



Image Analysis and Object Recognition

Exercise 2

Summer Semester 2024

(Course materials for internal use only!)

Computer Vision in Engineering – Prof. Dr. Rodehorst

M.Sc. Mariya Kaisheva

mariya.kaisheva@uni-weimar.de

Agenda

Topics:

- Assignment 1.** Image enhancement, Binarization, Morphological operators
- Assignment 2.** **Gradient of Gaussian filtering, Förstner interest operator**
- Assignment 3.** Shape detection based on Hough-voting
- Assignment 4.** Filtering in the frequency domain, Fourier descriptors for shape recognition
- Assignment 5.** Image segmentation using clustering
- Assignment 6.** Convolutional neural networks for image classification
- Final Project.** - *Will be announced during the last exercise class* -

Agenda

Start date and submission deadlines:

Assignment 1.	18.04.24 – 01.05.24
Assignment 2.	02.05.24 – 15.05.24
Assignment 3.	16.05.24 – 29.05.24
Assignment 4.	30.05.24 – 12.06.24
Assignment 5.	13.06.24 – 26.06.24
Assignment 6.	27.06.24 – 10.07.24
Final Project.	11.07.24 – 22.09.24

Wednesday by 23:00
(Central European Time)



Assignment 1: Sample Solution

Assignment 1: Overview

Topics:

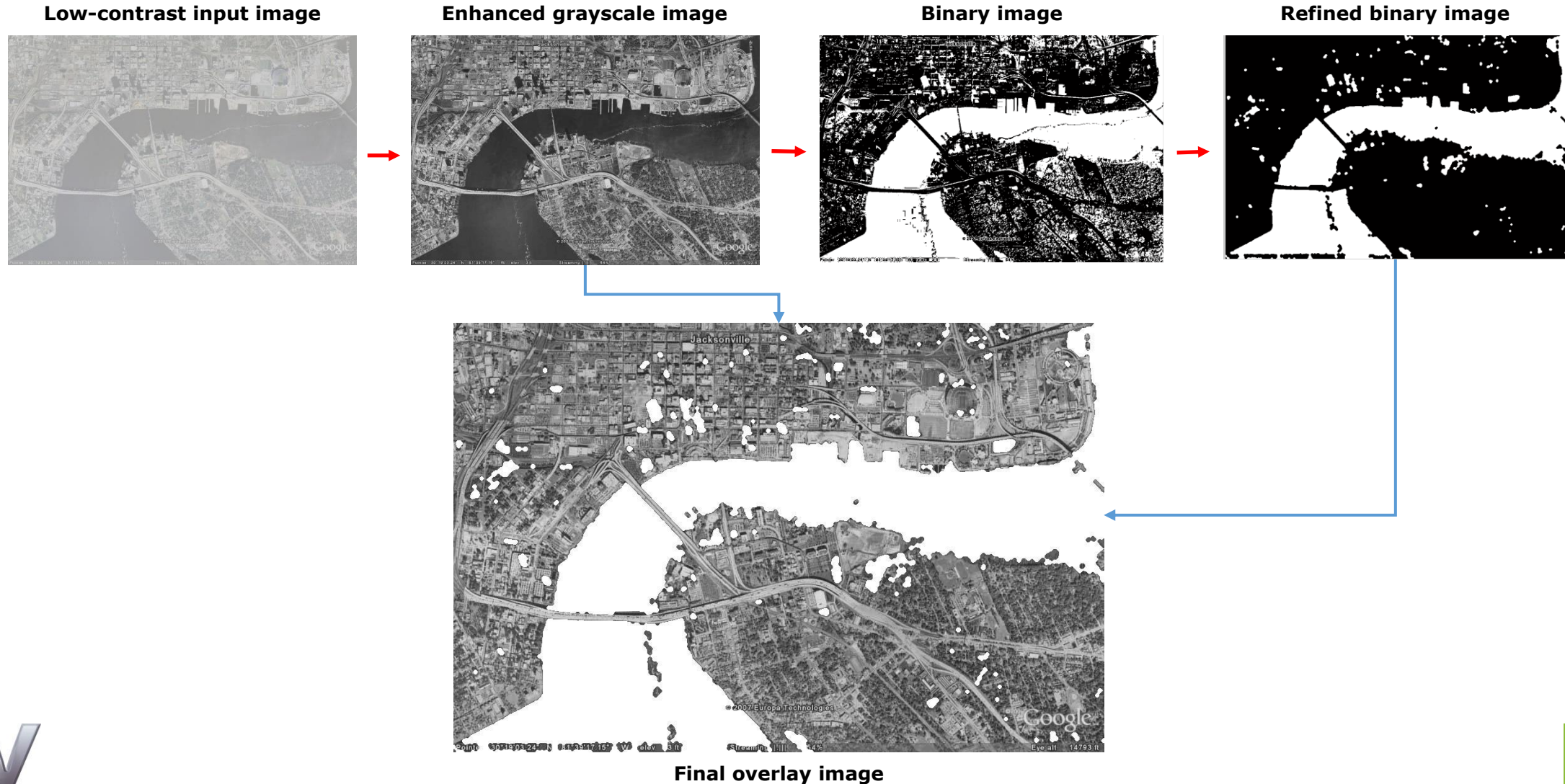
- Image enhancement → histogram stretching
- Global Thresholding → binary mask
- Morphological operators → opening, closing

Goal:

- Getting familiar with MATLAB
- Extracting image pixels representing foreground objects
→ e.g. extraction of the water regions



Assignment 1: Workflow



```

function Assignment1a                                     % task 4
% =====
input = rgb2gray(imread('input_sat_image.jpg'));
improved = Enhancing(input);
output = imoverlay(improved, Filtering(Thresholding(improved)), 'white');
figure, imshow(output), title('Output image');

function output = Enhancing(input)                       % task 1
% =====
figure, imshow(input), title('Input image');
figure, imhist(input), title('Grayvalue histogram');
output = imadjust(input);
figure, imhist(output), title('Stretched histogram');
figure, imshow(output), title('Enhanced image');

function output = Thresholding(input)                   % task 2
% =====
thresh = graythresh(input);
output = imcomplement(imbinarize(input, thresh));
figure, imshow(output), title('Binary mask');

function output = Filtering(input)                      % task 3
% =====
se = strel('disk', 7);
output = imclose(imopen(input, se), se);
figure, imshow(output), title('Morphological Filtering');

```

```
function Assignment1a                                     % task 4
% =====
input = rgb2gray(imread('input_sat_image.jpg'));
improved = Enhancing(input);
output = imoverlay(improved, Filtering(Thresholding(improved)), 'white');
figure, imshow(output), title('Output image');
```

```
function output = Enhancing(input)                       % task 1
% =====
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figure, imhist(input), title('Grayvalue histogram');
output = imadjust(input);
figure, imhist(output), title('Stretched histogram');
figure, imshow(output), title('Enhanced image');
```

```
function output = Thresholding(input)                   % task 2
% =====
thresh = graythresh(input);
output = imcomplement(imbinarize(input, thresh));
figure, imshow(output), title('Binary mask');
```

```
function output = Filtering(input)                     % task 3
% =====
se = strel('disk', 7);
output = imclose(imopen(input, se), se);
figure, imshow(output), title('Morphological Filtering');
```

convenience function also
available in Octave

(However, for this assignment we asked you to
implement contrast stretching on your own!)


```

function Assignment1a                                     % task 2
% =====
input = rgb2gray(imread('input_sat_image.jpg'));
improved = Enhancing(input);
output = imoverlay(improved, Filtering(Thresholding(improved)), 'white');
figure, imshow(output), title('Output image');

function output = Enhancing(input)                       % task 1
% =====
figure, imshow(input), title('Input image');
figure, imhist(input), title('Grayvalue histogram');
output = imadjust(input);
figure, imhist(output), title('Stretched histogram');
figure, imshow(output), title('Enhanced image');

function output = Thresholding(input)                   % task 2
% =====
thresh = graythresh(input);
output = imcomplement(imbinarize(input, thresh));
figure, imshow(output), title('Binary mask');

function output = Filtering(input)                      % task 3
% =====
se = strel('disk', 7);
output = imclose(imopen(input, se), se);
figure, imshow(output), title('Morphological Filtering');

```

The equivalent function
in Octave is **im2bw()**

```
function Assignment1b                                     % task 4
% =====
input = double(imread('input_sat_image.jpg')) / 255;
improved = Enhancing(mean(input, 3));
mask = Filtering(Thresholding(improved)); improved(mask) = 1;
figure, imshow(improved), title('Output image');

function output = Enhancing(input)                        % task 1
% =====
figure, imshow(input), title('Input image');
figure, imhist(input), title('Grayvalue histogram');
mn = min(min(input)); mx = max(input(:));
output = (input - mn) / (mx - mn);
figure, imhist(output), title('Stretched histogram');
figure, imshow(output), title('Enhanced image');

function output = Thresholding(input)                    % task 2
% =====
thresh = graythresh(input);
output = ~(input > thresh);
figure, imshow(output), title('Binary mask');

function output = Filtering(input)                       % task 3
% =====
se = ones(15, 15);
output = imclose(imopen(input, se), se);
figure, imshow(output), title('Morphological Filtering');
```

```
function Assignment1b                                     % task 4
%
input = double(imread('input_sat_image.jpg')) / 255;
improved = Enhancing(mean(input, 3));
mask = Filtering(Thresholding(improved)); improved(mask) = 1;
figure, imshow(improved), title('Output image');

function output = Enhancing(input)                       % task 1
%
figure, imshow(input), title('Input image');
figure, imhist(input), title('Grayvalue histogram');
mn = min(min(input)); mx = max(input(:));
output = (input - mn) / (mx - mn);
figure, imhist(output), title('Stretched histogram');
figure, imshow(output), title('Enhanced image');

function output = Thresholding(input)                   % task 2
%
thresh = graythresh(input);
output = ~(input > thresh);
figure, imshow(output), title('Binary mask');

function output = Filtering(input)                      % task 3
%
se = ones(15, 15);
output = imclose(imopen(input, se), se);
figure, imshow(output), title('Morphological Filtering');
```

work with **double values**
to avoid rounding errors

```
function Assignment1b                                     % task 4
% =====
input = double(imread('input_sat_image.jpg')) / 255;
improved = Enhancing(mean(input, 3));
mask = Filtering(Thresholding(improved)); improved(mask) = 1;
figure, imshow(improved), title('Output image');

function output = Enhancing(input)                       % task 1
% =====
figure, imshow(input), title('Input image');
figure, imhist(input), title('Grayvalue histogram');
mn = min(min(input)); mx = max(input(:));
output = (input - mn) / (mx - mn);
figure, imhist(output), title('Stretched histogram');
figure, imshow(output), title('Enhanced image');

function output = Thresholding(input)                   % task 2
% =====
thresh = graythresh(input);
output = ~(input > thresh);
figure, imshow(output), title('Binary mask');

function output = Filtering(input)                      % task 3
% =====
se = ones(15, 15);
output = imclose(imopen(input, se), se);
figure, imshow(output), title('Morphological Filtering');
```

same result with
different syntax

Octave

```
function Assignment1b
```

```
% task 4
```

```
% =====
```

```
input = double(imread('input_sat_image.jpg')) / 255;
```

```
improved = Enhancing(mean(input, 3));
```

```
mask = Filtering(Thresholding(improved)); improved(mask) = 1;
```

```
figure, imshow(improved), title('Output image');
```

```
function output = Enhancing(input)
```

```
% task 1
```

```
% =====
```

```
figure, imshow(input), title('Input image');
```

```
figure, imhist(input), title('Grayvalue histogram');
```

```
mn = min(min(input)); mx = max(input(:));
```

```
output = (input - mn) / (mx - mn);
```

```
figure, imhist(output), title('Stretched histogram');
```

```
figure, imshow(output), title('Enhanced image');
```

```
function output = Thresholding(input)
```

```
% task 2
```

```
% =====
```

```
thresh = graythresh(input);
```

```
output = ~(input > thresh);
```

```
figure, imshow(output), title('Binary mask');
```

```
function output = Filtering(input)
```

```
% task 3
```

```
% =====
```

```
se = ones(15, 15);
```

```
output = imclose(imopen(input, se), se);
```

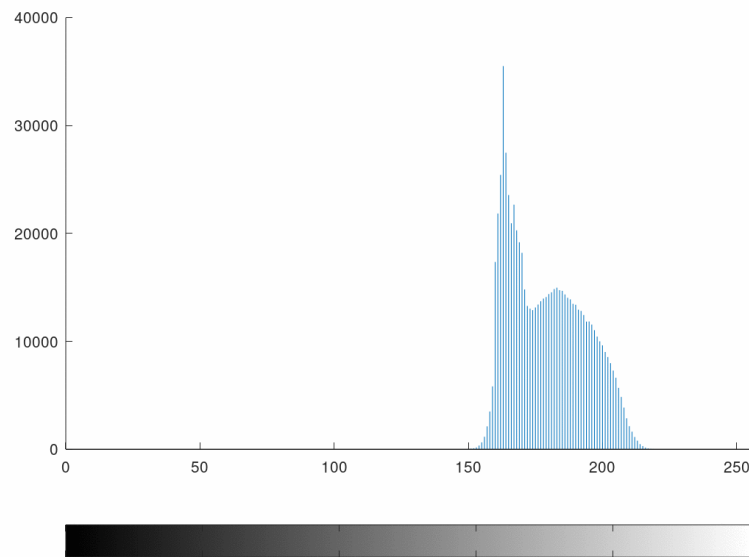
```
figure, imshow(output), title('Morphological Filtering');
```

logical values →

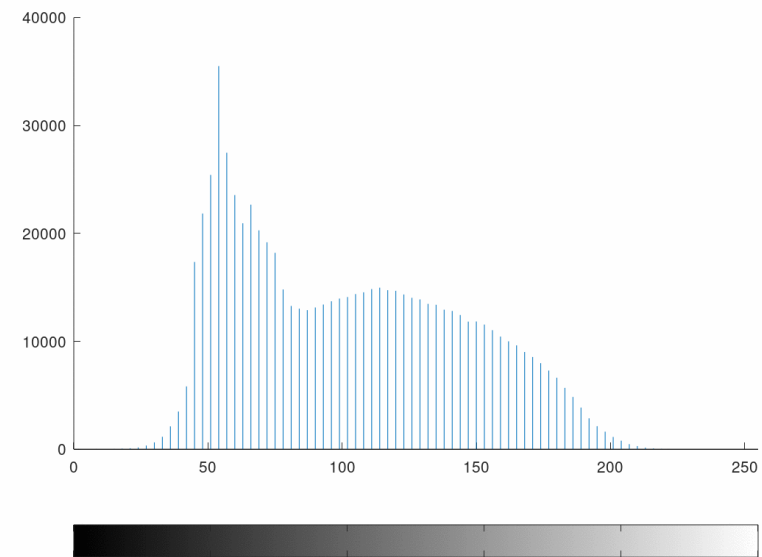


Assignment 1 – sample results

Initial Histogram



Histogram after Enhancement



Assignment 1 – sample results

Enhanced grayscale image



Influence of the size of the structuring element
on the refined binary mask



SE: disk, 5



SE: disk, 7

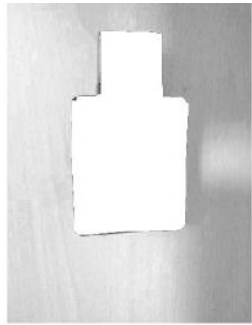


SE: disk, 10

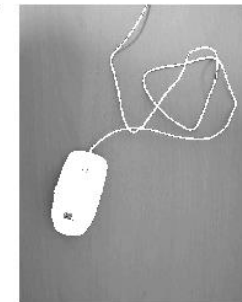
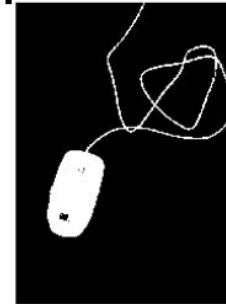
Assignment 1 – selected submission results



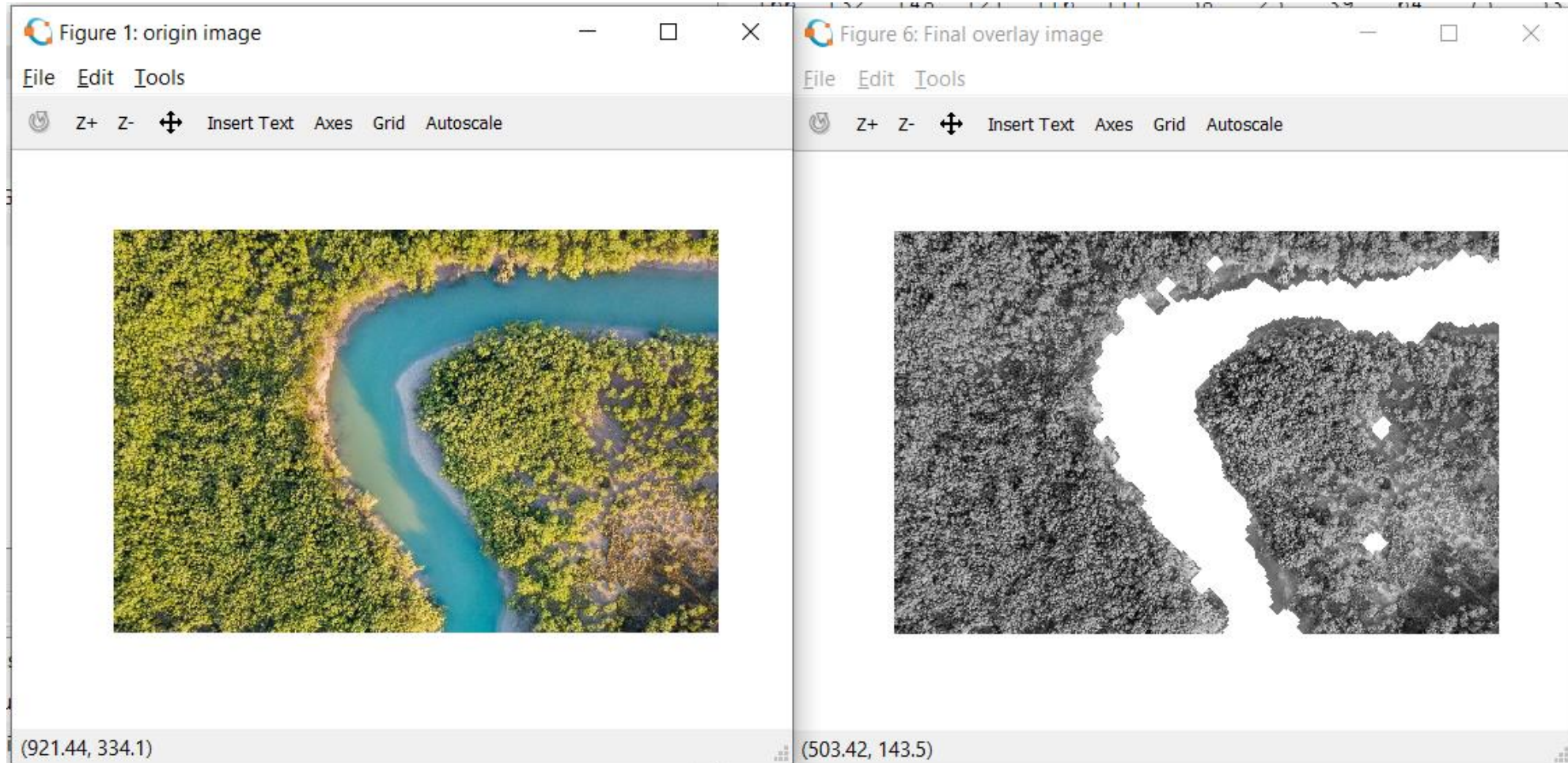
```
2 | image=enhance(0,15);  
63 | SE = strel ("square", 1);  
64 | newImage=imopen (image, SE);  
65 | %structural element shape and size for Closing.  
66 | SE = strel ("square", 35);  
67 | newImage=imclose(newImage, SE);
```



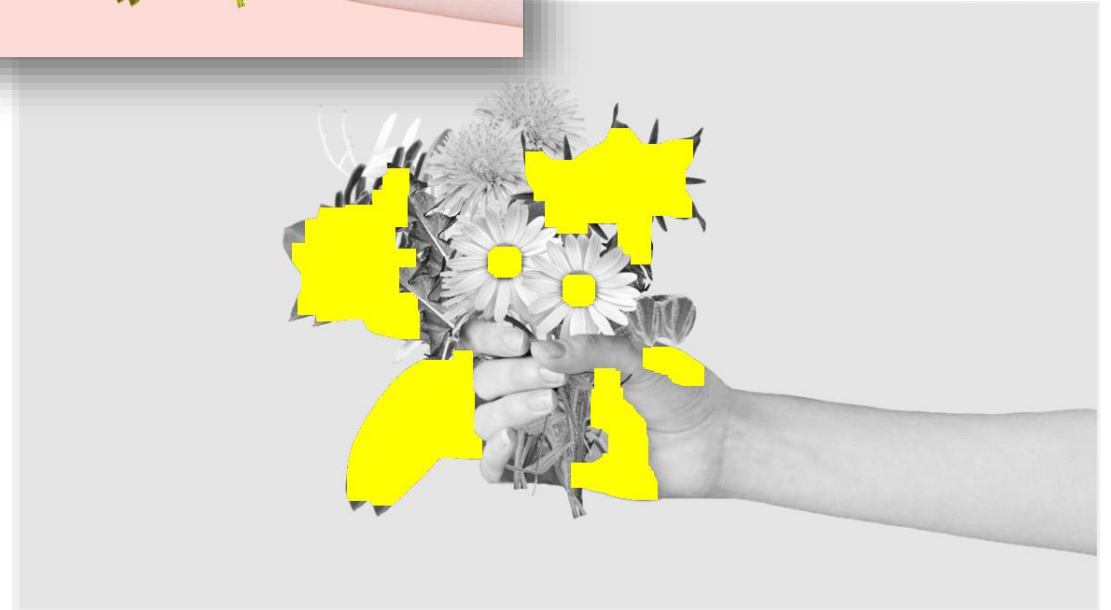
```
2 | image=enhance(0,0);  
63 | SE = strel ("square", 2);  
64 | newImage=imopen (image, SE);  
65 | %structural element shape and size for Closing.  
66 | SE = strel ("square", 2);  
67 | newImage=imclose(newImage, SE);
```



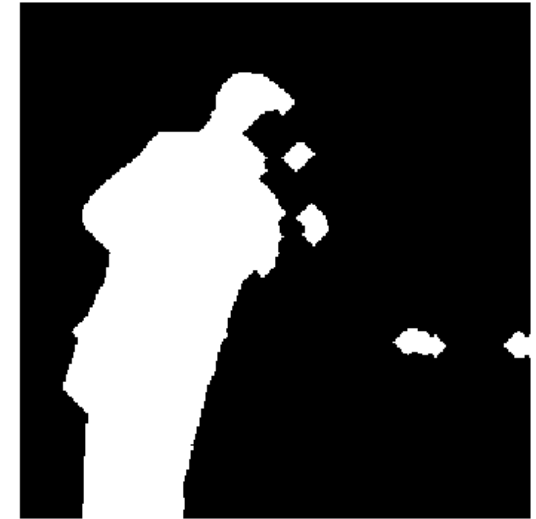
Assignment 1 – selected submission results



Assignment 1 – selected submission results



Assignment 1 – selected submission results





Assignment 2

Assignment 2: Overview

Topics:

- Image filtering with Gradient of Gaussian (GoG)
- Interest points

Goal:

- Learn how to perform image filtering
- Practice reducing noise and **simultaneously** deriving image gradients (intensity changes)
- Practice identifying points of interest with the help of image gradients

Assignment 2: Overview

Input:

- Provided image → *ampelmaennchen.png*
- Or a different image of your own choice

Tasks:

- **A:** Gradient of Gaussian (GoG) filtering
- **B:** Förstner operator

The topics of GoG filtering and Förstner operator will be covered in detail during **lecture number 5.**



Provided
input image

Assignment 2: Task A

GoG filtering:

- Compute **GoG-filter kernels** for filtering in x- and y- direction
- Apply the two filters G_x and G_y on the input image I using convolution to derive **two gradient images I_x and I_y**
- Compute and visualize the **gradient magnitude**

$$G = \sqrt{I_x^2 + I_y^2}$$



Input image I



Grayscale converted I

Note:

Compute grayscale image and scale it to double [0.0 , 1.0].

Assignment 2: Task A

GoG filtering:

- Compute **GoG-filter kernels** for filtering in x- and y- direction
- Apply the two filters G_x and G_y on the input image I using convolution to derive **two gradient images I_x and I_y**
- Compute and visualize the **gradient magnitude**

$$G = \sqrt{I_x^2 + I_y^2}$$



Gradient image I_x



Gradient image I_y

Assignment 2: Task A

GoG filtering:

- Compute **GoG-filter kernels** for filtering in x- and y- direction
- Apply the two filters G_x and G_y on the input image I using convolution to derive **two gradient images I_x and I_y**
- Compute and visualize the **gradient magnitude**

$$G = \sqrt{I_x^2 + I_y^2}$$



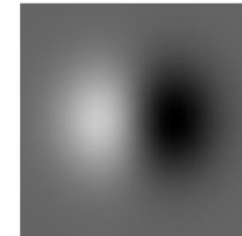
Gradient
magnitude

2D GoG filter computation

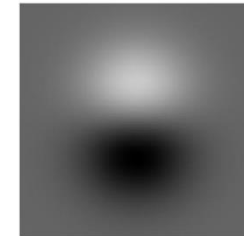
$$G_x = \frac{\partial G(x,y,\sigma)}{\partial x} = -\frac{x}{2\pi\sigma^4} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

general formula for computation of G_x

G_x



G_y

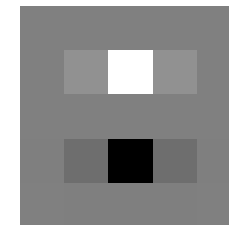
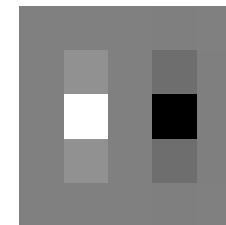


general
GoG filters
in x and y
directions

$$G_y = G_x^T$$

$$G_x = \begin{bmatrix} 0.0000 & 0.0001 & 0.0000 & -0.0001 & -0.0000 \\ 0.0002 & 0.0466 & 0.0000 & -0.0466 & -0.0002 \\ 0.0017 & 0.3446 & 0.0000 & -0.3446 & -0.0017 \\ 0.0002 & 0.0466 & 0.0000 & -0.0466 & -0.0002 \\ 0.0000 & 0.0001 & 0.0000 & -0.0001 & -0.0000 \end{bmatrix}$$

numerical example with $\sigma = 0.5$

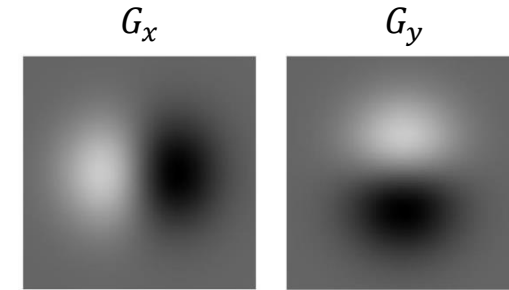


5x5 pixel
GoG filters
in x and y
directions

2D GoG filter computation

$$G_x = \frac{\partial G(x,y,\sigma)}{\partial x} = -\frac{x}{2\pi\sigma^4} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

general formula for computation of G_x



$$G_y = G_x^T$$

- 1) Define standard deviation, e.g. $\sigma = 0.5$
- 2) Filter kernel radius: $r = \lceil 3 \cdot \sigma \rceil = 2.0$
- 3) Define two arrays c_x and c_y with $(r \cdot 2 + 1)$ columns and rows for centered local coordinates

$$c_x = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix}; \quad c_y = c_x^T$$

- 4) Compute filter using c_x and c_y for x and y $\rightarrow \frac{\partial G(x,y,\sigma)}{\partial x} = -\frac{c_x}{2\pi\sigma^4} \exp\left(-\frac{(c_x^2 + c_y^2)}{2\sigma^2}\right)$

Assignment 2: Task B

Förstner interest operator:

- a. Compute the **autocorrelation matrix** M for each pixel using a 5×5 moving window
- b. Instead of storing M for each pixel, compute the **cornerness** w and **roundness** q from M and store these values in matrices W and Q . Plot these arrays.
- c. Derive a **binary mask** M_c of potential interest points by simultaneously applying thresholds, e.g. $t_w = 0.004$ and $t_q = 0.5$, on W and Q
- d. Plot an overlay of the initial input image and the **detected points**

Auto-correlation Matrix M

- Identification of corners
- Input: First order derivatives in x- and y-direction I_x and I_y (i.e. the result of Task A.b.)



Grayscale image



I_x (GoG)



I_y (GoG)

Auto-correlation Matrix M

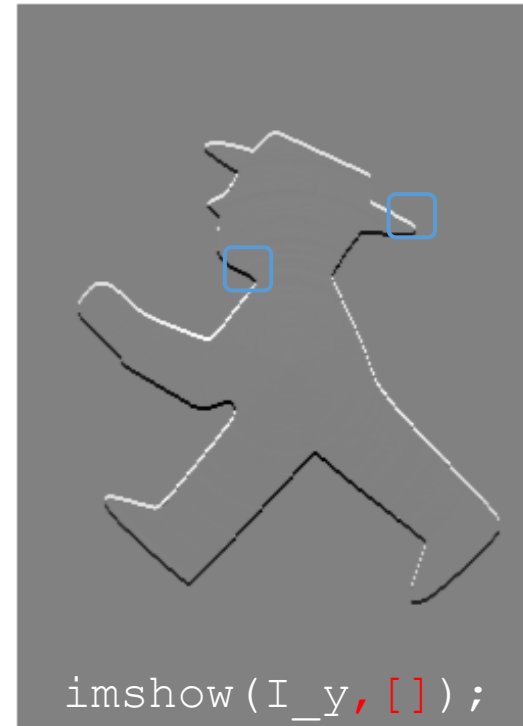
- Identification of **corners**
- Input: First order derivatives in x- and y-direction I_x and I_y (i.e. the result of Task A.b.)



Grayscale image



I_x (GoG)



I_y (GoG)

Auto-correlation Matrix M

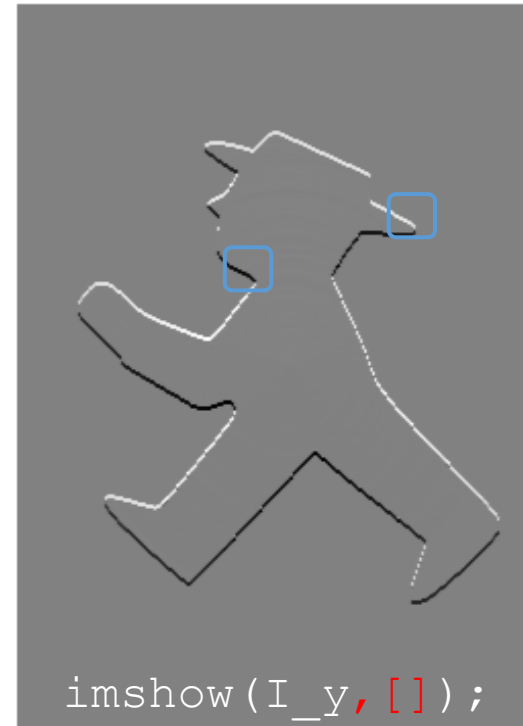
- Identification of **corners**
- Input: First order derivatives in x- and y-direction I_x and I_y (i.e. the result of Task A.b.)



Grayscale image



I_x (GoG)



I_y (GoG)



I_x^2, I_y^2
and
 $I_x I_y$
3 arrays
necessary for
the next
steps

Auto-correlation Matrix M

Computation of M for each pixel:

Definition: $w_N = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & \color{red}{1} & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow$ weights for the local neighborhood N

$$M = \sum_{x,y \in N} w_N(x,y) \cdot \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = w_N \star \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$\rightarrow M = \sum_{x,y \in N} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$ M contains the sum of all values of I_x^2 , I_y^2 and $I_x I_y$ in the local neighborhood N of 5x5 pixels

Auto-correlation Matrix M

For each pixel in the image (except boundaries):

1) Extract local image window w_N for I_x^2 , I_y^2 and $I_x I_y$

2) Compute M :

→ sum up extracted values I_x^2 , I_y^2 and $I_x I_y$

→ $\bar{I}_x^2 = \sum_N I_x^2$, also for \bar{I}_y^2 and $\bar{I}_x \bar{I}_y$

3) Build M (2×2 matrix) for each pixel

$$M = \begin{bmatrix} \bar{I}_x^2 & \bar{I}_x \bar{I}_y \\ \bar{I}_x \bar{I}_y & \bar{I}_y^2 \end{bmatrix}$$

Or: convolve I_x^2 , I_y^2 and $I_x I_y$ with w_N and then compute M for each pixel

Auto-correlation Matrix M

Cornerness:

$$w = \frac{\text{trace}(M)}{2} - \sqrt{\left(\frac{\text{trace}(M)}{2}\right)^2 - \det(M)}, \quad w > 0$$

Roundness:

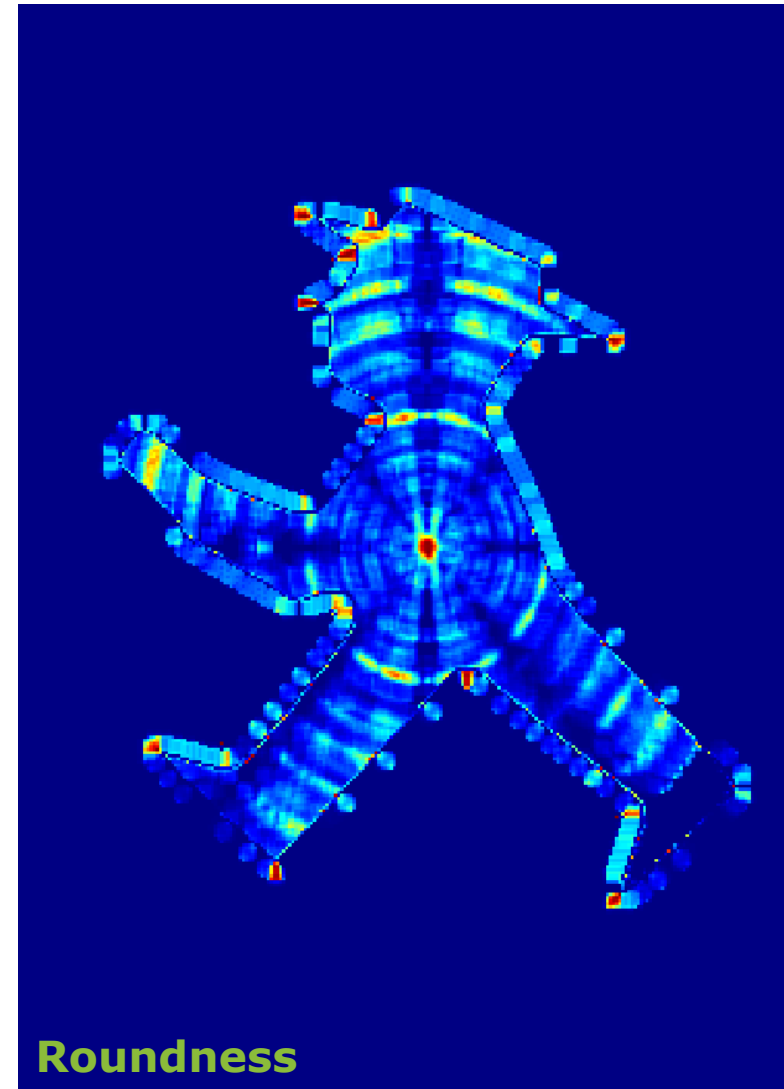
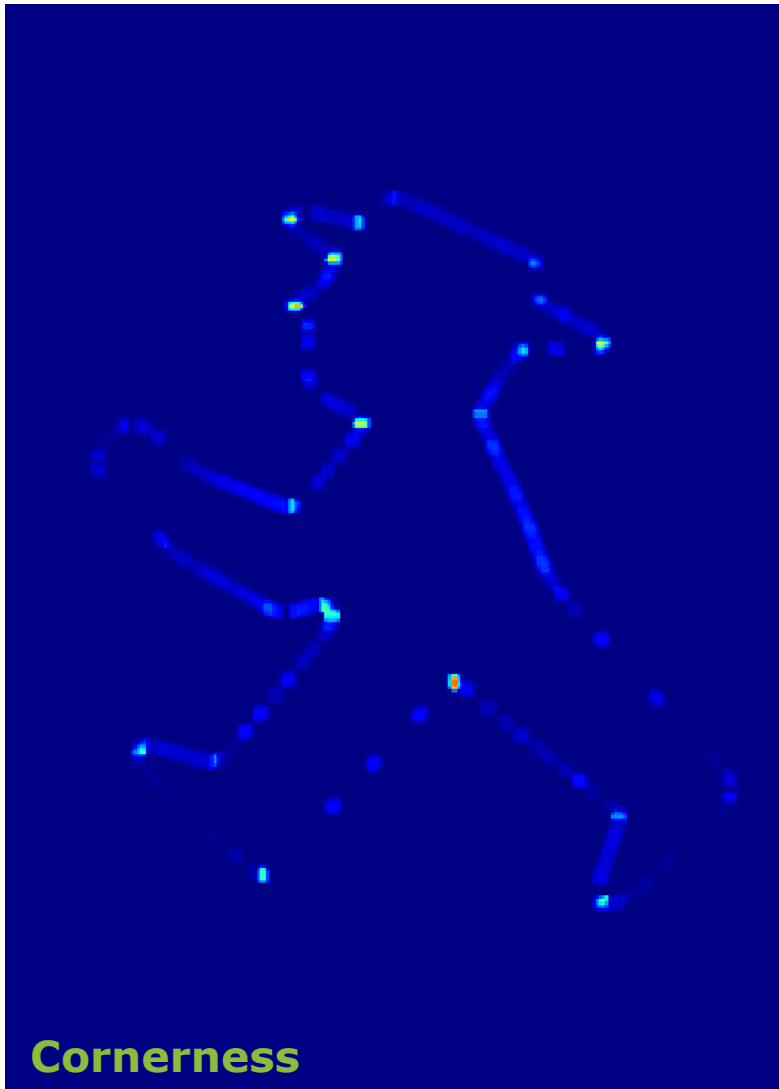
$$q = \frac{4 \cdot \det(M)}{\text{trace}(M)^2}, \quad 0 \leq q \leq 1$$

Find **corner point candidates** M_c , if

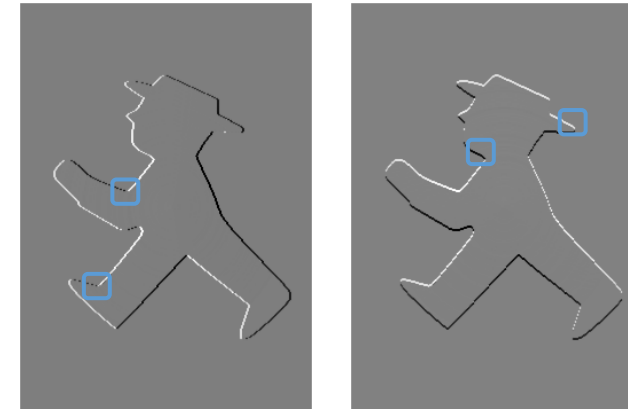
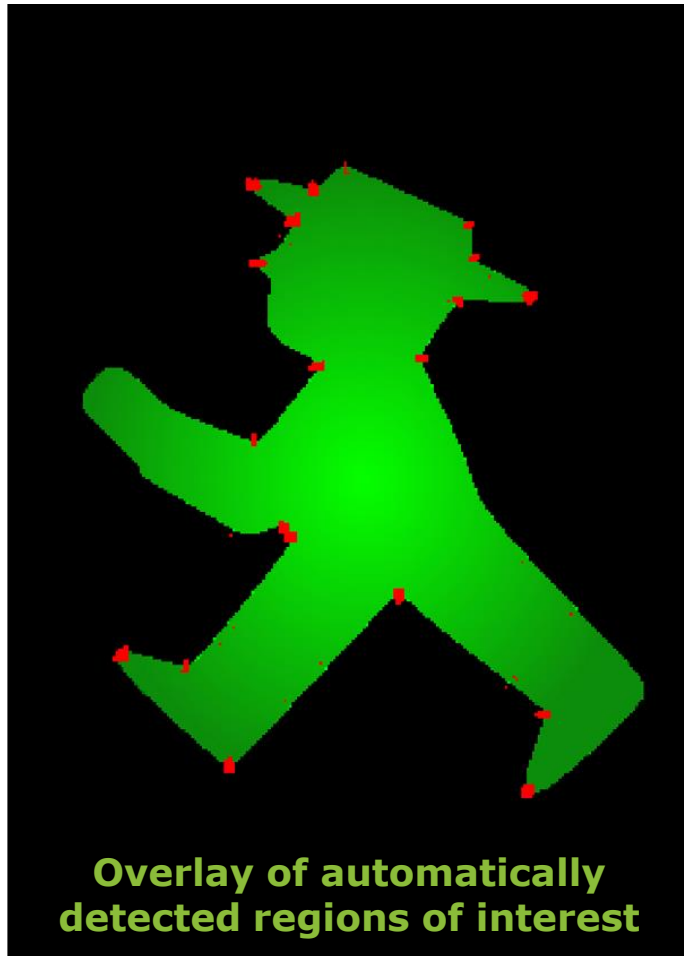
$$w > t_w \quad \text{and} \quad q > t_q$$

$$t_w = [0.001 \dots 0.01], t_q = [0.5 \dots 0.75]$$

Thresholded regions of w and q



Overlay of I with M_c



Gradient images I_x and I_y

□ areas expected to be detected as interest regions

Assignment 2: Task B

Förstner interest operator:

- Compute the **autocorrelation matrix** M for each pixel using a 5×5 moving window
- Instead of storing M for each pixel, compute the **corneriness** w and **roundness** q from M and store these values in matrices W and Q . Plot these arrays
- Derive a **binary mask** M_c of potential interest points by simultaneously applying thresholds, e.g. $t_w = 0.004$ and $t_q = 0.5$, on W and Q
- Plot an overlay of the initial input image and the **detected points**



Original input image I
overlaid with
detected interest points