SYSEN5200, MAE5920, CEE5252, EE5130, ORIE5142, SYSEN5210

Homework 5

You should describe any programs, spreadsheets or formulae used through program listings and sample output, or by printing out formulae in any spreadsheets.

On-campus students should drop their homework assignments in the box located on the second floor of Rhodes Hall. You may work with a partner and hand in a single homework with both your names and net IDs on it.

Off-campus students should use the electronic drop box to hand in a document named LastName_NetId_HomeworkNo. If you work in a pair then you should *each* hand in a copy of the homework with the file name YourLastName_YourNetId_OtherLastName_OtherNetId_HomeworkNo, and ensure that the names and netIDs of both students appear at the top of the file you hand in.

1. (40 pts) Consider a four-way stop intersection, in which cars arriving at the intersection from all four directions need to come to a full stop before proceeding into the intersection. When cars are present at multiple directions, then the drivers take turns, yielding right-of-way to the right. Suppose we are interested in the average waiting time of a vehicle at the intersection, the average number of vehicles waiting at the intersection at any given time, and the fraction of time that the intersection is full (has cars in all directions).

   In this problem you will do the preparation for modeling this intersection using discrete-event simulation. There is no need to write out the pseudo-code or do the actual manual simulation.

   (a) (20 pts) Describe the state of the system and the events that change the system state (you don't need to describe how the state is changed).

   (b) (20 pts) Give expressions for the statistics of interest (the average waiting time of a vehicle at the intersection, the average number of vehicles waiting at the intersection at any given time, and the fraction of time that the intersection is full) based on a single simulation run from time 0 to time T. Be sure to define any variables you use in your expressions.

2. (60 pts) Big Red Bikes is a small-scale bike-sharing business in Cornell. There are three stations that are approximately equally distanced from each other: station 1 at Rhodes Hall, Station 2 in Arts Quad, and Station 3 at Noyes. Students can use their Cornell ID to check out or return a bike at the kiosk next to any of the 3 stations. In general they only use the bikes to commute to their classes, so the bikes are usually checked out and returned at different stations. Suppose Cornell students are very lazy. If they walk to a station and there is no bike available, they will stand there and wait for someone else to return a bike; If they bike to a station and find it full, then they will wait there until someone else checks out a bike, making a dock available at that station. Suppose students can check out or return a bike (if bike/dock available) at a kiosk instantaneously. Each station has 3 docks, and at time 0 there are 2 bikes (and 1 empty dock) at each station.

   We wish to compute the total number of students who have completed their trips (NCS), fraction of time that a bike is available (waiting for customers in a dock) (FBA), the fraction of time that a dock is available (FDA), the average total time a student spends in the system (TIS), the average waiting time for a bike (WB), and the average waiting time for a dock (WD). To give you an example on some of those statistics, suppose you simulated the system for 100 minutes and had 20 students who have come to check out bikes. 15 of them have already returned the bike (NCS). If the sum of total time spent in the docks by the 6 bikes in the system is 300 minutes, then FBA$= 300/(100*6) = 0.5$. If the sum of the total times from each student arriving at a station (wishing to check out a bike) to him/her returning a bike is 120 minutes, then TIS$= 120/20 = 6$ minutes. If there is a cumulative total of 30 minutes spent by students waiting at an empty station for a bike, then WB$= 30/20 = 1.5$ minutes.

   To simulate the Big Red Bikes system, one can use the template given by *hw5template.xlsx*. In there we describe the state of the system using the following quantities:

   - **NB1, NB2, NB3**: The number of bikes available at stations 1, 2, 3, respectively.
   - **NWB1, NWB2, NWB3**: The number of students waiting at stations 1, 2, and 3 for available bikes.

- **NWD1, NWD2, NWD3**: The number of students waiting at stations 1, 2, and 3 for available docks.

We describe changes to the system state using the events:

- **Arrive(i) (A(i))**: Arrival of a student wanting to check out a bike at station i. The index i takes values 1, 2, 3.
- **Depart(i) (D(i))**: Arrival of a student riding a bike wanting to return the bike at station i. The index i takes values 1, 2, 3. (We call it a Departure because the student is attempting a departure from the system.)
- **End (E)**: End of simulation.

Let T be the duration of the simulation period. The following statistics variables are not necessary to run the simulation, but they help to calculate the statistics of interest. To differentiate from the state variables, we underline the statistics variables.

- **<u>NAR</u>**: The total number of students who have arrived to pick up bikes.
- **<u>NIS</u>**: The number of students who are currently in the system (either riding a bike, or waiting for a bike/dock).
- **<u>NCS</u>**: The total number of students who have completed their trips by successfully returning the bikes. Note that **<u>NCS</u>** = **<u>NAR</u>** - **<u>NIS</u>** at any time.
- **<u>IntNIS</u>**: A running total of the quantity
$$\int_0^T NIS(t)\, dt,$$
where $NIS(t)$ is the value of **NIS** at time t.
- **<u>IntNB</u>**: A running total of the quantity
$$\int_0^T NB1(t) + NB2(t) + NB3(t)\, dt,$$
where $NB1(t), NB2(t), NB3(t)$ is the value of **NB1**, **NB2**, **NB3** at time t.
- **<u>IntND</u>**: A running total of the quantity
$$\int_0^T (3 - NB1(t)) + (3 - NB2(t)) + (3 - NB3(t))\, dt.$$
This is also equal to 9T-**IntNB**.
- **<u>IntNWB</u>**: The total time that students wait for checking out a bike. A running total of the quantity
$$\int_0^T NWB1(t) + NWB2(t) + NWB3(t)\, dt,$$
where $NWB1(t)$, $NWB2(t)$, $NWB3(t)$ are the value of **NWB1**, **NWB2**, and **NWB3** at time t.
- **<u>IntNWD</u>**: The total time that students wait for returning a bike. Similar to **IntNWB**, but for **NWD1**, **NWD2**, and **NWD3** instead.

(a) (5 pts) Describe how you would calculated the statistics of interest based on the state variables given in the problem, given that your simulation period is T.

(b) (30 pts) Write precise pseudo code that describes how different events change the state of the system, and what future events should be scheduled at the occurrence of each event.

(c) (25 pts) Use *hw5template.xlsx* to manually simulate Big Red Bikes over 20 minutes. See the sheet *Data* for the arrival times of students at the stations and their trip informations (For example, at 10.9 minutes into the simulation, a student shows up at station 1 and tries to check out a bike to ride to station 3. The time it takes him/her to reach station 3 is 5.4 minutes.). There are two lines given in the template for checking your simulation. Make sure that you compute the statistics of interest in your manual simulation. Clearly report these statistics.