MARKOV CHAIN MONTE CARLO

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Markov Chain Monte Carlo (MCMC) refers to a collection of techniques used to draw samples from distributions of interest by simulating Markov chains who equilibrium distribution is, by design, precisely the distribution of interest.

First, why do we want samples from distributions? It will often be written something like, we can approximated expectations of functions with respect to distributions

$$\mathbb{E}_P[f(x)] = \int f(x)P(x)dx \approx \frac{1}{L}\sum_{\ell=1}^{L} f(x^{(\ell)})$$

via a set of samples drawn from $P(x)$, i.e. $\{x^{(\ell)}\}_{\ell=1}^{L}$, $x^{(\ell)} \sim P$. We will not go into the technical justification and rates of convergence of this approximation here (for those see [**? ? ?**]), instead we will simply point out the reasonableness of this approximation by illustrating a couple example choices of f and interpretations there-of.

First, consider $f(x) = x$. In this case $\mathbb{E}_P[f(x)] = \mathbb{E}_P[x]$ is the definition of the mean of x under P. One might wonder, if not already familiar with MCMC and Bayesian methods, why exactly calculating the mean of x under P is interesting given that in most textbook statistics and inference settings