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Group multi-criteria supplier selection using an extended VIKOR method with interval 2-tuple linguistic information

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ABSTRACT

How to select the suitable suppliers in the supply chain is critical for an organization's success and has attracted much attention of both researchers and practitioners. Supplier selection can be regarded as a complex group multiple criteria decision making problem requiring consideration of a number of alternative suppliers and quantitative and qualitative criteria. Additionally, decision makers cannot easily express their judgments on the alternatives with exact numerical values in many practical situations, and there usually exists uncertain and incomplete assessments. In response, this paper proposes an extended VIKOR method for group multi-criteria supplier selection with interval 2-tuple linguistic information. The feasibility and practicability of the proposed interval 2-tuple linguistic VIKOR (ITL-VIKOR) method are demonstrated through three realistic supplier selection examples and comparisons with the existing approaches. Results show that the ITL-VIKOR method being proposed is more suitable and effective to handle the supplier selection problem under vague, uncertain and incomplete information environment.

1. Introduction

Supply chain management is the strategic coordination of the supply chain for the purpose of integrating supply and demand management (Chu & Varma, 2012; Lee, Cho, & Kim, 2014). It aims to reduce supply chain risk and uncertainty, diminish production costs, and optimize inventory levels, business processes, and cycle times, thus resulting in increased competitiveness, customer satisfaction and profitability (Boran, Genç, Kurt, & Akay, 2009). A supply chain is a network of suppliers, manufacturing plants, warehouses, and distribution channels organized to extract raw materials, convert these raw materials into intermediate and finished products, and distribute the finished products to customers (Bidhandi, Yusuff, Ahmad, & Abu Bakar, 2009). With the competition between companies evolved to competition between supply chains, supplier selection becomes one of the most critical activities of any business organization in purchasing and supply chain management nowadays.

Supplier selection is the process of identifying the most suitable suppliers who are able to provide the buyer with the right products/services at the right price, in the right quantities and at the right time (Yu & Wong, 2015). A good supplier selection makes a significant difference to an organization's future to reduce operational costs and improve the quality of its end products (Zeydan, Çolpan, & Çobanoğlu, 2011). Through supplier selection it is also possible to establish a strategic relationship with suppliers to gain competitive advantages and to improve organizational performance (Lima, Osiro, & Carpinetti, 2014). Therefore, companies have to select the best-fit suppliers and build long-term and profitable relationships with them to achieve growth and progress in today's global competitive market.

Due to the factors such as globalization and accelerated technological change, the selection of appropriate suppliers has received a great deal of attention of both researchers and practitioners. In the literature, a variety of approaches have been suggested to construct effective selection systems, which include analytic hierarchy process (AHP) (Deng, Hu, Deng, & Mahadevan, 2014; Shaw, Shankar, Yadav, & Thakur, 2012), grey relational analysis (GRA) (Rajesh & Ravi, in press), technique for order preference by similarity to ideal solution (TOPSIS) (Lima et al., 2014), data envelopment analysis (DEA) (Kumar, Jain, & Kumar, 2014; Toloo & Nalchigar, 2011), decision-making trial and evaluation laboratory (DEMATEL) (Ho, Feng, Lee, & Yen, 2012), linear program (LP) (Jadidi, Zolfaghari, & Cavalieri, 2014; Sawik, 2014; Ware, Singh, & Banwet, 2014), and some hybrid methods (Abdollahi, Razmi, & Arvan, 2015; Singh,

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2014; Vahdani, Tavakkoli-Moghaddam, Mousavi, & Ghodratnama, 2013). In addition, a detailed review and classification of the supplier selection methods can be found in Chai, Liu, and Ngai (2013) and Ho, Xu, and Dey (2010). Under many conditions, however, exact data are inadequate to model real-life situations because of the complexity of supplier selection problems. Therefore, fuzzy set theory (Zadeh, 1965) was incorporated to deal with the vagueness and ambiguity in the real decision making process. For example, Rezaei, Fahim, and Tavasszy (2014) investigated supplier selection in the airline retail industry by using a funnel methodology, in which conjunctive screening method is used to reduce the initial set of potential suppliers and then fuzzy AHP is used to rank and select the most suitable supplier(s). Kannan, Jabbour, and Jabbour (2014) proposed a framework using fuzzy TOPSIS to evaluate green suppliers for a Brazilian electronics company based on the criteria of green supply chain management practices. Kannan, Govindan, and Rajendran (in press) proposed a multiple criteria decision making (MCDM) model called fuzzy axiomatic design to select the most suitable supplier for the plastic manufacturing company in Singapore. Roshandel, Miri-Nargesi, and Hatami-Shirkouhi (2013) presented the fuzzy hierarchical TOPSIS for supplier selection and evaluation in detergent production industry. On the other hand, Liou, Chuang, and Tzeng (2014) proposed a fuzzy integral-based model for supplier evaluation and improvement based on DEMATEL-based analytic network process (ANP) combined with the basic concepts of VIKOR method. Lee et al. (2014) integrated AHP and TOPSIS based on the fuzzy theory to determine the prior weights of multiple criteria and select the best-fit suppliers taking the subjective and vague preferences of decision makers into consideration. Karsak and Dursun (2014) proposed a novel fuzzy MCDM framework for supplier selection by integrating quality function deployment (QFD) and DEA which considers the impacts of inner dependence among supplier assessment criteria through constructing a house of quality. Kar (2014) provided an integrated approach for group decision support for the supplier selection problem by employing fuzzy AHP for group decision making and fuzzy goal programming for discriminant analysis. Büyüközkan and Çifçi (2012) proposed a hybrid MCDM model to evaluate green suppliers by combining fuzzy DEMATEL, fuzzy ANP and fuzzy TOPSIS. Wu (2009) suggested a hybrid method to deal with supplier selection problems with fuzzy numbers based on GRA and Dempster–Shafer theory. In Deng and Chan (2011) a MCDM methodology, using fuzzy set theory and Dempster–Shafer theory, based on the main idea of the TOPSIS, was developed to deal with supplier selection under uncertain environments.

Previous studies have made significant contributions to supplier selection; however, the majority of researchers concentrated on supplier selection methods applying linguistic value by using fuzzy logic to handle the uncertainty in real decision making situations. As a result, an approximation process must be developed to express the results in the initial expression domain, which produces a loss of information and hence a lack of precision in the final results (Herrera & Martínez, 2000; Liu, Liu, & Wu, 2013). Furthermore, decision makers are often unsure of their preferences during the supplier selection process because of the reasons such as time pressure, lack of experience and data. They often demonstrate different evaluations or opinions from one to another and produce different types of assessment information for a certain alternative concerning a given criterion, some of which may be imprecise, uncertain and incomplete. These different types of information are very hard to incorporate into the supplier selection by using the crisp and fuzzy logic approaches. Whereas, the interval 2-tuple linguistic representation model (Liu, You, & You, in press; Zhang, 2012) overcomes the above-mentioned weaknesses. The advantages of this model are that decision makers can express their preferences by the use of linguistic term sets with different granularity of uncertainty and/or semantics (multigranular linguistic contexts), and their judgments can be expressed with an interval 2-tuple from the predefined linguistic term set. Therefore, the approach based on the interval 2-tuple linguistic representation model will be more flexible and precise to deal with linguistic terms in solving the supplier selection problems.

In other way, many quantitative and qualitative criteria (or factors) should be taken into consideration when selecting suppliers for an organization, including price, quality, delivery, service, reputation, and so on (Boran et al., 2009; Guo, Zhu, & Shi, 2014; Yücel & Güneri, 2011). In order to select the optimum suppliers it is necessary to make balance among these tangible and intangible factors some of which may conflict and compete like low price versus high quality. Moreover, there may be multiple decision makers taking part in the evaluation of alternatives together during the supplier selection process. Therefore, supplier selection is a kind of group MCDM problem (Deng et al., 2014) and MCDM techniques can be utilized to solve supplier selection problem of an organization. The VIKOR method, a very useful technique for MCDM, was first developed by Opricovic (1998) to solve a discrete decision problem with noncommensurable and conflicting criteria. This method focuses on ranking and selecting from a set of alternatives, and determines compromise solutions for a problem with conflicting criteria, which can help the decision makers to reach a final decision (Opricovic & Tzeng, 2007). The main advantages of the VIKOR method are that it introduces the multi-criteria ranking index based on the particular measure of "closeness" to the ideal solution (Opricovic & Tzeng, 2004), and the obtained compromise solution provides a maximum group utility for the "majority" and a minimum individual regret for the "opponent" (Opricovic & Tzeng, 2007). Due to its characteristics and capabilities, the VIKOR method has been widely studied and applied in group decision making problems in recent years (Liu, Mao, Zhang, & Li, 2013; Liu, Ren, Wu, & Lin, 2014; Liu, You, Fan, & Chen, 2014; Sanayei, Farid Mousavi, & Yazdankhah, 2010; Vahdani, Mousavi, Hashemi, Mousakhani, & Tavakkoli-Moghaddam, 2013).

The background introduced above shows that it may be inappropriate to use fuzzy logic based methods for evaluation and selection of suppliers because of the loss of information in the linguistic information processing. Moreover, decision makers tend to use different linguistic term sets to express their judgments on the subjective criteria, and there usually exists uncertain and incomplete assessments. In response, this paper develops a new group MCDM model using interval 2-tuple linguistic variables and extended VIKOR method to solve the supplier selection problems under uncertain and incomplete information environment. The proposed method can not only avoid information distortion and loss which occur formerly in the linguistic information processing, but also model the diversity and uncertainty of the assessment information provided by decision makers in supplier selection. Furthermore, both conflicting quantitative and qualitative criteria in real-life applications can be considered simultaneously in the developed method. The rest of the paper is organized as follows. In Section 2, some basic concepts of interval 2-tuple linguistic variables are briefly reviewed. In Section 3, an extended VIKOR approach is proposed to solve the group multi-criteria supplier selection problem with interval 2-tuple linguistic information. Three numerical examples are provided in Section 4 to illustrate the proposed approach and finally, some conclusions and future research directions are summarized in Section 5.

2. Preliminaries

2.1. 2-Tuple linguistic variables

The 2-tuple linguistic representation model was initiated by Herrera and Martínez (2000) based on the concept of symbolic translation. It is used to represent the linguistic information by means of a pair of values called linguistic 2-tuple, (s, α), where s is a linguistic term from the predefined linguistic term set S and α is a numerical value representing the symbolic translation. For overcoming the restriction of the Herrera and Martínez model, Chen and Tai (2005) proposed a generalized 2-tuple linguistic model to deal with multi-granular linguistic term sets.

Definition 1. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set with granularity $g + 1$, and $\beta \in [0,1]$ a value representing the result of a symbolic aggregation operation. Then the generalized translation function Δ used to obtain the 2-tuple linguistic variable equivalent to β can be defined as follows (Chen & Tai, 2005; Tai & Chen, 2009):

$$\Delta : [0,1] \to S \times \left[-\frac{1}{2g}, \frac{1}{2g}\right), \tag{1}$$

$$\Delta(\beta) = (s_i, \alpha), \text{with} \begin{cases} s_i, i = \text{round}(\beta \cdot g), \\ \alpha = \beta - \frac{i}{g}, \ \alpha \in \left[-\frac{1}{2g}, \frac{1}{2g}\right), \end{cases} \tag{2}$$

where round$(\cdot)$ is the usual rounding operation, s_i has the closest index label to β and α is the value of the symbolic translation. The interval of α is determined by the number of linguistic terms in S.

Definition 2. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There exists a function Δ^{-1}, which is able to convert a 2-tuple linguistic variable into its equivalent numerical value $\beta \in [0,1]$. The reverse function Δ^{-1} is defined as follows (Chen & Tai, 2005; Tai & Chen, 2009):

$$\Delta^{-1} : S \times \left[-\frac{1}{2g}, \frac{1}{2g}\right) \to [0,1], \tag{3}$$

$$\Delta^{-1}(s_i, \alpha) = \frac{i}{g} + \alpha = \beta. \tag{4}$$

It is noteworthy that the conversion of a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation (Herrera & Martínez, 2000):

$$s_i \in S \Rightarrow (s_i, 0). \tag{5}$$

Definition 3. Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, then (Herrera & Martínez, 2000):

(1) If $k < l$ then (s_k, α_1) is smaller than (s_l, α_2);
(2) If $k = l$ then
  (a) if $\alpha_1 = \alpha_2$ then (s_k, α_1) is equal to (s_l, α_2);
  (b) if $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2);
  (c) if $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2).

Definition 4. Let $X = \{(r_1, \alpha_1), (r_2, \alpha_2), \ldots, (r_n, \alpha_n)\}$ be a set of 2-tuples and $w = (w_1, w_2, \ldots, w_n)^T$ be their associated weights, with $w_j \in [0,1]$, $j = 1, 2, \ldots, n$, $\sum_{j=1}^{n} w_j = 1$. The 2-tuple weighted average (TWA) is defined as (Herrera & Martínez, 2000):

$$\text{TWA}(X) = \Delta\left(\frac{1}{n}\sum_{j=1}^{n} w_j \Delta^{-1}(r_j, \alpha_j)\right) = \Delta\left(\frac{1}{n}\sum_{j=1}^{n} w_j \beta_j\right). \tag{6}$$

2.2. Interval 2-tuple linguistic variables

The interval 2-tuple linguistic representation model was further put forward by Zhang (2012) based on the definitions of Chen and Tai (2005). It can be defined as follows.

Definition 5. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. An interval 2-tuple linguistic variable is composed of two 2-tuples, denoted by $[(s_k, \alpha_1), (s_l, \alpha_2)]$, where $(s_k, \alpha_1) \leqslant (s_l, \alpha_2)$, $s_k(s_l)$ and α_1 (α_2) represent the linguistic label of the predefined linguistic term set S and symbolic translation, respectively. The interval 2-tuple that expresses the equivalent information to an interval value $[\beta_1, \beta_2]$ $(\beta_1, \beta_2 \in [0,1],\ \beta_1 \leqslant \beta_2)$ is derived by the following function (Zhang, 2012, 2013):

$$\Delta[\beta_1, \beta_2] = [(s_k, \alpha_1), (s_l, \alpha_2)] \quad \text{with} \begin{cases} s_k, k = \text{round}(\beta_1 \cdot g), \\ s_l, l = \text{round}(\beta_2 \cdot g), \\ \alpha_1 = \beta_1 - \frac{k}{g}, \alpha_1 \in \left[-\frac{1}{2g}, \frac{1}{2g}\right), \\ \alpha_2 = \beta_2 - \frac{l}{g}, \alpha_2 \in \left[-\frac{1}{2g}, \frac{1}{2g}\right). \end{cases} \tag{7}$$

On the contrary, there is always a function Δ^{-1} such that an interval 2-tuple can be transformed into an interval value $[\beta_1, \beta_2]$ $(\beta_1, \beta_2 \in [0,1],\ \beta_1 \leqslant \beta_2)$ as follows:

$$\Delta^{-1}[(s_k, \alpha_1), (s_l, \alpha_2)] = \left[\frac{k}{g} + \alpha_1, \frac{l}{g} + \alpha_2\right] = [\beta_1, \beta_2]. \tag{8}$$

Especially, if $s_k = s_l$ and $\alpha_1 = \alpha_2$, then the interval 2-tuple linguistic variable becomes a 2-tuple linguistic variable.

Definition 6. Let $\tilde{X} = \{[(r_1, \alpha_1), (t_1, \varepsilon_1)], [(r_2, \alpha_2), (t_2, \varepsilon_2)], \ldots, [(r_n, \alpha_n), (t_n, \varepsilon_n)]\}$ be a set of interval 2-tuples and $w = (w_1, w_2, \ldots, w_n)^T$ be their associated weights, with $w_j \in [0,1]$, $j = 1, 2, \ldots, n$, $\sum_{j=1}^{n} w_j = 1$. The interval 2-tuple weighted average (ITWA) operator is defined as (Zhang, 2012, 2013):

$$\begin{aligned} &\text{ITWA}([(r_1, \alpha_1), (t_1, \varepsilon_1)], [(r_2, \alpha_2), (t_2, \varepsilon_2)], \ldots, [(r_n, \alpha_n), (t_n, \varepsilon_n)]) \\ &\quad = \Delta\left[\sum_{j=1}^{n} w_j \Delta^{-1}(r_j, \alpha_j), \sum_{j=1}^{n} w_j \Delta^{-1}(t_j, \varepsilon_j)\right]. \end{aligned} \tag{9}$$

Definition 7. Let $\tilde{a} = [(r_1, \alpha_1), (t_1, \varepsilon_1)]$ and $\tilde{b} = [(r_2, \alpha_2), (t_2, \varepsilon_2)]$ be two interval 2-tuples, then

$$D(\tilde{a}, \tilde{b}) = \Delta\left[\frac{1}{2}\left(\left|\Delta^{-1}(r_1, \alpha_1) - \Delta^{-1}(r_2, \alpha_2)\right| + \left|\Delta^{-1}(t_1, \varepsilon_1) - \Delta^{-1}(t_2, \varepsilon_2)\right|\right)\right] \tag{10}$$

is called the distance between $\tilde{a}$ and $\tilde{b}$.

Definition 8. Let $\tilde{X} = \{[(r_1, \alpha_1), (t_1, \varepsilon_1)], [(r_2, \alpha_2), (t_2, \varepsilon_2)], \ldots, [(r_n, \alpha_n), (t_n, \varepsilon_n)]\}$ and $\tilde{X}' = \{[(r_1', \alpha_1'), (t_1', \varepsilon_1')], [(r_2', \alpha_2'), (t_2', \varepsilon_2')], \ldots, [(r_n', \alpha_n'), (t_n', \varepsilon_n')]\}$ be two sets of interval 2-tuples, then

$$D(\tilde{X}, \tilde{X}') = \Delta\sum_{j=1}^{n}\left[\frac{1}{2}\left(\left|\Delta^{-1}(r_j, \alpha_j) - \Delta^{-1}\left(r_j', \alpha_j'\right)\right| + \left|\Delta^{-1}(t_j, \varepsilon_j) - \Delta^{-1}\left(t_j', \varepsilon_j'\right)\right|\right)\right] \tag{11}$$

is called the distance between $\tilde{X}_1$ and $\tilde{X}_2$.

3. Interval 2-tuple linguistic VIKOR method for supplier selection

In this section, we present an extended VIKOR method to solve group multi-criteria supplier selection problems in which the criteria weights take the form of 2-tuple linguistic information, and the criteria values take the form of interval 2-tuple linguistic information.

Suppose that a supplier selection problem has H decision makers $DM_h (h = 1, 2, \ldots, H)$, P alternatives A_p $(p = 1, 2, \ldots, P)$, and Q

decision criteria C_q ($q = 1,2,\ldots,Q$). Each decision maker DM_h is given a weight $\lambda_h > 0$ ($h = 1,2,\ldots,H$) satisfying $\sum_{h=1}^{H}\lambda_h = 1$ to reflect his/her relative importance in the group decision making process. Let $D_h = \left(d_{pq}^h\right)_{P\times Q}$ be the linguistic decision matrix of the hth decision maker, where d_{pq}^h is the linguistic information provided by DM_h on the assessment of A_p with respect to C_q. Let $v_h = \left(v_1^h, v_2^h, \ldots, v_Q^h\right)^T$ be the linguistic weight vector given by the hth decision maker, where v_q^h is the linguistic variable assigned to C_q by DM_h. In addition, decision makers may use different linguistic term sets to express their assessments. Based upon these assumptions or notations, the procedure of interval 2-tuple linguistic VIKOR (ITL-VIKOR) method for group multi-criteria supplier selection can be defined as the following steps:

Step 1: Convert the linguistic decision matrix $D_h = \left(d_{pq}^h\right)_{P\times Q}$ into interval 2-tuple linguistic decision matrix $\tilde{R}_h = \left(\tilde{r}_{pq}^h\right)_{P\times Q} = \left(\left[\left(r_{pq}^h, 0\right), \left(t_{pq}^h, 0\right)\right]\right)_{P\times Q}$, where $r_{pq}^h, t_{pq}^h \in S$, $S = \{s_i | i = 0,1,2,\ldots,g\}$ and $r_{pq}^h \leqslant t_{pq}^h$.

Suppose that DM_h provides his assessments in a set of five linguistic terms $S = \{s_0 = \textit{Very poor}, s_1 = \textit{Poor}, s_2 = \textit{Medium}, s_3 = \textit{Good}, s_4 = \textit{Very good}\}$. The linguistic information provided in the linguistic decision matrix D_k can be converted into corresponding interval 2-tuple linguistic assessments according to the following ways:

- A certain grade such as *Poor*, which can be written as $[(s_1,0), (s_1,0)]$.
- An interval such as *Poor-Medium*, which means that the assessment of an alternative with respect to the criterion under consideration is between *Poor* and *Medium*. This can be written as $[(s_1,0), (s_2,0)]$.
- No judgment, which means the decision maker is not willing to or cannot provide an assessment for an alternative with respect to the criterion under consideration. In other words, the assessment by this decision maker could be anywhere between *Very poor* and *Very good* and can be expressed as $[(s_0,0), (s_4,0)]$.

Step 2: Convert the linguistic weight vector $v_h = \left(v_1^h, v_2^h, \ldots, v_Q^h\right)^T$ into 2-tuple linguistic weight vector $w_h = [(w_1^h, 0), (w_2^h, 0), \ldots, \left(w_Q^h, 0\right)]^T$ by Eq. (5), where $w_q^h \in S$, $S = \{s_i | i = 0,1,2,\ldots,g\}$.

Step 3: Aggregate the decision makers' opinions to construct a collective interval 2-tuple linguistic decision matrix $\tilde{R} = (\tilde{r}_{pq})_{P\times Q}$, where

$$\tilde{r}_{pq} = [(r_{pq},\alpha_{pq}),(t_{pq},\varepsilon_{pq})] = \mathrm{ITWA}\left(\left[\left(r_{pq}^1,0\right),\left(t_{pq}^1,0\right)\right],\left[\left(r_{pq}^2,0\right),\left(t_{pq}^2,0\right)\right],\ldots,\left[\left(r_{pq}^H,0\right),\left(t_{pq}^H,0\right)\right]\right) = \Delta\left[\sum_{h=1}^{H}\lambda_h\Delta^{-1}\left(r_{pq}^h,0\right),\sum_{h=1}^{H}\lambda_h\Delta^{-1}\left(t_{pq}^h,0\right)\right],$$
$$p = 1,2,\ldots,P,\ q = 1,2,\ldots,Q. \tag{12}$$

Step 4: Aggregate the criteria weights provided by decision makers to determine the aggregated 2-tuple linguistic weight vector $w = [(w_1,\alpha_1), (w_2,\alpha_2),\ldots,(w_Q,\alpha_Q)]^T$, where

$$(w_q,\alpha_q) = \Delta\left[\sum_{h=1}^{H}\lambda_h\Delta^{-1}\left(w_q^h,0\right)\right],\quad q = 1,2,\ldots,Q. \tag{13}$$

Step 5: Determine the positive ideal solution (PIS) and the negative ideal solution (NIS) as:

$$r^+ = \left(r_1^+, r_2^+, \ldots, r_Q^+\right), \tag{14}$$

$$r^- = \left(r_1^-, r_2^-, \ldots, r_Q^-\right), \tag{15}$$

where

$$r_q^+ = \left(r_q^+,\alpha_q^+\right) = \left\{\begin{array}{l}\max\limits_p\{(t_{pq},\varepsilon_{pq})\},\ \text{for benefit criteria}\\ \min\limits_p\{(r_{pq},\alpha_{pq})\},\ \text{for cost criteria}\end{array}\right\},\quad q = 1,2,\ldots,Q, \tag{16}$$

$$r_q^- = \left(r_q^-,\alpha_q^-\right) = \left\{\begin{array}{l}\min\limits_p\{(r_{pq},\alpha_{pq})\},\ \text{for benefit criteria}\\ \max_p\{(t_{pq},\varepsilon_{pq})\},\ \text{for cost criteria}\end{array}\right\},\quad q = 1,2,\ldots,Q. \tag{17}$$

Step 6: Compute the normalized 2-tuple linguistic distance $(\bar{d}_{pq},\alpha_{pq})$ by Eq. (18).

$$(\bar{d}_{pq},\alpha_{pq}) = \Delta\left(\frac{\Delta^{-1}d\left(r_q^+,\tilde{r}_{pq}\right)}{\Delta^{-1}d\left(r_q^+,\tilde{r}_q^-\right)}\right),\quad p = 1,2,\ldots,P,\ q = 1,2,\ldots,Q, \tag{18}$$

where

$$d\left(r_q^+,\tilde{r}_{pq}\right) = \Delta\left[\frac{1}{2}\left(\left|\Delta^{-1}\left(r_q^+,\alpha_q^+\right) - \Delta^{-1}(r_{pq},\alpha_{pq})\right| + \left|\Delta^{-1}\left(r_q^+,\alpha_q^+\right) - \Delta^{-1}(t_{pq},\varepsilon_{pq})\right|\right)\right], \tag{19}$$

$$d\left(r_q^+,r_q^-\right) = \Delta\left(\left|\Delta^{-1}\left(r_q^+,\alpha_q^+\right) - \Delta^{-1}\left(r_q^-,\alpha_q^-\right)\right|\right). \tag{20}$$

Step 7: Compute the 2-tuples (S_p,α_p) and (R_p,α_p), $p = 1,2,\ldots,P$, by the relations

$$(S_p,\alpha_p) = \Delta\left(\sum_{q=1}^{Q}\frac{\Delta^{-1}(w_q,\alpha_q)\cdot\Delta^{-1}(\bar{d}_{pq},\alpha_{pq})}{\sum_{q=1}^{Q}\Delta^{-1}(w_q,\alpha_q)}\right), \tag{21}$$

$$(R_p,\alpha_p) = \Delta\left(\max_q\left(\frac{\Delta^{-1}(w_q,\alpha_q)\cdot\Delta^{-1}(\bar{d}_{pq},\alpha_{pq})}{\sum_{q=1}^{Q}\Delta^{-1}(w_q,\alpha_q)}\right)\right). \tag{22}$$

Step 8: Compute the 2-tuples (O_p,α_p), $p = 1,2,\ldots,P$, by the relation

$$(O_p,\alpha_p) = \Delta\left(\mu\frac{\Delta^{-1}(S_p,\alpha_p) - \Delta^{-1}(S^*,\alpha^*)}{\Delta^{-1}(S^-,\alpha^-) - \Delta^{-1}(S^*,\alpha^*)} + (1-\mu)\frac{\Delta^{-1}(R_p,\alpha_p) - \Delta^{-1}(R^*,\alpha^*)}{\Delta^{-1}(R^-,\alpha^-) - \Delta^{-1}(R^*,\alpha^*)}\right), \tag{23}$$

where $(S^*,\alpha^*) = \min_p(S_p,\alpha_p)$, $(S^-,\alpha^-) = \max_p(S_p,\alpha_p)$, $(R^*,\alpha^*) = \min_p(R_p,\alpha_p)$, $(R^-,\alpha^-) = \max_p(R_p,\alpha_p)$ and v is introduced as a weight for the strategy of maximum group utility, whereas $1\text{-}v$ is the weight of the individual regret. The value of v is set to 0.5 in this study.

Step 9: Rank the alternatives by sorting the 2-tuples (S_p,α_p), (R_p,α_p) and (O_p,α_p) in increasing order. The results are three ranking lists.

Step 10: Propose a compromise solution, the alternative $(A^{(1)})$, which is the best ranked by the measure (O_p, α_p) (minimum) if the following two conditions are satisfied:

C1. Acceptable advantage: $\Delta^{-1}(O(A^{(2)}), \alpha(A^{(2)})) - \Delta^{-1}(O(A^{(1)}), \alpha(A^{(1)})) \geqslant 1/(P-1)$, where $A^{(2)}$ is the alternative with second position in the ranking list by (O_p,α_p).

C2. Acceptable stability in decision making: The alternative $A^{(1)}$ must also be the best ranked by (S_p,α_p) or/and (R_p,α_p). This compromise solution is stable within a decision making process, which could be: "voting by majority rule" (when $\mu > 0.5$ is needed), or "by consensus" $\mu \approx 0.5$, or "with veto" ($\mu < 0.5$).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied or
- Alternatives $A^{(1)},A^{(2)},\ldots,A^{(M)}$ if the condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $\Delta^{-1}(O(A^{(M)}),\alpha(A^{(M)}))-\Delta^{-1}(O(A^{(1)}),\alpha(A^{(1)}))<1/(P-1)$ for maximum M.

4. Illustrative examples

In this section, three real-life examples are provided to demonstrate and validate the application of the proposed method for solving the supplier selection problems.

4.1. Example 1

A tertiary care university hospital, which is located in Shanghai, China, has carried out a risk analysis on the general anesthesia process (Liu, Liu, Liu, & Mao, 2012) because its higher level of risk, and the results showed that the most important failure modes were mainly caused by a key anaesthetic equipment. Hence, this tertiary care hospital needs to determine a most appropriate supplier for the equipment in order to improve the safety of general anesthesia process. After preliminary screening, five suppliers, named as A_1, A_2, A_3, A_4 and A_5, have remained as alternatives for further evaluation. An expert committee of four decision makers, DM_1, DM_2, DM_3 and DM_4, has been formed to select the best supplier. The selection decision is made on the basis of the following four criteria: technical capability (C_1), delivery performance (C_2), product quality (C_3), and product price (C_4).

The four decision makers employ different linguistic term sets to evaluate the alternatives with respect to the above criteria. Specifically, DM_1 provides his assessments in the set of 5 labels, A; DM_2 provides his assessments in the set of 7 labels, B; DM_3 provides her assessments in the set of 9 labels, C; DM_4 provides his assessments in the set of 5 labels, D. Additionally, the relative importance of the criteria was rated by the four decision makers with a set of 5 linguistic terms, E. These linguistic term sets are denoted as follows:

$$A=\{a_0=Very\ poor\ (VP),\ a_1=Poor\ (P),\ a_2=Medium\ (M),\\ a_3=Good\ (G)\}\ a_4=Very\ good\ (VG)\},$$

$$B=\{b_0=Very\ poor\ (VP),\ b_1=Poor\ (P),\ b_2=Medium\ poor\ (MP),\\ b_3=Medium\ (M),\ b_4=Medium\ good\ (MG),\ b_5=Good\ (G),\\ b_6=Very\ good\ (VG)\}.$$

$$C=\{c_0=Extreme\ poor\ (EP),\ c_1=Very\ poor\ (VP),c_2=Poor\ (P),\\ c_3=Medium\ poor\ (MP),\ c_4=Medium\ (M),\\ c_5=Medium\ good\ (MG),\ c_6=Good\ (G),\\ c_7=Very\ good\ (VG),\ c_8=Extreme\ good\ (EG)\},$$

$$D=\{d_0=Very\ poor\ (VP),\ d_1=Poor\ (P),\ d_2=Medium\ (M),\\ d_3=Good\ (G)\}\ d_4=Very\ good\ (VG)\},$$

$$E=\{e_0=Very\ unimportant\ (VU),\ e_1=Unimportant\ (U),\\ e_2=Medium\ (M),\ e_3=Important\ (I),\ e_4=Very\ important\ (VI)\}.$$

The assessments of the five alternatives on each criterion and the criteria weights provided by the four decision makers are presented in Tables 1 and 2, where ignorance information is highlighted and shaded. Considering their different domain knowledge and expertise, the four decision makers are assigned the following relative weights: 0.15, 0.20, 0.30, and 0.35 in the supplier selection decision process.

Next, we utilize the proposed ITL-VIKOR method to derive the most desirable alternative, which includes the following steps:

Step 1: Convert the linguistic decision matrix shown in Table 1 into interval 2-tuple linguistic decision matrix $\tilde{R}_h=\left(\left[\left(r^h_{pq},0\right),\left(t^h_{pq},0\right)\right]\right)_{5\times4}$. Taking DM_1 as an example, we can get the interval 2-tuple linguistic decision matrix $\tilde{R}_1$ as shown in Table 3.

Step 2: Convert the linguistic weight vector shown in Table 2 into 2-tuple linguistic weight vector $w_h=\left[(w^h_1,0),(w^h_2,0),(w^h_3,0),(w^h_4,0)\right]^T$, which is presented in Table 4.

Step 3: The evaluations of the five alternatives obtained from the four decision makers are aggregated by Eq. (12) and the collective interval 2-tuple linguistic decision matrix is provided in Table 5.

Step 4: The criteria weights provided by the four decision makers are aggregated by Eq. (13) and the aggregated 2-tuple linguistic weights are listed in the last row of Table 5.

Step 5: Delivery performance, product quality and technical capability are benefit criteria and price is a cost criterion. Thus the PIS and NIS of the collective interval 2-tuple linguistic decision matrix are determined as:

$$r^+=(\Delta(0.8917),\Delta(0.9667),\Delta(0.9292),\Delta(0.5375)),\\ r^-=(\Delta(0.3958),\Delta(0.5000),\Delta(0.4958),\Delta(0.8375)).$$

Step 6: The normalized 2-tuple linguistic distances $(\bar{d}_{pq},\alpha_{pq})$ are computed by Eq. (18) and shown in Table 6.

Steps 7 and 8: The 2-tuples (S_p,α_p), $(R_p,\ \alpha_p)$ and $(O_p,\ \alpha_p)$ $p=1,2,\ldots,5$ are calculated by Eqs. (21) and (22) and the results are shown in Table 7.

Step 9: Rank the alternatives in accordance with the 2-tuples (S_p,α_p), (R_p,α_p) and (O_p,α_p), referring to Definition 3. The ranking lists of the five alternatives are presented in Table 8.

Step 10: As we can see from Table 8, the ranking order of the five alternatives is $A_5\succ A_3\succ A_4\succ A_1\succ A_2$ in accordance with the 2-tuples of (O_p,α_p). Thus, the most desirable alternative is A_5.

Table 1
Linguistic assessments of alternatives provided by the four decision makers.

Decision makers	Alternatives	Criteria			
		C_1	C_2	C_3	C_4
DM_1	A_1	M-G	G	G	M-G
	A_2	G	M		VG
	A_3	M-G	M-G	G	G
	A_4	G	VG	M-G	M-G
	A_5	G-VG	VG	G	G
DM_2	A_1	VG	M	MG	G
	A_2		M-MG	MG	VG
	A_3	G	G	MG-G	MG-G
	A_4	MG	MG	G	G
	A_5	G	G	G-VG	M
DM_3	A_1	MG-VG	MG	VG	MG
	A_2	G	M	MG	G
	A_3	M-G	G		G
	A_4	M	G	M	G
	A_5	G	G-EG	G-VG	M-MG
DM_4	A_1	G	M	G-VG	G
	A_2	G	M-G	M-G	G
	A_3	G	VG	G-VG	G
	A_4		G	G	G
	A_5	G-VG	VG	VG	M

Table 2
Linguistic assessments of criteria weights.

Criteria	Decision makers			
	DM_1	DM_2	DM_3	DM_4
C_1	VI	I	I	VI
C_2	I	I	I	I
C_3	VI	VI	VI	VI
C_4	M	I	M	I

Table 3
Interval 2-tuple linguistic decision matrix of DM_1.

Alternatives	Criteria			
	C_1	C_2	C_3	C_4
A_1	$[(a_2,0), (a_3,0)]$	$[(a_3,0), (a_3,0)]$	$[(a_3,0), (a_3,0)]$	$[(a_2,0), (a_3,0)]$
A_2	$[(a_3,0), (a_3,0)]$	$[(a_2,0), (a_2,0)]$	$[(a_0,0), (a_4,0)]$	$[(a_4,0), (a_4,0)]$
A_3	$[(a_2,0), (a_3,0)]$	$[(a_2,0), (a_3,0)]$	$[(a_3,0), (a_3,0)]$	$[(a_3,0), (a_3,0)]$
A_4	$[(a_3,0), (a_3,0)]$	$[(a_4,0), (a_4,0)]$	$[(a_2,0), (a_3,0)]$	$[(a_2,0), (a_3,0)]$
A_5	$[(a_3,0), (a_4,0)]$	$[(a_4,0), (a_4,0)]$	$[(a_3,0), (a_3,0)]$	$[(a_3,0), (a_3,0)]$

Table 4
2-Tuple linguistic criteria weights.

Criteria	Decision makers			
	DM_1	DM_2	DM_3	DM_4
C_1	$(e_4,0)$	$(e_3,0)$	$(e_3,0)$	$(e_4,0)$
C_2	$(e_3,0)$	$(e_3,0)$	$(e_3,0)$	$(e_3,0)$
C_3	$(e_4,0)$	$(e_4,0)$	$(e_4,0)$	$(e_4,0)$
C_4	$(e_2,0)$	$(e_3,0)$	$(e_2,0)$	$(e_3,0)$

Table 5
Aggregated interval 2-tuple linguistic decision matrix.

Alternatives	Criteria			
	C_1	C_2	C_3	C_4
A_1	$\Delta[0.725, 0.838]$	$\Delta[0.575, 0.575]$	$\Delta[0.771, 0.858]$	$\Delta[0.692, 0.729]$
A_2	$\Delta[0.600, 0.800]$	$\Delta[0.500, 0.621]$	$\Delta[0.496, 0.733]$	$\Delta[0.838, 0.838]$
A_3	$\Delta[0.654, 0.767]$	$\Delta[0.817, 0.854]$	$\Delta[0.508, 0.929]$	$\Delta[0.733, 0.767]$
A_4	$\Delta[0.396, 0.746]$	$\Delta[0.771, 0.771]$	$\Delta[0.654, 0.692]$	$\Delta[0.729, 0.767]$
A_5	$\Delta[0.767, 0.892]$	$\Delta[0.892, 0.967]$	$\Delta[0.854, 0.925]$	$\Delta[0.538, 0.575]$
Weights	$\Delta(0.70)$	$\Delta(0.60)$	$\Delta(0.80)$	$\Delta(0.51)$

To illustrate the effectiveness of the proposed ITL-VIKOR, we used the above case study to analyze some existing supplier selection approaches, which include fuzzy VIKOR (Sanayei et al., 2010), fuzzy TOPSIS (Chen, Lin, & Huang, 2006), intuitionistic fuzzy TOPSIS (IF-TOPSIS) (Boran et al., 2009), and interval ELECTRE (Vahdani, Jabbari, Roshanaei, & Zandieh, 2010) methods. Table 9 exhibits the ranking results of all the alternative suppliers as derived using these approaches.

From the results given in Table 9, it can be observed that all the five methods suggest suppliers A_5 and A_2 as the first and last choices respectively. This demonstrates the validity of the suggested model. In addition, the rankings of alternatives by the fuzzy VIKOR are exactly the same as those by the fuzzy TOPSIS:$A_5 \succ A_1 \succ A_3 \succ A_4 \succ A_2$. Both the two methods suggest supplier A_1 as the second choice and supplier A_4 as the fourth choice, while supplier A_3 ranks the second and A_1 ranks the fourth by the proposed ITL-VIKOR method. This inconsistency can be understood by the fact that the fuzzy VIKOR and the fuzzy TOPSIS methods are based on the extension principle, which may produce the consequent loss of information and hence the lack of precision in the final results (Herrera & Martínez, 2000; Liu, Li, You, & Chen, in press). The main advantage of the proposed supplier selection method is its computational model that offers linguistic results in the initial expression domain in a precise way. In other words, using Eqs. (1) and (2), we can express the final results in the original linguistic domain adopted by each decision maker. Taking the decision maker DM_2 as an example, the final results can be expressed by 2-tuples derived from the linguistic term set B with 7 labels, which are listed in Table 7.

Table 6
Normalized 2-tuple linguistic distances.

Alternatives	Criteria			
	C_1	C_2	C_3	C_4
A_1	$\Delta(0.223)$	$\Delta(0.839)$	$\Delta(0.264)$	$\Delta(0.576)$
A_2	$\Delta(0.387)$	$\Delta(0.871)$	$\Delta(0.726)$	$\Delta(1.000)$
A_3	$\Delta(0.366)$	$\Delta(0.281)$	$\Delta(0.486)$	$\Delta(0.708)$
A_4	$\Delta(0.647)$	$\Delta(0.420)$	$\Delta(0.591)$	$\Delta(0.701)$
A_5	$\Delta(0.126)$	$\Delta(0.080)$	$\Delta(0.091)$	$\Delta(0.063)$

Table 7
The 2-tuples (S_p, α_p), (R_p, α_p) and (O_p, α_p) of the five alternatives.

	A_1	A_2	A_3	A_4	A_5
(S_p, α_p)	$\Delta(0.623)$ $(b_4, -0.044)$	$\Delta(0.766)$ $(b_5, -0.067)$	$\Delta(0.387)$ $(b_2, 0.054)$	$\Delta(0.536)$ $(b_3, 0.036)$	$\Delta(0.089)$ $(b_1, -0.078)$
(R_p, α_p)	$\Delta(0.257)$ $(b_2, -0.076)$	$\Delta(0.267)$ $(b_2, -0.066)$	$\Delta(0.138)$ $(b_1, -0.029)$	$\Delta(0.174)$ $(b_1, 0.007)$	$\Delta(0.034)$ $(b_0, 0.034)$
(O_p, α_p)	$\Delta(0.873)$ $(b_5, 0.040)$	$\Delta(1.000)$ $(b_6, 0.000)$	$\Delta(0.445)$ $(b_3, -0.055)$	$\Delta(0.63)$ $(b_4, -0.037)$	$\Delta(0.000)$ $(b_0, 0.000)$

The IF-TOPSIS method uses the concept of intuitionistic fuzzy set and suggests that supplier A_1 is better than supplier A_4. However, a close look at the values of the criteria for the suppliers A_1 and A_4 in Table 6 reveals that supplier A_4 is comparatively better than supplier A_1 in the case of two criteria (i.e., C_1 and C_3). And both the two criteria have higher weights compared with the other criteria. Thus, proposing supplier A_4 as the third choice and supplier A_1 as the fourth choice which is given by the proposed method seems more logic than that proposed by the IF-TOPSIS method. Furthermore, the proposed method makes a provision to deal with the quantitative criteria, if such criteria are present in the problem. But this was missing in the IF-TOPSIS method. In addition, using the interval ELECTRE method leads to supplier A_1 as the second choice and supplier A_4 as the fourth choice. However, both suppliers A_1 and A_3 rank the second and cannot be distinguished from each other according to the interval ELECTRE. Also, the interval ELECTRE utilizes interval numbers to evaluate the suppliers with respect to each criterion; but in some situations decision makers are hard to express their evaluations in this way especially for the qualitative criteria.

4.2. Example 2

The first example only considers the subjective evaluations of decision makers in the supplier selection process, but in some situations, both quantitative and qualitative criteria are existed in a particular supplier selection problem. Thus, in what follows, an example of international supplier selection from Wu (2009) is cited to further demonstrate the proposed ITL-VIKOR method. The same problem was also solved by Deng and Chan (2011) using a fuzzy dempster MCDM method. International supplier selection is a very important strategic decision involving decisions balancing a

Table 8
Ranking of the alternative suppliers.

	A_1	A_2	A_3	A_4	A_5
By (S_p, α_p)	4	5	2	3	1
By (R_p, α_p)	4	5	2	3	1
By (O_p, α_p)	4	5	2	3	1

Table 9
Ranking comparisons for Example 1.

Alternatives	ITL-VIKOR	Fuzzy VIKOR	Fuzzy TOPSIS	IF-VIKOR	Interval ELECTRE
A_1	4	2	2	3	2
A_2	5	5	5	5	5
A_3	2	3	3	2	2
A_4	3	4	4	4	4
A_5	1	1	1	1	1

number of conflicting criteria, and these decisions are becoming ever more complex with the increase in outsourcing, offshore sourcing, and various electronic businesses. A group of three experts was identified in the example to aid in the supplier selection (whose weighting vector $\lambda = (0.203, 0.281, 0.516)^T$). The four criteria considered in the global supplier selection are: product late delivery (C_1), cost (C_2), risk factor (C_3), and supplier service performance (C_4). Product late delivery rate and cost are crisp values as outlined in Table 10, but risk factor and supplier service performance are fuzzy data for each alternative supplier. Here, the following linguistic term set is adopted to express the subjective evaluations of the three decision makers: $S = \{s_0 = Very\ low(VL), s_1 = Low\ (L), s_2 = Medium\ (M), s_3 = High\ (H), s_4 = Very\ high\ (VH)\}$.

Now, we implement the proposed method to get the most suitable alternative:

Step 1: The linguistic evaluations of C_3 and C_4 for different suppliers are converted into 2-tuple linguistic variables by Eq. (5) and the objective data of C_1 and C_2 for the six suppliers are normalized first using the linear normalization method (Shih, Shyur, & Lee, 2007) ($r_{pq} = x_{pq}/\max_p\{x_{pq}\}$) and then converted into 2-tuple linguistic variables by Eqs. (1) and (2).

Table 10
Alternative suppliers for Example 2 (Wu, 2009).

Decision makers	Alternatives	Criteria			
		C_1	C_2	C_3	C_4
DM_1	A_1	60	40	L	H
	A_2	60	40	M	M
	A_3	70	80	L	VH
	A_4	50	30	M	M
	A_5	90	130	VH	VL
	A_6	80	120	VL	VL
DM_2	A_1	60	40	M	H
	A_2	60	40	H	M
	A_3	70	80	L	VH
	A_4	50	30	M	M
	A_5	90	130	H	L
	A_6	80	120	L	VL
DM_3	A_1	60	40	M	H
	A_2	60	40	L	L
	A_3	70	80	L	H
	A_4	50	30	M	H
	A_5	90	130	V	L
	A_6	80	120	L	VL

Table 11
2-Tuple linguistic decision matrix of DM_1 for Example 2.

Alternatives	Criteria			
	C_1	C_2	C_3	C_4
A_1	$\Delta(0.667)$	$\Delta(0.308)$	$(s_1,0)$	$(s_3,0)$
A_2	$\Delta(0.667)$	$\Delta(0.308)$	$(s_2,0)$	$(s_2,0)$
A_3	$\Delta(0.778)$	$\Delta(0.615)$	$(s_1,0)$	$(s_4,0)$
A_4	$\Delta(0.556)$	$\Delta(0.231)$	$(s_2,0)$	$(s_2,0)$
A_5	$\Delta(1.000)$	$\Delta(1.000)$	$(s_4,0)$	$(s_0,0)$
A_6	$\Delta(0.889)$	$\Delta(0.923)$	$(s_0,0)$	$(s_0,0)$

Table 12
Aggregated 2-tuple linguistic decision matrix for Example 2.

Alternatives	Criteria			
	C_1	C_2	C_3	C_4
A_1	$(s_3, -0.083)$	$(s_1, 0.058)$	$(s_2, -0.051)$	$(s_3, 0)$
A_2	$(s_3, -0.083)$	$(s_1, 0.058)$	$(s_2, -0.059)$	$(s_1, 0.121)$
A_3	$(s_3, 0.028)$	$(s_2, 0.115)$	$(s_1, 0)$	$(s_3, 0.121)$
A_4	$(s_2, 0.056)$	$(s_1, -0.019)$	$(s_2, 0)$	$(s_3, -0.121)$
A_5	$(s_4, 0)$	$(s_4, 0)$	$(s_4, -0.07)$	$(s_1, -0.051)$
A_6	$(s_4, -0.111)$	$(s_4, -0.077)$	$(s_1, -0.051)$	$(s_0, 0)$

Taking DM_1 as an example, we can get the 2-tuple linguistic decision matrix R_1 as shown in Table 11. It may be mentioned here that the 2-tuple linguistic variables are special cases of the interval 2-tuple linguistic variables.

Steps 2 and 4: According to the method suggested in Deng and Chan (2011), the weights of the four criteria are determined as follows:

$$w_1 = 0.252, \quad w_2 = 0.307, \quad w_3 = 0.155,$$
$$w_4 = 0.286.$$

Step 3: The objective information and subjective evaluations of the six alternatives are aggregated by Eq. (12) and the collective 2-tuple linguistic decision matrix is obtained as shown in Table 12.

Step 5: Product late delivery, cost, and risk factor are cost criteria and supplier service performance is a benefit criterion. Thus the PIS and NIS of the collective 2-tuple linguistic decision matrix are obtained as:

$$r^+ = ((s_1, 0.043), (s_1, -0.103), (s_1, -0.051), (s_3, 0.121)),$$
$$r^- = ((s_2, 0.028), (s_1, -0.114), (s_4, -0.07), (s_0, 0)).$$

Step 6: The normalized 2-tuple linguistic distances $(\bar{d}_{pq}, \alpha_{pq})$ are computed by Eq. (18) as shown in Table 13.

Steps 7 and 8: The 2-tuples (S_p, α_p), (R_p, α_p) and (O_p, α_p), $p = 1, 2, \ldots, 5$ are calculated by Eqs. (21) and (22) and the results are presented in Table 14.

Step 9: Rank the alternatives in accordance with the 2-tuples (S_p, α_p), (R_p, α_p) and (O_p, α_p), referring to Definition 3. The results are shown in Table 15 and compared with fuzzy VIKOR (Sanayei et al., 2010) and the methods suggested by Wu (2009) and Deng and Chan (2011).

For the international supplier selection problem, all of the four methods give the top two ranks to suppliers A_1 and A_4. This is because the first ranked alternative has no advantage to be a single solution. According to the results of the proposed model, the first

Table 13
Normalized 2-tuple linguistic distances for Example 2.

Alternatives	Criteria			
	C_1	C_2	C_3	C_4
A_1	$(s_1, 0)$	$(s_0, 0.1)$	$(s_1, 0.092)$	$(s_1, -0.111)$
A_2	$(s_1, 0)$	$(s_0, 0.1)$	$(s_1, 0.082)$	$(s_2, 0.074)$
A_3	$(s_2, 0)$	$(s_2, 0)$	$(s_0, 0.07)$	$(s_0, 0)$
A_4	$(s_0, 0)$	$(s_0, 0)$	$(s_2, -0.088)$	$(s_1, 0.028)$
A_5	$(s_4, 0)$	$(s_4, 0)$	$(s_4, 0)$	$(s_3, 0.021)$
A_6	$(s_3, 0)$	$(s_4, -0.1)$	$(s_0, 0)$	$(s_4, 0)$

condition (C1) is not satisfied: $\Delta^{-1}(O(A^{(2)}), \alpha(A^{(2)})) - \Delta^{-1}(O(A^{(1)}), \alpha(A^{(1)})) < \frac{1}{6-1}$, which indicates that both the suppliers A_1 and A_4 are good alternatives and can be considered as the best compromise solutions in this example.

4.3. Example 3

There are still some situations, in which there are only objective data existed in a supplier selection problem, and transformation of crisp values into fuzzy information for decision making is also important (Yue & Jia, 2013). Therefore, to demonstrate the proposed method for supplier selection with only quantitative data, an example from Yue (2013) is considered. A textile company located in Shijiazhuang, China is desired to select the most suitable supplier for its purchasing decision regarding a main product that affects the production process considerably. After pre-evaluation, four potential suppliers, recorded as A_1, A_2, A_3, and A_4, are included in the evaluation process. Five decision makers, $DM_k(k = 1,2,3,4,5)$ make decision for this selection. These experts from different departments or institutions include a production manager, a quality manger, a material manager, a planning manager and a research and development manager. The weight vector of the decision makers is $\lambda = (0.197, 0.250, 0.163, 0.242, 0.148)^T$. Four criteria are considered as: price/cost (C_1), technical level (C_2), supply capacity (C_3) and product quality (C_4). The quantitative values of the supplier selection criteria are given in Table 16. Suppose that the following linguistic term set is employed to express the decision information:

$$S = \{s_0 = Very\ low,\ s_1 = Low,\ s_2 = Medium\ low,\ s_3 = Medium,\ s_4 = Medium\ high,\ s_5 = High,\ s_6 = Very\ high\}.$$

Next, various steps of the proposed method are carried out to derive the most desirable supplier:

Steps 1 and 3: The quantitative data of the four criteria for the four suppliers are normalized first using the linear normalization method (Shih et al., 2007) and then converted into 2-tuple linguistic variables by Eqs. (1) and (2). As a result, the collective 2-tuple linguistic decision matrix is obtained as expressed in Table 17.

Steps 2 and 4: By aggregate the crisp criteria weights given by the four decision makers (Yue, 2013) using weighted average, the weights of criteria are determined as follows:

Table 15
Ranking comparisons for Example 2.

Alternatives	ITL-VIKOR	Fuzzy VIKOR	Wu (2009)	Deng and Chan (2011)
A_1	1	1	2	2
A_2	4	4	4	3
A_3	3	3	3	4
A_4	2	2	1	1
A_5	6	6	6	6
A_6	5	5	5	5

$$w_1 = 0.282,\quad w_2 = 0.265,\quad w_3 = 0.280,\quad w_4 = 0.173.$$

Step 5: In this example, cost is a cost criterion and technical level, supply capacity and product quality are benefit criteria. Thus the PIS and NIS of the collective 2-tuple linguistic decision matrix are determined as:

$$r^+ = ((s_1, 0.043), (s_1, -0.103), (s_1, -0.051), (s_3, 0.121)),$$
$$r^- = ((s_2, 0.028), (s_1, -0.114), (s_4, -0.07), (s_0, 0)).$$

Step 6: The normalized 2-tuple linguistic distances $(\bar{d}_{pq}, \alpha_{pq})$ are computed by Eq. (18) and shown in Table 18.

Steps 7 and 8: The 2-tuples (S_p, α_p), (R_p, α_p) and (O_p, α_p), $p = 1, 2, \ldots, 5$ are calculated by Eqs. (21) and (22) and the results are shown in Table 19.

Step 9: Rank the alternatives in accordance with the 2-tuples (S_p, α_p), (R_p, α_p) and (O_p, α_p) in increasing order. Table 20 displays the results of ranking produced by ITL-VIKOR and traditional VIKOR, extended TOPSIS (Yue, 2013) and traditional TOPSIS methods.

For this supplier selection problem, all of the listed methods give the top rank to supplier A_1 and the lowest rank to supplier A_4. The ranking order of alternatives by the proposed ITL-VIKOR exactly matches with the traditional VIKOR. That is, in the crisp situation, there is a very good agreement between ranking results of the proposed ITL-VIKOR and the classical VIKOR. This demonstrates the validity of the suggested approach. In addition, the obtained ranking orders by the extended TOPSIS and the traditional TOPSIS are exactly the same but they are different from the ones derived from the VIKOR-based methods. This can be explained by the characteristics of VIKOR and TOPSIS. First, the aggregation approaches of both the two models are different. The VIKOR introduces an aggregating function, representing the distance from the ideal solution, while the TOPSIS method is based on the principle that the optimal point should have the shortest distance from the positive ideal solution and the farthest from the negative ideal solution. Second, the two models use different kinds of normalization to eliminate the units of the criteria functions: the VIKOR method uses linear normalization but the TOPSIS method uses vector normalization.

Table 14
The 2-tuples (S_p, α_p), (R_p, α_p) and (O_p, α_p) of the six alternatives for Example 2.

	A_1	A_2	A_3	A_4	A_5	A_6
(S_p, α_p)	$(s_1, -0.064)$	$(s_1, 0.059)$	$(s_1, 0.04)$	$(s_1, -0.107)$	$(s_4, -0.065)$	$(s_3, 0.001)$
(R_p, α_p)	$(s_0, 0.063)$	$(s_1, -0.086)$	$(s_1, -0.096)$	$(s_0, 0.08)$	$(s_1, 0.057)$	$(s_1, 0.036)$
(O_p, α_p)	$(s_0, 0.027)$	$(s_1, 0.062)$	$(s_1, 0.028)$	$(s_0, 0.034)$	$(s_4, 0)$	$(s_3, 0.091)$

Table 16
Alternative suppliers for Example 3 (Yue, 2013).

Decision makers	Alternatives	Criteria			
		C_1	C_2	C_3	C_4
DM_1	A_1	1055	89	93	78
	A_2	1038	90	76	88
	A_3	1029	78	69	97
	A_4	1040	92	89	65
DM_2	A_1	1055	65	89	100
	A_2	1038	78	88	92
	A_3	1029	75	90	69
	A_4	1040	92	89	71
DM_3	A_1	1055	66	78	71
	A_2	1038	76	81	63
	A_3	1029	71	64	67
	A_4	1040	70	68	76
DM_4	A_1	1055	92	98	97
	A_2	1038	89	93	95
	A_3	1029	83	81	80
	A_4	1040	90	80	77
DM_5	A_1	1055	71	80	75
	A_2	1038	68	99	80
	A_3	1029	77	70	80
	A_4	1040	82	79	73

Table 17
Aggregated 2-tuple linguistic decision matrix for Example 3.

Alternatives	Criteria			
	C_1	C_2	C_3	C_4
A_1	$(s_6, 0)$	$(s_5, 0.061)$	$(s_6, 0)$	$(s_6, 0)$
A_2	$(s_6, -0.016)$	$(s_6, -0.06)$	$(s_6, -0.017)$	$(s_6, -0.012)$
A_3	$(s_6, -0.025)$	$(s_5, 0.059)$	$(s_5, 0.028)$	$(s_5, 0.074)$
A_4	$(s_6, -0.014)$	$(s_6, 0)$	$(s_6, -0.078)$	$(s_5, 0.003)$

Table 18
Normalized 2-tuple linguistic distances for Example 3.

Alternatives	Criteria			
	C_1	C_2	C_3	C_4
A_1	$(s_6, 0)$	$(s_6, -0.015)$	$(s_0, 0)$	$(s_0, 0)$
A_2	$(s_2, 0.013)$	$(s_3, 0.064)$	$(s_1, -0.045)$	$(s_0, 0.076)$
A_3	$(s_0, 0)$	$(s_6, 0)$	$(s_6, 0)$	$(s_3, 0.068)$
A_4	$(s_3, -0.077)$	$(s_0, 0)$	$(s_3, 0.06)$	$(s_6, 0)$

Table 19
The 2-tuples (S_p, α_p), (R_p, α_p) and (O_p, α_p) of the four alternatives for Example 3.

	A_1	A_2	A_3	A_4
(S_p, α_p)	$(s_3, 0.043)$	$(s_2, -0.039)$	$(s_4, -0.023)$	$(s_3, -0.051)$
(R_p, α_p)	$(s_2, -0.052)$	$(s_1, -0.017)$	$(s_2, -0.054)$	$(s_1, 0.007)$
(O_p, α_p)	$(s_5, 0.022)$	$(s_0, 0)$	$(s_6, -0.007)$	$(s_2, -0.022)$

Table 20
Ranking comparisons for Example 3.

Alternatives	ITL-VIKOR	VIKOR	Extended TOPSIS	TOPSIS
A_1	3	3	2	2
A_2	1	1	1	1
A_3	4	4	4	4
A_4	2	2	3	3

The three examples presented above have demonstrated and validated the proposed method as a potential decision making method for dealing with the supplier selection problems under different situations. From the analysis, it can be concluded that the ranking orders of the alternative suppliers given by the proposed method are more accurate and reliable. Comparing with other supplier selection methods in prior literature, the ITL-VIKOR method proposed in this paper has the following advantages:

- The proposed method has exact characteristic in linguistic information processing. It can effectively avoid the loss and distortion of information which occur formerly in the linguistic information processing.
- Multiple and conflicting criteria including quantitative as well as qualitative can be considered and evaluated by using the extended VIKOR method during the supplier selection process. The proposed method is a general method and can consider any number of quantitative and qualitative supplier selection criteria.
- Performance criteria of suppliers and their relative importance weights are evaluated in a linguistic manner rather than in precise numerical values. This enables the decision makers to express their judgments more realistically and makes the assessment easier to be carried out.
- The diversity and uncertainty of decision makers' assessment information can be well reflected and modeled using interval 2-tuple linguistic variables. And it provides an organized method to combine expert knowledge and experience for use in a supply chain management study.
- The proposed method is an appropriate and efficient method for the supplier selection problem. By considering three ranking lists, (S_p, α_p), (R_p, α_p) and (O_p, α_p), and by adopting the coefficient v as well as the acceptable advantage, the proposed model becomes more flexible and convenient for fine adjustments according to the decision maker's preferences.

5. Conclusions

In this paper, we study the supplier selection problem in supply chain management under incomplete and uncertain information environment. And an extended VIKOR method for group decision making with interval 2-tuple linguistic variables is employed to deal with the supplier selection problems in the supply chain system. During the supplier selection process, the alternatives' information and performances are usually incomplete and uncertain. Different decision makers may demonstrate different opinions because of their different expertise and backgrounds. Thus, in the proposed supplier selection method, interval 2-tuple linguistic variables, a new representation of uncertain information, is used to express the decision makers' diversity assessments; the VIKOR method, a recently introduced MCDM method, is utilized to handle the multiple conflicting criteria and select the optimum suppliers. The new model, combing the benefits of interval 2-tuple and VIKOR, can provide valuable assistance for the best supplier selection decision making problems.

Three empirical examples have been given to illustrate the effectiveness of the proposed method. We further tested the validity and demonstrated the advantages of the proposed methodology by comparing with the current methods in the literature. From experimental results, it can be concluded that the ITL-VIKOR method for group decision making with interval 2-tuple linguistic information is more suitable to address the supplier selection problem of an organization under uncertain and incomplete information environment. In addition, with the help of this model, top management can make full use of the decision information available in the group multi-criteria supplier selection to establish an effective supply chain for an organization to reduce operational costs and improve the quality of its end products.

As far as future research is concerned, the following directions are recommended. Firstly, the criteria are assumed to be independent in the proposed method when modeling the supplier selection problem. In actual cases, however, various criteria are often hierarchically structured and may have interdependent relationships to some extent. Accordingly in the future, a modified model that could construct the network relationship and cope with the interdependence among different criteria should be developed. Secondly, the weighted average is used in information aggregation of the proposed model. But depending on the particular type of aggregation operator used, the rankings of the alternatives may be dissimilar and lead to different decisions. In future, we expect to further extend the proposed method by adding other characteristics in the decision making process, such as weighted geometric mean, hybrid average and generalized mean operators, and more complex structures. Finally, the proposed method for supplier selection in uncertain linguistic environment offers a general procedure for multi-criteria group decision making. For future research, it would be interesting to explore the application of the approach presented in here to deal with other management decision making problems.

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