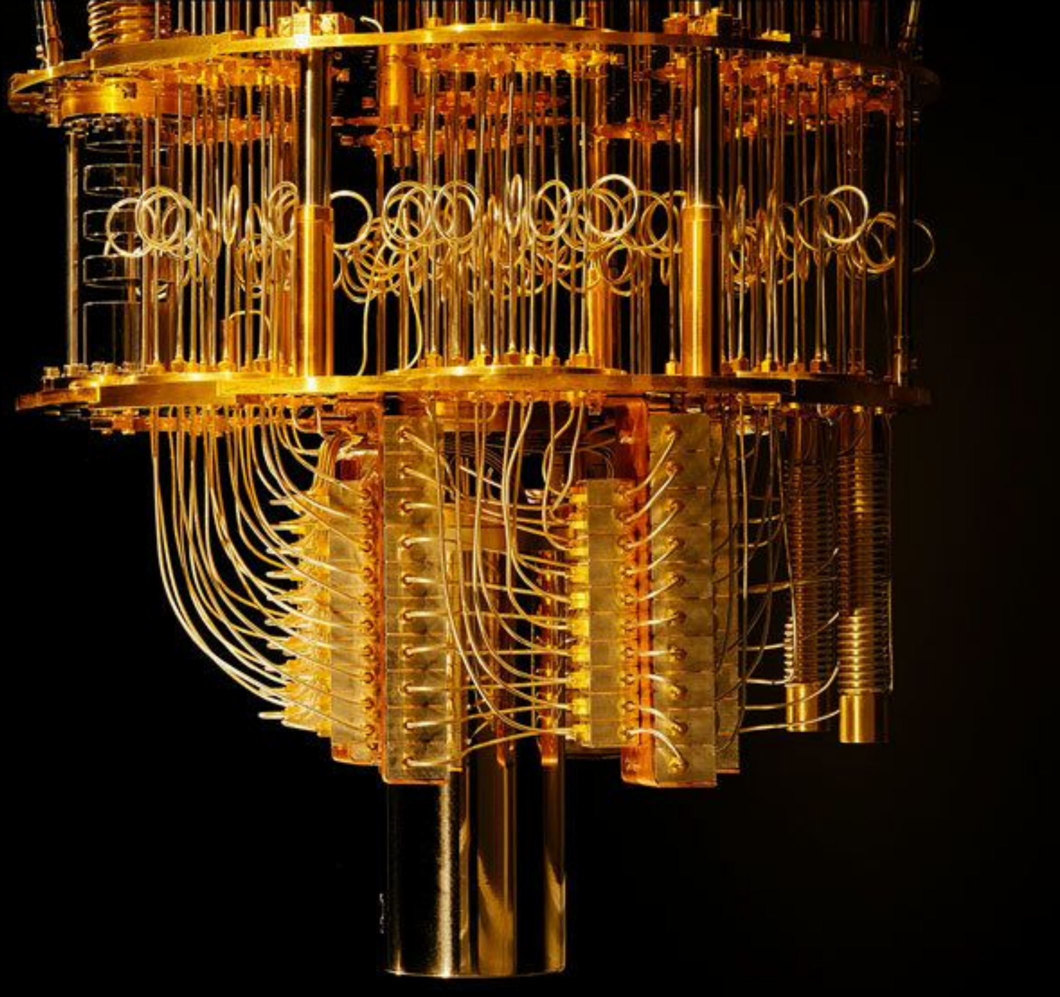
Quantum Simulation Training



Maxime Gaignard - Barkay Guttel - Vincent Jouanny - Benoît Neukelmance UGA - M2 Nano

Table of contents

I.

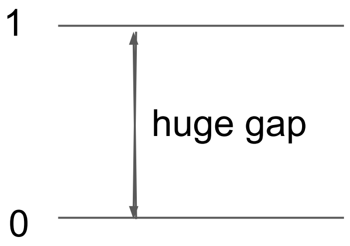
Classical computer

classical bits : 0 or 1

semiconductor physics :
-band theory

classical algorithm

classical error correction :
-bit flip : 0 → 1



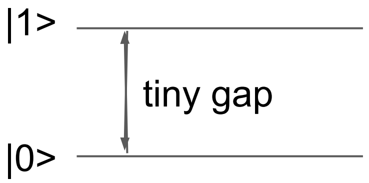
Quantum computer

quantum bits : $\alpha|0\rangle+\beta|1\rangle$

quantum physics :
-superposition of states
-entanglement

quantum algorithm

quantum error correction :
-bit flip : $\alpha|0\rangle+\beta|1\rangle \rightarrow \alpha|1\rangle+\beta|0\rangle$
- phase flip : $\alpha|0\rangle+\beta|1\rangle \rightarrow \alpha|0\rangle-\beta|1\rangle$



I. **quantum bit** = 2 level system : $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$, with $|\alpha|^2+|\beta|^2=1$

measurement in a basis : computational basis $\{|0\rangle ; |1\rangle\}$
Hadamard basis $\{|+\rangle ; |-\rangle\}$

$|+\rangle = (|0\rangle+|1\rangle)/\sqrt{2}$
$|-\rangle = (|0\rangle-|1\rangle)/\sqrt{2}$

Example : measurement of $|\psi\rangle$ in the computational basis

Result	0	1
Probability	$|\alpha|^2$	$|\beta|^2$

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I. Quantum logic gate :

1 qubit

- Pauli gates :

Pauli-X 

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ *bit flip*

Pauli-Y 

$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

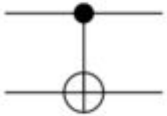
Pauli-Z 

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ *phase flip*

- Hadamard gate : 

$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ *change of basis*

2 qubits

- CNOT gate : 
$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

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I. Density matrix : $\rho = |\psi\rangle\langle\psi|$

1 qubit case : $\rho \longrightarrow$ 2x2 matrix

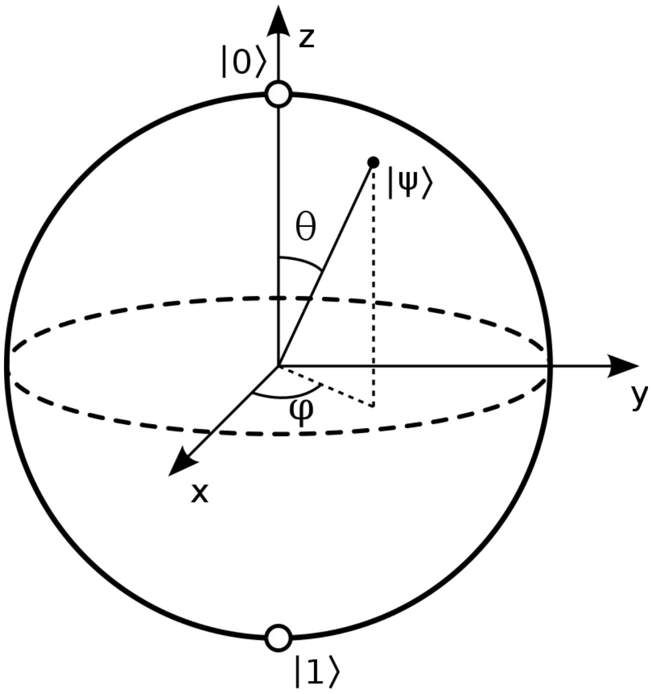
$$\rho = (I_2 + \alpha X + \beta Y + \gamma Z)/2$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \longrightarrow \text{Bloch vector}$$

<u>*Example :*</u> $\rho_{|0\rangle} = |0\rangle\langle 0| = (I_2 + Z)/2$

$$\text{Bloch vector} \longrightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Bloch sphere :



I. **Stabilizer :** Gate S such that $S|\psi\rangle = |\psi\rangle$

- For 1 qubit

	|0>	**|1>**	**|+>**	**|->**
S	Z	-Z	X	-X

- For 2 qubits

<table>
<tr><th></th><th>|00></th><th>|11></th><th>|++></th><th>|--></th></tr>
<tr><td>S</td><td colspan="2">Z_1Z_2</td><td colspan="2">X_1X_2</td></tr>
</table>

With : $X_1 = X \otimes I_2$, $X_2 = I_2 \otimes X$, $Z_1 = Z \otimes I_2$, $Z_2 = I_2 \otimes Z$

I. How to avoid errors ?

Classical computer	Quantum computer
Adding redundancy $0 \rightarrow 000$ $1 \rightarrow 111$	No cloning theorem (1982)
Error correction code $010 \rightarrow 0$ $011 \rightarrow 1$	Quantum error correction

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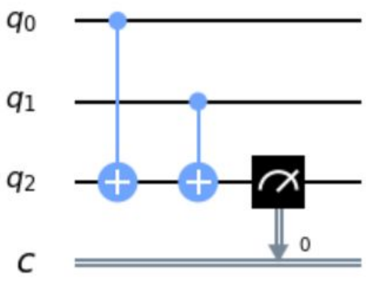
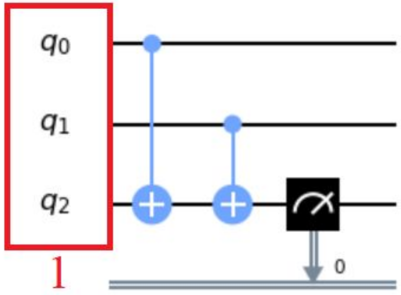
II. ZZ stabilizer: 2 qubits

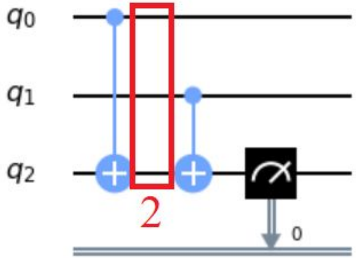




$$|\Psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$|\Psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

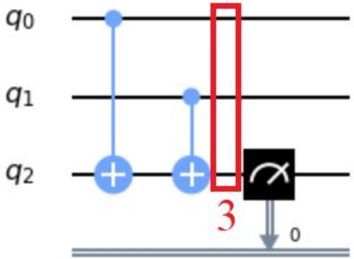
$$|\Psi_c\rangle = |0\rangle$$



$$|\Psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$|\Psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$$|\Psi_c\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

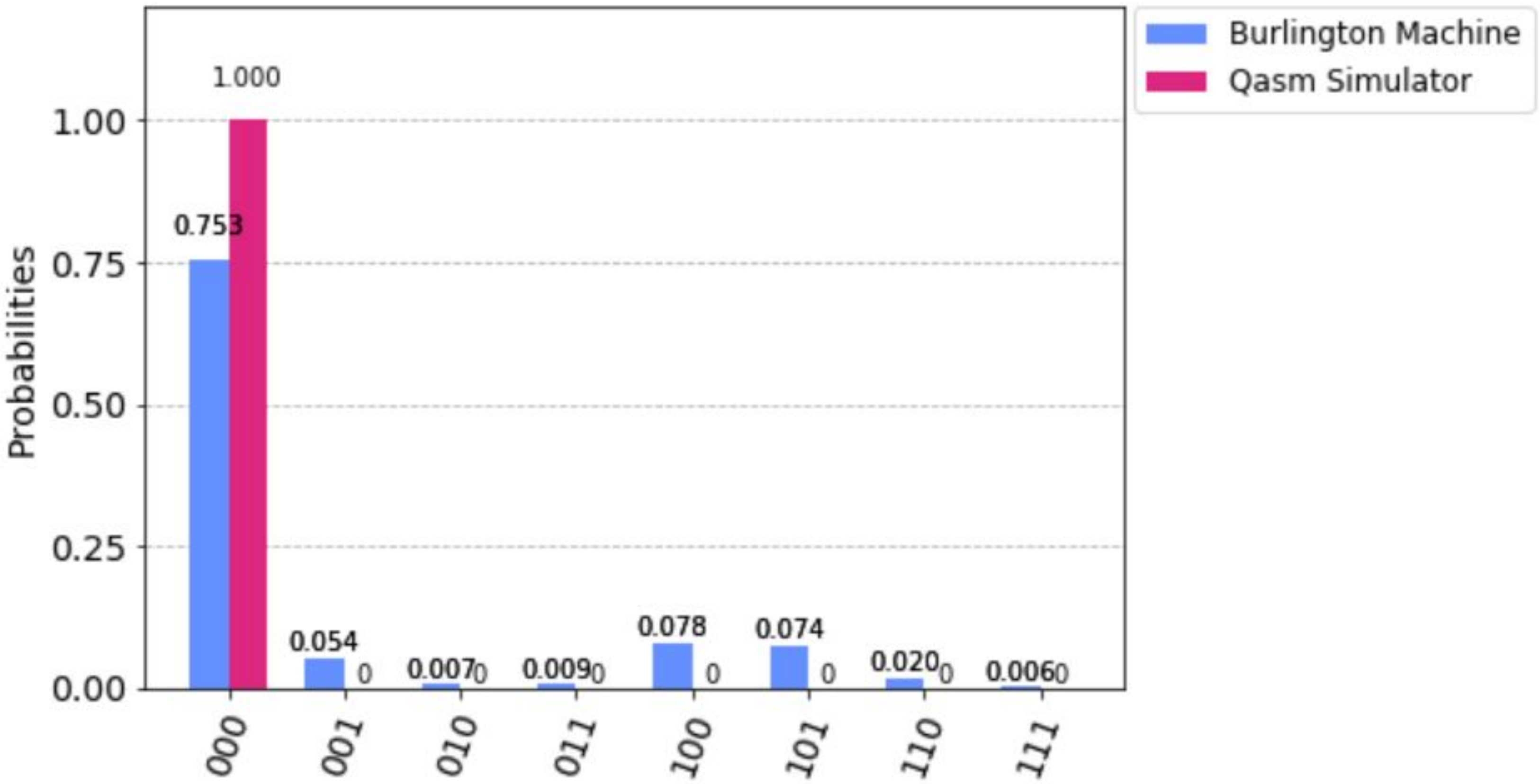


$$|\Psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$|\Psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$$|\Psi_c\rangle = (\alpha_1\alpha_2 + \beta_1\beta_2)|0\rangle + (\alpha_1\beta_2 + \beta_1\alpha_2)|1\rangle$$

II. Ideal case simulation and real device extended results



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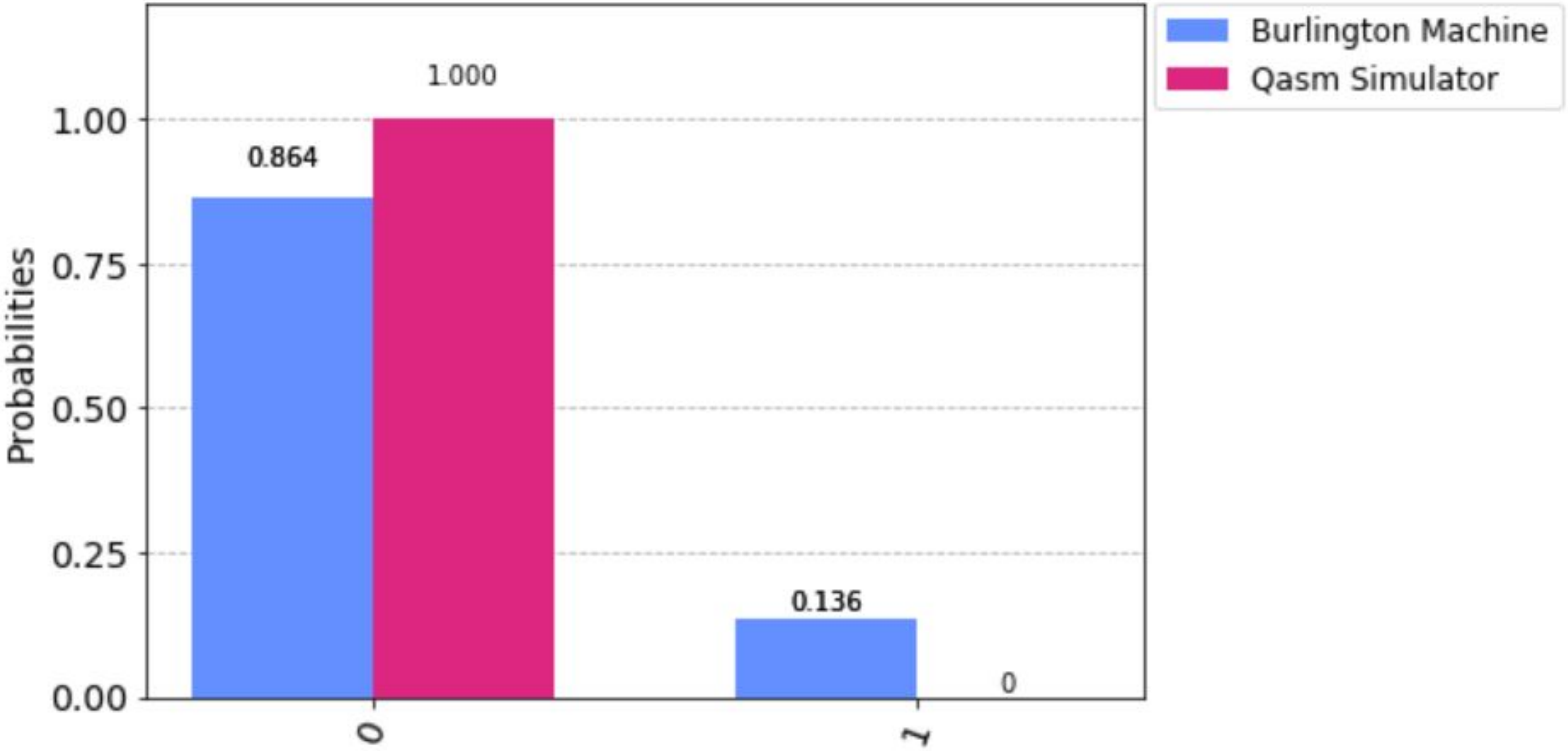
II. Ideal case simulation and real device results



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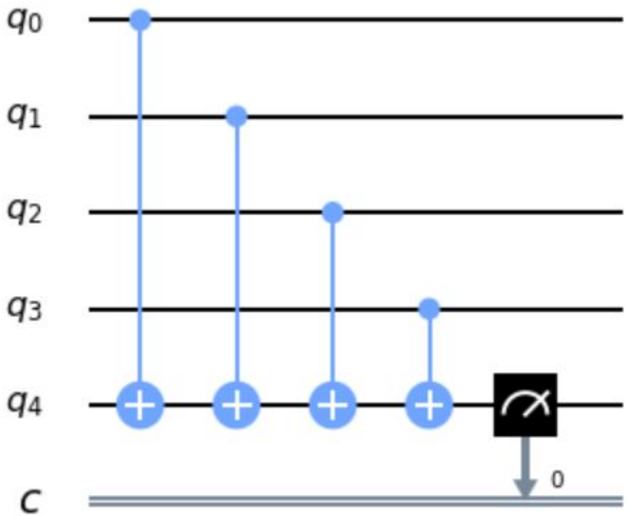
II. ZZ stabilizer: 4 qubits



After all CNOT gates:

$$|\Psi_c\rangle = \Big((\alpha_1\alpha_2\alpha_3\alpha_4) + (\beta_1\beta_2\beta_3\beta_4) + \sum(\alpha_i\alpha_j\beta_k\beta_l)\Big)|0\rangle + \Big(\sum(\alpha_i\alpha_j\alpha_k\beta_l) + \sum(\alpha_i\beta_j\beta_k\beta_l)\Big)|1\rangle$$

i,j,k,l $\in \{1, 2, 3, 4\}$

a$\neq b; a, b \in \{i, j, k, l\}$

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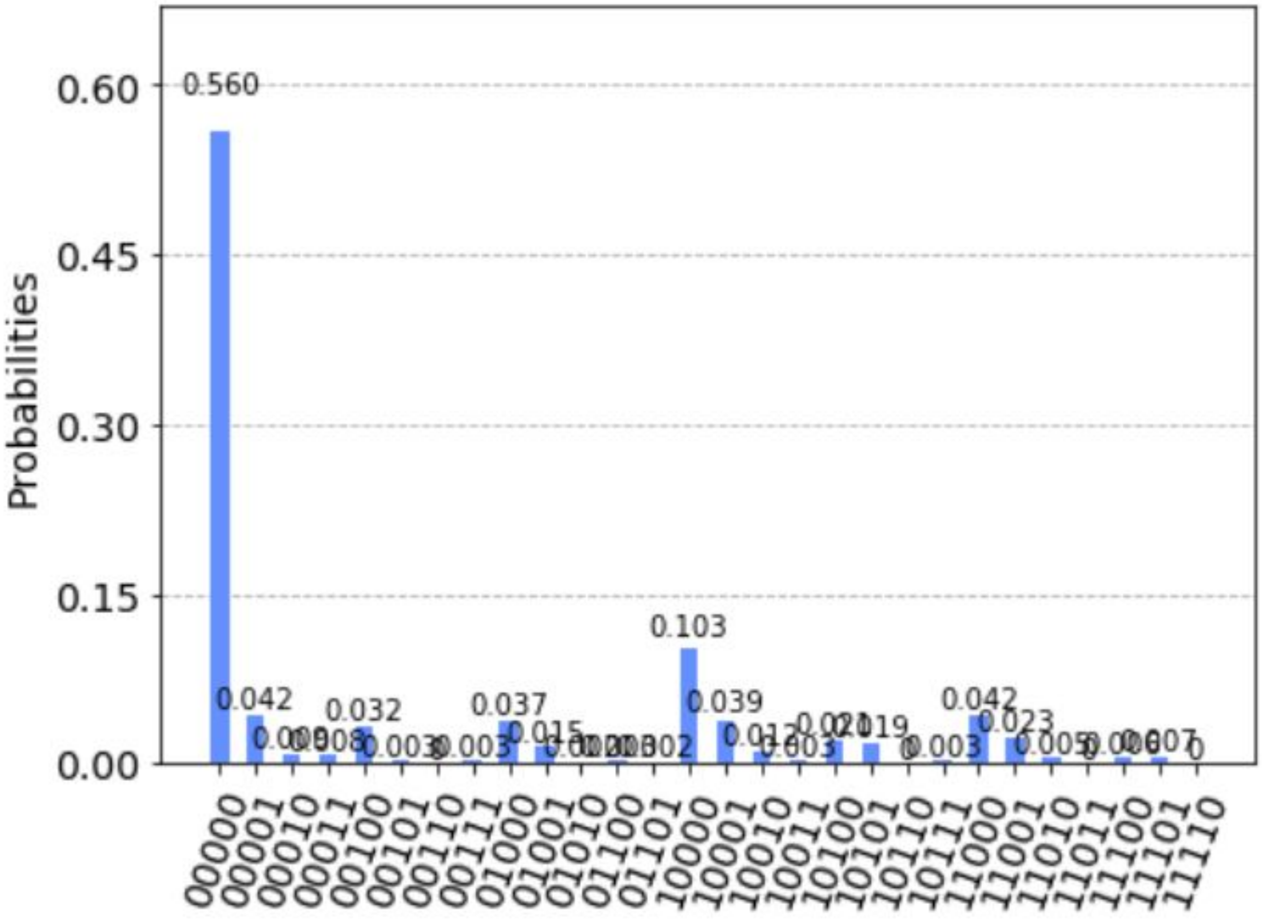
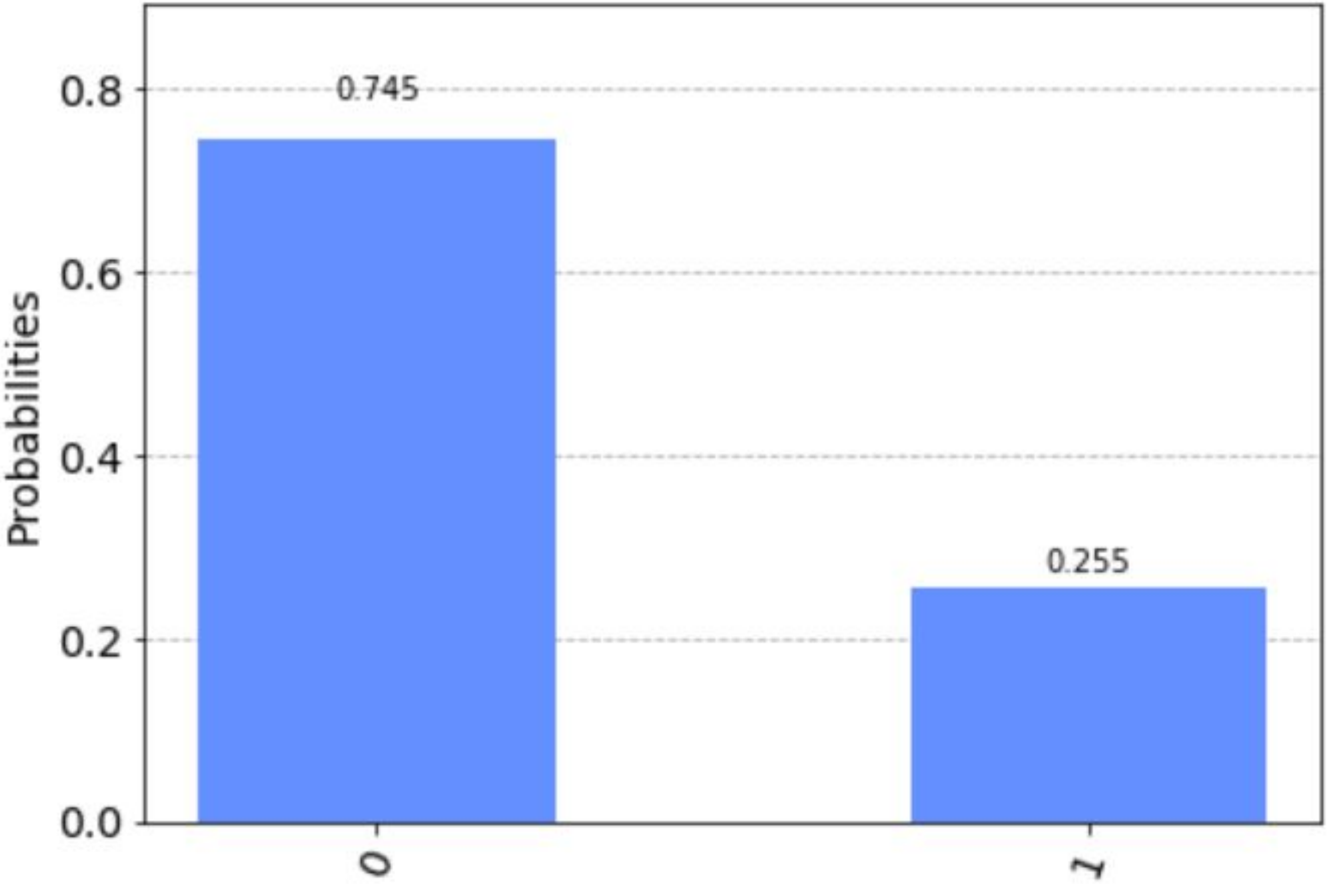
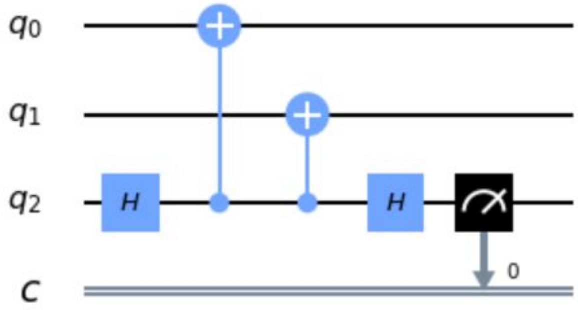
II. Real device extended results



II. Real device results



II. XX stabilizer: 2 qubits

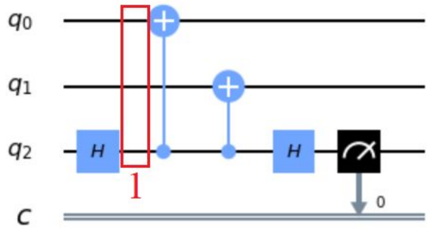


$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

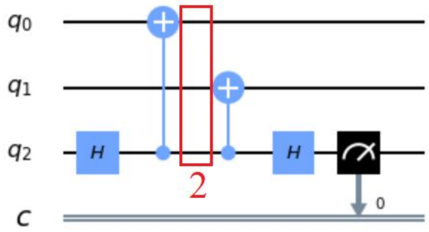
$$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$$



$$|\Psi_1\rangle = \alpha_1|+\rangle + \beta_1|-\rangle$$

$$|\Psi_2\rangle = \alpha_2|+\rangle + \beta_2|-\rangle$$

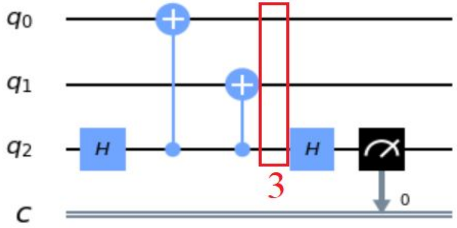
$$|\Psi_c\rangle = |+\rangle$$



$$|\Psi_1\rangle = \alpha_1|+\rangle + \beta_1|-\rangle$$

$$|\Psi_2\rangle = \alpha_2|+\rangle + \beta_2|-\rangle$$

$$|\Psi_c\rangle = \alpha_1|+\rangle + \beta_1|-\rangle$$

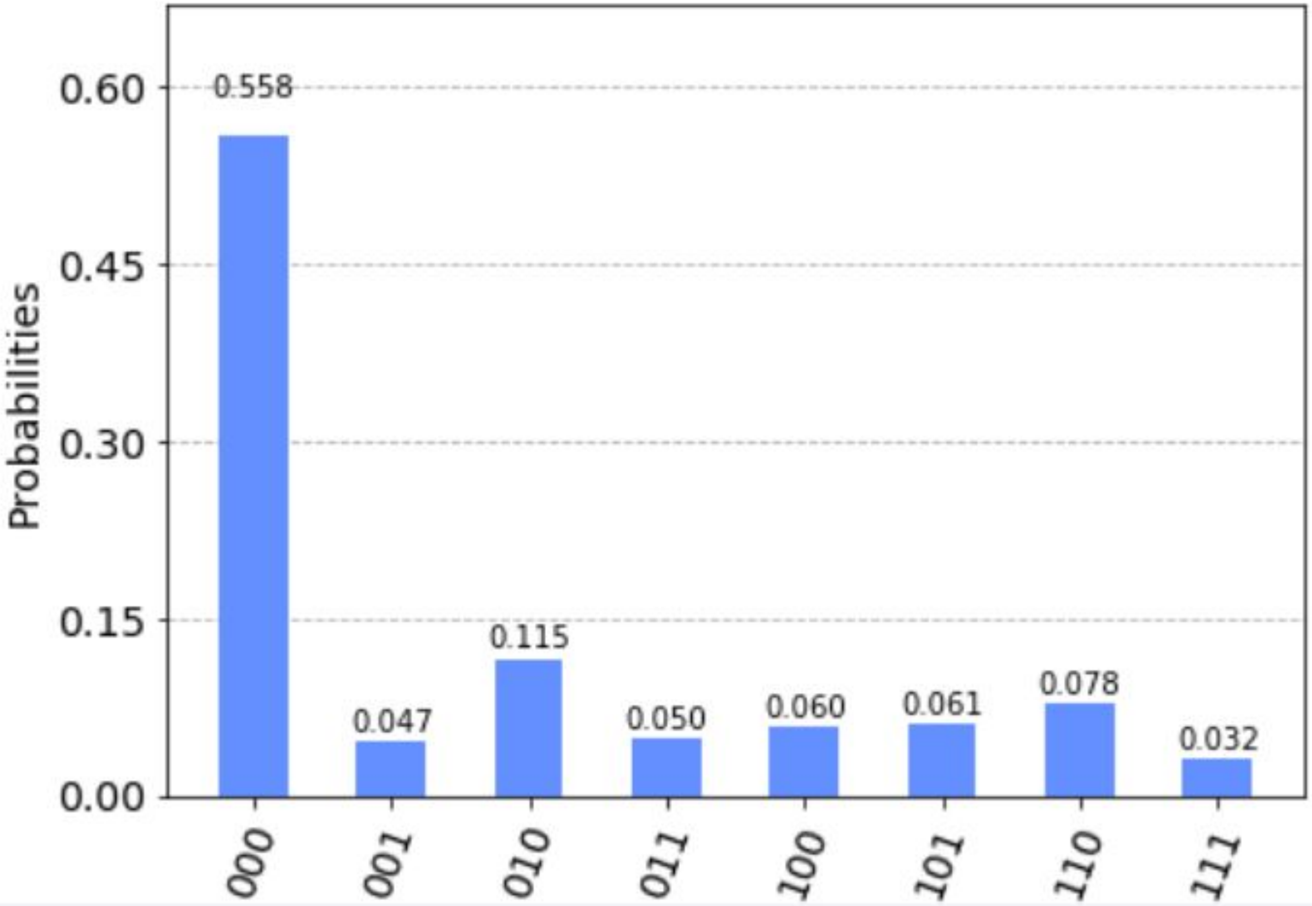


$$|\Psi_1\rangle = \alpha_1|+\rangle + \beta_1|-\rangle$$

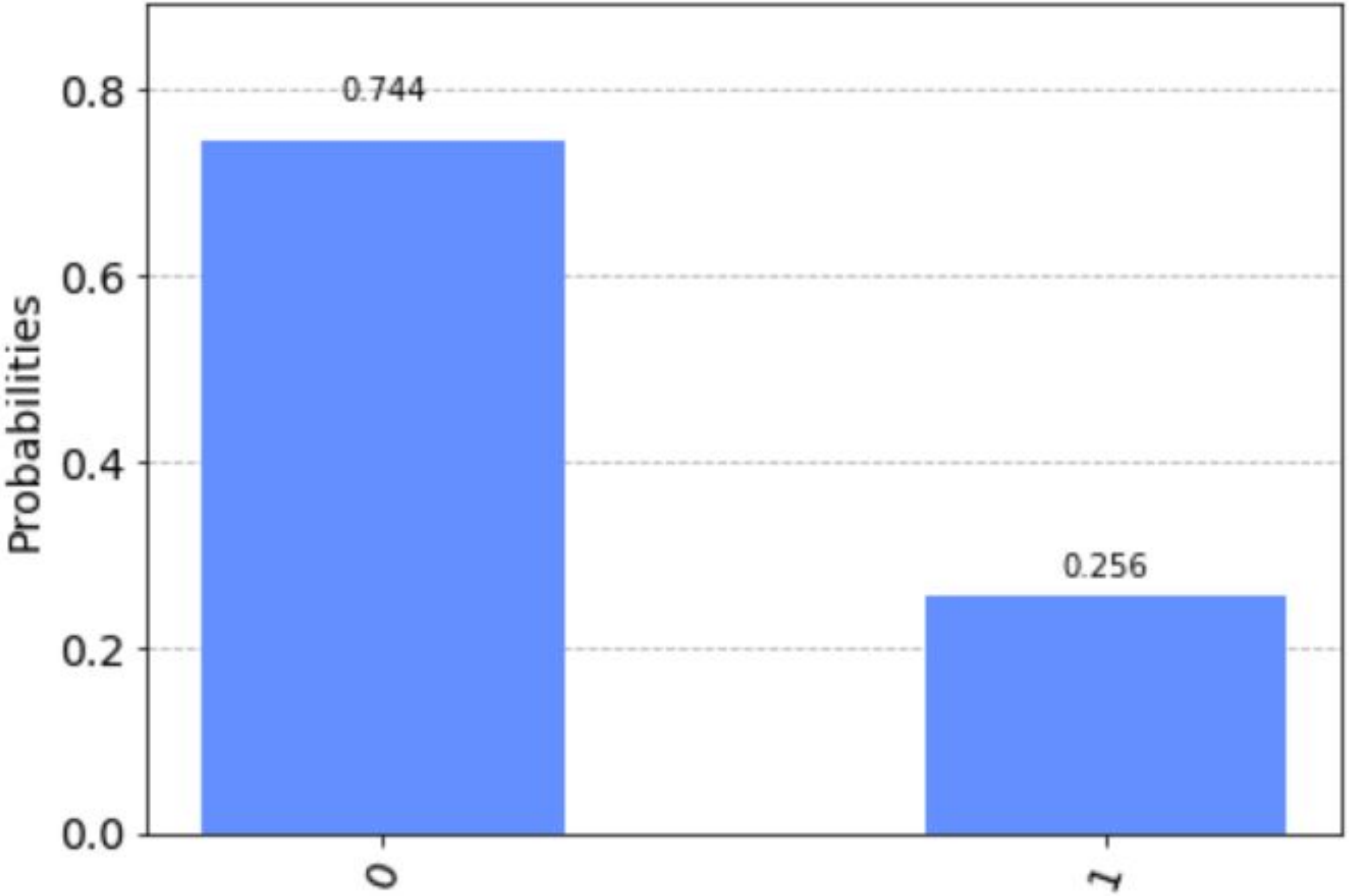
$$|\Psi_2\rangle = \alpha_2|+\rangle + \beta_2|-\rangle$$

$$|\Psi_c\rangle = (\alpha_1\alpha_2 + \beta_1\beta_2)|+\rangle + (\beta_1\alpha_2 + \alpha_1\beta_2)|-\rangle$$

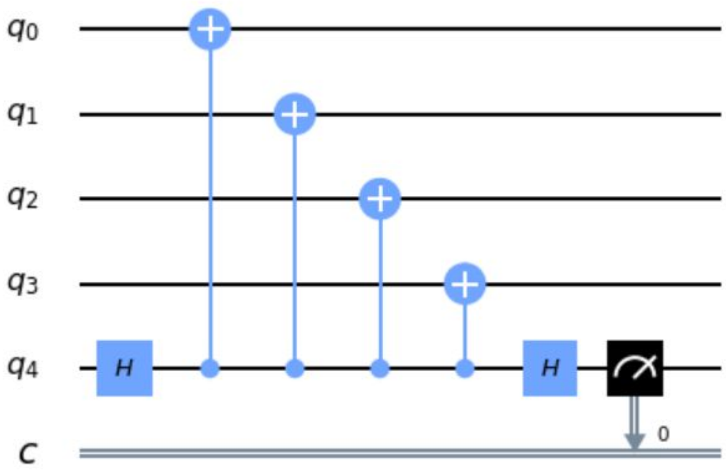
II. Real device extended results



II. Real device results



II. XX stabilizer: 4 qubits



After all CNOT gates:

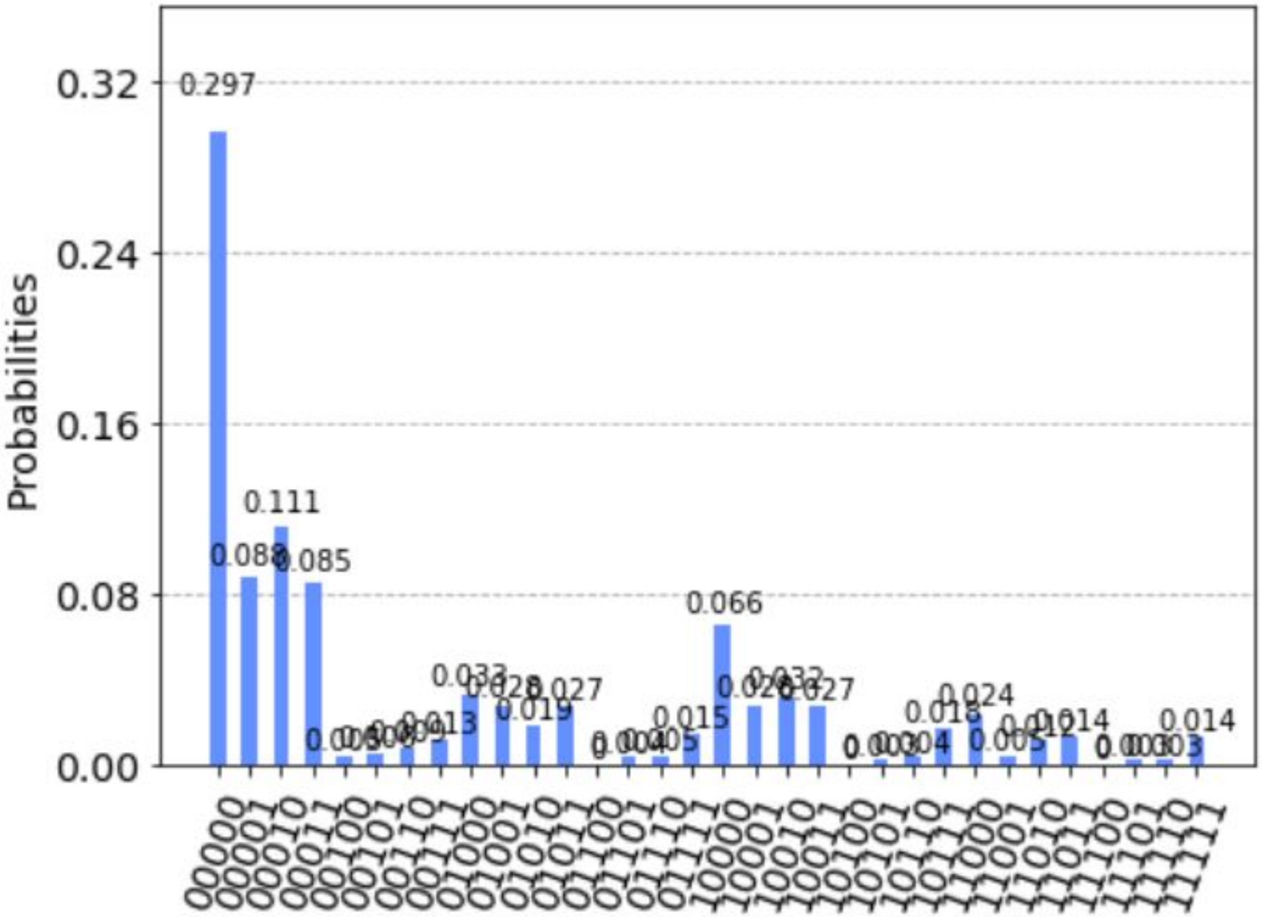
$$|\Psi_c\rangle = \left(\alpha_1\alpha_2\alpha_3\alpha_4 + \beta_1\beta_2\beta_3\beta_4 + \sum\left(\alpha_i\alpha_j\beta_k\beta_l\right)\right)|+\rangle$$

$$+ \left(\sum\left(\alpha_i\alpha_j\alpha_k\beta_l\right) + \sum\left(\alpha_i\beta_j\beta_k\beta_l\right)\right)|-\rangle$$

$$\text{i,j,k,l} \in \{1, 2, 3, 4\}$$

$$\text{a} \neq b; a, b \in \{i, j, k, l\}$$

II. Real device extended results



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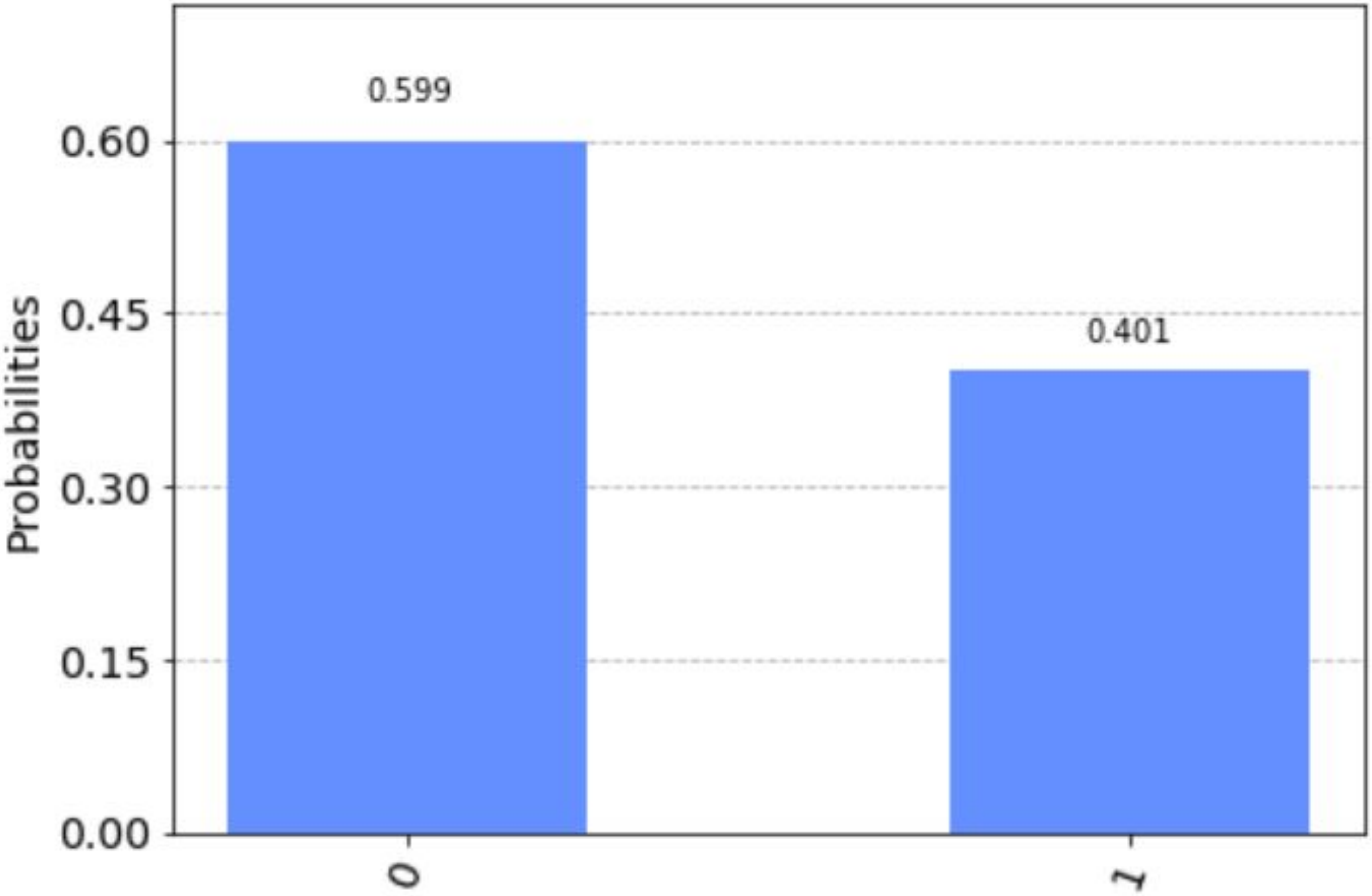
II. Real device results



III. Repetition code

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Principle of the repetition code

Classical error correction

Repetition of the same bit to form a new logical bit.

$$0 \xrightarrow{Logical} 000 \xrightarrow{Error} 100 \xrightarrow{Correction} 0$$
$$1 \xrightarrow{Logical} 111 \xrightarrow{Error} 101 \xrightarrow{Correction} 1$$

Going Quantum

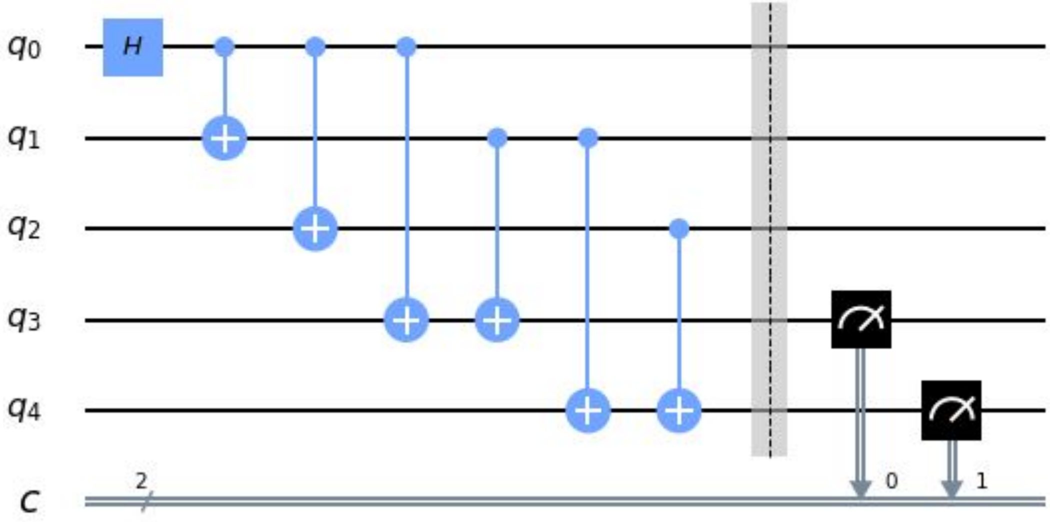
No Cloning Theorem

There is no unitary operator, $\boldsymbol{U}$, such that:

$$\boldsymbol{U}(|\phi\rangle\,|e\rangle) \rightarrow |\phi\rangle\,|\phi\rangle$$

Idea: Use Bell states and Stabilizers.

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|0\rangle \xrightarrow{Encoding} |\psi\rangle_L = \alpha\,|000\rangle + \beta\,|111\rangle$$



Principle of the repetition code

1. Bell state encoding:

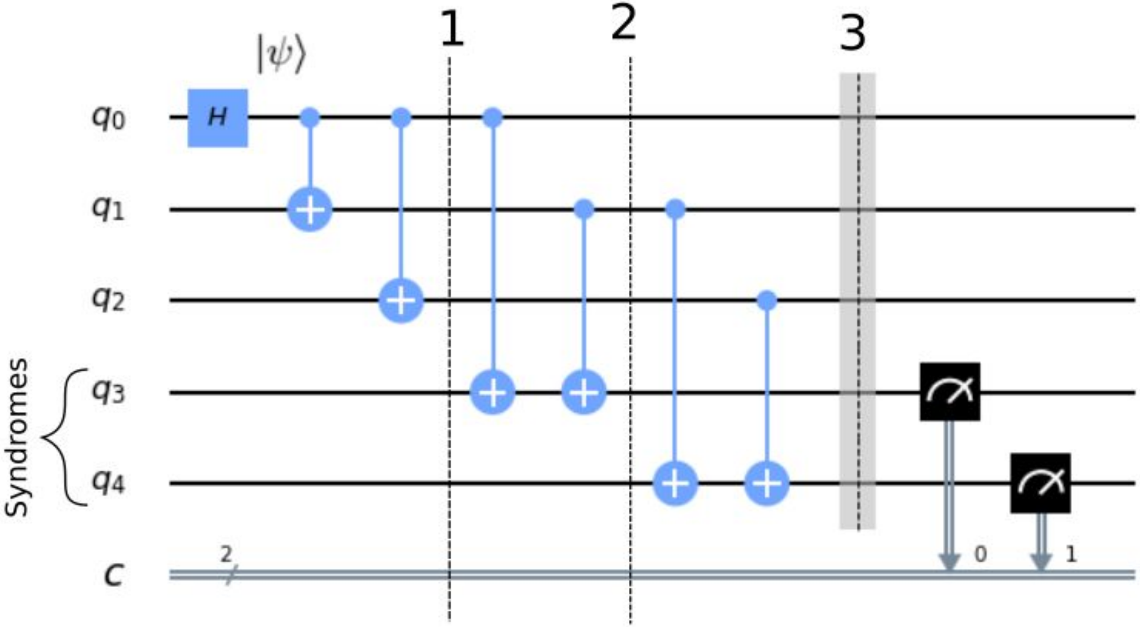
$$|\psi_1\rangle = \alpha\,|000\rangle + \beta\,|111\rangle$$

2. First stabilizer measurement

$$|\psi_{S_1}\rangle = Z_1 Z_2\,|\psi_1\rangle$$

3. Second stabilizer measurements

$$|\psi_{S_2}\rangle = Z_2 Z_3\,|\psi_1\rangle$$



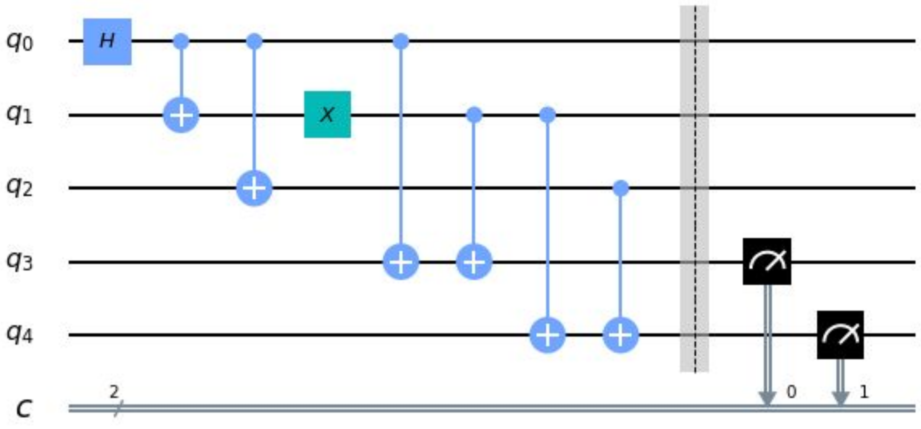
Introducing error on single qubits:

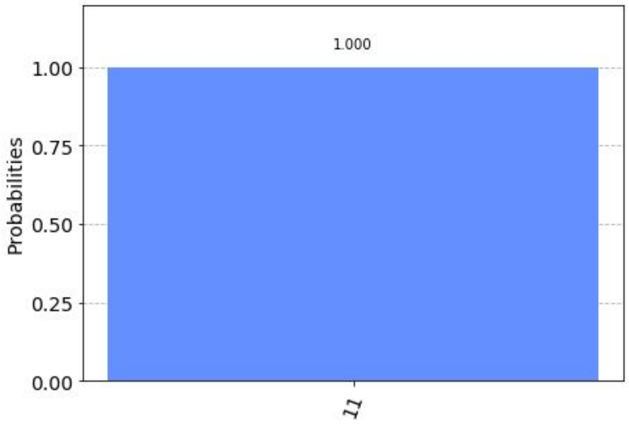
	Z_1Z_2	Z_2Z_3
$\mathcal{E}(q_0)$	1	0
$\mathcal{E}(q_1)$	1	1
$\mathcal{E}(q_2)$	0	1

We know where the error come from !

Running the program on the IBMQ simulator

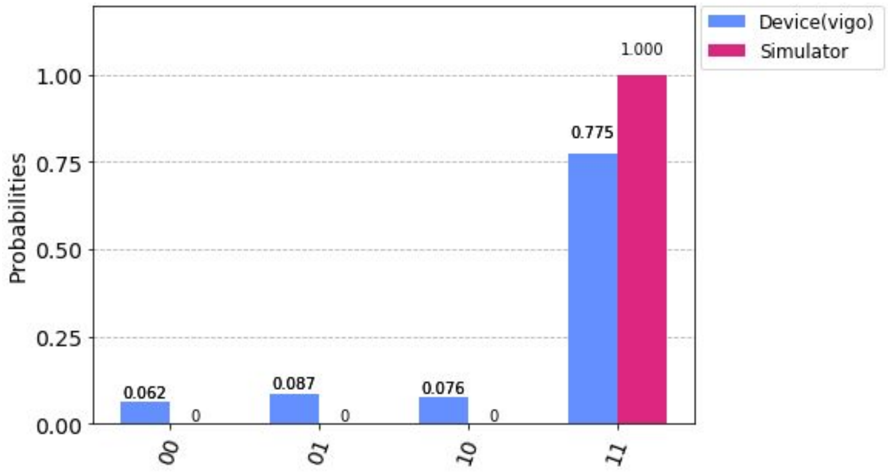
Aer simulator

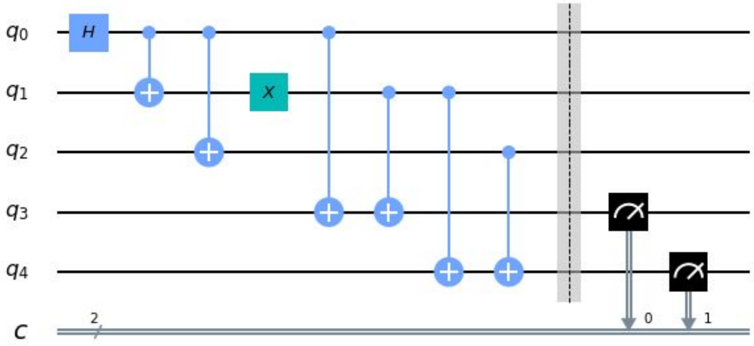




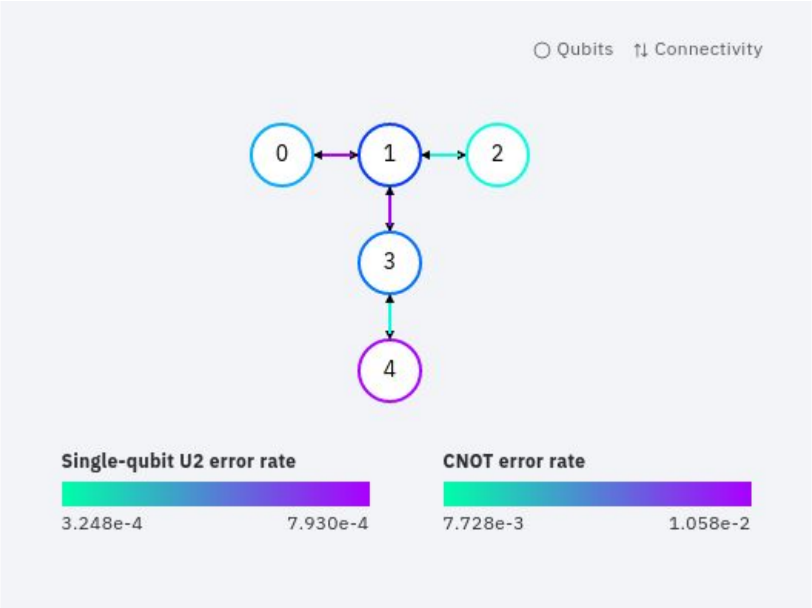
It’s what we expect from an error occuring on qubit 1.

Running the code on a real machine



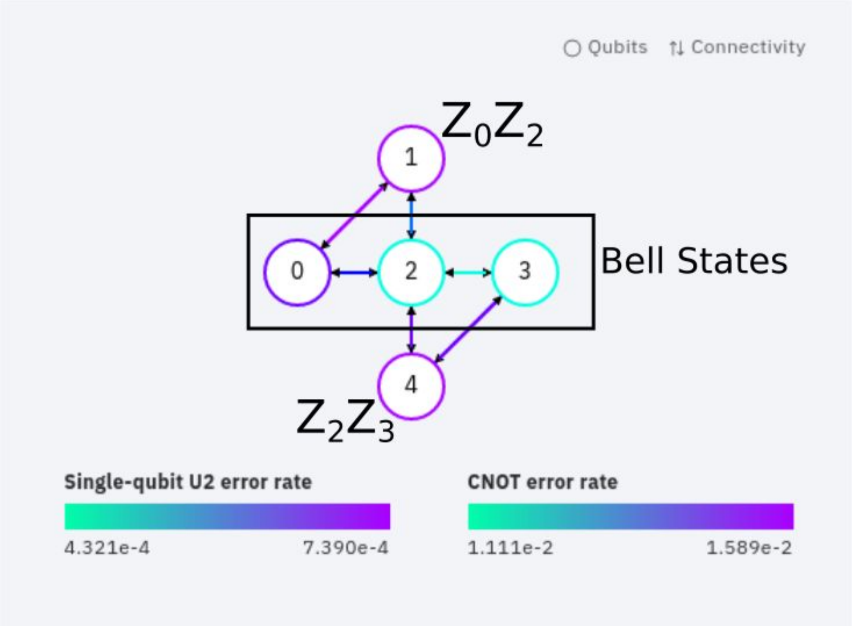


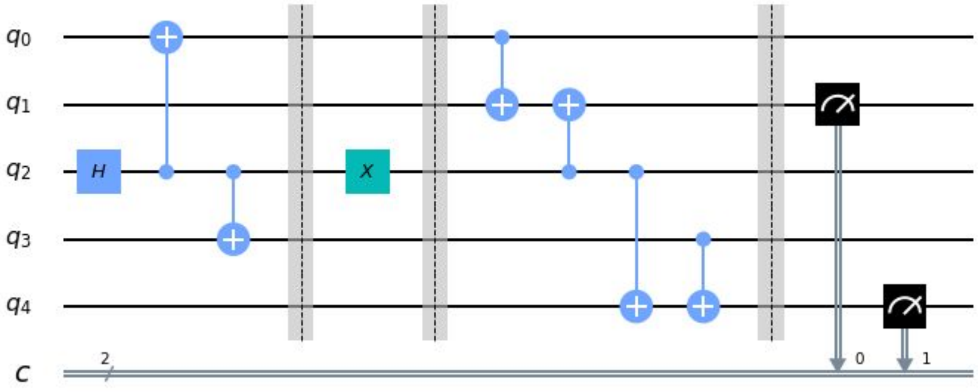
Why is it that bad ?

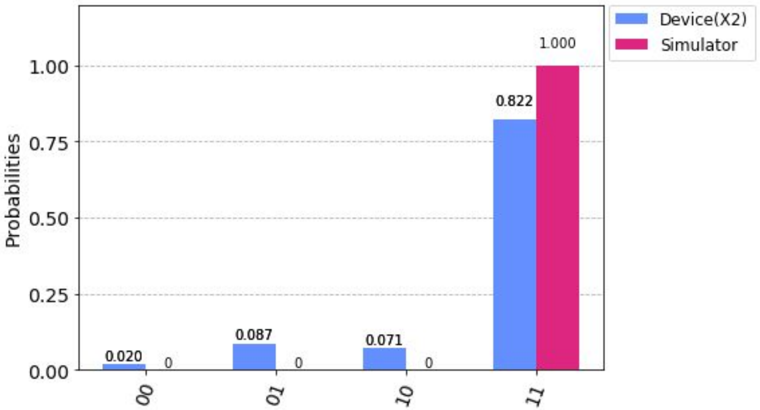


What can we improve ?

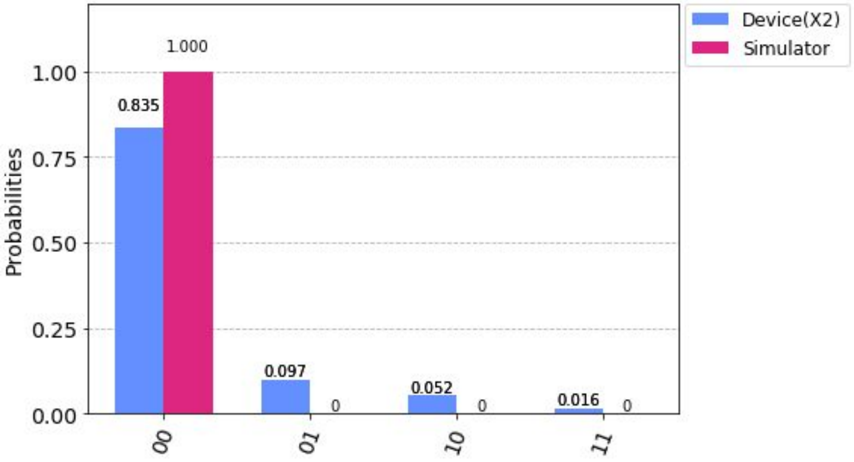
We can use a different architecture, to improve the connectivity.

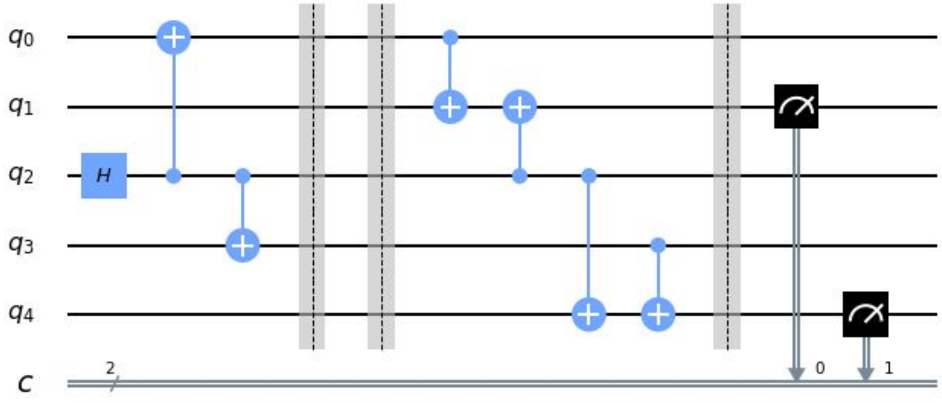






Running the circuits without artificial error

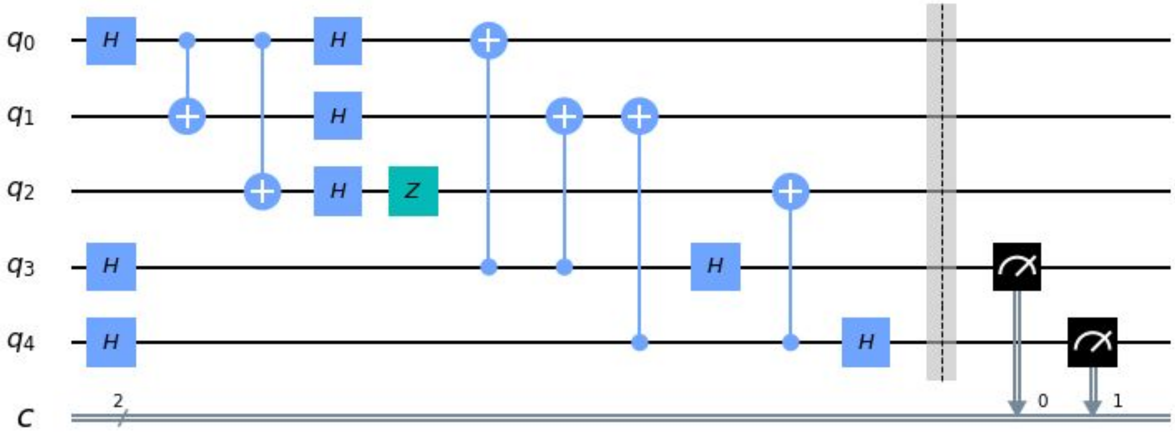


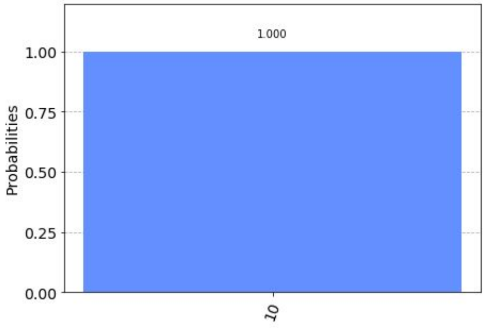


Phase Correction

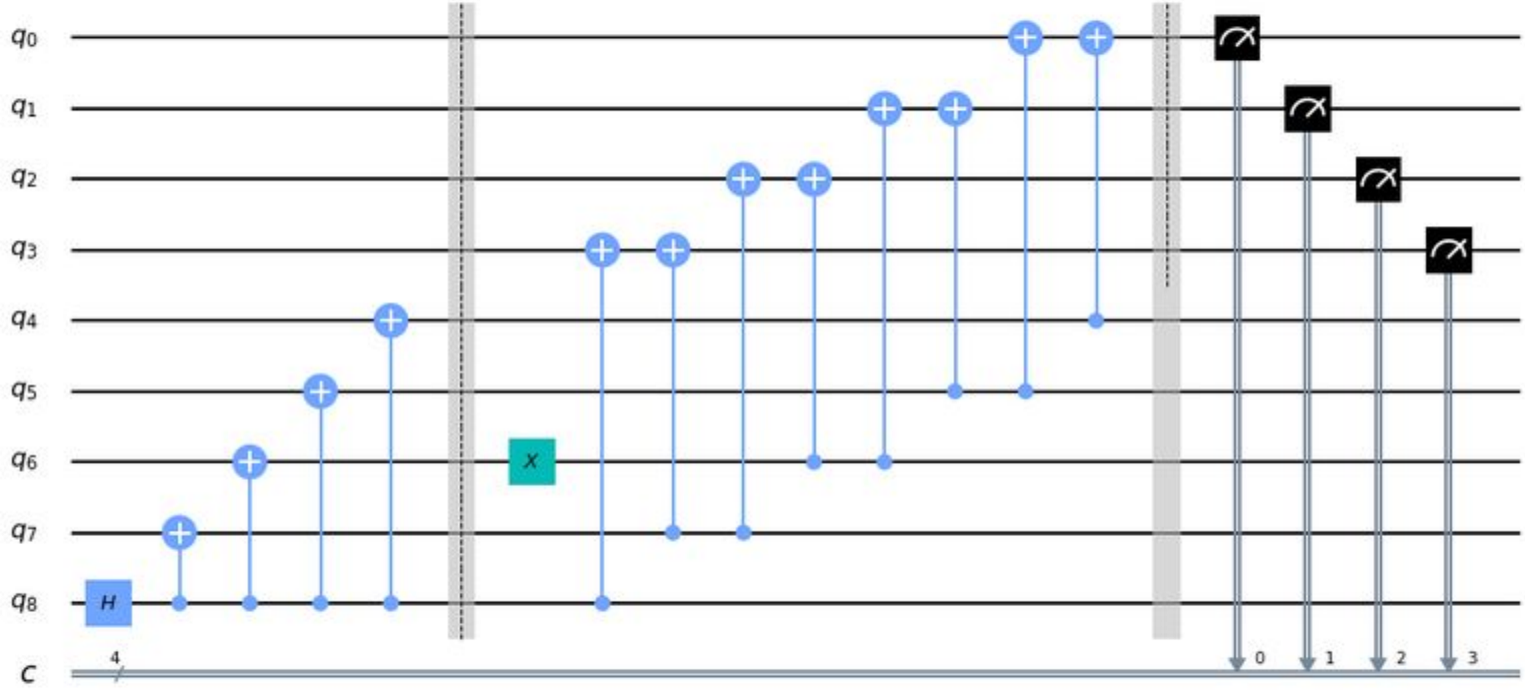
1. Encoding (GHZ state)
2. We go to Hadamard space.
3. Artificially induced phase error.
4. X_0X_1 and X_1X_2 stabilizer measurements.
5. We go back to the computational basis

➔ $X_0X_1 = 0$ $X_1X_2 = 1$ there is a phase error on qubit 2.



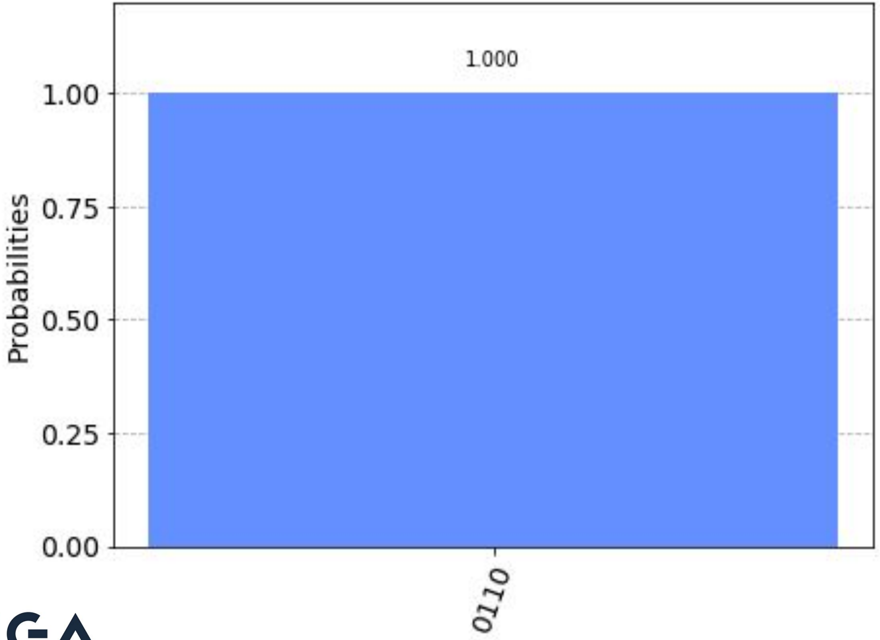


5 qubits code



Analysis of the 5 qubits code

Let's now start from the result



$Z_4 Z_5$	$Z_5 Z_6$	$Z_6 Z_7$	$Z_7 Z_8$
0	1	1	0

The stabilizers Z_5Z_6 and Z_6Z_7 both detect an error.

➔ There is an error on qubit 6.

Advantage of the 5 qubits code

We can now detect single and double errors.

	Z_4Z_5	Z_5Z_6	Z_6Z_7	Z_7Z_8
$\mathcal{E}(q_4)$	1	0	0	0
$\mathcal{E}(q_5)$	1	1	0	0
$\mathcal{E}(q_6)$	0	1	1	0
$\mathcal{E}(q_7)$	0	0	1	1
$\mathcal{E}(q_8)$	0	0	0	1
$\mathcal{E}(q_4, q_5)$	0	1	0	0
$\mathcal{E}(q_4, q_6)$	1	1	1	0
$\mathcal{E}(q_4, q_7)$	1	0	1	1
$\mathcal{E}(q_4, q_8)$	1	0	0	1
$\mathcal{E}(q_5, q_6)$	1	0	1	0
$\mathcal{E}(q_5, q_7)$	1	1	1	1
$\mathcal{E}(q_5, q_8)$	1	1	0	1
$\mathcal{E}(q_6, q_7)$	0	1	0	1
$\mathcal{E}(q_6, q_8)$	1	0	0	1
$\mathcal{E}(q_7, q_8)$	0	0	1	0

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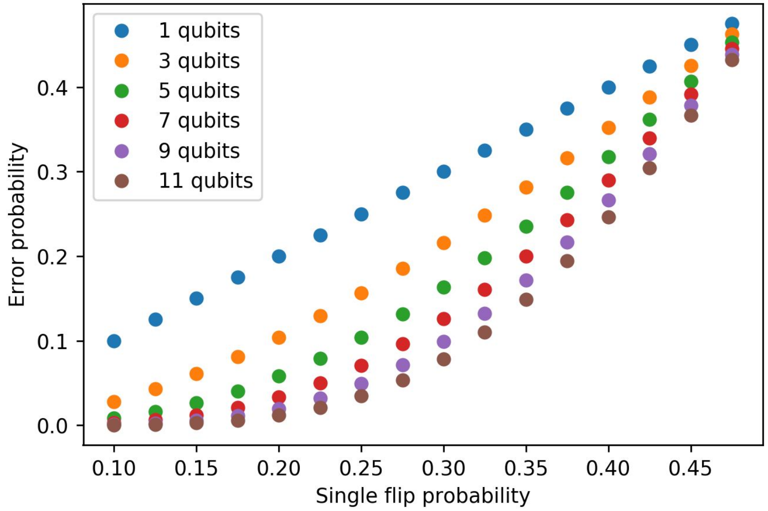
Simple error model

An error happens when more than half of the qubits are flipped.

$$p_{error} = \sum_{n=(N+1)/2}^{N} \binom{N}{n} p_{flip}^n (1 - p_{flip})^{N-n}$$

p_{flip} is the probability that one spin flips



The more qubits there are, the less error probability there is.

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Conclusion

+. We can detect errors and correct them using Bell states, stabilizer measurements and syndromes.

+. The more qubits there are, the less the system is prone to an error.

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IV. Minimal Surface Code

Surface Codes

- Easy to scale up.
- One logical qubit is encoded using many physical qubits.
- The more physical qubits (and stabilizing measurements) used, the more robust the logical qubit is.

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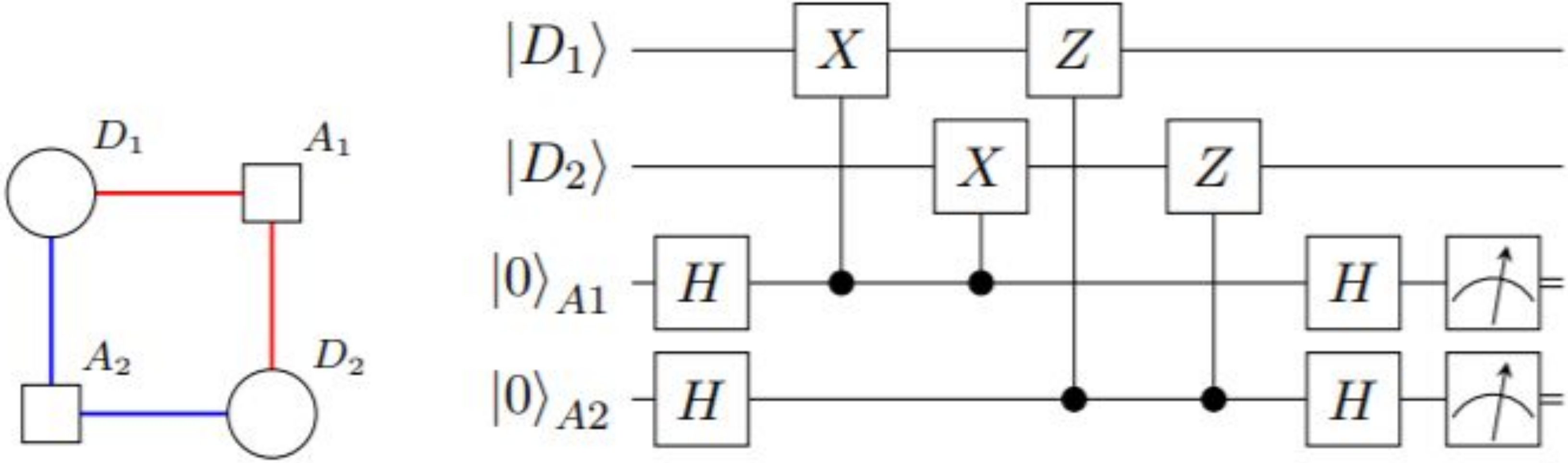
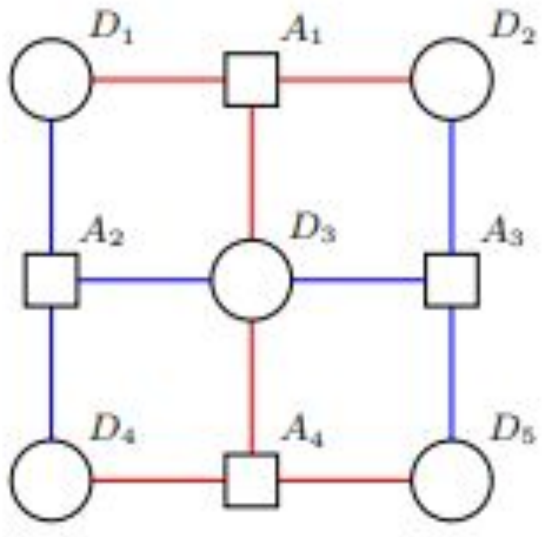
Fundamental Plaquette



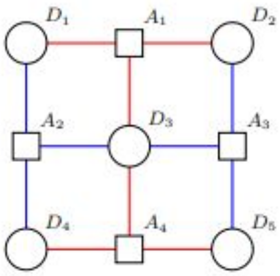
(a) (b)

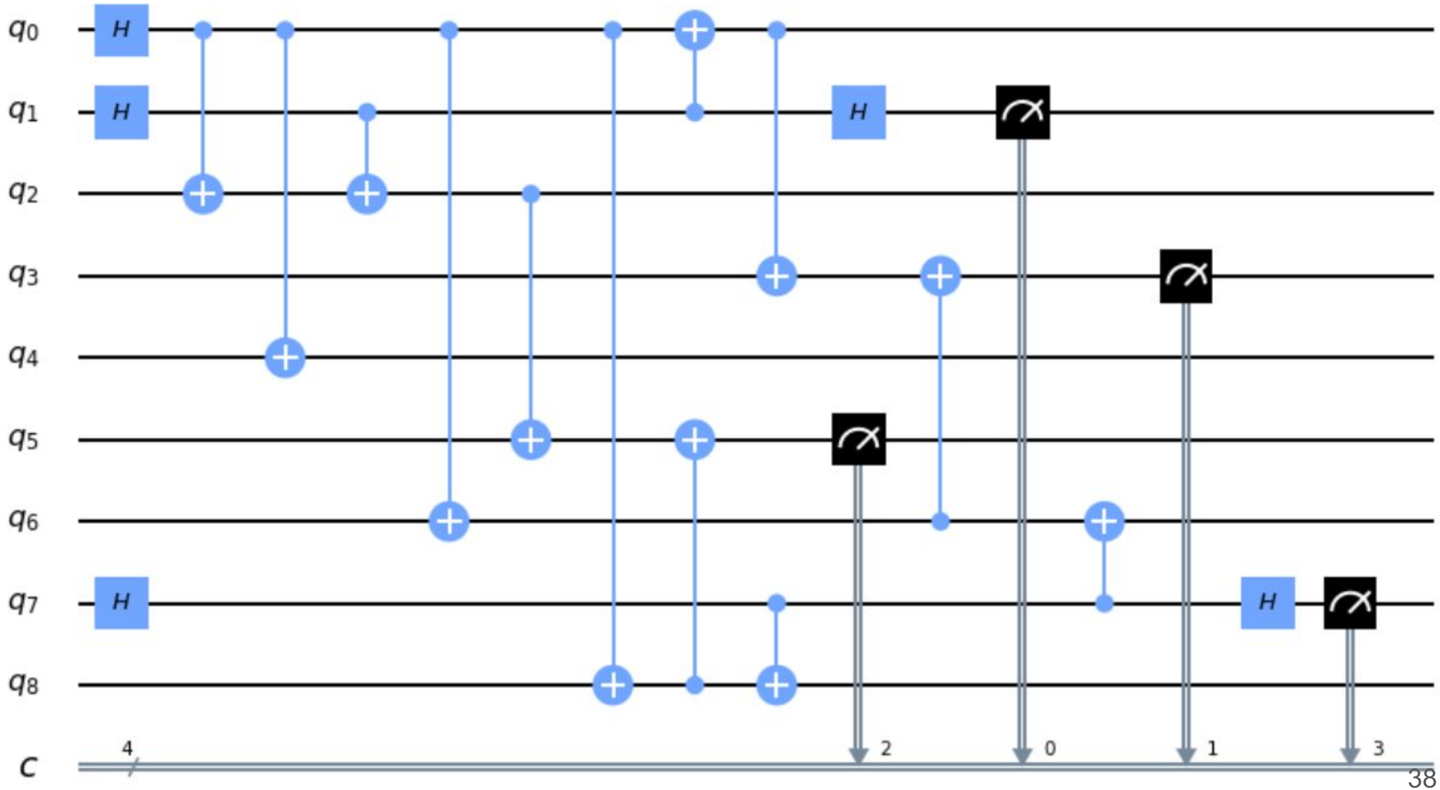
Joschka Roffe’s introductory guide: arXiv:1907.11157v1

4-plaquettes code: [[5,1,2]]

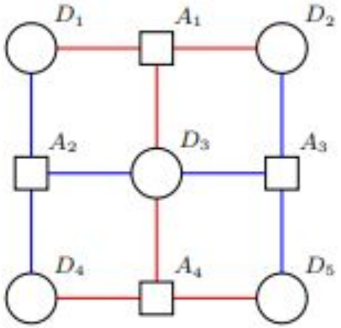


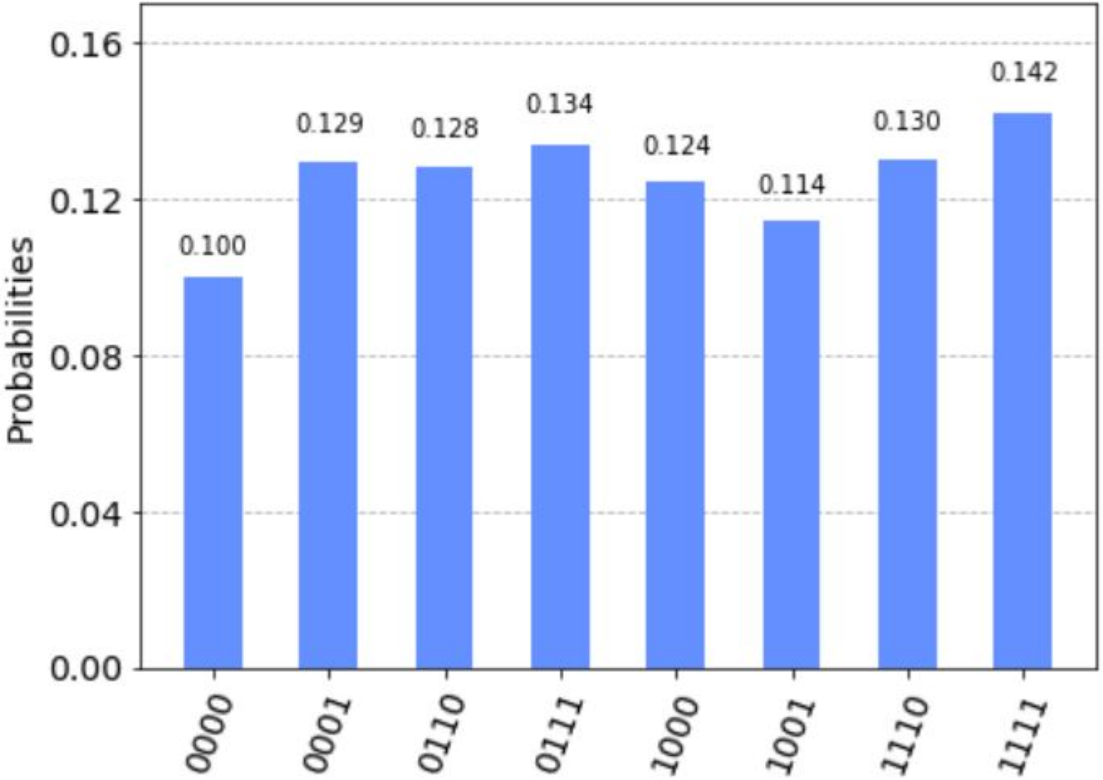
4-plaquettes code encoding





4-plaquettes code results without noise





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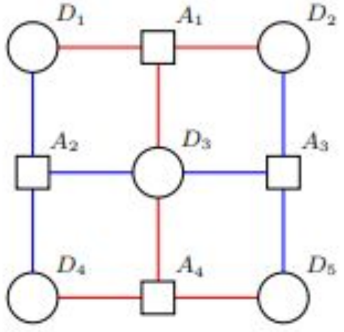
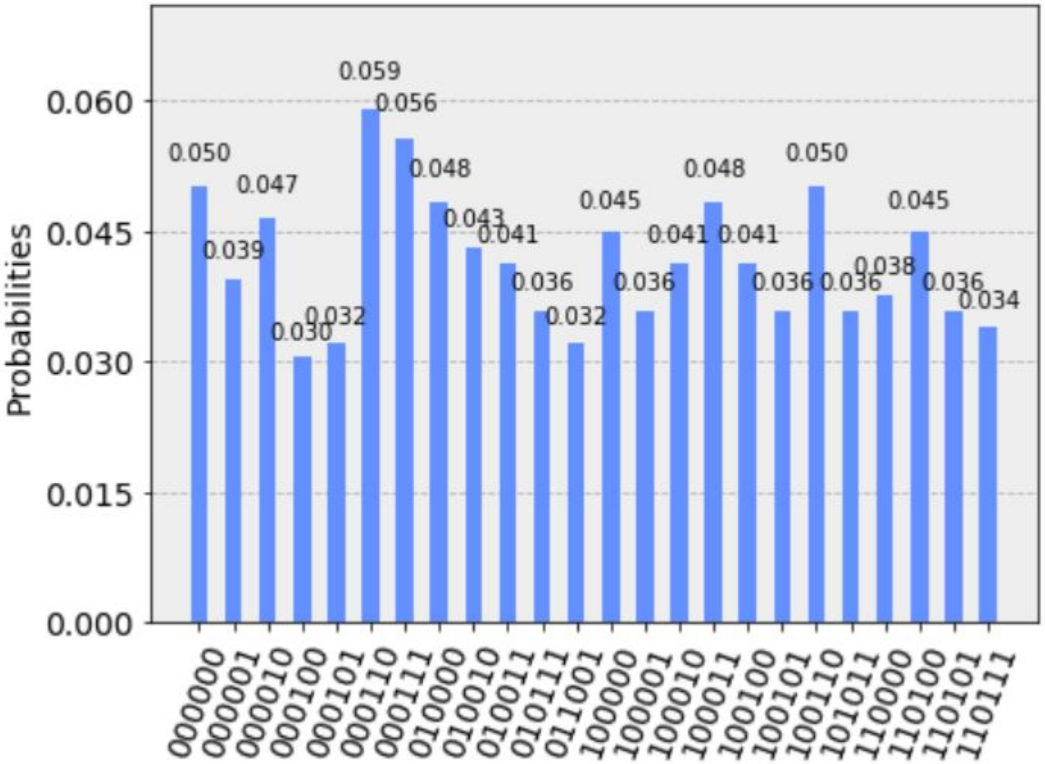
4-plaquettes code filtered results with noise





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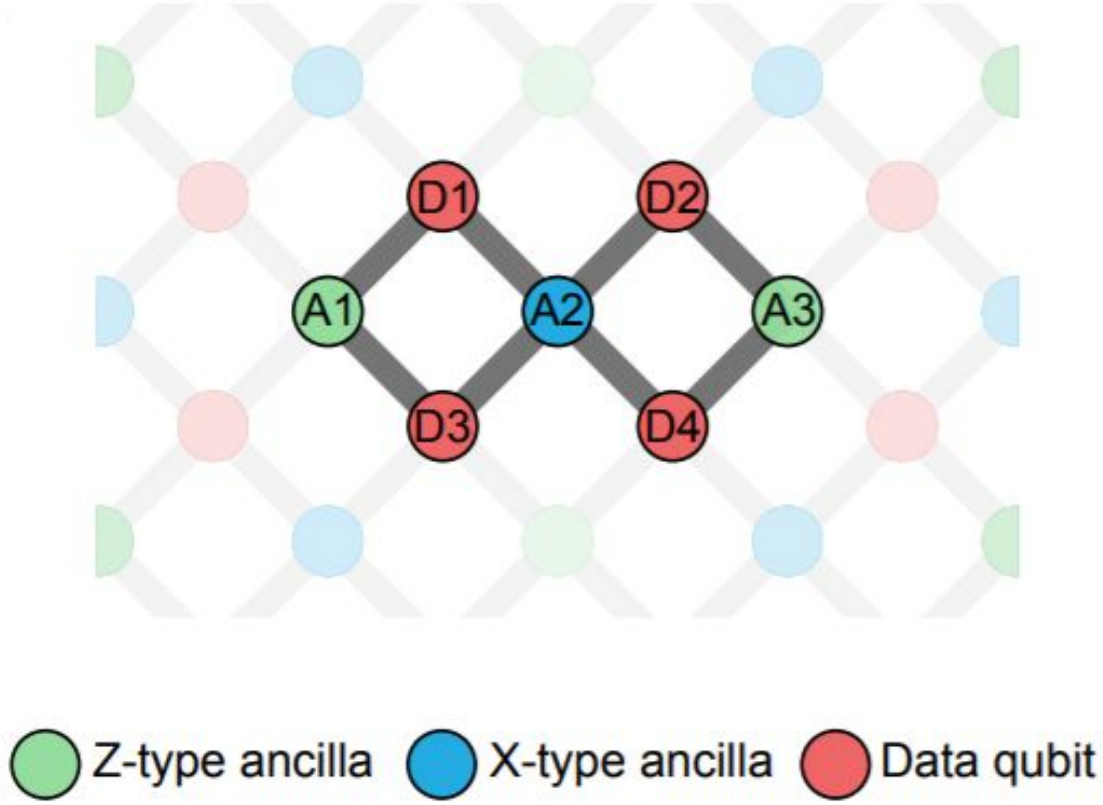
Experimental Results by Andersen et al.



arXiv:1912.09410

Logical qubit definition

- <u>All</u> physical qubits must be measured.
- Stabilizing measurements do not affect a logical qubit state.

$$Z_L = Z_{D1}Z_{D2}, \quad \text{or} \quad Z_L = Z_{D3}Z_{D4}, \tag{2}$$

$$X_L = X_{D1}X_{D3}, \quad \text{or} \quad X_L = X_{D2}X_{D4}, \tag{3}$$

$$|0\rangle_L = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle),$$

$$|1\rangle_L = \frac{1}{\sqrt{2}}(|0101\rangle + |1010\rangle).$$

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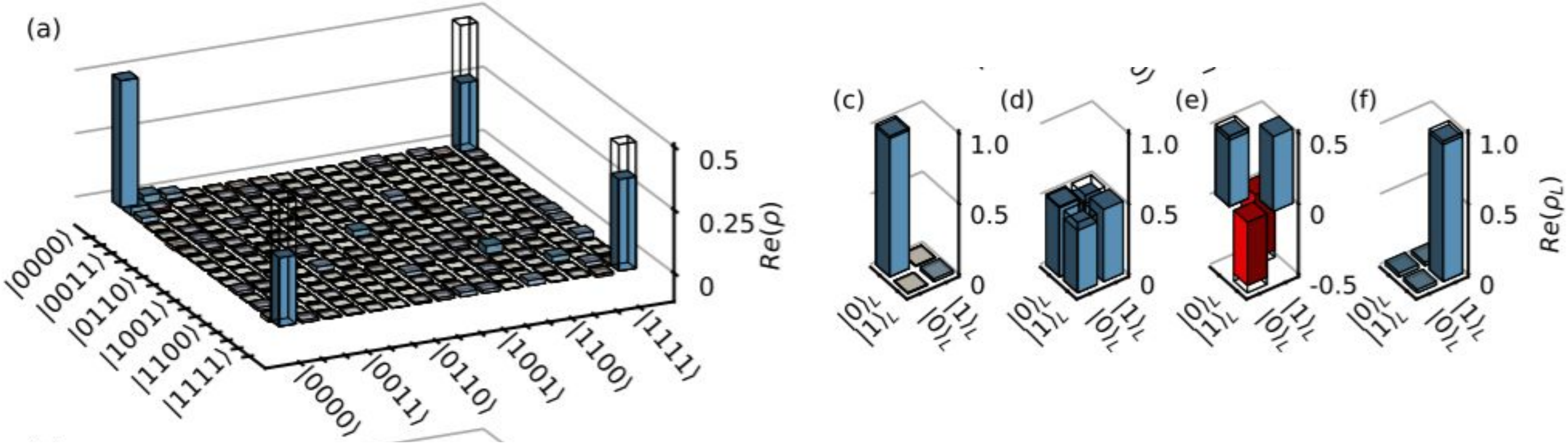
Fidelity measurements



Fidelity: 70.3% 98.2% 94.2% 94.8% 97.3%