Play Selection in American Football

Daniel Bestard, Hans-Peter Hoellwirth, Akhil Lohia, Michael Cameron

Barcelona Graduate School of Economics

April 5, 2017

• Subconsciously, sport players compute probabilities

- Subconsciously, sport players compute probabilities
- They use estimates of distances, angles and probabilities of events occurring

- Subconsciously, sport players compute probabilities
- They use estimates of distances, angles and probabilities of events occurring
- Aim: provide a framework for optimising decisions in American football

- Subconsciously, sport players compute probabilities
- They use estimates of distances, angles and probabilities of events occurring
- Aim: provide a framework for optimising decisions in American football
- Use of DP to come up with the optimal policies. Two different implementations:

- Subconsciously, sport players compute probabilities
- They use estimates of distances, angles and probabilities of events occurring
- Aim: provide a framework for optimising decisions in American football
- Use of DP to come up with the optimal policies. Two different implementations:
 - Exact solution is feasible under some assumptions

- Subconsciously, sport players compute probabilities
- They use estimates of distances, angles and probabilities of events occurring
- Aim: provide a framework for optimising decisions in American football
- Use of DP to come up with the optimal policies. Two different implementations:
 - Exact solution is feasible under some assumptions
 - For more general cases, approximations of the expected reward-to-go function are provided (API and OPI)

Parameters of the dynamic programming algorithm

- State of the system:
 - x_i: yards to the goal line
 - y_i: yards to the first down
 - d: number of downs

Parameters of the dynamic programming algorithm

- State of the system:
 - x_i : yards to the goal line
 - y_i: yards to the first down
 - d: number of downs
- Policies or actions that players can take:
 - P: pass
 - R: run
 - U: punt
 - K: kick

3 / 9

(BGSE) DP in Football April 5, 2017

Parameters of the dynamic programming algorithm

- State of the system:
 - x_i: yards to the goal line
 - y_i: yards to the first down
 - d: number of downs
- Policies or actions that players can take:
 - P: pass
 - R: run
 - U: punt
 - K: kick
- Rewards:
 - Touchdown: 6.8
 - Field goal: 3
 - Safety: -2
 - Opposition score $= -\frac{6.8x}{100}$

April 5, 2017

3 / 9

Exact Dynamic Programming

DP Equation

$$\mu^k(i) = arg \max_{u \in U} \left[\sum_{i \in S} p_{ij}(u) (g(i, u, j) + J^{\mu^{k-1}}(j)) \right] \ orall i \in S$$

- $p_{ij}(u)$: transition probabilities
- g(i, u, j): reward function
- $J^{\mu^{k-1}}(j)$: reward-to-go function

J is computed exactly using the 15250 possible states of the system.

4□ > 4□ > 4 = > 4 = > = 900

Simulations

- We create a reasonable class of policies and implement it.
- Policies are compared by calculating the points from one drive.
- Simulations for an optimal heuristic policy are run from the starting state of $(x_i, y_i, d) = (80, 10, 1)$.
- Example of a simulation:

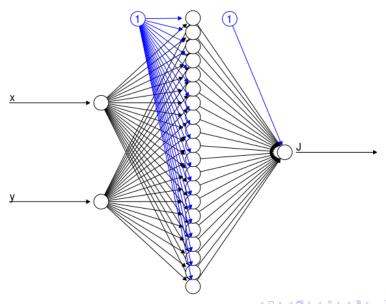
$$\begin{bmatrix} 25 \\ 10 \\ 1 \end{bmatrix} \longrightarrow_0^P \begin{bmatrix} 17 \\ 2 \\ 2 \end{bmatrix} \longrightarrow_0^R \begin{bmatrix} 14 \\ 10 \\ 1 \end{bmatrix} \longrightarrow_0^P \begin{bmatrix} 10 \\ 6 \\ 2 \end{bmatrix} \longrightarrow_0^P \begin{bmatrix} 10 \\ 6 \\ 3 \end{bmatrix} \longrightarrow_0^R \begin{bmatrix} 8 \\ 4 \\ 4 \end{bmatrix} \longrightarrow_3^K T$$

(BGSE) DP in Football April 5, 2017

API and OPI Algorithm

- Two different ways to make the approximations
 - API: Many training sample points, few iterations
 - OPI: Few training sample points, many iterations
- Algorithm:
- **1** Start an initial policy μ_0
- For each k:
 - Given μ_k , simulate N_s sample trajectories
 - $oldsymbol{arrho}$ Fit the neural network using the sample data to estimate the J
 - Update policy

Neural Network



Policy Update

Approximated DP algorithm

$$\mu^{k}(i) = arg \max_{u \in U} \left[\sum_{j \in S} p_{ij}(u) (g(i, u, j) + \widetilde{J}^{\mu^{k-1}}(j)) \right]$$



Super Bowl XLIX

Seahawks should have run!

(BGSE) DP in Football