LINEAR REGRESSION MODELS W4315

INTRO SURVEY QUESTIONS

September 7, 2009

Instructor: Frank Wood (10:35-11:50)

1. (0 points)

(a) What is your field of study?

(b) What is your intended profession?

(c) Why are you taking this course?

(d) How do you see yourself using regression?

(e) What would you like to learn in this course?

2. (0 points) Find the maximum value of x that satisfies the equation

$$-x^2 + ln(x) = a,\ x > 0.$$

Give a simplified algebraic answer.

3. (0 points)

(a) Given an algebraic solution to the following matrix equation

$$\mathbf{Ax} - \mathbf{b} = \mathbf{0}$$

(b) What conditions does your answer put on the matrix $\mathbf{A}$?

4. (0 points) If $X \sim \text{Poisson}(\lambda)$ and $\lambda \sim \text{Gamma}(\alpha, \beta)$ how is $X|\alpha, \beta$ distributed? Hint, remember:

$$\begin{aligned}
P(a|c) &= \int P(a|b)P(b|c)db \\
P(X|\lambda) &= \frac{1}{X!}\lambda^X e^{-\lambda} \\
P(\lambda|\alpha, \beta) &= \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}
\end{aligned}$$

Don't simplify the $\Gamma()$'s in the final result.

5. (0 points)

Let $\theta_i \sim \text{Exp}(\beta)$, $1 \leq i \leq n$ be samples from an exponential distribution. Remember that for an exponential distribution $P(\theta) = \frac{1}{\beta}e^{-\frac{\theta}{\beta}}$ for $\theta > 0$. Also remember that for the exponential distribution $E(\Theta) = \beta$ and $V(\theta) = \beta^2$ where $E()$ stands for expectation and $V()$ for variance.

(a) Derive the maximum likelihood estimator $\hat{\beta}$ for β given observations $\{\theta_i\}_{i=1}^n$.

(b) Is this estimator biased or unbiased? Show work. Reminder: the definition of bias is $B(\hat{\beta}) = \beta - E(\hat{\beta})$.

(c) Derive the sample variance of the estimator. Remember $V(aX) = a^2V(X)$.

(d) Given a set of samples $\{\theta_i\}_{i=1}^n$ as above, illustrate the small sample symmetric confidence interval for $\hat{\beta}$. Remember $T = \sqrt{n}\frac{\hat{\beta}-\beta}{\hat{\sigma}} \sim T_{n-1}$ where T_{n-1} stands for a student's-T distribution with $n-1$ degress of freedom. The sample variance is given by $\hat{\sigma}^2 = \frac{1}{n-1}\sum_i(\theta_i - \hat{\beta})^2$ but this expansion is unecessary and provided only for familiarity. What you should derive is c in this statement

$$\hat{\beta} - c \leq \beta \leq \hat{\beta} + c$$

with probability .9. A T-distribution CDF table is provided.