14D006 Stochastic Models and Optimization

Problemset 1

Daniel Bestard Delgado, Michael Cameron, Hans-Peter Höllwirth, Akhil Lohia

February 24, 2017

1 Machine Maintenance

2 Discounted Cost

In the framework of the basic problem, consider the case where the cost is of the form

$$\mathbb{E}_{\{w_k\}}[\alpha^N g_N(x_N) + \sum_{k=0}^{N-1} \alpha^k g_k(x_k, u_k, w_k)]$$

where $\alpha \in (0,1)$ is a discount factor.

Let $J_k^{'}$ be the optimal value of the $(N-k)$-tail problem with cost function $g_N^{'}(x_N) = \alpha^N g_N(x_N)$ and $g_k^{'}(x_k) = \alpha^k g_k(x_k, u_k, w_k)$. Then we have

$$
\begin{aligned}
J_N^{'}(x_N) &= g_N^{'}(x_N) \\
&= \alpha^N g_N(x_N) \\
\alpha^{-N} J_N^{'}(x_N) &= g_N(x_N)
\end{aligned}
$$

$$
\begin{aligned}
J_k^{'}(x_k) &= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k}\left[g_k^{'}(x_k, u_k, w_k) + J_{k+1}^{'}(f_k(x_k, u_k, w_k))\right] \\
&= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k}\left[\alpha^k g_k(x_k, u_k, w_k) + J_{k+1}^{'}(f_k(x_k, u_k, w_k))\right] \\
\alpha^{-k} J_k^{'}(x_k) &= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k}\left[g_k(x_k, u_k, w_k) + \alpha^{-k} J_{k+1}^{'}(f_k(x_k, u_k, w_k))\right] \\
&= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k}\left[g_k(x_k, u_k, w_k) + \alpha\alpha^{-(k+1)} J_{k+1}^{'}(f_k(x_k, u_k, w_k))\right]
\end{aligned} \tag{1}
$$

Now let $J_k(x_k) = \alpha^{-k} J^{'}(x_k)$ and so we get the DP-like algorithm

$$
\begin{aligned}
J_N(x_N) &= \alpha^{-N} J^{'}(x_N) \\
&= g_N(x_N)
\end{aligned}
$$

$$
\begin{aligned}
J_k(x_k) &= \alpha^{-k} J^{'}(x_k) \\
&= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k}\left[g_k(x_k, u_k, w_k) + \alpha\alpha^{-(k+1)} J_{k+1}^{'}(f_k(x_k, u_k, w_k))\right] \\
&= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k}\left[g_k(x_k, u_k, w_k) + \alpha J_{k+1}(f_k(x_k, u_k, w_k))\right]
\end{aligned} \tag{2}
$$

3 Multiplicative Cost

4 Knapsack Problem

5 Traveling Repairman Problem