BGSE MSc in Data Science - Stochastic Models and Optimization
Instructor: Mihalis G. Markakis

Problem Set 1

Problem 1 (Machine Maintenance - [B05] Exercise 1.3)

Suppose that we have a machine that is either running or is broken down. If it runs throughout one week it makes a gross profit of \$100. If it fails during the week, the gross profit is zero. If it is running and at the start of the week and we perform preventive maintenance, the probability that it will fail during the week is 0.4. If we do not perform such maintenance, the probability of failure is 0.7. However, maintenance will cost \$20. When the machine is broken down at the start of the week, it may either be repaired at a cost of \$40, in which case it will fail during the week with probability 0.4, or it may be replaced at a cost of \$90 by a new machine that is guaranteed to work properly through its first week of operation. Reformulate the problem within the DP framework, and find the optimal repair-replacement-maintenance policy that maximizes the total expected profit over four weeks, assuming a new machine at the start of the first week.

Problem 2 (Discounted Cost - [B05] Exercise 1.6)

In the framework of the basic problem, consider the case where the cost is of the form

$$\mathbb{E}_{\{w_k\}}\Big\{\alpha^N g_N(x_N) + \sum_{k=0}^{N-1} \alpha^k g_k(x_k, u_k, w_k)\Big\},$$

where $\alpha \in (0,1)$ is a discount factor. Develop a DP-like algorithm for this problem.

Problem 3 (Multiplicative Cost - [B05] Exercise 1.9)

In the framework of the basic problem, consider the case where the cost has the multiplicative form

$$\mathbb{E}_{\{w_k\}}\Big\{g_N(x_N) \cdot g_{N-1}(x_{N-1}, u_{N-1}, w_{N-1}) \cdots g_0(x_0, u_0, w_0)\Big\}.$$

Develop a DP-like algorithm for this problem assuming that $g_k(x_k, u_k, w_k) > 0$, for all x_k, u_k, w_k, and k.

Hint: One cannot simply minimize the logarithm of total expected cost because $\log \mathbb{E}\{yz\} \neq \mathbb{E}\{\log(yz)\} = \mathbb{E}\{\log(y)\} + \mathbb{E}\{\log(z)\}$.

Problem 4 (**Knapsack Problem - [B05] Exercise 1.10**)

Assume that we have a vessel whose maximum weight capacity is z and whose cargo is to consist of different quantities of N different items. Let v_i denote the value of the i^{th} type of item, w_i the weight of the i^{th} type of item, and x_i the number of items of type i that are loaded in the vessel. The problem is to find the most valuable cargo, i.e., to maximize $\sum_{i=1}^{N} x_i v_i$ subject to the constraints $\sum_{i=1}^{N} x_i w_i \leq z$ and $x_i \in \mathbb{N}$. Reformulate the problem within the DP framework.

Problem 5 (**Traveling Repairman Problem - [B05] Exercise 1.24**)

A repairman must service N sites, which are located along a line and are sequentially numbered $1, 2, \ldots, N$. The repairman starts at a given site s with $1 < s < N$, and is constrained to service only sites that are adjacent to the ones serviced so far, i.e., if he has already serviced sites $i, i+1, \ldots, j$, then he may service only site $i-1$ (assuming $1 < i$) or site $j+1$ (assuming $j < N$). There is a waiting cost of c_i for each time period that site i has remained unserviced and there is a travel cost t_{ij} for servicing site j right after site i. Reformulate the problem within the DP framework.

Note: Whenever you are asked to reformulate a problem within the DP framework, you have to clearly identify the primitives of the problem, i.e., state, control, uncertainty (if any), dynamics, cost structure, constraints (if any), and then write the DP algorithm (initialization, recursion) based on these primitives.