Assignment No. – 2 **Bhagyashree Deshpande**

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Ans: X = 600

Mean = 45+10

Standard Deviation = 8

```
from scipy import stats

round(1 - stats.norm.cdf(60, loc = 55, scale = 8), 5)

0.26599
```

∴ Option B

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.

   A. More employees at the processing center are older than 44 than between 38 and 44.

   Ans: False.

   Since mean = 38 and standard deviation = 6, most of the ages are lying between 32 and 44.

```
from scipy import stats

#Z score for 44

round(1 - stats.norm.cdf(44, loc = 38, scale = 6), 5)

0.15866 = 63 people out of 400
#Z score between 38 and 44

stats.norm.cdf(44, loc = 38, scale = 6) - stats.norm.cdf(38, loc = 38, scale = 6)

0.3413447460685429 = 137 people out of 400
```

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True.

```
from scipy import stats

stats.norm.cdf(30, loc = 38, scale = 6)

0.09121121972586788
```

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

   Ans: As we know that if $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ are two independent random variables, then $X+Y \sim N(\mu_1+ \mu_2 , \sigma_1^2+ \sigma_2^2)$ and $X*Y \sim N(\mu_1* \mu_2 , \sigma_1^2* \sigma_2^2)$.

   Similarly, if Z = aX + bY, where X and Y are defined as above, i.e., Z is a linear combination of X and Y, then $Z \sim N(a\mu_1+ b\mu_2 , a^2\sigma_1^2+ b^2\sigma_2^2)$

   $\therefore$ The equations

   $2X_1 \sim N(2\mu, 4\sigma^2)$

   $X_1 + X_2 \sim N(\mu+\mu, \sigma^2+ \sigma^2) \sim N(2\mu, 2\sigma^2)$

   $2X_1 – (X_1 + X_2) \sim N(4\mu, 6\sigma^2)$

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9
B. 80.2, 119.8
C. 22, 78
D. 48.5, 151.5
E. 90.1, 109.9

Ans: Since it is 99%, we need to add 0.5% on either side.
i.e., 0.005 = 0.99+0.005 = 0.995

```
from scipy import stats

#Z value of 0.005

stats.norm.ppf(0.005)

-2.575829303548901
#Z value of 0.995

stats.norm.ppf(0.995)

2.5758293035489004
```

Z = (x – Mean)/SD
x = SD*Z+Mean
x = 20*Z+100
A = (20 * (-2.57)) + 100 = 48.5
B = (20 * 2.57) + 100 = 151.5
Option D

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

   Ans:

```
import numpy as np

from scipy import stats

from scipy.stats import norm

Mean = 5+7

print('Mean Profit is Rs', Mean*45,'Million')

Mean Profit is Rs 540 Million
print('Range is Rs',(stats.norm.interval(0.95, 540, 225)),'in Millions')

Range is Rs (99.00810347848784, 980.9918965215122) in Millions
```

B. Specify the 5th percentile of profit (in Rupees) for the company

Ans: From Z table we know that 5th percentile value is -1.645

X = SD + Mean

```
import numpy as np

X = 540 + (-1.645) * (225)

print('5th percentile of profit (in Million Rupees) is', np.round(X,))

5th percentile of profit (in Million Rupees) is 170.0
```

C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Making loss. i.e., X < 0

```
from scipy import stats

# Division 1

stats.norm.cdf(0,5,3)

0.0477903522728147
# Division 2

stats.norm.cdf(0,7,4)

0.040059156863817086
```

Division 2 will face more loss.