 39.9 \text{ g/mol}$
Unknowns: number of moles n
mass m
Ideal Gas Law: $PV = nRT$
$m = nM$

Solution: $PV = nRT$, so $n = \frac{PV}{RT}$

$$V = 20.0 \text{ L} = (20.0 \text{ L})(1.0 \times 10^{-3} \text{ m}^3\text{/L})$$
$$= 0.0200 \text{ m}^3$$
$$n = \frac{(101.3 \times 10^3 \text{ Pa})(0.0200 \text{ m}^3)}{(8.31 \text{ Pa·m}^3\text{/mol·K})(273 \text{ K})}$$
$$= 0.89 \text{ mol}$$
$$m = (0.89 \text{ mol})(39.9 \text{ g/mol}) = 35.5 \text{ g}$$

Appendix D

Practice Problems

9. A tank of helium gas used to inflate toy balloons is at 15.5×10^6 Pa pressure at 293 K. Its volume is 0.020 m^3. How large a balloon would it fill at 1.00 atmosphere and 323 K?

10. What is the mass of helium gas in Practice Problem 9? The mass of helium is 4.00 grams per mole.

11. Two hundred liters of hydrogen gas at 0°C are kept at 156 kPa. The temperature is raised to 95°C and the volume is decreased to 175 L. What is the pressure of the gas now?

12. The molecular weight of air is about 29 grams/mole. What is the volume of one kilogram of air at standard atmospheric pressure and 20°C?

Real Gases

An ideal gas is a model. The model assumes that gas particles are point masses and exert no forces on each other unless they are in contact. The particles that make up real gases, however, have a definite size and attract one another, even when not in contact. It is perhaps surprising, then, that real gases obey the ideal gas law to high precision, except at very high pressures and very low temperatures. Real gases behave this way because their particles are very small in comparison to the distances between them, and because the forces between the particles are very weak.

Consider a gas such as oxygen. At room temperature and atmospheric pressure, the volume of the particles is very small compared to the total volume of the gas. The particles move so fast that the tiny attractive forces among the particles have practically no effect.

Now consider oxygen gas at −80°C. At this low temperature, the kinetic energy of the slower particles is almost equal to the potential energy resulting from the attractive forces. When the particles approach each other, they are drawn together. The gas volume shrinks faster than the equation $PV = nRT$ predicts.

A second effect is seen when the pressure is high. The distance between the particles is reduced. The volume taken up by the gas particles is now a considerable fraction of the total gas volume. Because it is more difficult to compress the gas, increases in pressure do not cause proportional volume decreases. The gas volume shrinks more slowly than predicted for an ideal gas.

An ideal gas could never change to a liquid. As a real gas is cooled and compressed, the thermal energy of the particles and the separation between them decrease. The particles collide slowly and often. As a result of the forces of attraction, particles stick together. A liquid or a solid begins to form. At some combination of temperature and pressure, determined by the properties of the gas particles, every real gas changes to a liquid or a solid.

Reviewing Concepts

1. If you made a barometer and filled it with a liquid one third as dense as mercury, how high would the level of the liquid be on a day of normal atmospheric pressure?
2. What happens when the pressure acting on a gas is held constant but the temperature of the gas is changed?
3. What happens when the temperature of a gas remains constant and pressure is changed?
4. If a balloon filled with air is at rest, then the average velocity of the particles is zero. Does that mean the assumptions of the kinetic theory are not valid? Explain.
5. What causes atmospheric pressure?
6. State standard atmospheric pressure in four different units.
7. Why does the air pressure in automobile tires increase when the car is driven on a hot day?
8. When an air cylinder with a movable piston is placed in a refrigerator, the volume inside the cylinder shrinks. How could you increase the volume to its original size without removing the piston from the refrigerator?
9. Describe how a real gas differs from an ideal gas. What are some of the consequences of these differences?

Applying Concepts

10. **a.** Explain why liquid rises in a straw when you drink a soda.

    b. What would be the longest soda straw you could use? Assume you could remove all air from the straw.
11. You lower a straw into water and place your finger over its top. As you lift the straw from the water, you notice water stays in it. Explain. If you lift your finger from the top, the water runs out. Why?
12. Explain why you could not use a straw to drink a soda on the moon.
13. A tornado produces a region of extreme low pressure. If a house is hit by a tornado, is it likely to explode or implode? Why?
14. Suppose real gas particles had volume but no attractive forces. How would this real gas differ from an ideal gas? How would it differ from other real gases?

15. Explain how Charles's experiments with gases predicted the value of the absolute zero of temperature.

16. Suppose you are washing dishes. You place a glass, mouth downward, over the water and lower it slowly.

    a. What do you see?

    b. How deep would you have to push the glass to compress the air to half its original volume? Try part **a.** Do not try part **b.**

17. Once again at the sink, you lift a filled glass above the water surface with the mouth below the surface and facing downward.

    a. The water does not run out. Why?

    b. How tall would the glass have to be to keep the water from running out?

Problems

18. Weather reports often give atmospheric pressure in inches of mercury. The pressure usually ranges between 28.5 and 31.0 inches. Convert these two values to Torr, millibars, and kPa.

19. a. Find the force exerted by air at standard atmospheric pressure on the front cover of your textbook.

    b. What mass would exert the same force?

20. A tire gauge at a service station indicates a pressure of 32.0 psi (lb/in^2) in your tires. One standard atmosphere is 14.7 psi. What is the absolute pressure of air in your tires in kPa?

21. How high a column of alcohol (density 0.9 that of water) could be supported by atmospheric pressure?

22. Two cubic meters of a gas at 30.0°C are heated at constant pressure until the volume is doubled. What is the final temperature of the gas?

23. The pressure acting on 50.0 m^3 of air is 1.01×10^5 Pa. The air is at -50.0°C. The pressure acting on the air is increased to 2.02×10^5 Pa. Then the air occupies 30.0 m^3. What is the temperature of the air at this new volume?

24. The pressure acting on 50.0 cm^3 of a gas is reduced from 1.2 atm to 0.30 atm. What is the new volume of the gas if the temperature does not change?

25. A tank containing 30.0 m^3 of natural gas at 5.0°C is heated at constant pressure by the sun to 30.0°C. What is its new volume?

26. Fifty liters of gas are cooled to 91.0 K at constant pressure. Its new volume is 30.0 liters. What was the original temperature?

27. Suppose the lungs of a scuba diver are filled to a capacity of 6.0 liters while the diver is 8.3 m below the surface of a lake. To what volume would the diver's lungs (attempt to) expand if the diver suddenly rose to the surface?

28. A bubble of air with volume 0.050 cm^3 escapes from a pressure hose at the bottom of a tank filled with mercury. When the air bubble reaches the surface of the mercury, its volume is 0.500 cm^3. How deep is the mercury?

29. At 40.0 K, 0.100 m^3 of helium is at 408 kPa. The pressure exerted on the helium is increased to 2175 kPa while its volume is held constant. What is the temperature of the helium now?

30. A 20-L sample of neon is at standard atmospheric pressure at 300°C. The sample is cooled in dry ice to −79°C at constant volume. What is its new pressure?

31. A 50.0-cm^3 sample of air is at standard pressure and −45.0°C. The pressure on the gas sample is doubled and the temperature adjusted until the volume is 30.0 cm^3. What is the temperature?

32. At 40.0 K, 10.0 m^3 of nitrogen is under 4.0×10^2 kPa pressure. The pressure acting on the nitrogen is increased to 2000 kPa. Its volume stays the same. What is the temperature of the nitrogen?

33. The markings on a thermometer are worn off, so a student creates new degree markings, which she calls °S. She then measures the volume of a gas held at constant pressure at three temperatures, finding 30 L at 90°S, 45 L at 120°S, and 60 L at 150°S. What is absolute zero on the S scale?

34. How many particles are in one cubic centimeter of air (6.02×10^{23} particles/mole) at standard pressure at 20°C temperature?

35. Physicists can, with proper equipment, obtain vacuums with pressures of 1.0×10^{-11} Torr.
   a. What fraction of atmospheric pressure is this?
   b. Using the results from Problems 34 and 35a, find the number of particles in a cubic centimeter of this vacuum.

36. Two hundred grams of argon (39.9 gram/mole) are sold in a bottle at 5.0 atm and 293 K. How many liters are in the bottle?

37. A 2.00-L tank of gas is designed to hold gas at 20.0 atm and 50.0°C. What mass of methane (16.0 gram/mole) can be put into the tank?

38. The pressure on 20.0 liters of gas is 120.0 kPa. If the temperature is 23.0°C, how many molecules are present?

39. The specific gravity of mercury is 13.6; that is, mercury is 13.6 times more dense than water. If a barometer were constructed using water rather than mercury, how high (in meters) would the water rise under normal atmospheric pressure?

Topic 6

Radio Transmissions

Do you sometimes listen to your favorite radio station while doing homework? If you close your eyes while listening, you might think the announcer is right there in the room with you. Do you ever think about how the sounds you're hearing get to your ears? **Figure 1** illustrates how the sound waves travel from the announcer to you.

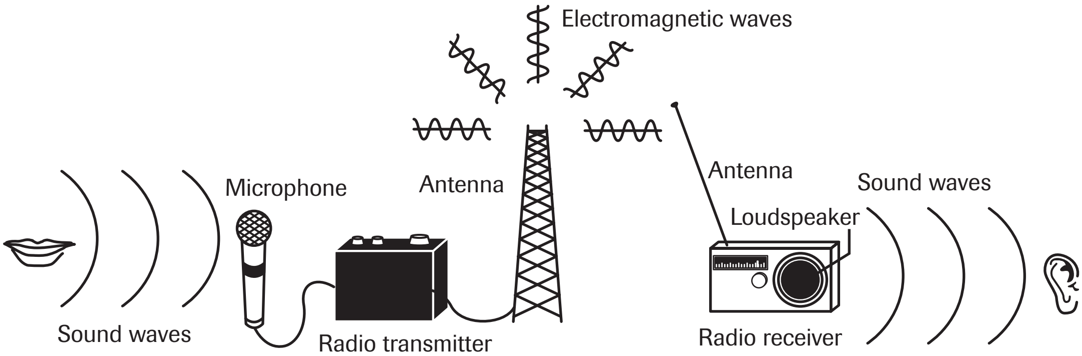


FIGURE 1 The radio broadcast transmitting system has three major components—a microphone, a transmitter, and an antenna. A radio receiving system also has three major components—an antenna, a receiver, and a loudspeaker. In some radio receivers the antenna is inside the case.

As you recall from Chapter 25, a dynamic microphone consists of a diaphragm connected to a coil of wire that is free to move in a magnetic field. When sound waves vibrate the diaphragm, the coil moves in a magnetic field, producing a small, induced electromotive force (*EMF*) in the coil. The induced *EMF* is called a signal. As shown in **Figure 2,** the signal has the same wave properties, such as frequency, as the sound wave that produced it, and its amplitude is proportional to the amplitude of the sound wave.

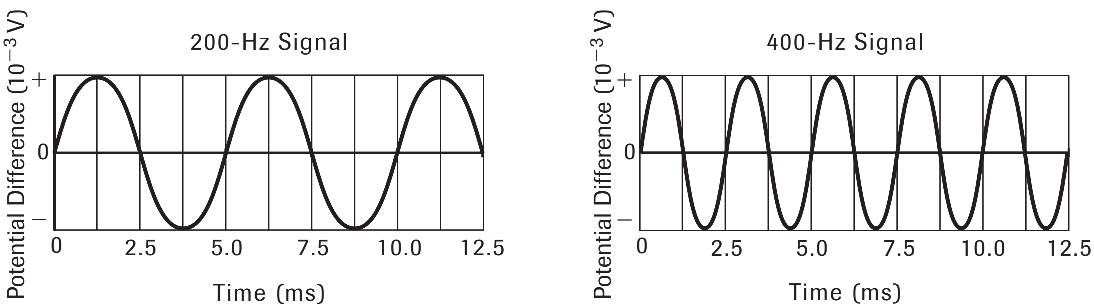


FIGURE 2 The signals produced by a 200-Hz and an 400-Hz sound wave have the same frequencies as the sound waves.

Signals produced by sound waves with frequencies in the range of human hearing (20–20 000 Hz) could be amplified and broadcast as electromagnetic waves. However, broadcasting electromagnetic waves with this frequency has two drawbacks. First, waves broadcast from local stations would interfere with each other. Second, antennas that transmit and receive electromagnetic waves in this frequency range would be over 10^5 km high. Following the Practice Problems below, we will see how these drawbacks can be overcome.

Practice Problems

1. Find the frequency, period, and wavelength of a sound wave produced by a source vibrating at 440 Hz. The speed of sound in air at 20°C is 343 m/s.
2. Find the frequency, period, and wavelength of an electromagnetic wave generated by a 440-Hz signal. Recall that an electromagnetic wave moves at the speed of light.
3. The optimum antenna height is half the wavelength of the electromagnetic wave it is designed to receive. What optimum antenna height would be needed to receive electromagnetic waves produced by a 440-Hz signal?

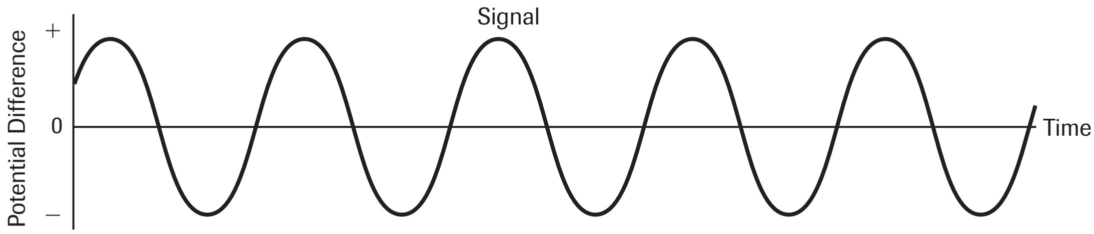
Modulation

To eliminate the problems of interference and antenna length, radio waves are broadcast at frequencies much higher than the signal frequencies. AM radio stations broadcast at 500–1600 kHz and FM radio stations at 88–108 MHz. The terms AM and FM are abbreviations for amplitude-modulated and frequency-modulated. In AM radio, the signal is used to vary the amplitude of the radio wave. In FM radio, the signal is used to vary the frequency of the radio wave.

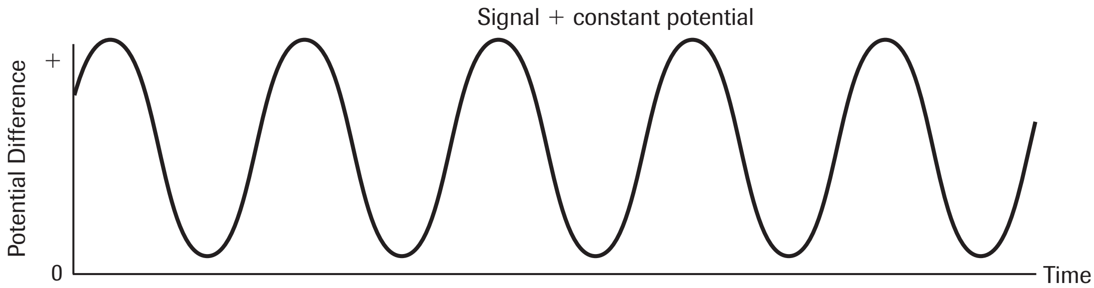
AM Radio

An AM radio wave is produced by a signal varying the amplitude of a high-frequency carrier wave produced in the transmitter. Every radio station operates at a specific frequency that defines the carrier wave. **Figure 3** illustrates amplitude modulation of a carrier wave by a single-frequency signal.

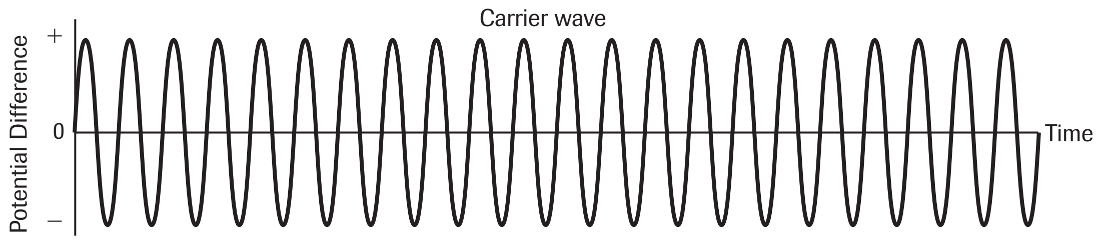
FIGURE 3 A signal modulates the amplitude of a carrier wave, thus producing an AM carrier wave.



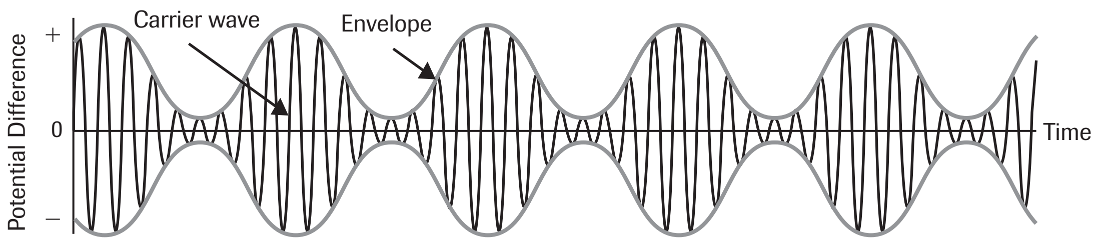
a A single-frequency sound wave produces a single-frequency signal.



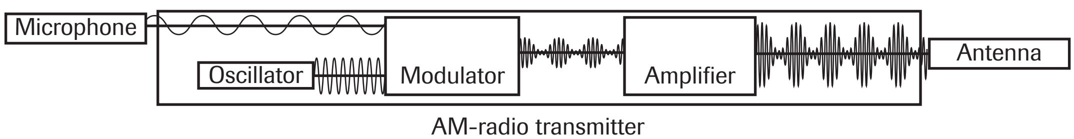
b The transmitter adds a constant potential difference to the signal to produce a signal with positive voltage.



c The carrier wave is produced by a high-frequency oscillator. Its frequency is the frequency assigned to the broadcasting station. In radio broadcasting, the frequency of the carrier wave is usually 10^3–10^5 times as high as the frequency of the signal.



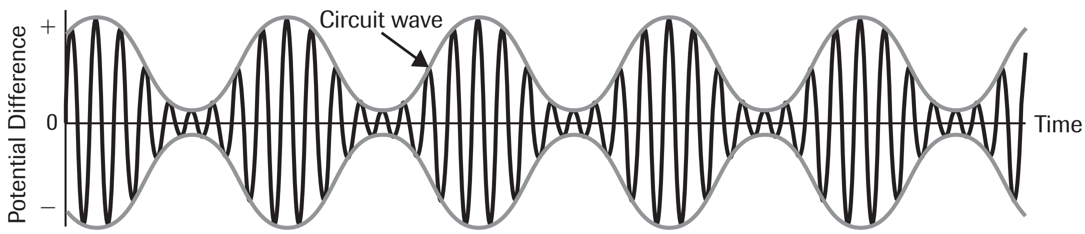
d An amplitude-modulated wave is produced by the modulator, a device that multiplies the amplitude of the carrier wave at each instant by the amplitude of the signal at that instant. The “envelope,” or outline of the amplitude-modulated wave, has the characteristics of the original signal.



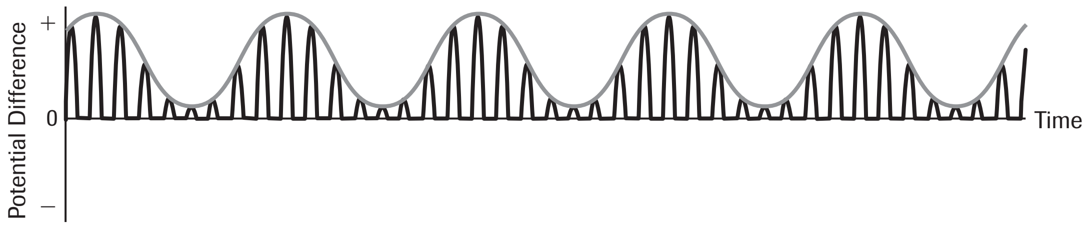
e The components of an AM radio transmitter and their functions

To broadcast AM radio, the AM carrier wave drives an electric current in an antenna, which radiates the AM radio wave in all directions. As you recall from Chapter 26, a radio receiver selects (called tuning the radio) the carrier wave of a particular station by a coil and capacitor circuit connected to the antenna. The capacitance is adjusted until the oscillation frequency of the circuit matches the frequency of the carrier wave. The resulting circuit wave, which matches the carrier wave, is then amplified. To obtain the signal from the carrier wave, the AM radio receiver must remove the carrier wave. This process is called demodulation and is illustrated in **Figure 4.**

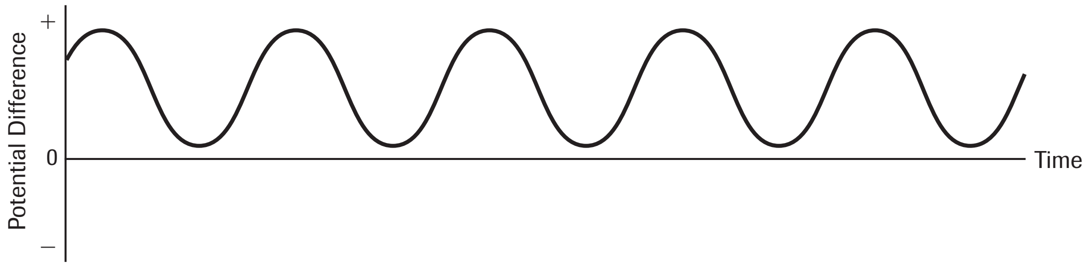
FIGURE 4 A signal is obtained from a circuit wave by demodulation, a process that rectifies and removes the high-frequency component of the circuit wave.



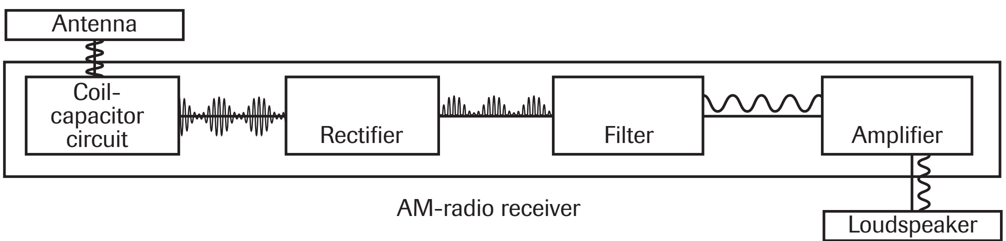
a The characteristics of the circuit wave of the receiver match those of the carrier wave of the transmitter.



b As the circuit wave passes through a rectifier, such as a diode, the parts of the wave with negative voltage are eliminated.



c A resistor and capacitor form a filter that removes the high-frequency component from the circuit wave, leaving only the signal.



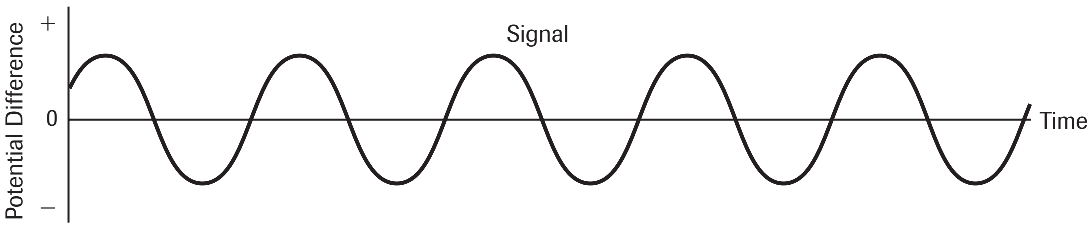
d The components of an AM radio receiver and their functions

The amplified signal from the demodulator then drives a loudspeaker. As you recall from Chapter 24, a loudspeaker consists of a wire-wound coil attached to a paper cone. The coil is positioned in the magnetic field of a permanent magnet. The signal sends a small current through the coil. The magnetic field exerts a force on the coil because the coil is carrying a current. The force pushes the coil in or out of the field, depending on the current's direction. The motion of the coil vibrates the paper cone, which produces sound waves in the air. Because the receiver signal that drives the loudspeaker matches the signal produced by the microphone in the broadcasting station, the characteristics of sound waves produced by the loudspeaker match those of the sound waves that struck the microphone.

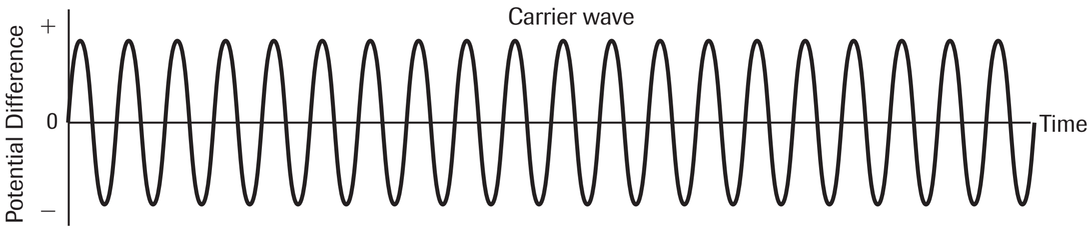
FM Radio

In frequency modulation, the signal modulates the frequency of a radio wave by varying the frequency of a high-frequency carrier wave produced in the transmitter. **Figure 5** illustrates frequency modulation by a single-frequency signal.

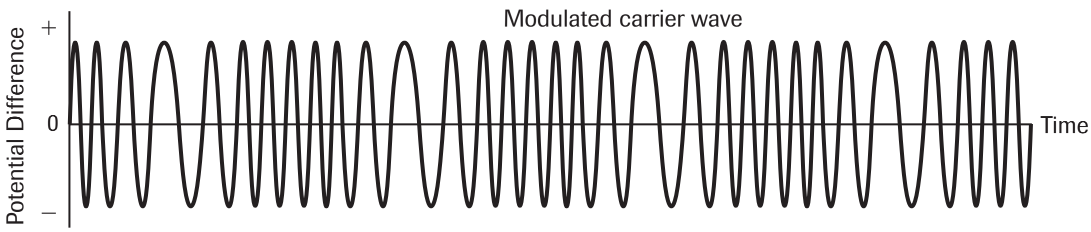
FIGURE 5 A signal modulates the frequency of a carrier wave to produce an FM carrier wave.



a A single-frequency sound wave produces a single-frequency signal.



b A high-frequency oscillator supplies a carrier wave of constant amplitude and frequency.



c The change in the frequency of the carrier wave at each instant is directly proportional to the signal voltage (its amplitude) at that instant.

Similar to AM radio broadcasting, the FM carrier wave generates radio waves that an antenna radiates. The FM radio receiver detects the radio waves, generates an FM circuit wave, and then extracts the signal from the circuit wave.

Reviewing Concepts

1. How does the signal affect the carrier wave in an AM radio transmitter?
2. How does the signal affect the carrier wave in an FM radio transmitter?

Applying Concepts

3. Suppose a modulator adds the signal and carrier waves instead of multiplying them.
   - **a.** Using the signal and carrier waves shown in **Figure 3b** and **Figure 3c,** sketch the wave that this modulator would produce.
   - **b.** How does the wave produced by adding the signal and carrier waves compare to the wave produced by multiplying the signal and carrier waves?
4. Devices such as microphones and loudspeakers are classified as energy converters, that is, they convert the input energy to another form.
   - **a.** What are the forms of the input and output energy of a microphone?
   - **b.** What are the forms of the input and output energy of a loudspeaker?
5. Which type of modulation allows the transmitter to generate radio signals at maximum power? Recall from Chapter 25 that the average power dissipated in an AC circuit is proportional to the square of the amplitude of the voltage.
6. Television signals modulate the carrier at frequencies as high as 4 MHz. Could TV be broadcast in the AM band? Explain.
7. Interpret the importance of wave behaviors and characteristics in the radio industry.
8. Compare the characteristics of electromagnetic waves used in radio with sound waves and their characteristics.

Topic 7

Relativity

Since the time that the theory of relativity was developed by Albert Einstein, it has become one of the most important theories in physics. Relativity actually includes two separate theories, special relativity and general relativity. Special relativity concerns motion at constant velocity, and general relativity includes accelerated motion and motion in a gravitational field. Special relativity does not need complicated mathematics, but general relativity does. For this reason, we will consider only special relativity.

While working as a clerk in the Patent Office in Bern, Switzerland, Einstein published three papers in 1905. The first, on the photoelectric effect, won him the 1921 Nobel prize. The second explained the random motion of particles, called Brownian motion. The third, "On the Electrodynamics of Moving Bodies," presented the special theory of relativity. Although Einstein gained fame among scientists and the public, relativity was ignored by some physicists and opposed by others. Nevertheless, experiments have always confirmed its predictions. Because relativity theory explains all motion so well and is built on such simple foundations, it has been accepted as one of the foundations of modern physics.

Galilean Relativity

You know that the observed curve of a trajectory depends on the observer. Suppose an airplane passenger tosses a ball straight up while the plane is flying horizontally at a constant 200 m/s. To the passenger on the plane, the horizontal velocity of the ball is zero. But to a person on the ground viewing the ball through a telescope, the horizontal velocity of the ball is 200 m/s. Each person could define a frame of reference, a coordinate system having an x-axis and a y-axis. Newton's laws are valid for these two frames of reference, and could be used to analyze the motion. The only difference in the motion of the ball between the two frames is the horizontal velocity of the ball. To the passenger on the plane, the velocity is zero; to the person on the ground, it is 200 m/s. The relative velocity of the two frames is 200 m/s. They are said to be equivalent because Newton's laws can be used to analyze motion in either frame.

Do Newton's laws work in other frames of reference? Suppose the plane accelerates. If a ball is at rest on the floor of the plane, the passenger sees the ball begin to roll backward. The passenger sees the ball accelerated even though no force is exerted on it in the direction of the acceleration. The passenger would believe that Newton's laws were violated. A person on the ground, however, sees the ball moving with constant velocity. To this observer Newton's laws still apply. The two frames of reference are not equivalent because one is accelerated.

Newton's laws apply to nonaccelerated frames of reference. The fact that relative motion doesn't affect the laws of motion is called Galilean relativity, in honor of Galileo, the Italian astronomer and physicist. Galilean relativity also applies if the ball has a horizontal velocity in both frames of reference. Suppose the ball is thrown forward in the plane. The passenger might measure the horizontal velocity as 10 m/s. The observer on the ground would measure the velocity as 200 m/s + 10 m/s = 210 m/s. Thus velocities add. That is, the velocity of the ball in the stationary frame is the sum of the relative velocity of the two frames of reference plus the velocity of the ball in the moving frame.

Light

Does Galilean relativity work for light as well as moving bodies? When Einstein was only 16, he asked himself what a light wave would look like if the observer were moving alongside it at the speed of light. The electric fields would now be stationary. But stationary electric fields do not induce magnetic fields. Thus, Einstein reasoned, the laws of electromagnetism do not hold in this frame of reference.

There was another problem with light. Nineteenth-century physicists, believing that mechanics was the foundation of physics, tried to build mechanical models of everything, including electromagnetic waves. Sound, they knew, is a pressure wave traveling through a medium. What, they asked, is the medium for a light wave? They theorized that there was a substance, called ether, in which the electric and magnetic fields oscillated. Ether was at rest, and all bodies moved through it. It was then the preferred frame of reference and had some remarkable properties. For example, Earth moved through ether without resistance.

Michelson's Experiments

In 1887, the U.S. physicist Albert A. Michelson, built an interferometer, as shown in **Figure 1,** to measure Earth's motion through ether. A light beam was split in two by a partially silvered mirror. One light path moved parallel to the motion of Earth, and the other path moved perpendicular to it. The two beams were reflected to mirrors and recombined, then observed at an eyepiece. If the light beams took the same time traveling the two paths, they would interfere constructively, producing a bright spot. If one beam took half an oscillation period less time, the interference would be destructive, and a dark area would result.

Michelson assumed that light moved through ether as a swimmer moved through a river. The time needed to swim a certain distance would depend on whether the swimmer moves with, against, or perpendicular to the current. Earth was supposed to be moving through the ether, dragging the interferometer with it. So, the time needed for light to travel from the half-silvered mirror to the reflecting mirror and back should depend on whether the light is moving parallel or perpendicular to the motion of the ether. The interferometer could be rotated to bring first one arm and then the other arm parallel to the motion of Earth through the ether. Thus, the interference should change from dark to light as the instrument was rotated. But no change was observed. Michelson was disappointed and confused by this result. He repeated the experiment with U.S. physicist E.W. Morley, using a much more sensitive instrument. Again, the motion of Earth through ether could not be detected. Many explanations were offered for this puzzling result. None was convincing.

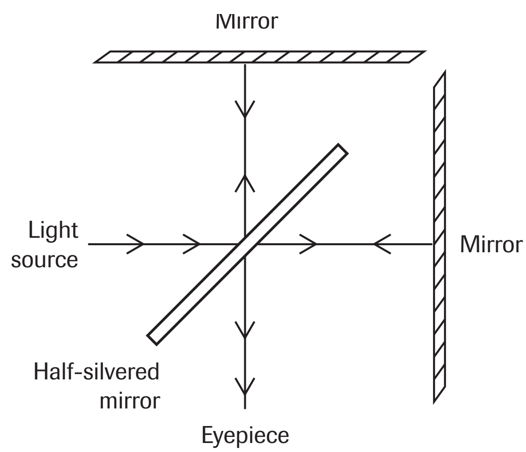


FIGURE 1 A. A. Michelson's interferometer

Einstein's Two Postulates

Einstein presented two postulates in his 1905 paper on relativity. He called one postulate the principle of relativity. It stated that all frames of reference are equivalent for electromagnetism as well as mechanics. That is, there is no preferred reference frame. There is no frame absolutely at rest. The second postulate stated that light always moves at the same speed in all frames of reference. The speed of light is independent of the motion of the source and the motion of the observer. Light emitted by the headlight of an airplane moves at 3×10^8 m/s with respect to the plane and 3×10^8 m/s with respect to the ground. Einstein stated that as a result of these two postulates, there was no need for an ether.

If a particle moving at half the speed of light emits a gamma ray, the ray moves away from the particle at the speed of light. The gamma ray also moves at the speed of light when measured by an observer on Earth. The velocity of light does not add like a vector. Einstein's ideas make us rethink our understanding of time, length, momentum, and energy. Some of this new thinking challenges our "logical" minds.

The Meaning of Time

What is time? Einstein said time is something measured by a clock. The ticks clock makes are separated by t_0 seconds. The time between the ticks is not the same for a person who is moving with respect to the clock. In the next section, you learn that an observer moving at velocity v measures a longer time t between ticks.

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Because c is the speed of light, the denominator is always smaller than one, so the time t measured by the moving observer is always longer than the time t_0 measured by an observer moving with the clock. This result is called **time dilation** because t is longer (dilated) than t_0.

Because all motion is relative, whether the clock is moving with respect to the observer or the observer is moving with respect to the clock, to an observer not moving with the clock, the intervals between ticks seen dilated.

Suppose the clock is on a moving satellite. An observer on Earth sees the clock running slowly. If the clock is on Earth and the observer is on the satellite, that observer would find Earth's clock was running slowly. Time dilation agrees with the principle of relativity. It shows that there is no preferred frame of reference in which clocks give the correct time. No frame is absolutely at rest.

One consequence of time dilation is the Twin Effect or Twin Paradox illustrated in **Figure 2.** One twin boards a very fast spaceship while the other stays at home. Moving clocks, including biological clocks that control aging, run slow. Therefore, the twin in the spaceship ages more slowly than the twin who stays at home.

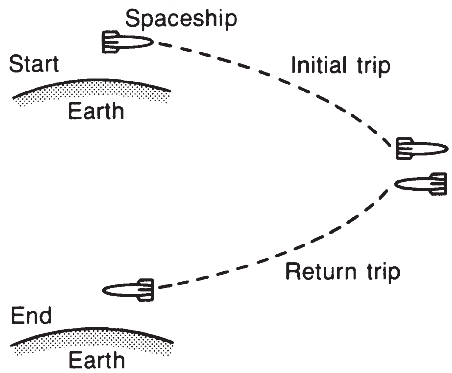


FIGURE 2 Twin Effect

Additional Topics

When they meet again after the trip, the one who remained at home is older. But, you may say, all reference frames are equivalent. Why does the twin in the spaceship not see the Earth twin speeding away, and thus find that the Earth twin ages more slowly? The key to the paradox is the requirement that the twins must be reunited. The traveling twin returns, reversing the direction of motion. You can tell which twin moved, and thus which twin aged less.

If a spaceship starts and stops, it is in a frame of reference that undergoes acceleration. Special relativity does not apply to frames that accelerate. General relativity must be used to handle the acceleration. The result, however, is the same. The twin effect was tested in 1971. A very precise atomic clock was carried in commercial airplanes around the globe. When the clock returned, it was slow by the interval predicted by Einstein's theory.

Time Dilation

Suppose a flash lamp and a mirror are separated by a distance d, as in **Figure 3a.** The lamp is a timing device. It makes regular flashes, like a clock makes ticks, to mark time intervals. The lamp is designed so that the time between flashes is the time needed for the light flash to be reflected from the mirror and return to the lamp. An observer in the same frame of reference would find that the time t_0 for the flash to travel to the mirror and back to the lamp is equal to the distance $2d$ divided by c, the speed of light. That is $t_0 = 2d/c$.

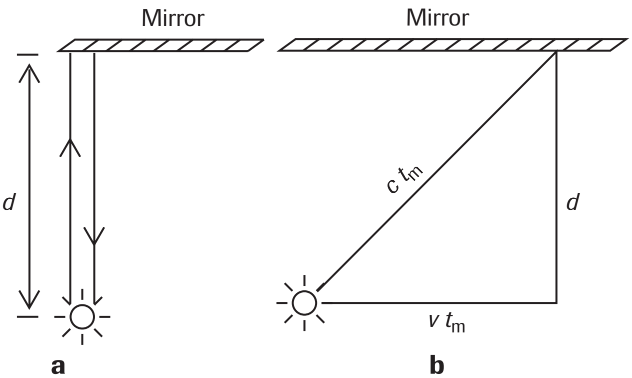


FIGURE 3 The lamp and mirror are at rest **(a).** The lamp and mirror are moving to the right, while the observer is at rest **(b).**

Now suppose that the lamp and mirror are moving to the right, as in **Figure 3b.** What would an observer at rest measure for the travel time of the flash? The speed of light is c, as in all frames of reference. But to the observer at rest the light takes a longer path, the hypotenuse of a right triangle. Let t_m be the time for the flash to travel to the mirror as measured by an observer at rest. Then the distance the light travels is ct_m. During this time, the whole apparatus moves a distance vt_m to the right. Since $d = ct_0/2$, we can use the Pythagorean theorem to find t_m.

$$\left(\frac{ct_0}{2}\right)^2 + (vt_m)^2 = (ct_m)^2$$

$$t_m = \frac{ct_0/2}{\sqrt{c^2 - v^2}}$$

The return trip from the mirror to the detector takes the same time. The total trip, t, then, takes $2t_m$:

$$t = \frac{ct_0}{\sqrt{c^2 - v^2}} = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Appendix D

Length Contraction

How do you measure the length of an object? You usually place a meterstick next to the object and measure the object along the meterstick. But what if the object is moving? The locations of both ends of the object must be marked at the same time. If two observers are moving at different velocities, however, they measure time differently. Thus an observer moving with the object and an observer moving with the meterstick would not agree on the time the locations of the ends were marked.

If the length of an object measured at rest is L_0, then an observer moving along the length at velocity v measures

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}.$$

The quantity under the radical is always less than one. Therefore, $L < L_0$. That is, the length of an object is shorter when measured by a moving observer. For example, suppose the object is 1 meter long, $L_0 = 1.00$ m. If an observer is moving at 3/5 the speed of light, then $v/c = 3/5 = 0.6$, and

$$L = (1.00 \text{ m})\sqrt{1 - \frac{v^2}{c^2}} = 0.80 \text{ m}.$$

A Test of Relativity Theory

The two predictions of special relativity, time dilation and length contraction, have been tested and found to be correct. The experiment used muons, medium-mass elementary particles that are short-lived, whose half-life at rest is 1.53×10^{-6} s (see Chapter 30). When cosmic rays strike nuclei in molecules 7 km or higher above the surface of Earth the collisions produce many high-speed particles, some of which decay into muons. The experiment was done with muons moving at 2.984×10^8 m/s, which is 99.52% the speed of light. How far could a muon travel at this speed during a half-life?

$$\begin{aligned} d &= vt \\ &= (2.984 \times 10^8 \text{ m/s})(1.53 \times 10^{-6} \text{ s}) \\ d &= 456 \text{ m} \end{aligned}$$

If a muon is produced 7 km above Earth's surface and travels only 456 m in one half-life, then it could not reach the surface. Yet many do.

The half-life of a muon is like the units of time measured by a clock, and moving clocks run slower than stationary clocks. To an observer on Earth, a muon's half-life t is

$$\begin{aligned} t &= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \\ &= \frac{1.53 \times 10^{-6} \text{ s}}{\sqrt{1 - \frac{(2.984 \times 10^8 \text{ m/s})^2}{(2.998 \times 10^8 \text{ m/s})^2}}} \\ t &= 1.59 \times 10^{-5} \text{ s}. \end{aligned}$$

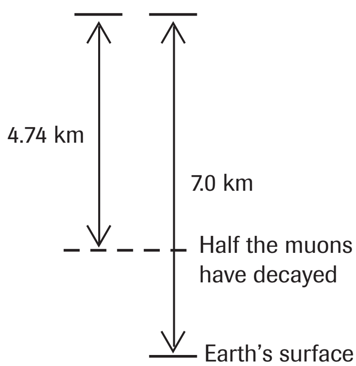


FIGURE 4 A muon travels 4.74 km during one half-life, as measured by an observer on Earth. After this time, half the muons have decayed. Some of the others do not decay before they have traveled 7.0 km. These particles reach Earth's surface.

In this time, a muon can travel a distance

$$
\begin{aligned}
d &= vt \\
&= (2.984 \times 10^8 \text{ m/s})(1.59 \times 10^{-5} \text{ s}) \\
d &= 4.74 \text{ km.}
\end{aligned}
$$

Therefore many muons live long enough to reach Earth's surface. This result can be understood another way. To an observer moving with the muon, Earth and its atmosphere are moving, and moving lengths contract. Thus the 7.00-km distance to Earth is contracted to

$$
\begin{aligned}
L &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
&= (7.00 \text{ km}) \sqrt{1 - \frac{(2.984 \times 10^8 \text{ m/s})^2}{(2.998 \times 10^8 \text{ m/s})^2}} \\
L &= 676 \text{ km.}
\end{aligned}
$$

Because the distance muons can travel in one half-life is 474 m, in less than two half-lives muons can travel 676 m through Earth's atmosphere.

This actual experiment was done by measuring the number of muons traveling at 99.52% the speed of light at the top of New Hampshire's Mt. Washington, 1907 m above sea level, and the number of muons at sea level. The ratio of these numbers was in excellent agreement with the predictions of the theory of special relativity.

Adding Relative Velocities

Suppose a spaceship is moving at speed u. It fires an object at speed v, as measured by an astronaut on the ship. How fast would the object be moving as seen by an observer on Earth? According to Galilean relativity, the velocity v of the object measured by an observer at rest, is $v' = v + u$. For example, if a spaceship is moving at velocity $u = 0.60c$, and the object it fired moves at $v = 0.50c$, then the velocity of the object measured by an observer on Earth is $v' = v + u = 0.50c + 0.60c = 1.10c$. Therefore, according to Galilean relativity, the velocity of the object measured by an observer on Earth is faster than the speed of light. In Einstein's theory, no object can reach this speed.

Einstein used time dilation and length contraction to develop a new rule for relative velocities:

$$
v' = \frac{v + u}{1 + \frac{uv}{c^2}}.
$$

Using Einstein's rule, the observer on Earth would find

$$
v' = \frac{(0.60c) + (0.50c)}{1 + \left(\frac{0.60c}{c}\right)\left(\frac{0.50c}{c}\right)} = \frac{1.10c}{1.30} = 0.85c.
$$

At low speeds, the new rule agrees with the one from Galilean relativity. When u and v are both much less than c, the denominator is very close to 1, and $v' = v + u$ to a very good approximation.

Appearance of Fast-Moving Objects

As you have read, one of the consequences of relativity is that the length of a fast-moving object shrinks in the direction of motion. Another consequence is that to an observer at rest, a fast moving object appears to be slightly rotated. A person directly viewing the front of an object can also see a side, as shown in **Figure 5.**

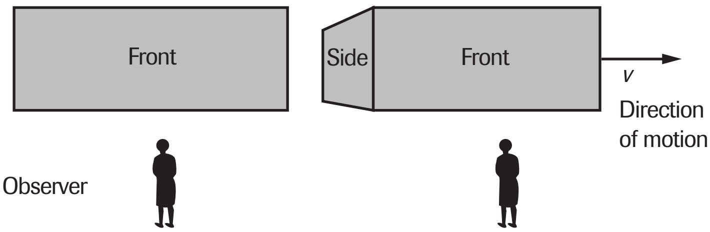


FIGURE 5 Viewing the rectangular solid at rest, an observer sees only the front, which is directly before him. However, when the object is traveling at velocity comparable to the speed of light, the observer sees a side as well as the front.

Overhead views of the moving rectangular solid at times t_0 and t_1 are shown in **Figure 6.** Vertex A cannot be seen because a light ray cannot travel from the observer through the solid to vertex A. If the solid is moving to the right at a velocity close to the speed of light, however, it moves out of the path of the light ray, allowing the observer to see vertex A.

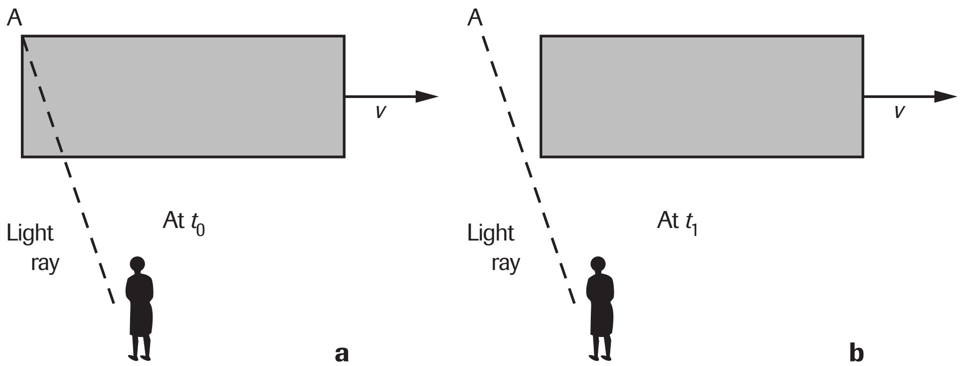


FIGURE 6 Overhead view of rectangular solid moving at velocity v (near the speed of light) at times t_0 **(a)** and t_1 **(b).** Because vertex A can be seen, the object appears to be rotated.

The Energy of Rapidly-Moving Objects

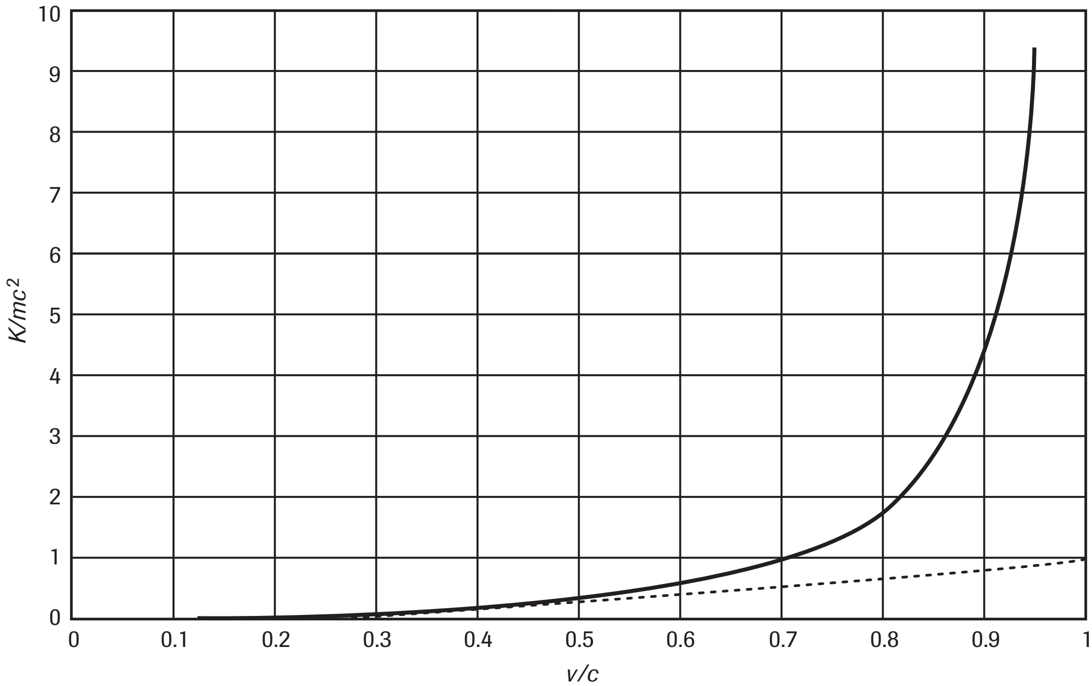
According to Newton, if you do work on an object, you may increase its kinetic energy. Einstein showed that the familiar equation for kinetic energy, $K = \frac{1}{2}mv^2$, is not correct when the object is moving near the speed of light. Rather, Einstein showed, the kinetic velocity is given by the equation

$$K = mc^2\left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right]$$

Does this equation for kinetic energy disagree with Newton when v is small? To find out, we will need an algebraic approximation for the term in the equation that involves the square root. That term can be written as $(1 - x)^n$, where $x = (v/c)^2$ and $n = -\frac{1}{2}$. According to the Binomial Theorem, $(1 - x)^n \cong 1 - nx$. So, in this equation $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \cong 1 - \frac{1}{2}(v^2/c^2)$. Therefore, when v is much less than c, $K = mc^2\,(1 - \frac{1}{2}(v^2/c^2) - 1) = \frac{1}{2}mv^2$. Thus, when v is small, Einstein's equation agrees with Newton's.

Figure 7 shows both Einstein's (solid line) and Newton's (dashed line) expressions for the ratio K/mc^2 for velocities from zero to 95% of the speed of light.

FIGURE 7 As the speed of an object approaches the speed of light, its kinetic energy becomes extremely large, much larger than Newton's non-relativistic physics would predict.



This relationship for kinetic energy has been tested in all high-energy particle accelerators, both linear accelerators and synchrotrons (Chapter 30). It takes more and more work to achieve an increase in kinetic energy when the particles speed approaches that of light.

You know that kinetic energy is just one form of energy. There are many kinds of potential energy. It is often useful to define the total energy of a system as the sum of kinetic and potential energies due to forces such as gravitation, springs, etc., that are exerted within the system. Einstein recognized that he could define the total energy of a fast-moving particle as

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = K + mc^2$$

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That is, the total energy of a particle is equal to its kinetic energy plus mc^2. Einstein called that term the *rest energy* of the particle. It is a form of potential energy, the potential energy of the mass itself. Any particle, even one that is not moving, has a rest energy given by $E_0 = mc^2$.

Thus mass itself is a form of energy. Consider a flashlight battery. When it is new, its energy is in the form of chemical energy. If you connect it to a bulb in a circuit, that chemical energy will be changed into electrical energy, and then into light and thermal energy. Eventually, the battery will become "discharged," the bulb will dim, and eventually become dark and cold. All the available chemical energy of the battery will be gone. According to Einstein, its mass will be reduced. While this is true, because the energy change is just a few hundred joules, the mass change will be so small as to be undetectable by even the most sensitive scales. Only in reactions involving the strong nuclear force (Chapters 30, 31) can the change in mass be measured. For example, when a uranium nucleus splits into two smaller nuclei and several neutrons, the energy released is enormous and the change in mass is very small but measurable.

The Realm of Relativity

Einstein's special relativity does not contradict the laws of Newton. They agree for objects moving at ordinary velocities. Relativistic effects are observable only for objects moving almost as fast as light does. Because we have little experience with such high velocities, we do not usually observe time dilation, length contraction, and mass gain.

Relativity forces us to examine our ideas about motion closely. For example, we have looked at the measurement of time and distance more closely and found surprising new results. Examining familiar ideas often leads to a new appreciation of our world and, sometimes, to new theories.

Reviewing Concepts

1. State the two postulates of relativity.
2. A spaceship moves at half the speed of light and shines one laser beam forward and another beam backward. As seen by an observer at rest, does the beam shining forward appear to move faster than the one shining backward? Explain.
3. One student travels in a plane moving to the left at 100 m/s. A student on the ground drives a car to the right at 30 m/s. A third student is standing still. Identify and describe the relative motion between the students. What is the speed of light for each reference frame?
4. Suppose, in another world, the speed of light were only 10 m/s, the speed of a fast runner. Describe some effects on athletes in this world.

5. In the world of question 4, a contestant rides a bicycle. Would she see the bicycle as shorter than it was at rest? What would stationary people who are observing the bicycle see?

6. An airplane generates sound waves. When it flies at the speed of sound, it catches up with the waves, producing a sonic boom.

   a. Could an object radiating light in a vacuum ever catch up with its waves? Explain.

   b. Suppose the object moves at 90% the speed of light in a vacuum. Then it enters water. If its speed did not change, could it now catch up with the light waves it emits in the water? Explain.

7. An astronaut aboard a spaceship has a quartz watch, a meterstick, and a mass on a spring. What changes would the astronaut observe in these items as the spaceship moves at half the speed of light? Explain.

8. The astronaut in question 7 measures the period of an oscillator consisting of a mass on a spring. Would the frequency of oscillation change? Explain.

9. Under what circumstances could a 40-year-old man have a 50-year-old daughter?

10. Science books often say that "matter can never be created nor destroyed." Is this correct? If not, how should it be modified?

Problems

11. A rocket, 75 m long, moves at $v = 0.50c$. What is its length as measured by an observer at rest?

12. A spaceship traveling at $v = 0.60c$ passes near Earth. A 100.0-m long soccer field lies below its path. What is the length of the field as measured by the crew aboard the ship?

13. A certain star is 30.0 light-years away. One light-year is the distance light travels in one year. From the point of view of a person on Earth, how long would it take a spaceship traveling at $v = 0.866c$ to make the one-way trip? Ignore the time needed for acceleration and braking.

14. From the point of view of the astronauts in the spaceship in problem 13, how long would the round-trip require?

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15. A spaceship is 98 m long. How fast would it have to be moving to appear only 49 m long?

16. The lifetime of a pion at rest is 2.6×10^{-8} s. What is the lifetime, measured by an observer at rest, of a pion traveling at $0.80c$?

17. The rest energy of a pion is 140 MeV. What is its kinetic energy when it moves at $0.80c$? Its total energy?

18. When a pion decays, it emits a muon. If the pion decays at rest, the muon is emitted at velocity $0.80c$. If the pion is moving at $0.50c$, and the muon is emitted in the same direction as the pion, what is the velocity of the muon as seen by an observer at rest?

19. A spaceship is moving at velocity $0.40c$. A meteor is moving toward it at $0.60c$, as measured by the crew on the ship. What is the speed of the meteor as measured by an observer at rest?

20. What is the equivalent energy in joules of 1.0 kg of apples?

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