Identifying Causal Effects in Network Experiments with Non-compliance

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Example with Potential for Indirect Treatment Effects

Crepon et al. (2013; QJE)

- Large-scale job-seeker assistance program in France.
- Randomized offers of intensive job placement services.

Displacement Effects of Labor Market Policies

"Job seekers who benefit from counseling may be more likely to get a job, but at the expense of other unemployed workers with whom they compete in the labor market. This may be particularly true in the short run, during which vacancies do not adjust: the unemployed who do not benefit from the program could be partially crowded out."

Two Key Ingredients for Studying Social Interactions

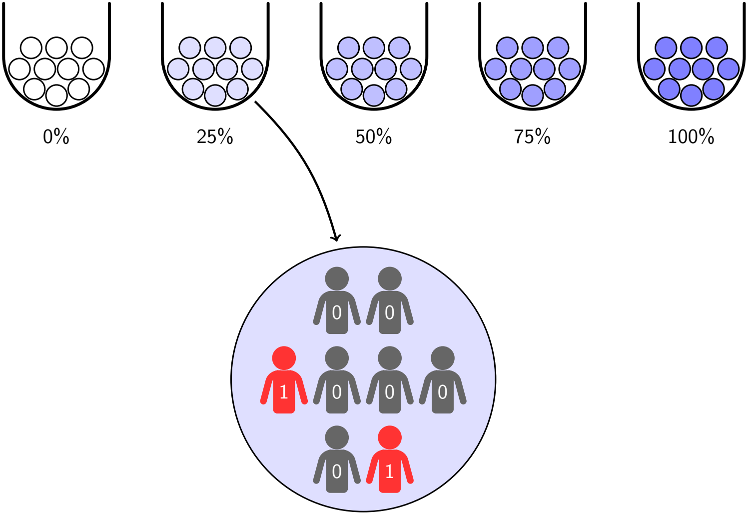
Partial Interference Assumption

- Each individual belongs to a single, known group g.
- Potential spillovers within but not between groups.

Randomized Saturation Design

- Saturation $\equiv$ fraction of individuals offered treatment.
- Groups randomly assigned saturation $s_g \in \mathcal{S}$.
- Individuals randomly assigned treatment offer $z_{ig} \in \{0, 1\}$.

Randomized Saturation Design



Beyond Intent-to-treat Effects

- Literature on spillovers in randomized saturation experiments assumes perfect compliance or considers only intent-to-treat.
- Treatment take-up is often low (e.g. 35% in Crepon et al.) making it difficult to detect indirect effects.
- Can we use this exogenous variation in individual-level offers and group-level saturation to learn causal effects?

For today: one-sided non-compliance. . .

Simplest Possible Example: Pairs of Individuals

Randomized Saturation

Offer treatment to both, one, or neither member of a given pair.

Notation

- $g = 1, \ldots, G$ indexes groups
- $i = 1, \ldots, N_g$ indexes individuals within group g
- z_{ig} is the treatment offer and D_{ig} the take-up of (i, g)
- $\widetilde{z}_{ig}$ and $\widetilde{D}_{ig}$ are defined analogously for (i, g)'s partner

Potential Outcomes

$Y_{ig}(D_{ig}, \widetilde{D}_{ig})$: no spillovers between pairs, possibly spillovers within

Pairs Example: Direct and Indirect Effects

Direct: Change (i, g)'s Treatment

- $\mathbb{E}[Y_{ig}(1,0) - Y_{ig}(0,0)] \quad \leftarrow$ Untreated Partner
- $\mathbb{E}[Y_{ig}(1,1) - Y_{ig}(0,1)] \quad \leftarrow$ Treated Partner

Indirect: Change Partner's Treatment

- $\mathbb{E}[Y_{ig}(0,1) - Y_{ig}(0,0)] \quad \leftarrow (i, g)$ Untreated
- $\mathbb{E}[Y_{ig}(1,1) - Y_{ig}(0,0)] \quad \leftarrow (i, g)$ Treated

Question

With perfect compliance we can recover all four effects. But what if compliance is imperfect?

Principal Strata with One-sided Non-compliance

Potential Treatments

$D_{ig}(z_{ig}, \widetilde{z}_{ig})$ is an equilibrium action in the game played by pair g.

	$(z, \widetilde{z})$	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
n	Never-taker	0	0	0	0
c	Complier	0	1	0	1
ℓ	"Loner"	0	1	0	0
f	"Follower"	0	0	0	1

Strategic Take-up

Types ℓ and f are strategic: their take-up depends not only on their own treatment offer, but on their partner's offer.

Local Average Treatment Effects – No Strategic Takeup

Compliers with Compliers

$$\mathbb{E}\left[Y(1,0) - Y(0,0)|(c,c)\right] \quad \leftarrow \text{Direct Effect, Untreated Partner}$$

$$\mathbb{E}\left[Y(1,1) - Y(0,1)|(c,c)\right] \quad \leftarrow \text{Direct Effect, Treated Partner}$$

$$\mathbb{E}\left[Y(0,1) - Y(0,0)|(c,c)\right] \quad \leftarrow \text{Indirect Effect when Untreated}$$

$$\mathbb{E}\left[Y(1,1) - Y(1,0)|(c,c)\right] \quad \leftarrow \text{Indirect Effect when Treated}$$

Compliers with Never-takers

$$\mathbb{E}\left[Y(1,0) - Y(0,0)|(c,n)\right] \quad \leftarrow \text{Direct Effect, Untreated Partner}$$

$$\mathbb{E}\left[Y(0,1) - Y(0,0)|(n,c)\right] \quad \leftarrow \text{Indirect Effect when Untreated}$$

Example: What Goes Wrong with Strategic Takeup?

Adding a Strategic Type

Suppose that, in addition to compliers (c) and never-takers (n), the population contains "loner" types (ℓ) who only take up treatment when they are offered but their partner is not.

Exclusion Restriction

Rule out spillovers in inducement: $\widetilde{z}_{ig}$ only affects D_{ig} through $\widetilde{D}_{ig}$.

Possible Pairings

$(t_{ig}, \widetilde{t}_{ig}) \in \{(c,c),(c,n),(c,\ell),(n,c),(n,n),(\ell,c)\}$

Example: What Goes Wrong with Strategic Takeup?

- The only observable moment that involves $Y(1,1)$ is

$$\mathbb{P}(\mathbf{d}_{11}|\mathbf{z}_{11})\mathbb{E}(Y|\mathbf{d}_{11},\mathbf{z}_{11}) = \mathbb{P}(c,c)\mathbb{E}[Y(1,1)|(c,c)]$$

- The only observable moments that involve $Y(0,1)$ are

$$\begin{aligned}
\mathbb{P}(\mathbf{d}_{01}|\mathbf{z}_{11})\mathbb{E}(Y|\mathbf{d}_{01},\mathbf{z}_{11}) &= \mathbb{P}(n,c)\mathbb{E}[Y(0,1)|(n,c)] + \mathbb{P}(\ell,c)\mathbb{E}[Y(0,1)|(\ell,c)] \\
\mathbb{P}(\mathbf{d}_{01}|\mathbf{z}_{01})\mathbb{E}(Y|\mathbf{d}_{01},\mathbf{z}_{01}) &= \mathbb{P}(c,c)\mathbb{E}[Y(0,1)|(c,c)] + \mathbb{P}(c,\ell)\mathbb{E}[Y(0,1)|(c,\ell)] \\
&\quad + \mathbb{P}(n,c)\mathbb{E}[Y(0,1)|(n,c)] + \mathbb{P}(\ell,c)\mathbb{E}[Y(0,1)|(\ell,c)]
\end{aligned}$$

- No well defined group of individuals for whom we can learn the direct effect $Y(1,1) - Y(0,1)$ under strategic takeup.

Testable Implication of No Strategic Takeup

If there are no strategic types in the population, then we must have

$$\mathbb{P}(D_{ig} = 1 | z_{ig} = 1, \widetilde{z}_{ig} = 0) = \mathbb{P}(D_{ig} = 1 | z_{ig} = 1, \widetilde{z}_{ig} = 1).$$

Violations of this condition indicate the presence of type ℓ and/or type f individuals in the population.

Fully General Version

Notation

Let $\widetilde{\mathbf{z}}_{ig}$ be the vector of treatment offers and $\widetilde{\mathbf{D}}_{ig}$ the vector of treatment take-up for everyone in g *except* person i.

Potential Outcomes

$Y_{ig}(D_{ig}, \widetilde{\mathbf{D}}_{ig})$ may depend on treatment take-up of everyone in g.

Potential Treatments

$D_{ig}(z_{ig}, \widetilde{\mathbf{z}}_{ig})$ may depend on treatment offers of everyone in g.

But this is too general to make progress...

Restrictions

Notation

Let $\bar{D}_g$ be the average of D_{ig} over all individuals in group g.

1. Large Groups

Contribution of D_{ig} to $\bar{D}_g$ is negligible.

2. Aggregate Interactions

$$Y_{ig}(D_{ig}, \widetilde{\mathbf{D}}_{ig}) = Y_{ig}(D_{ig}, \bar{D}_g)$$

3. No Strategic Takeup

$$D_{ig}(z_{ig}, \widetilde{\mathbf{z}}_{ig}) = D_{ig}(z_{ig})$$

For this talk, continue to assume one-sided non-compliance...

Restrictions Continued...

4. Correlated Random Coefficients Model

$$Y_{ig}(D_{ig}=0,\bar{D}_g)=\kappa^0_{ig}+\kappa^1_{ig}h_1(\bar{D}_g)+\cdots+\kappa^p_{ig}h_p(\bar{D}_g)$$
$$Y_{ig}(D_{ig}=1,\bar{D}_g)=\lambda^0_{ig}+\lambda^1_{ig}h_1(\bar{D}_g)+\cdots+\lambda^p_{ig}h_p(\bar{D}_g)$$

- $\boldsymbol{\kappa}$ and $\boldsymbol{\lambda}$ may be *correlated* with $\bar{D}_g$
- p can be as large as # of saturations

Example:

$$Y_{ig}(0,\bar{D}_g)=\kappa^0_{ig}+\kappa^1_{ig}\bar{D}_g$$
$$Y_{ig}(1,\bar{D}_g)=\lambda^0_{ig}+\lambda^1_{ig}\bar{D}_g$$

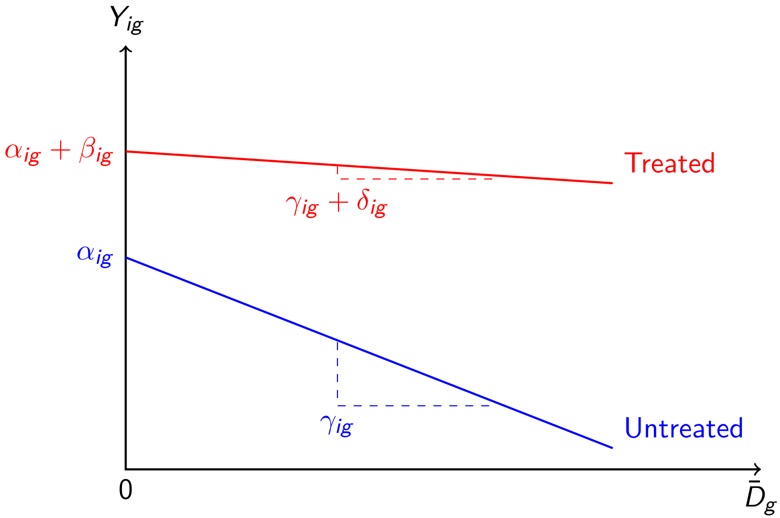
Example: Linear Correlated Random Coefficients Model

Re-write as a Single Equation:

$$Y_{ig} = \alpha_{ig} + \beta_{ig} D_{ig} + \gamma_{ig} \bar{D}_g + \delta_{ig} D_{ig} \bar{D}_g$$

- Both D_{ig} and $\bar{D}_g$ are endogenous
- $(\alpha_{ig}, \beta_{ig}, \gamma_{ig}, \delta_{ig})$ likely correlated with D_{ig} and $\bar{D}_g$
- Instruments: offers z_{ig}, saturations s_g, interaction $z_{ig} \times s_g$
- Can we recover means of $\alpha_{ig}, \beta_{ig}, \gamma_{ig}, \delta_{ig}$?

Hypothetical Example with Displacement Effects



Why is strategic take-up a problem?

Wald Estimand *within group*

$\mathbb{E}_g[Y(1, \bar{D}_g) - Y(0, \bar{D}_g)|\text{complier}] = \mathbb{E}_g\left[\beta|\text{complier}\right] + \mathbb{E}_g\left[\gamma|\text{complier}\right]\bar{D}_g$

What if there is strategic takeup?

Wald Estimator unchanged, but definition of *complier* is specific to the saturation s_g assigned to g.

Why is this a problem?

Changing s_g changes *both* $\bar{D}_g$ and $\mathbb{E}_g[\beta|\text{complier}], \mathbb{E}_g[\gamma|\text{complier}]$ since it changes who is a complier.

Why Not Just Run 2SLS?

- Homogeneous 1st-stage: 2SLS $\rightarrow_p$ $\mathbb{E}$[random coefs.]
  - Heckman & Vytlacil (1998)
  - Wooldridge (1997, 2003, 2008)
- Our 1st-stage is *heterogeneous*:
  - Effect of z_{ig} on D_{ig} depends on whether (i, g) is a complier
  - Effect of s_g on $\bar{D}_g$ depends on fraction of compliers in g
- Alternatives under (restricted) 1st-stage heterogeneity:
  - Florens et al. (2008)
  - Masten & Torgovitsky (2016)
- Re-write our model in so that we can build on these results...

Re-writing the Model

No Strategic Takeup

$D_{ig} = c_{ig} \times z_{ig}$, where $c_{ig} = 1$ iff (i, g) is a complier.

Substituting into the Model

$$\begin{aligned} Y_{ig} &= \alpha_{ig} + \beta_{ig}(c_{ig} \times z_{ig}) + \gamma_{ig}\bar{D}_g + \delta_{ig}(c_{ig} \times z_{ig})\bar{D}_g \\ &= \alpha_{ig} + \widetilde{\beta}_{ig} z_{ig} + \gamma_{ig}\bar{D}_g + \widetilde{\delta}_{ig} z_{ig}\bar{D}_g \end{aligned}$$

Transformed Regression

$$\begin{aligned} Y_{ig} &= X_{ig}'\boldsymbol{\theta}_{ig} \\ X_{ig} &= (1, z_{ig}, \bar{D}_g, z_{ig}\bar{D}_g)' \\ \boldsymbol{\theta}_{ig} &= (\alpha_{ig}, \widetilde{\beta}_{ig}, \gamma_{ig}, \widetilde{\delta}_{ig})' \end{aligned}$$

A Conditional OLS Estimand

Crucial Point

Under our assumptions, X_{ig} is conditionally independent of $\boldsymbol{\theta}_{ig}$ given c_g, the fraction of compliers in g.

Intuition

Large group size $\Rightarrow \bar{D}_g \approx (c_g \times s_g) \Rightarrow$ variation in $\bar{D}_g$ *given* c_g comes only from the randomly assigned saturation.

$$\begin{aligned} Y_{ig} = X_{ig}'\boldsymbol{\theta}_{ig} \implies \mathbb{E}\left[X_{ig}Y_{ig}|c_g\right] &= \mathbb{E}[X_{ig}X_{ig}'\boldsymbol{\theta}_{ig}|c_g] \\ \mathbb{E}\left[X_{ig}Y_{ig}|c_g\right] &= \mathbb{E}[X_{ig}X_{ig}'|c_g]\mathbb{E}[\boldsymbol{\theta}_{ig}|c_g] \\ \mathbb{E}[\boldsymbol{\theta}_{ig}|c_g] &= \mathbb{E}[X_{ig}X_{ig}'|c_g]^{-1}\mathbb{E}[X_{ig}Y_{ig}|c_g] \end{aligned}$$

$$\boxed{\mathbb{E}\left[\boldsymbol{\theta}_{ig}\right] = \int \mathbb{E}[X_{ig}X_{ig}'|c_g]^{-1}\mathbb{E}[X_{ig}Y_{ig}|c_g]\, dF(c_g)}$$

A Conditional OLS Estimator

$$\boxed{\mathbb{E}\left[\boldsymbol{\theta}_{ig}\right] = \int \mathbb{E}[X_{ig}X_{ig}'|c_g]^{-1}\mathbb{E}[X_{ig}Y_{ig}|c_g]\, dF(c_g)}$$

In Practice

- Fraction of compliers c_g is consistently estimable.
- Sample analogues, kernel-weighted OLS

Relation to Underlying Parameters

$$\mathbb{E}[\boldsymbol{\theta}_{ig}] = \left(\mathbb{E}[\alpha_{ig}], \mathbb{E}\left[\beta_{ig}c_{ig}\right], \mathbb{E}[\gamma_{ig}], \mathbb{E}\left[\delta_{ig}c_{ig}\right]\right)'$$

Iterated Expectations

$$\mathbb{E}[\beta_{ig}|c_{ig}=1] = \frac{\mathbb{E}\left[\beta_{ig}c_{ig}\right]}{\mathbb{P}[c_{ig}=1]}, \quad \mathbb{E}[\delta_{ig}|c_{ig}=1] = \frac{\mathbb{E}\left[\delta_{ig}c_{ig}\right]}{\mathbb{P}[c_{ig}=1]}$$

Linear Correlated Random Coefficients Model

$$\boxed{Y_{ig} = \alpha_{ig} + \beta_{ig} D_{ig} + \gamma_{ig} \bar{D}_g + \delta_{ig} D_{ig} \bar{D}_g}$$

Identified Parameters

- Treatment on the Treated: $\mathbb{E}\left[\beta_{ig} | D_{ig} = 1\right], \mathbb{E}\left[\delta_{ig} | D_{ig} = 1\right]$
- Spillover on the Untreated: $\mathbb{E}\left[\alpha_{ig}\right], \mathbb{E}\left[\gamma_{ig}\right]$
- Can also use an auxiliary regression to recover means of $(\alpha_{ig}, \gamma_{ig})$ for compliers and for never-takers

Where do we use the saturations?

- Recall: $X_{ig} = (1, z_{ig}, \bar{D}_g, z_{ig} \bar{D}_g)$.
- Cannot invert $\mathbb{E}[X_{ig} X_{ig}' | c_g]$ unless there is variation in s_g.

Conclusion

Summary

- Crucial but testable assumption: no strategic takeup
- Random coefficients model of aggregate interactions
- Identification of direct and indirect causal effects

Didn't Talk About Today

Tests for strategic takeup: (1) simple regression-based test, (2) non-parametric bootstrap-randomization test.

In Progress

- Asymptotic analysis of group size vs. number of groups
- Generalization to two-sided non-compliance
- Simulations and Empirical Example