Play Selection in American Football

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- Use of DP to come up with the optimal policies. Two different implementations:
  - Exact solution is feasible under some assumptions
  - For more general cases, approximations of the expected reward-to-go function are provided (API and OPI)

Parameters of the dynamic programming algorithm

- State of the system:
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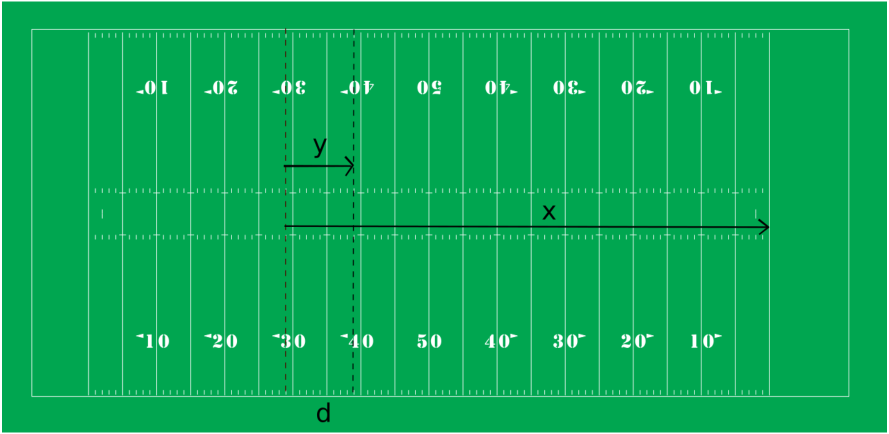
- Rewards:
  - Touchdown: 6.8
  - Field goal: 3
  - Safety: -2
  - Opposition score $= -\frac{6.8x}{100}$

The Football Model



Exact Dynamic Programming

DP Equation

$$\mu^k(i) = arg \max_{u \in U} \left[\sum_{j \in S} p_{ij}(u)(g(i,u,j) + J^{\mu^{k-1}}(j)) \right]$$

- $p_{ij}(u)$: transition probabilities
- $g(i,u,j)$: reward function
- $J^{\mu^{k-1}}(j)$: reward-to-go function

J is computed exactly using the 15250 possible states of the system.

Heuristic Algorithm

- We create a reasonable class of policies and implement it.
- Policies are compared by calculating the points from one drive.
- Simulations are run from the starting state of $(x_i, y_i, d) = (80, 10, 1)$.

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- Simulations are run from the starting state of $(x_i, y_i, d) = (80, 10, 1)$.
- Example of simulation:

$$\begin{bmatrix}25\\10\\1\end{bmatrix} \longrightarrow_0^P \begin{bmatrix}17\\2\\2\end{bmatrix} \longrightarrow_0^R \begin{bmatrix}14\\10\\1\end{bmatrix} \longrightarrow_0^P \begin{bmatrix}10\\6\\2\end{bmatrix} \longrightarrow_0^P \begin{bmatrix}10\\6\\3\end{bmatrix} \longrightarrow_0^R \begin{bmatrix}8\\4\\4\end{bmatrix} \longrightarrow_3^K T$$

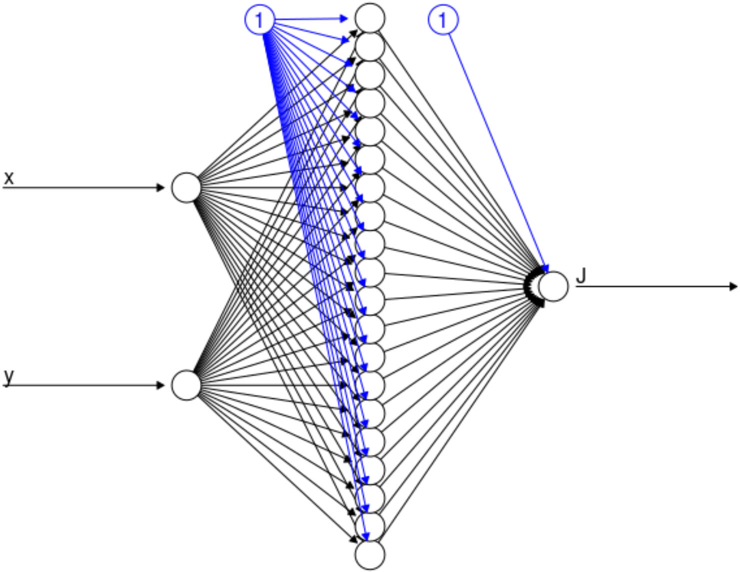
Approximate Dynamic Programming

- In more general frameworks, the exact reward-to-go function cannot be computed
- Two different ways for approximating:
  - API: Many training sample points, few iterations
  - OPI: Few training sample points, many iterations

API and OPI Algorithm

1. Start an initial policy μ_0
2. For each k:
    1. Given μ_k, generate N_e sample trajectories
    2. Fit the neural network to estimate the reward-to-go function
    3. Update policy

Neural Network



Policy Update

Approximated DP algorithm

$$\mu^k(i) = arg \max_{u \in U} \left[\sum_{j \in S} p_{ij}(u)(g(i,u,j) + \widetilde{J}^{\mu^{k-1}}(j)) \right]$$

Super Bowl XLIX

Seahawks should have run!