## Play Selection in American Football

Daniel Bestard, Hans-Peter Hoellwirth, Akhil Lohia, Michael Cameron

Barcelona Graduate School of Economics

April 5, 2017

• Subconsciously, sport players compute probabilities

- Subconsciously, sport players compute probabilities
- They use estimates of distances, angles and probabilities of events occurring

- Subconsciously, sport players compute probabilities
- They use estimates of distances, angles and probabilities of events occurring
- Aim: provide a framework for optimising decisions in American football

- Subconsciously, sport players compute probabilities
- They use estimates of distances, angles and probabilities of events occurring
- Aim: provide a framework for optimising decisions in American football
- Use of DP to come up with the optimal policies. Two different implementations:

- Subconsciously, sport players compute probabilities
- They use estimates of distances, angles and probabilities of events occurring
- Aim: provide a framework for optimising decisions in American football
- Use of DP to come up with the optimal policies. Two different implementations:
  - Exact solution is feasible under some assumptions

- Subconsciously, sport players compute probabilities
- They use estimates of distances, angles and probabilities of events occurring
- Aim: provide a framework for optimising decisions in American football
- Use of DP to come up with the optimal policies. Two different implementations:
  - Exact solution is feasible under some assumptions
  - For more general cases, approximations of the expected reward-to-go function are provided (API and OPI)

# Parameters of the dynamic programming algorithm

- State of the system:
  - x<sub>i</sub>: yards to the goal line
  - y<sub>i</sub>: yards to the first down
  - d: number of downs

# Parameters of the dynamic programming algorithm

- State of the system:
  - x<sub>i</sub>: yards to the goal line
  - y<sub>i</sub>: yards to the first down
  - d: number of downs
- Policies or actions that players can take:
  - P: pass
  - R: run
  - U: punt
  - K: kick

# Parameters of the dynamic programming algorithm

- State of the system:
  - $x_i$ : yards to the goal line
  - y<sub>i</sub>: yards to the first down
  - d: number of downs
- Policies or actions that players can take:
  - P: pass
  - R: run
  - U: punt
  - K: kick
- Rewards:
  - Touchdown: 6.8
  - Field goal: 3
  - Safety: -2
  - Opposition score  $= -\frac{6.8x}{100}$

# **Exact Dynamic Programming**

#### **DP** Equation

$$\mu^k(i) = arg \max_{u \in U} \left[ \sum_{i \in S} p_{ij}(u) (g(i, u, j) + J^{\mu^{k-1}}(j)) \right] \ orall i \in S$$

- $p_{ij}(u)$ : transition probabilities
- g(i, u, j): reward function
- $J^{\mu^{k-1}}(j)$ : reward-to-go function

J is computed exactly using the 15250 possible states of the system.

#### Simulations

- We create a reasonable class of policies and implement it.
- Policies are compared by calculating the points from one drive.
- Simulations for an optimal heuristic policy are run from the starting state of  $(x_i, y_i, d) = (80, 10, 1)$ .
- Example of a simulation:

$$\begin{bmatrix} 25 \\ 10 \\ 1 \end{bmatrix} \longrightarrow_0^P \begin{bmatrix} 17 \\ 2 \\ 2 \end{bmatrix} \longrightarrow_0^R \begin{bmatrix} 14 \\ 10 \\ 1 \end{bmatrix} \longrightarrow_0^P \begin{bmatrix} 10 \\ 6 \\ 2 \end{bmatrix} \longrightarrow_0^P \begin{bmatrix} 10 \\ 6 \\ 3 \end{bmatrix} \longrightarrow_0^R \begin{bmatrix} 8 \\ 4 \\ 4 \end{bmatrix} \longrightarrow_3^K T$$

# Policy Update

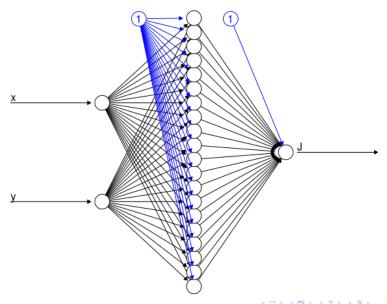
## Approximated DP algorithm

$$\mu^{k}(i) = arg \max_{u \in U} \left[ \sum_{j \in S} p_{ij}(u) (g(i, u, j) + \widetilde{J}^{\mu^{k-1}}(j)) \right]$$

# API and OPI Algorithm

- Algorithm:
  - **1** Start an initial policy  $\mu_0$
  - **2** For each  $k \in \{1, ..., K\}$ :
    - **1** Given  $\mu_{k-1}$ , simulate  $N_s$  sample trajectories
    - 2 Fit the neural network using the sample data to estimate the  $J^{\mu^{k-1}}$
    - **3** Update policy to get  $\mu^k$
- Two different ways to make the approximations
  - API: Many training sample points, few iterations
  - OPI: Few training sample points, many iterations

## Neural Network



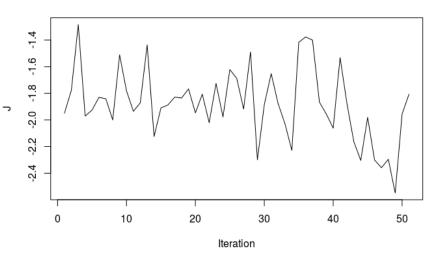
# Policy Update

## Approximated DP algorithm

$$\mu^{k}(i) = arg \max_{u \in U} \left[ \sum_{j \in S} p_{ij}(u) (g(i, u, j) + \widetilde{J}^{\mu^{k-1}}(j)) \right]$$

4回 > 4回 > 4 回

### **Expected Reward from Best Policy**



# Super Bowl XLIX

Seahawks should have run!