

Name: \_\_\_\_\_

# UNIT 1 LEARNING GUIDE – SEQUENCES

## INSTRUCTIONS:

Using a pencil, complete the following questions as you work through the related lessons.

**Show ALL of your work**, as is explained in the lessons. Do your best and always ask questions if there is anything that you don't understand.

### 1.1 INTRODUCTION TO SEQUENCES

1. Find the common difference in each sequence.

**Ex.** 10, 21, 32, 43, 54, 65 ...

**Common difference = 11**

a. 1, 8, 15, 22, 29, 36 ...

b. 12, 8, 4, 0, -4, -8 ...

c. 75, 25, -25, -75, -125 ...

2. Find the common difference then determine the value of the given term in the arithmetic sequence given.

a) 35, 31, 27, 23, ... $a_5 =$	b) _____, -44, -75, -106, -137 $a_1 =$	c) -7, -9, _____, -13, -15 ... $a_3 =$
d) -4, -24, -44, -64, ... $a_6 =$	e) -30, -40, -50, -60, $a_9 =$	f) 8, 13, 18, _____, 28... $a_4 =$

## 1.2 ARITHMETIC SEQUENCES

1. Use the formula for arithmetic sequences to determine the value of each term.

$$a_n = a_1 + (n - 1)d$$

**Ex.** Find the 16<sup>th</sup> term in the sequence 6, 9, 12, 15, 18 ...

$$a_n = a_{16} \quad n = 16 \quad d = 3 \quad a_1 = 6$$

$$a_n = a_1 + (n - 1)d$$

$$a_{16} = 6 + (16 - 1)3$$

$$a_{16} = 6 + (15)3$$

$$a_{16} = 6 + 45$$

$$a_{16} = 51$$

- a. Find the 35<sup>th</sup> term in the sequence 55, 59, 63, 67, 71 ...

$$a_n = \quad n = \quad d = \quad a_1 =$$

- b. Find the 21<sup>th</sup> term in the sequence -13, -11, -9, -7, -5 ...

$$a_n = \quad n = \quad d = \quad a_1 =$$

- c. Find the 45<sup>th</sup> term in the sequence 20, 10, 0, -10, -20, -30...

$$a_n = \quad n = \quad d = \quad a_1 =$$

2. Write an equation for the  $n^{\text{th}}$  term of the following sequences.

$$a_n = a_1 + (n - 1)d$$

Ex. -8, -6, -4, -2, 0 ...

a) 55, 70, 85, 100, 115, ...

$$a_n = a_1 + (n - 1)d$$

$$a_1 = -8 \quad d = 2$$

$$a_n = -8 + (n - 1)2$$

b. 12, 6, 0, -6, -12 ...

c. -45, -40, -35, -30, -25 ...

3. Use the formula for arithmetic sequences to solve the following problems.

$$a_n = a_1 + (n - 1)d$$

a. If  $a_{32} = 640$  and the common difference is 4, find the first term in the series.

b. If  $a_{17} = -3$  and the common difference is 5, find the first term in the series.

c. If  $a_{29} = 1541$  and the common difference is 35, find the first term in the series.

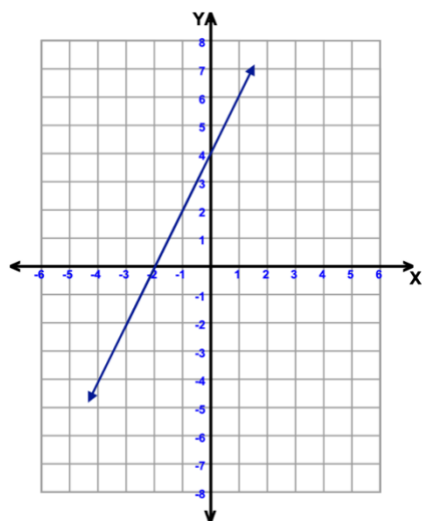
4. Solve the following word problems.
- Determine the arithmetic sequence whose third term is 16 and 7th term exceeds the 5th term by 12
  - How many multiples of 4 lie between 10 and 350?
  - Kali deposited \$5 in the first week of a year and then deposited \$ 1.75 more per week thereafter. If the  $n$ th week, her weekly savings become \$ 20.75, find  $n$

- d. Lanie has decided to add strength training to her exercise program. Her trainer suggests that she add weight lifting for 5 minutes during her routine for the first week. Each week thereafter, she is to increase the weight lifting time by 2 minutes. If Lanie continues with this increase in weight lifting time, how many minutes will she be devoting to weight lifting in week 10?
  
  
  
  
  
  
  
  
  
  
- e. The summer Olympics occur every four years. Starting with 2024, in which year will the 16th summer Olympics occur?
  
  
  
  
  
  
  
  
  
  
- f. The sum of the 4th and 8th terms of an arithmetic sequence is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the arithmetic sequence.
  
  
  
  
  
  
  
  
  
  
- g. Raoul started work in 1990 at an annual salary of \$ 5000 and received an increment of 200 each year. In which year did his income reach \$ 8000?

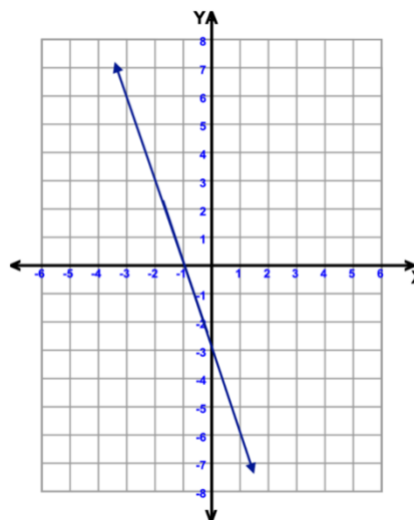
## 1.3 LINEAR RELATIONS

1. Identify each graph as a linear relation or an arithmetic sequence, then determine the equation for each graph. *Reminder:* The equation for a linear relation takes the form  $y = mx + b$ , while the equation for an arithmetic sequence takes the form  $a_n = a_1 + (n - 1)d$ .

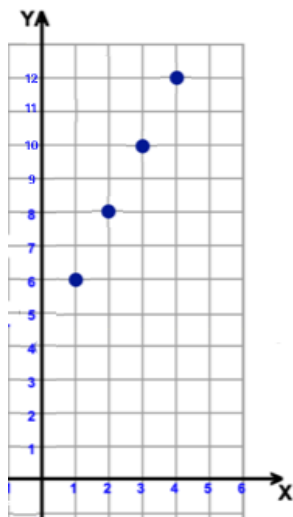
a.



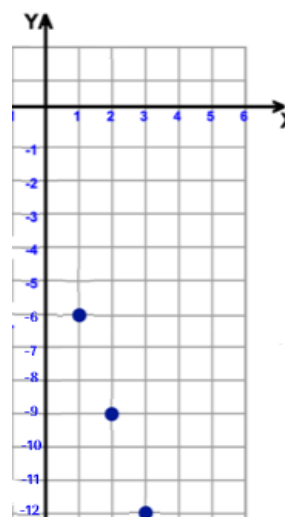
b.



c.



d.



1.4 ARITHMETIC SERIES
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1. Determine the sum of each of the following arithmetic series

a. $6 + 14 + 22 + \dots$ up to 20 terms	b. $\sqrt{2} + \sqrt{8} + \sqrt{18} + \dots$ up to 13 terms
c. $\frac{7}{2} + \frac{13}{4} + 3 + \dots$ up to 10 terms	d. $-40 - 33 - 26 - \dots$ up to 31 terms
e. $-\frac{5}{9} - \frac{11}{18} - \frac{2}{3} + \dots$ up to 17 terms	f. $74 + 61 + 48 + \dots$ up to 19 terms

2. The eleventh term of an arithmetic sequence is 30 and the sum of the first eleven terms is 55. What is the common difference?

3. How many terms of the arithmetic sequence 2, 8, 14, 20, ... are required to give a sum of 660?



4. What is the sum of the eleventh to thirtieth terms (inclusive) of the arithmetic sequence 7, 12, 17, 22, ... ?

5. The side lengths of a quadrilateral form an arithmetic sequence. If the perimeter of this quadrilateral is 74 cm and the longest side is 29 cm. What are the other side lengths?

1.5 GEOMETRIC SEQUENCES
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1. Determine if the sequence given is arithmetic or geometric. Find the ratio (r) or difference (d) in each. Give the next 3 terms in each sequence.

a. 28, 18, 8, ...

b. -16, -6, 4, ...

c. -2, -10, -50,

d. -3, -1,  $-\frac{1}{3}$ , ...

e. -1, 6, -36, ...

f.  $1, \frac{1}{2}, 0, \dots$

2. Find the common ratio r for each of the following geometric sequences algebraically.

a. 3, -12, 48, ...

b. 0.01, 0.06, 0.36, ...

c.  $2x^8, 8x^6, 32x^4, \dots$

d.  $1, x + 3, x^2 + 6x + 9, \dots$

3. Find the indicated term in each sequence...

a. 4, 12, 36, ...  
9<sup>th</sup> term

b. -2, 6, -18, ...  
9<sup>th</sup> term

c. -20, -10, -5, ...  
9<sup>th</sup> term

d.  $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \dots$   
12<sup>th</sup> term

e.  $3x^2, 6x^4, 12x^6, \dots$   
7<sup>th</sup> term

f)  $1, 2x, (2x)^2, \dots$   
5<sup>th</sup> term

4. The fifth term of a geometric sequence is 1875. If the first term is 3, find the common ratio.

5. The first term of a geometric sequence is 5 and the sixth term is 160. What is the common ratio?

6. Which term of the geometric sequence 1, 3, 9, ... has a value of 19 683?

7. The fourth term of a geometric sequence is 27 and the seventh term is 1. What is the first term?

8. If the geometric sequences 162, 54, 18, ..... and  $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}, \dots$  have their  $n$ th term equal, find the value of  $n$ .

9. The following three terms  $x$ ,  $x + 5$ ,  $x + 15$  form a geometric sequence, find the value of  $x$ .

10. In 2011, the number of students in a small school is 132. It is estimated that the student population will increase by 4% each year. Estimate the student population in 2020.

1.6 GEOMETRIC SERIES
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1. Find the designated sum of the geometric series.

a)  $S_7$  of  $4+8+16+32+\dots$

b)  $S_8$  of  $1-6+36-216+\dots$

c)  $S_{12}$  of  $2-4+8-16 \dots$

d)  $S_8$  of  $3+15+75+375+\dots$

e)  $S_6$  of  $\frac{3}{5} + \frac{9}{10} + \frac{27}{20} + \frac{81}{40} + \dots$

f)  $S_{10}$  of  $-2 + 6 + -18 + 54 + \dots$

2. The sum of the first two terms of a geometric sequence is 15, and the sum of the second and third terms is 60. Determine the first three terms of the sequence algebraically.

3. The third term of a geometric series is 24 and the fourth term is 36. Determine the sum of the first 10 terms.

4. 4. A virus goes through a computer, infecting files. If one file was infected initially and the total number of files infected doubles every minute, how many total files will be infected in 18 minutes?

5. A new shopping mall is gaining in popularity. Every day since it opened, the number of shoppers is 20% more than the number of shoppers the day before. The total number of shoppers over the first 4 days is 671. How many shoppers were at the mall on the first day? Round your final answer to the nearest integer

6. Rosie went on a hiking trip. The first day she walked 18 kilometers. Each day since, she walked 90% of what she walked the day before. What is the total distance Rosie has traveled by the end of the 10th day? Round your final answer to the nearest kilometer.



## 1.7 INFINITE GEOMETRIC SERIES

1. Determine whether the infinite geometric series is convergent or divergent.

a.  $729 + 243 + 81 + \dots$  \_\_\_\_\_

b.  $343 + 49 + 7 + \dots$  \_\_\_\_\_

c.  $2 + 5 + 12.5 + \dots$  \_\_\_\_\_

d.  $4 - 14 + 49 + \dots$  \_\_\_\_\_

2. Find the sum of each of these convergent geometric series.

a.

$$1 + \frac{10}{11} + \frac{100}{121} \dots$$

b.

$$1 - 0.99 + 0.99^2 - 0.99^3 \dots$$

c.

$$1 + 0.8 + 0.64 \dots$$

d.

$$5 + \frac{15}{4} + \frac{45}{16} + \dots$$

e.

$$1 + \frac{8}{9} + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^3 \dots$$

f.

$$1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 \dots$$

3. Determine the common ratio of the infinite geometric series describes below.

a. First term: 1, Sum: 2.25

b. First term: 96, Sum: 64

c. First term: -4, Sum:  $-\frac{16}{5}$

d. First term: 1, Sum: 2.5

4. A child on a swing is given a big push. She travels 14 feet on the first back-and-forth swing but only  $\frac{5}{6}$  as far on each successive back-and-forth swing. How far (total distance) does she travel before the swing stops?

5. A ball is thrown 12 meters in the air (so that the initial up-and-down distance is 24 meters). The ball rebounds 95% of the distance it falls. What is the total vertical distance travelled by the ball before it stops bouncing?

6. A side of a square is 12 cm. The midpoints of its sides are joined to form an inscribed square, and this process is continued. Find the sum of the perimeters of the squares if this process is continued without end (round answer to two decimal places).

### 1.8 SIGMA NOTATION

1. Write out each of the following series.

a.

$$\sum_{n=1}^6 n^4$$

b.

$$\sum_{k=1}^{\infty} -4 \cdot 2^{k-1}$$

c.

$$\sum_{k=3}^7 \frac{k+1}{k}$$

d.

$$\sum_{k=0}^{\infty} 2^{k+1} x^k$$

2. Express each of these sums using sigma notation.

a.  $1^2 + 2^2 + 3^2 + 4^2 + 5^2$

b.  $13 + 20 + 27 \dots$

c.  $3 + 6 + 9 + 12 + 15 + 18$

d.  $x^3 + x^5 + x^7 + x^9 + \dots + x^{199}$

3. Find the sum of these series. For a) and c), assume that the series includes both terms.

a.

$$\sum_{n=1}^{25} 5n - 2$$

b.

$$\sum_{k=1}^{\infty} 3\left(\frac{2}{5}\right)^k$$

c.

$$\sum_{k=7}^{32} 2k + 3$$

d.

$$\sum_{k=1}^{10} 2^{k-1}$$

e.

$$\sum_{k=0}^{\infty} 5\left(\frac{1}{4}\right)^k$$

f.

$$\sum_{n=1}^{\infty} 4(0.2)^n$$

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## UNIT 1 – ANSWER KEY

### SECTION 1.1

1. a. 7 b. -4 c. -50

2. a. -4, 19 b. -13, -31 c. -2, -11 d. -20, -104 e. -10, -110 f. 5, 23

### SECTION 1.2

1. a. 191 b. 27 c. -420

2. a.  $a_n = 55 + (n - 1)15$  b.  $a_n = 12 + (n - 1)(-6)$  c.  $a_n = -45 + (n - 1)5$

3. a. 516 b. -83 c. 561

4. a. 4, 10, 16 .... b. 85 c. 10 d. 23 e. 2084 f. -13, -8, -3 g. 16

### SECTION 1.3

1. a. Linear equation,  $y = 2x + 4$

b. Linear equation,  $y = -3x - 3$

c. Arithmetic sequence,  $a_n = 6 + (n - 1)2$

d. Arithmetic sequence,  $a_n = -6 + (n - 1)(-3)$

### SECTION 1.4

1. a. 1640 b.  $91\sqrt{2}$  c.  $\frac{95}{4}$  d. 2015 e. -17 f. -817

2. 5 3. 15 4. 2090 5. 29 cm, 22 cm, 15 cm, 8 cm

## SECTION 1.5

1. a. Arithmetic  $d = -10$  Terms:  $-2, -12, -22$       b. Arithmetic  $d = 10$  Terms:  $14, 24, 34$   
     c. Geometric  $r = 5$  Terms:  $-250, -1250, -6250$       d. Geometric  $r = \frac{1}{3}$  Terms:  $-\frac{1}{9}, -\frac{1}{27}, -\frac{1}{81}$   
     e. Geometric  $r = -6$  Terms:  $216, -1296, 7776$       f. Arithmetic  $d = -\frac{1}{2}$  Terms:  $-\frac{1}{2}, -1, -\frac{3}{2}$
2. a.  $-4$  b.  $6$  c.  $4/x^2$  d.  $x + 3$
3. a.  $26\,244$  b.  $-13\,122$  c.  $-5/64$  d.  $1/354294$  e.  $192x^{14}$  f.  $16x^4$  or  $(2x)^4$       4.  $5$
5.  $2$       6.  $10$       7.  $729$       8.  $5$       9.  $5$       10.  $188$

## SECTION 1.6

- 1 a.  $508$  b.  $-239\,945$  c.  $-2730$  d.  $292\,968$  e.  $399/32$  f.  $29\,524$
2.  $3, 12, 48$
3.  $58025/48$
4.  $262143$  5.  $125$  6.  $117$

## SECTION 1.7

1. a. convergent b. convergent c. divergent d. divergent
2. a.  $11$  b.  $0.5025$  c.  $5$  d.  $20$  e.  $9$  f.  $4$       3. a.  $.56$  b.  $-1/2$  c.  $-1/4$  d.  $0.6$       4.  $84'$   
 5.  $480$       6.  $163.88$

## SECTION 1.8

- 1 a.  $1^4 + 2^4 + 3^4 + 4^4 + 5^4 + 6^4$       b.  $-4 - 8 - 16 - 32 - \dots$
- c.  $\frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6} + \frac{8}{7}$       d.  $2 + 4x + 8x^2 + 16x^3 + \dots$
2. a.  $\sum_{n=1}^5 n^2$  b.  $\sum_{x=2}^{\infty} (7x - 1)$  c.  $\sum_{r=1}^6 3r$  d.  $\sum_{w=1}^{99} x^{2w+1}$
3. a.  $1575$  b.  $2$  c.  $1092$  d.  $1023$  e.  $20/3$  f.  $1$