LINEAR REGRESSION MODELS W4315

HOMEWORK 5 QUESTIONS

October 15, 2009

Due: 10/22/29

Instructor: Frank Wood (10:35-11:50)

1. (15 points) In order to get a maximum likelihood estimate of the parameters of a Box-Cox transformed simple linear regression model ($Y_i^\lambda = \beta_0 + \beta_1 X_i + \epsilon_i$), we need to find the gradient of the likelihood with respect to its parameters (the gradient consists of the partial derivatives of the likelihood function w.r.t. all of the parameters). Derive the partial derivatives of the likelihood w.r.t all parameters assuming that $\epsilon_i \sim N(0, \sigma^2)$. (N.B. the parameters here are $\lambda, \beta_0, \beta_1, \sigma$)

(Extra Credit: Given this collection of partial derivatives (the gradient), how would you then proceed to arrive at final estimates of all the parameters? Hint: consider how to increase the likelihood function by making small changes in the parameter settings.)

2. (10 points) Problem 4.22 in the book.

3. (10 points) Do problem 5.3 in the book.

4. (10 points) Do problem 5.15 in the book.

5. (10 points) Do problem 5.17 in the book.

6. (15 points) Do problem 5.24 in the book.

7. (15 points) Do problem 5.29 in the book.

8. (15 points) Consider the following linear regression model:

$$\mathbf{Y} = \mathbf{X}\beta + \epsilon$$

where $\mathbf{X}$ is the design matrix whose first column consists of all ones, β is a vector including all the regression parameters, ϵ is the error term which follows the standard Gaussian assumption (independent, equal-variance normal distribution, i.e. $\epsilon \sim \mathbf{N}(\mathbf{0}, \sigma^2\mathbf{I})$ where $\mathbf{I}$ is identity matrix). Derive the maximum likelihood estimator for β and σ using matrix calculation.