LINEAR REGRESSION MODELS W4315

HOMEWORK 1 QUESTIONS

September 15, 2009

Instructor: Frank Wood (10:35-11:50)

1. (25 points) Let $Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$ be a linear regression model with distribution of error terms unspecified (but with mean $E(\epsilon) = 0$ and variance $V(\epsilon_i) = \sigma^2$ (σ^2 finite) known). Show that $s^2 = MSE = \frac{\sum(Y_i - \hat{Y}_i)^2}{n-2}$ is an unbiased estimator for σ^2. $\hat{Y}_i = b_0 + b_1 X_i$ where $b_0 = \bar{Y} - b_1 \bar{X}$ and $b_1 = \frac{\sum_i ((X_i - \bar{X})(Y_i - \bar{Y}))}{\sum_i (X_i - \bar{X})^2}$

2. (25 points) Derive the maximimum likelihood estimators $\hat{\beta}_0, \hat{\beta}_1$, and $\hat{\sigma}^2$ for parameters β_0, β_1, and σ^2 for the normal linear regression model (i.e. $\epsilon_i \sim_{iid} N(0, \sigma^2)$).

3. (50 points) Do problem 1.19 in the book.