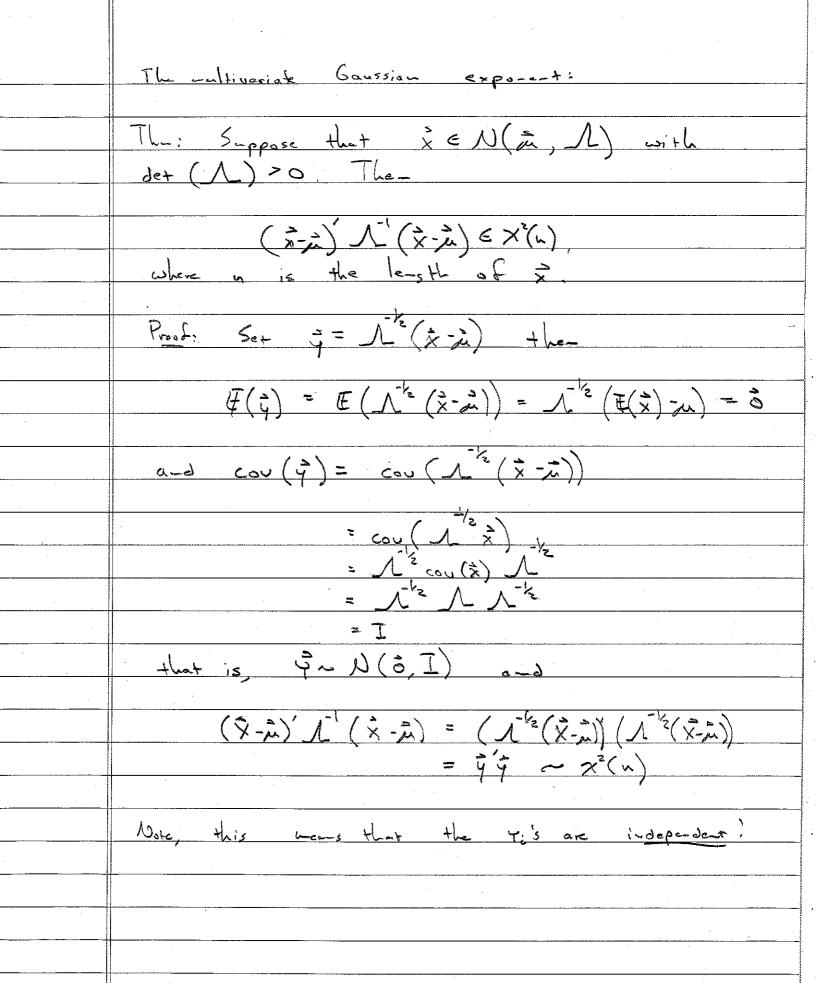
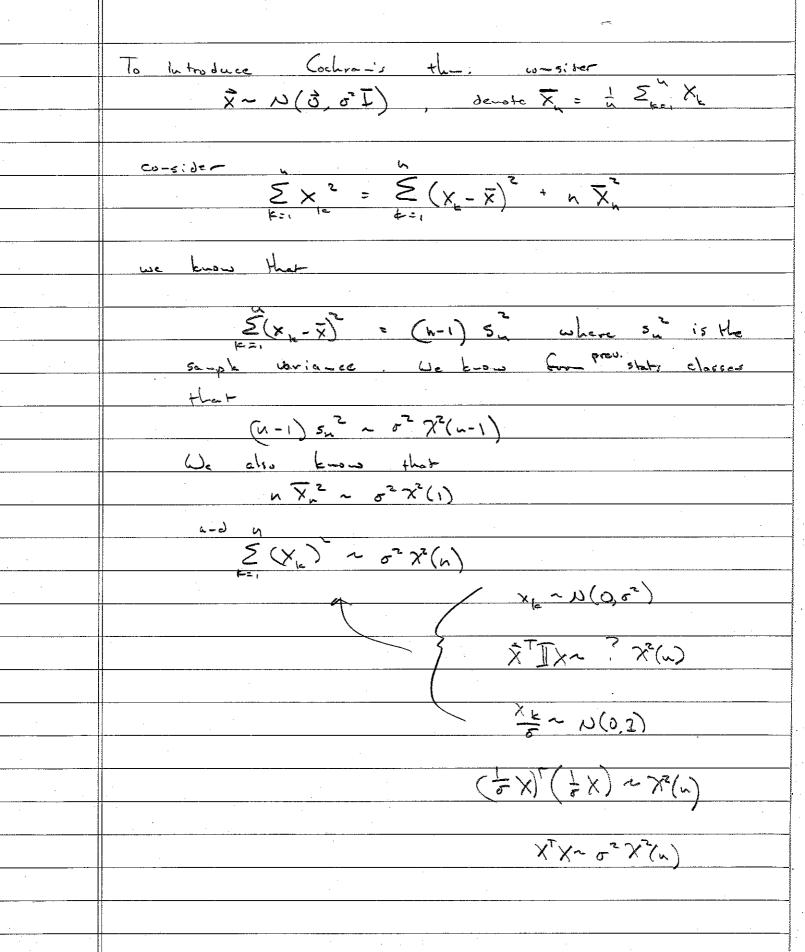
·	Cochran's Theore - , Proof, take - in large part
	from Gut pg 139.
<u> </u>	- Quadratic Forms are important
	- least squares
	AVOUA
	regre si-
·	- Bosic idea: split san of squares into a number
	of quadratic forms, each of which corresponds
	to some cause of variation
	- exa-ple: crop yield us
	b) a - o - + of water and irregation
	e) units of sunlight
	d) eta
	- Result: each quadratic for- corresponds to
	one cause with one final for the
	residual form that measures the random errors
	involved in the experiment
	- Cochrais the says that all of these
·	quadratic forms are independent and 72 distributed
	- This ca- be used to test hypotheses about the effects of (or influence d) different inputs on
-	the aut put
	part.
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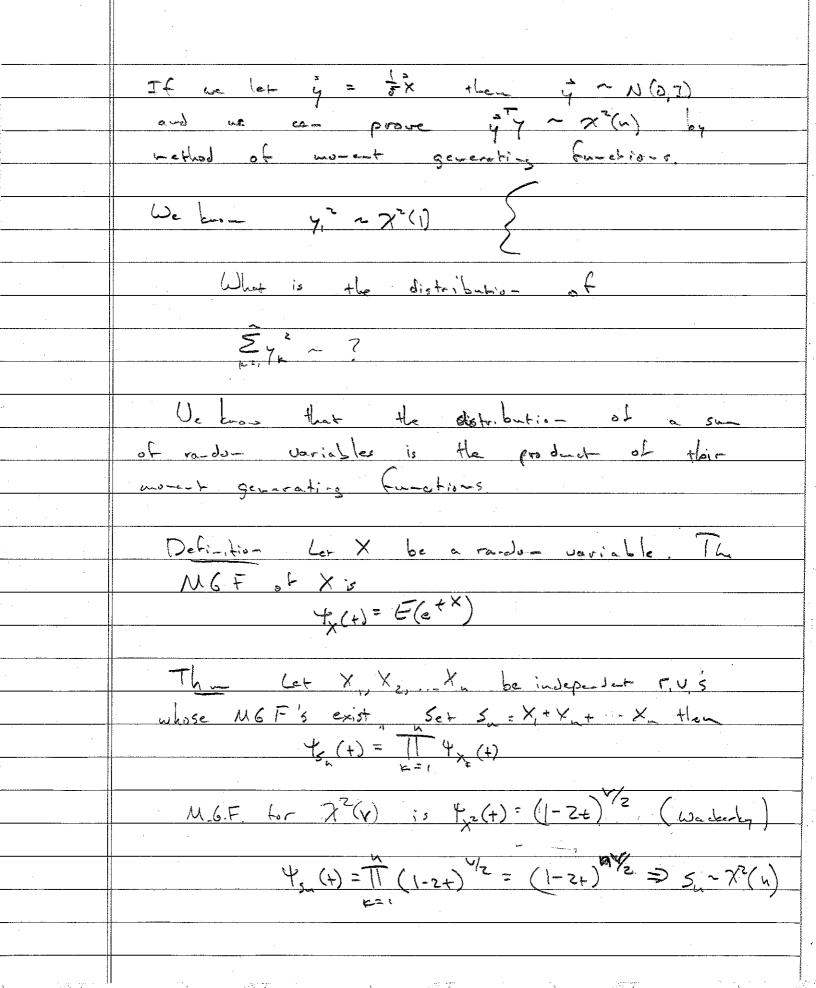


	Kenenber
	what is A-12?
	Assume A is a symmetric matrix (all graduation
	for-s are symmetric and recall that symmetric
	matrices can be diagonalized
-	C'AC = D
	where D is a diagonal working and C is
	an arthogonaltrix, i.e, c'=c' &> c'c=I
	The diagonal elements of D am the
	eige-values, D D. of A. w
	Clearly det A = det D = II >, and
	trace A: trace (CDC') = trace (D)
	D=D's casy to define it's just than
	S= [+1] If we set
	$\hat{D} = [+JJ:] \qquad \text{set}$ $\hat{D} = [+JJ:] \qquad \text{set}$
	then we have
	B'=BB= CPC'CPC'= CPPC'=
	CDc'=A
	so B is the square rook of A.
	ie B= A'E
	Additionally (A= (c'oc)
	$(A^{-1})^{1/2} = (A^{1/2})^{-1}$ $(D^{-1}C/CD) = I$
	10 (C'DC) - C'D'C
	some sort and holds
. arti	
. "	



V. 47.7

V. 100



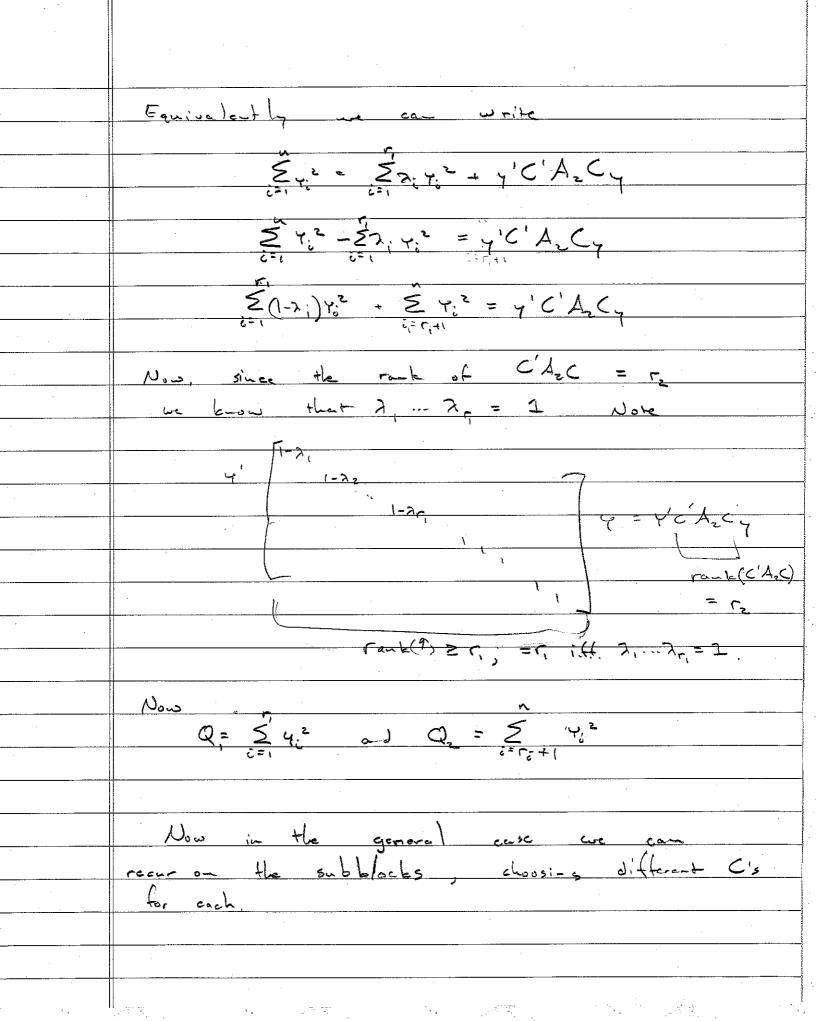
 $\sum_{k=1}^{\infty} \left(x_{k} - \overline{x}_{k} \right)^{2} + \sqrt{x}_{k}^{2}$ is equivalent to $\vec{x}'\vec{\perp}\vec{x} = \vec{x}(\vec{x}-\vec{x})\vec{x} + \vec{x}(\vec{x}-\vec{x})\vec{x}$ and from this we can arrive up the fact Cochracis The easily. Toward's proving Cochran's Thu. we start with the following lemma (Gut) Let x, x2, --- x be real --- bers. Suppose that I x; 2 can be split into a sum of non-negative definite quadratic forms, that is, \(\sum_{\chi^2} = \Q_1 + Q_2 + \ldots + \Q_2 \) where Q = x'A; x and rank Q = rank (A;) = c Vo If & n=u then I on orthogo-al matrix C st. > = Cy

Q= 10-12+1 + 10-12+2 + X2

Note: each of the gued forms contains different non-overlapping sets of yis, and that He # of ter-s in each Qi is ri We start with the case u=2. The general case can be proved by induction. Proof for == 2: By discipline we have $0 = \sum_{i=1}^{N} x_i^2 = x'A_i \times - x'A_i \times = 0, +0,$ where A and Az are non-negative definite (. Symmetric: they are quadratic forms) matrices
with ranks , and re. (+ + z = m = 150 by assiption) note we know that A is symmetric, if A is positive seni- definite it 6'AB 20 46 #0 We know that symmetric matrices can be or thogo-ally diago-alized is C'AC=1. If A is pos. seni Jet then b'Ab = b'(c'Ac)b = b'Ab = 0 where 6 = Cb This was that 1 is a so pos se-i deli-ite which nears that A -ust have all positive entries (A has all positive any vis)

A. Santa Santa

	Because A, is symmetric it can be orthogally
	diago-alized (check Lin Alg. review Pg 77)
	C'A, C = D
-	
	where Dis diago-al and the diago-al elements
	of which are the eigenvelves of A. Since
	rowke (A)= r we know that rowle (A) = # wa-
	zero eigen values in D (non-singular transforms
	preserve rome, is roule (BA) = rome (A) if B
	von sing for rack is number of linearly independent
	col's or row's)
.:	
·	Since rank (A,)=r, and A, is symmetric
	then we have 2,, 2, positive eigenvalues and
	N-17 7 - values equal to zero.
	Set x = Cy
	Q = \(\int \) \(\int
·	
	= y CA, Cy + y CA2Cy
	= you + y' C A2 Cy
	= \frac{2}{2}, \tau_{1}^{2} + \tau_{1}^{2} \tau_{2}^{2} \tau_{1}^{2}
· · · · · · · · · · · · · · · · · · ·	



	Now, Cochrais The is almost funediate.
C.T.:	Let X, X2, X be independent N(0, 52) T. V.S
	and suppore
	*I+= \(\hat{z} \times \cdot z = Q + Q \(\hat{z} + \cdot \cdo
	where Q Q2,, Qe are non-negative deli
	$Q_i = \frac{1}{2}A_i \times i = 1 \dots k$ Set ra-k $A_i = r_i$
	Sch ra-k A:= T:
	IF r, + r2 + r = ~
	then:
	a) Q, Q, are i-dependent and b) Q: ~ 2 x2(r), i=1, k
	By the learn a an orthogonal waters C s.t.
	X= CY yrelds
	Q = 4 2 +
	8= 1 (14) 5
	Pr = Turker + Yu
	Where T To are independent N(O, or) Ty's and
	each occurs only in one term.

(Leek
$$\hat{x} \sim \mathcal{N}(0, \sigma T)$$

$$\hat{x} = C\hat{y} \qquad \mathcal{E}(\hat{y}) = \mathcal{E}(C'\hat{x}) = \hat{\sigma}$$

$$Co(\hat{y}) = Cou(C'\hat{x}) = Cou(X)C'$$

$$= \sigma^2 \in Tc' = \sigma^2 T$$

Now for represent the Love

$$SST0 = Y' \left[T - (\frac{1}{N})T\right]Y$$

$$SSE = Y' \left[T - (\frac{1}{N})T\right]Y$$

$$SSE = Y' \left[T - (\frac{1}{N})T\right]Y$$

$$CST0 = SSE + SSE$$

$$Y - (X\hat{y}) \sim \mathcal{N}(0, \sigma^2 T)$$

$$(Y - Y\hat{y})' \left[(Y - X\hat{y}) - (X\hat{y})' \left[T + (X\hat{y})'(X\hat{y})\right]\right]$$

$$= Y' \left[Y - Y' \left[X\hat{y} - (X\hat{y})' \left[T + (X\hat{y})'(X\hat{y})\right]\right]$$