Warning

The math you are about to see won't be learned until later on in your math development, thus watching the following video may cause unintentional learning, spontaneous brain development and random bouts of understanding. Proceed at your own risk.

click here to view the full question

Solving Equations:

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x - 2 = 0 \qquad x - 1 = 0$$

$$x = 2 \qquad x = 1$$

Simplifying Rational Expressions:

$$\frac{x^2+5x+4}{x^2+3x-4}$$

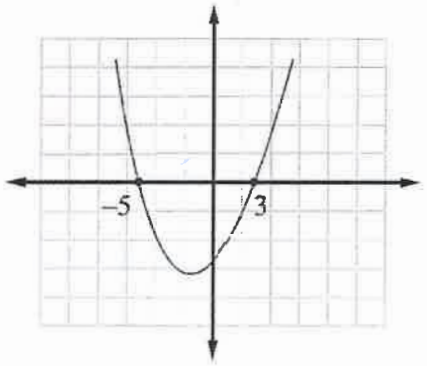
$$= \frac{(x+1)\cancel{(x+4)}}{(x-1)\cancel{(x+4)}}$$

$$= \frac{x+1}{x+1}$$

Graphically

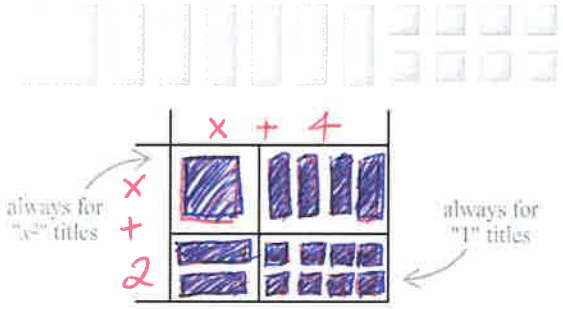
$$f(x) = x^2 + 2x - 15$$

$$f(x) = (x+5)(x-3)$$



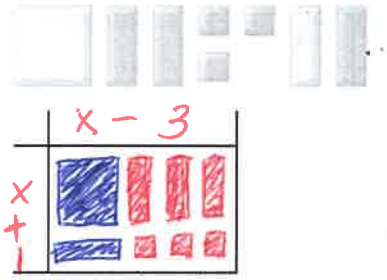
$ax^2 + bx + c \rightarrow a = 1 \rightarrow x^2 + bx + c$

Example Factor $x^2 + 6x + 8$

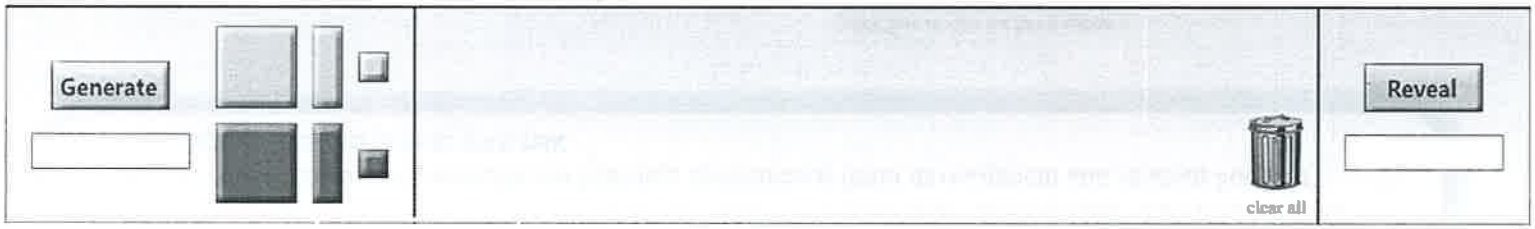


Check by foil $(x+2)(x+4)$
$= x^2 + 4x + 2x + 8$
$= x^2 + 6x + 8$

Example Factor $x^2 - 2x - 3$



Check by foil $= x^2 - 3x + x - 3$
$= x^2 - 2x - 3$

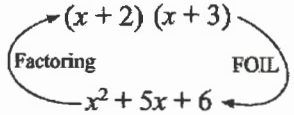


Factoring Trinomials When a is 1

Page 3 - Factoring is the Opposite of Expanding

Hot Tip

Factoring or expanding are just changing the form of the expression. It is still the same expression.



Evaluate for $x = 1$

$(1+2)(1+3)$
$(3)(4)$
12

$1^2+5(1)+6$
$1+5+6$
12

Example 1

$(x + 2)(x + 3)$

$x^2+3x+2x+6$
x^2+5x+6
x^2+bx+c
$b=5=2+3$
$c=6=2\times3$

Example 2

$(x - 1)(x + 4)$

$x^2+4x-x-4$
x^2+3x-4
x^2+bx+c
$b=3=-1+4$
$c=-4=-1\times4$

$x^2 + bx + c$

$(x \pm \_\_)\ (x \pm \_\_)$

+ to ☐
× to ☐

Factoring Trinomials When a is 1

Page 4 - Steps to Factoring a Trinomial Where a=1

Step-By-Step

1.) Write out the factor pairs of "c"
- If "c" is + and "b" is +, then we only need + pairs
- If "c" is + and "b" is (-), then we only need (-) pairs
- If "c" is (-) include both possible ± pairs

2.) Look for a factor pair that adds to "b"
- Remember to take the sign into account
- If no pair adds to "b", it can't be factored

3.) Factor
- Use factor pair discovered above
- Always of form $(x \pm \_)(x \pm \_)$

4.) Check your answer
- Expand using FOIL
- There is no excuse to factor incorrectly
- Always, always check your answer

$x^2 + bx + c$

Example

$x^2 - 6x - 16$

Factors	*Sum*
-1, 16	-1+16 = 15
1, -16	1 -16 = -15
-2, 8	-2+8 = 6
2, -8	2 - 8 = -6
-4, 4	-4+4 = 0

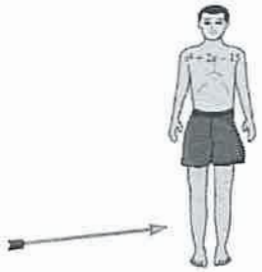
$(x+2)(x-8)$

Check: $x^2 - 8x + 2x - 16$

$x^2 - 6x - 16$

Factoring Trinomials When a is Not 1

Page 1 - Look for Common Factors



Hot Tip

If "a" doesn't equal 1 see if there is a common factor that can be removed from all terms.

Example 1: Factor

$3x^2 + 6x - 45$

Common Factor: 3

$3(x^2+2x-15)$

~~-1, 15~~

~~1, -15~~

-3, 5 ✓

$3(x-3)(x+5)$

Example 2: Factor

$2x^3 - 12x^2 - 14x$

Common Factor: 2x

$2x(x^2-6x-7)$

~~-1, 7~~

1, -7 ✓

$2x(x+1)(x-7)$

Hot Tip

A binomial can be a common factor.

Example 1: Factor Fully

$x^2(x-3)-2x(x-3)-24(x-3)$

$(x-3)\left(\frac{x^2(x-3)}{(x-3)}-\frac{2x(x-3)}{(x-3)}-\frac{24(x-3)}{(x-3)}\right)$

$(x-3)(x^2-2x-24)$

$(x-3)(x+4)(x-6)$

$x^2y-2xy-24y$
$y(x^2-2x-24)$
$y=x-3$

Factors	*Sum*	
$-1, 24$	23	x
$1, -24$	-23	x
$-2, 12$	10	x
$2, -12$	-10	x
$-4, 6$	2	x
$4, -6$	-2	✓

Remember

We can treat a binomial just as if it was a variable in a term, just like it was a "y".

Factoring Trinomials When a is Not 1

Page 3 - Steps to Factoring by Grouping

Step-By-Step

1.) Look for common factors
- Factor any common factors out first
- Now we will just deal with trinomial in the square brackets
- If no common factor, no square brackets

2.) Write out the factor pairs of "ac"
- First multiply "*a*" and "*c*"
- Look for a factor pair of "*ac*" that adds to "*b*".
- If no pair adds to "*b*", it can't be factored

3.) Split up "*bx*" into the factor pair
- $ax^2 + f_1x + f_2x + c$
- The order of f_1 and f_2 is important
- If further steps don't work, switch the order

4.) Look for common factors in the groups
- The first two terms are the first group
- The second two terms are the second group
- Factor out the GCF in each group

5.) Factor out the common binomial factor
- We now have a common binomial factor
- If there isn't one, switch the order of f_1 and f_2
- Remove the square brackets and we are done

6.) Check your answer
- There is no excuse to factor incorrectly
- Always, always check your answer

Hot Tip

Use Factor by grouping when 'a' is not 1 in $ax^2 + bx + c$

$6x^3 + 15x^2 - 36x$

$3x[2x^2 + 5x - 12]$

$2(-12) = -24$

$3x[2x^2 + 8x - 3x - 12]$

$3x[2x(x+4) - 3(x+4)]$

$3x[(x+4)(2x-3)]$

$3x(x+4)(2x-3)$

Factors	*Sum*	
1, -24	-23	×
-1, 24	23	×
2, -12	-10	×
-2, 12	10	×
3, -8	-5	×
-3, 8	5	✓

Check:

$3x(2x-3)(x+4)$

$(6x^2 - 9x)(x+4)$

$(6x^3 + 24x^2 - 9x^2 - 36x)$

$6x^3 + 15x^2 - 36x$

OR

$3x(2x-3)(x+4)$

Advanced Factoring
Page 1 - Difference of Squares

Example 1:

$x^2 - 16$

$x^2+0x-16$

$(x-4)(x+4)$

Factors	Sum	
$-1, 16$	15	×
$1, -16$	-15	×
$-2, 8$	6	×
$2, -8$	-6	×
$-4, 4$	0	✓

Example 2:

$4x^2 - 9$

$4x^2+0x-9$

$4x^2+6x-6x-9$

$2x(2x+3)-3(2x+3)$

$(2x+3)(2x-3)$

Factors	Sum	
$-3, 12$	9	×
$3, -12$	-9	×
$-4, 9$	5	×
$4, -9$	-5	×
$-6, 6$	0	✓

Definition

A difference of squares is an expression that takes the form a^2-b^2 and can always be factored into $(a-b)(a+b)$

Example 3:

$9z^2 - 49$

$(3z-7)(3z+7)$

Check: $9z^2+21z-21z-49$

$9z^2-49$ ✓

$a^2 = 9z^2$

$\sqrt{a^2} = \sqrt{9z^2}$

$a = 3z$

$b^2 = 49$

$\sqrt{b^2} = \sqrt{49}$

$b = 7$

Example 3:

$x^4 - 81$

$(x^2-9)(x^2+9)$

$(x-3)(x+3)(x^2+9)$

Question

What do the following polynomials have in common?

$x^2 + 4x + 4$

$(x+2)(x+2)$

Factors	*Sum*
1, 4	5 ✗
2, 2	4 ✓

$9x^2 + 6x + 1$

$9x^2+3x+3x+1$

$3x(3x+1)+(3x+1)$

$(3x+1)(3x+1)$

$(3x+1)^2$

Factors	*Sum*
1, 9	10 ✗
3, 3	6 ✓

Definition

A perfect square trinomial when factored has identical binomial factors, and thus can be expressed as $(a+b)^2$.

$9x^2 - 30xy + 25y^2$

$ac = (9)(25) = 225$

$9x^2 - 15xy - 15xy + 25y^2$

$3x(3x-5y) - 5y(3x-5y)$

$(3x-5y)(3x-5y)$

$(3x-5y)^2$

Factors	*Sum*
−1, −225	−226 ✗
−3, −75	−78 ✗
−5, −45	−50 ✗
−15, −15	−30 ✓

Example

$12x^2 - 75$

$a^2 - b^2 = (a-b)(a+b)$

$3(4x^2 - 25)$

$3((2x)^2 - 5^2)$

$3(2x-5)(2x+5)$

Example

$5x^2 - 720$

$5(x^2 - 144)$

$5(x^2 - 12^2)$

$5(x-12)(x+12)$

Example

$-8x^2 + 56x - 98$

$-2(4x^2 - 28x + 49)$

$-2(4x^2 - 14x - 14x + 49)$

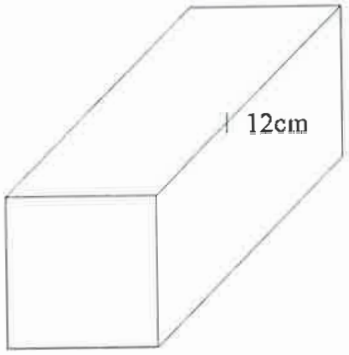
$-2(2x(2x-7) - 7(2x-7))$

$-2(2x-7)(2x-7)$

$-2(2x-7)^2$

Example

Given that the volume of a square-based rectangular prism is given by the equation $V = 108x^2 - 288x + 192$, determine the expression that represents the side length of the square if the longest side is 12cm.



$V = 108x^2 - 288x + 192$

$V = 12(9x^2 - 24x + 16)$

$V = 12(9x^2 - 12x - 12x + 16)$

$V = 12(3x(3x-4) - 4(3x-4))$

$V = 12(3x-4)^2$

Volume = length x width x height

= ( )( )( )

Therefore, the side length is $(3x-4)$ cm.