SWING UP AND STABILIZATION OF ROTARY INVERTED PENDULUM

Lab report

submitted by

Bhagat Ram Labana(22m1083)
Shubham Dey(22m1084)

Master of technology
Control and Computing

Under the guidance of
prof. Debasattam Pal

Department of electrical engineering

INDIAN INSTITUTE OF TECHNOLOGY,BOMBAY

February,2023

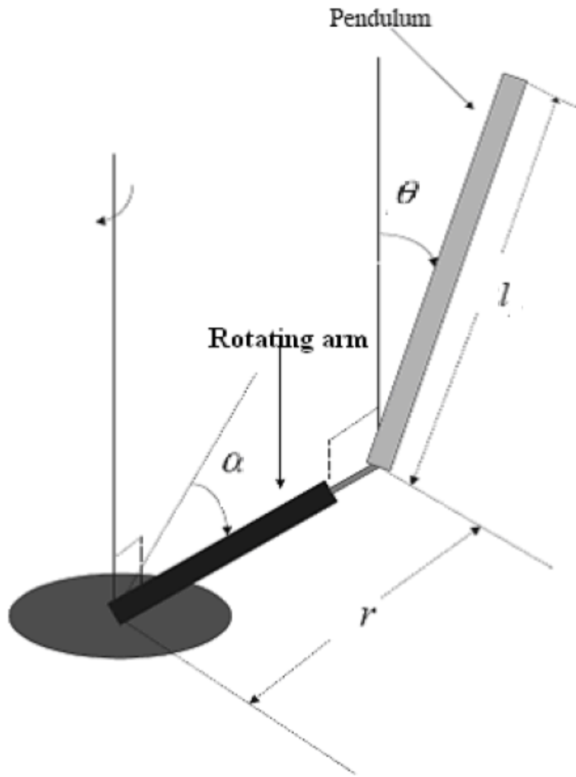
0.1 Swing Up Rotary inverted pendulum

Aim:

The aim of the experiment is to swing up the rotary inverted pendulum and then stabilize it in the upright position. Swing up of the pendulum is done by the energy control. and the LQR Technique is used to stablize the pendulam in upright position.

0.1.1 Background information on the topic of the lab:

In this experiment the Design of the optimum controller for the nonlinear rotary inverted pendulum (RIP) using linear quadratic regulator (LQR) is done. First we tried to make the pendulum swing in the upright position from the downward position. Swingup is done using the energy control. To perform energy control it is necessary to understand how the energy is influenced by the acceleration of the pivot. The Using LQR (Linear quadratic control) the stabilization is done. LQR, an optimal control technique is generally used for the control of linear dynamical systems. Non linear system states are fed to LQR which is designed using linear state-space equations. inverted pendulum, is a highly non-linear unstable system. Here the objective of the controller is to control the system in such a way that the arm reaches A desired position and the inverted pendulum stabilizes in the upright position. MATLAB-SIMULINK model For the implementation of control plans have been developed. A Model switch controller is used to switch the control of the system from the swing up to the Stabilizaton LQR controller and from LQR to Swing up control in certain conditions.



Schematic representation of the plant

In this experiment, we used the simulink MODELs to make rotary inverted pendulum and did simulation on it.

0.1.2 Swing Up:

Inputs to Algorithm: Θ , $\dot{\Theta}$, α , $\dot{\alpha}$. here Θ is the angle made by the pendulum with vertical and α is the angle made with the horizontal.
Algorithm output: torque,

The swing up controls general framework includes, the retrieving the angle and the angular velocity of the rotary arm and pendulum both. with these quantity we are able to find the energy of the pendulum at every position. the equation involved to find the energy of the pendulum at each position is given by

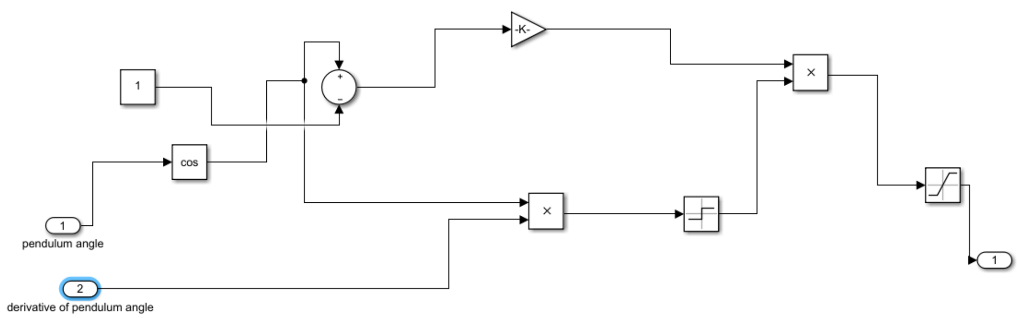
$$E = \frac{1}{2}J\dot{\Theta}^2 + mglcos(\Theta) \tag{1}$$

And the control law for the pendulum is given by the acceleration applied.

$$u = sat_{ng}(k(E - E_0)sign(\dot{\Theta}cos(\Theta)) \tag{2}$$

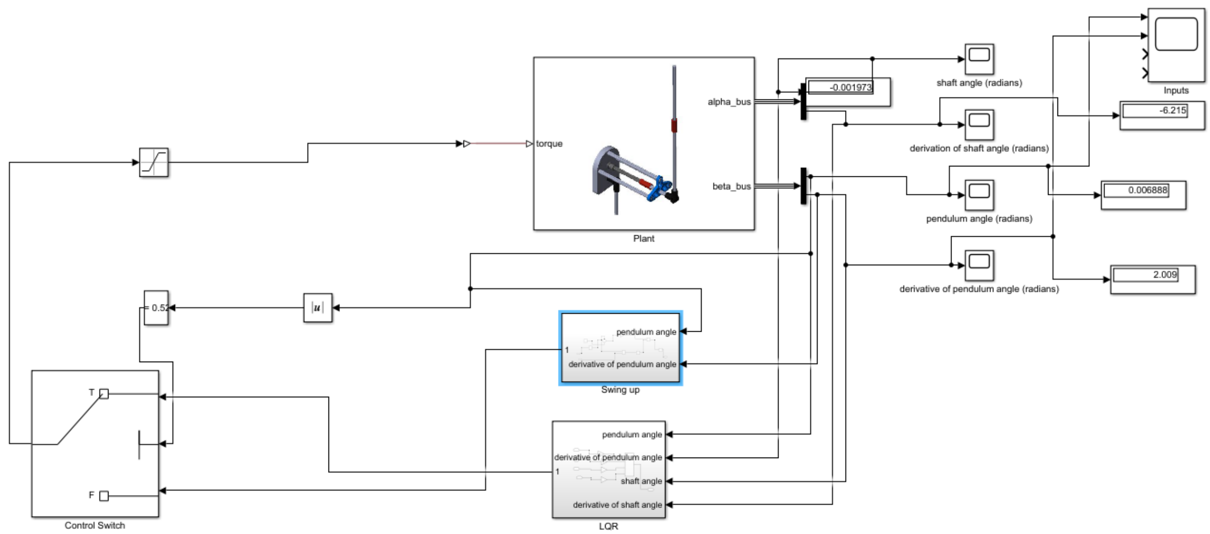
These above two equation has been implemented in the simulink model given below.

Simulink Model For Swing Up



Simulink Model For Energy Control

0.1.3 Simulation and Modelling



0.1.4 Stabilization using Linear quadratic regulator:

Inputs to Algorithm: Θ , $\dot{\Theta}$, α , $\dot{\alpha}$.
Algorithm output: Torque

The LQR Theory states that a nth order stabilizable system $\dot{x(t)} = Ax(t) + Bu(t)$ such that for all $t \geq 0, x(0) = x_0$ where $x(t) \in R^n$ is the state vector and $u(t) \in R^m$ is the input vector, determine the matrix gain $K \in R^{m*n}$ such that for the static, full-state feedback control law u(t) = –Kx(t), K is given by equation (4) where R is found out by Minimizing the cost functional J.

$$J = \int_0^\infty ((x(t))^T Q(x(t)) + (u(t))^T Ru(t))dt \tag{3}$$

$$K = R^{-1} B^T P \tag{4}$$

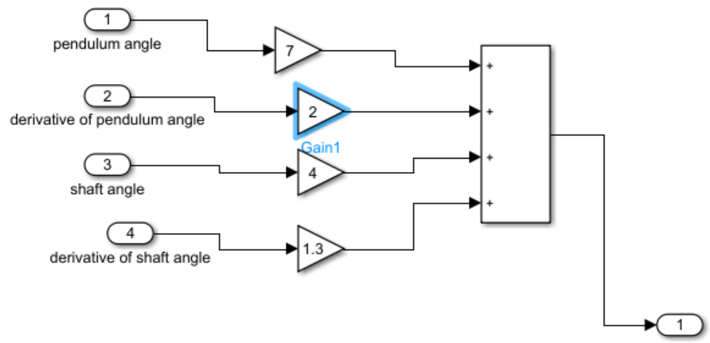
In equation(4) P is a non negative definite matrix satisfying the matrix Riccati equation,

$$A^T P + PA + Q - PBR^{-1} B^T P = 0 \tag{5}$$

Gain K Calulation Code

m = 3.3*10^-3;
$Jeq = 1.23 * 10^-4$;
$r = 0.109$;
$J = 1.1 * 10^-4$;
$l = 0.1832$;
$g = 9.81$;
$b = 0.0$;
$Rm = 3.30$;
$kb = 0.02797$;
$kt = 0.02797$;
$Er = 2 * m * g * l$;
$n = 2$;
$k_swing = 1.2$;
$AA = J * m * (r^2) + J * m * (l^2) + J * Jeq$;
$aa = (r * ((m * l)^2) * g)/AA$;
$bb = kt * kb * (J + m * l^2)/AA * Rm$;
$cc = (g * m * l) * (Jeq + (m * r^2))/AA$;
$dd = (m * r * l * kt * kb)/(AA * Rm)$;
$mm = ((J + m * (l^2)) * kt)/(AA * Rm)$;
$nn = (m * l * kt * r)/(AA * Rm)$;
$A = [0010; 0001; 0aa - bb0; 0ccdd0]$;
$B = [0; 0; mm; -nn]$;
$Q = diag([201015])$;
$R = 1$;
$K = lqr(A, B, Q, R)$;

Simulink Model For K gain Matrix



Simulink for K gain matrix

0.1.5 Procedure to do tuning:

- First of all we added path to the simulink model which was provided and we got to know we have to add few other blocks to implement rotary inverted pendulum .
- We tried to swing up the pendulum with energy control. First some dummy torque was given but with that pendulum was not going to upright position. then we applied some constant torque upto 75 degrees and then torque was made zero.In this case pendulum was going to upright position but we were not able to stabilize it using LQR stabilization method. then we tried to swing up the pendulum with energy control. with energy control we were able to swing up it with some oscillations.
- After swing up using energy control we implemented LQR and Calculated the The gain matrix K . Which comes into the action when the inverted pendulum lies within the desired boundary range of 30 degrees.
- Finally in the switching control we provided the condition that if the pendulum angle is lies outside the desired boundary range then swing up control takes place otherwise LQR stabilization will come into the action .In this way inverted pendulum was performed.

0.1.6 CONCLUSION:

Design of an optimal control technique in inverted pendulum was performed using three control actions which included swing up energy control,LQR stabilization an switching control.These control actions have been shown in MATLAB-SIMULINK model and the working of each control action has been discussed in this paper.

0.1.7 REFERENCES:

1. M. Siva Kumar, B. Dasu and G. Ramesh, ”Design Of LQR Based Stabilizer For Rotary Inverted Pendulum System” IJCTA, 9(29), 2016, pp. 21-29

2. K.J. Astrom, K. Furuta, ”Swinging up a pendulum by energy control”, Automatica 36 (2000) 287-295

3. Navin John Mathew, K. Koteswara Rao, N. Sivakumaran, ”Swing Up and Stabilization Control of a Rotary Inverted Pendulum”,IFAC DY-COPS 2013 December 18-20, 2013. Mumbai, India