

# Problemset 1

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## 1 Machine Maintenance

## 2 Discounted Cost

In the framework of the basic problem, consider the case where the cost is of the form

$$\mathbb{E}_{\{w_k\}}[\alpha^N g_N(x_N) + \sum_{k=0}^{N-1} \alpha^k g_k(x_k, u_k, w_k)]$$

where  $\alpha \in (0, 1)$  is a discount factor.

Let  $J_k^*$  be the optimal value of the  $(N - k)$ -tail problem with cost function  $g_N^*(x_N) = \alpha^N g_N(x_N)$  and  $g_k^*(x_k) = \alpha^k g_k(x_k, u_k, w_k)$ . Then we have

$$\begin{aligned}
J_N^*(x_N) &= g_N^*(x_N) \\
&= \alpha^N g_N(x_N) \\
\alpha^{-N} J_N^*(x_N) &= g_N(x_N) \\
J_k^*(x_k) &= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k} [g_k^*(x_k, u_k, w_k) + J_{k+1}^*(f_k(x_k, u_k, w_k))] \\
&= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k} [\alpha^k g_k(x_k, u_k, w_k) + J_{k+1}^*(f_k(x_k, u_k, w_k))] \\
\alpha^{-k} J_k^*(x_k) &= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k} [g_k(x_k, u_k, w_k) + \alpha^{-k} J_{k+1}^*(f_k(x_k, u_k, w_k))] \\
&= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k} [g_k(x_k, u_k, w_k) + \alpha \alpha^{-(k+1)} J_{k+1}^*(f_k(x_k, u_k, w_k))]
\end{aligned} \tag{1}$$

Now let  $J_k(x_k) = \alpha^{-k} J^*(x_k)$  and so we get the DP-like algorithm

$$\begin{aligned}
J_N(x_N) &= \alpha^{-N} J^*(x_N) \\
&= g_N(x_N) \\
J_k(x_k) &= \alpha^{-k} J^*(x_k) \\
&= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k} [g_k(x_k, u_k, w_k) + \alpha \alpha^{-(k+1)} J_{k+1}^*(f_k(x_k, u_k, w_k))] \\
&= \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k} [g_k(x_k, u_k, w_k) + \alpha J_{k+1}(f_k(x_k, u_k, w_k))]
\end{aligned} \tag{2}$$

### 3 Multiplicative Cost

In the framework of the basic problem, consider the case where the cost has the multiplicative form

$$\mathbb{E}_{\{w_k\}} [g_N(x_N) g_{N-1}(x_{N-1}, u_{N-1}, w_{N-1}) \dots g_0(x_0, u_0, w_0)]$$

assuming that  $g_k(x_k, u_k, w_k) > 0$ , for all  $x_k, u_k, w_k$ , and  $k$ .

### 4 Knapsack Problem

### 5 Traveling Repairman Problem