

Course: 15MA102 - ENGINEERING MATHEMATICS

Event: B.Tech Even Semester- May 2017



USN NO:

II Semester B. Tech Examinations May 2017

Course Title: ENGINEERING MATHEMATICS-II

Duration: 3 Hours **Time:** 10:00 am - 1:00 pm Course Code:15MA102 Date: 11.05.2017 Max. Marks: 60

Note:

- 1. Each question carries 12 marks.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

SECTION-1

1. (a) Solve $z^3 - 4 = 0$.

[3 marks]

1. (b) Assuming that $|z_2|=1$ and $\bar{z_1}z_2\neq 1$, prove that

$$\left| \frac{z_1 - z_2}{1 - \bar{z_1} z_2} \right| = 1$$

[3 marks]

1. (c) State and prove the necessary conditions for a function to be complex differentiable.

[6 marks]

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OR

2. (a) Solve $z^8 = 1$ for z and factor it into quadratic factors with real coefficients. [7 marks]

2. (b) Suppose that v is a harmonic conjugate of u in a domain D and also that u is a harmonic conjugate of v in D. Show how it follows that both u(x, y) and v(x, y) must be constant throughout D.
[5 marks]

SECTION-2

3. (a) Evaluate the following integral:

i)
$$\frac{\cosh bt - \cos at}{t}$$
;

ii)
$$\int_0^\infty te^{-2t}\cos tdt;$$

[6 marks]

3. (b) Find the Laplace transform of the following periodic functions:

$$f(t) = \begin{cases} t & 0 \le t \le \pi \\ 2\pi - t & \pi \le t \le 2\pi \end{cases}$$

and $f(t+2\pi) = f(t)$.

[6 marks]

OR

- 4. (a) Find the Laplace transform of i) $e^{-3t} \sin 5t \sin 3t$ ii) $\int_0^\infty e^{-t} \frac{\sin t}{t}$ [6 marks]
- 4. (b) Solve using Laplace transform, $\frac{d^2y}{dx^2} 3\frac{dy}{dx} + 2y = 4$, given that y(0) = 2, y'(0) = 3. [6 marks]

SECTION-3

- 5. (a) With proper justification, determine if the following set V, forms a vector space over \mathbb{R} :
 All f satisfying f(x) = f(1-x) for all x. [6 marks]
- 5. (b) Let V be the vector space consisting of all real valued functions defined on the real line. Determine if the following subset of V is dependent or independent. Compute the dimension of the subspace spanned by the subset.

$$\{\cos^2 x, \sin^2 x\}$$

[2 marks]

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5. (c) Find an orthonormal basis for the subspace spanned by $x_1 = (1, 1, 1), x_2 = (1, 0, 1),$ [4 marks] $x_3 = (3, 2, 3).$

- 6. (a) When is a non empty set V, a vector space over a field \mathbb{F} ?
- 6. (b) Prove the following statement in a real euclidean space: $\langle x, y \rangle = 0$ if and only if ||x + y|| = ||x - y||.
- 6. (c) Find an orthonormal basis for the subspace spanned by $x_1 = (1, -1, 1, -1), x_2 = (5, 1, 1, 1),$ $x_3 = (-3, -3, 1, -3).$

SECTION-4

- 7. (a) Obtain the Fourier series of periodic function $f(x) = \frac{\pi x}{2}$ in $0 < x < 2\pi$. Hence deduce that $1-\frac13+\frac15-\frac17+\ldots=\frac\pi4$. 7. (b) Find the Fourier series of $f(x)=x^2-2$, when -2< x<2. [6 marks]
- [6 marks]
- 8. (a) Find the Fourier series of

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

8. (b) Obtain the Fourier series expansion of f(x), given that $f(x) = kx(\pi - x)$ on $0 < x < 2\pi$, where k is a constant.

SECTION-5

9. (a) Use the Fourier Integral Theorem to show that

$$\int_0^\infty \frac{\cos ax}{a^2 + 1} da = \frac{\pi}{2} e^{-x} (x > 0)$$

[6 marks]

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9. (b) Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$$

Hence, deduce the value of $\int_0^\infty \frac{\sin s}{s} ds$. OR

[6 marks]

10. (a) Find the Fourier transform of

$$f(x) = \begin{cases} a^2 - x^2 & |x| < a \\ 0 & |x| > a > 0 \end{cases}.$$

Hence, show that $\int_0^\infty \frac{\sin x - x \cos x}{x^3} dx = \frac{\pi}{4}$

[6 marks]

10. (b) Find the Fourier transform of $e^{-a^2x^2}$, a > 0.

[6 marks]