1. a) neither

b) valid

c)

| P | Q | R | A= Q∨R | B= P⇒Q | C= BYPHAVR | ¬C |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 |

unsatisfiable

d) neither

e)

| P | Q | R | S | A= P⇒Q | B= Q⇒R | C= R⇔S | D= S∧¬P | ANDACD |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |

unsatisfiable

2. a. $\sim(P \wedge Q)$

$\sim P \vee \sim Q$

$\{\sim P, \sim Q\}$

b. $\sim(P \vee Q)$

$\sim P \wedge \sim Q$

$\{\sim P\}, \{\sim Q\}$

c. $(P \vee Q) \Rightarrow R$

$\sim(P \vee Q) \vee R$

$\sim P \wedge \sim Q \vee R$

$\{\sim P\}, \{\sim Q, R\}$

d. $\sim((P \vee Q) \Rightarrow R)$

$\sim(\sim P \wedge \sim Q \vee R)$

$P \vee Q \wedge \sim R$

$\{P, Q\}, \{\sim R\}$

3. "Pigs are animals": $\boxed{\forall x\, Pig(x) \Rightarrow Animal(x)}$

"The head of a pig is the head of an animal": $\boxed{\forall x,y\, Pig(x) \wedge HeadOf(x,y) \Rightarrow Animal(y)}$

4. a) "John did something to annoy Mary

Annoy(x,y): where x did something to annoy Mary

John(x): where x is John

Mary(x): where x is Mary

$\boxed{\exists x \exists y: John(x) \wedge Mary(y) \wedge Annoy(x,y)}$

b. "If you push anything hard enough, it will fall over"

Thing(x): x is a thing

PushHard(x): x is pushed hard

FallOver(x): x falls over

$\boxed{\forall x: Thing(x) \wedge PushHard(x) \Rightarrow FallOver(x)}$

c. "If I am ugly, you are a monkey's uncle"

I(x): x is me

You(x): x is you

Ugly(x): x is ugly

MonkeyUncle(x): x is a monkey's uncle

$\boxed{\exists x \exists y: I(x) \wedge You(y) \wedge Ugly(x) \Rightarrow MonkeyUncle(y)}$

d. "It takes two to start a fight"

StartFight(x,y): x and y start a fight together

$\boxed{\forall x \forall y: (x \neq y) \wedge (StartFight(x,y))}$

5. a) $P(A, \sim FA, G, \sim FG, CT)$

$P(A|\sim FA, G) \cdot P(\sim FA) \cdot P(G|CT, \sim FG) \cdot P(\sim FG|CT) \cdot P(CT)$

$1 \qquad 1-fa \qquad x \qquad 1-n \qquad m$

$1 \cdot (1-fa) \cdot x \cdot (1-n) \cdot m = \boxed{(1-fa) \cdot x \cdot (1-n) \cdot m}$

b) $P(G|CT) = P(G|FG, CT) \cdot P(FG|CT) \cdot P(G|\sim FG, CT) \cdot P(\sim FG|CT)$

$y \qquad n \qquad x \qquad 1-n$

$\boxed{y \cdot n \cdot x \cdot (1-n)}$

d) $P(CT|\sim FA, \sim FG) = \boxed{\frac{P(CT, \sim FA, \sim FG)}{P(\sim FA, \sim FG)}}$