UNIT-2 Linear Programming :-

Standard form of linear programming Problem

Geometry of linear programming problems

Definitions and Theorems

Solution of a System of linear Simultaneous Equatio[ns]

Pivotal reduction of a general system of equations

Motivation to the Simplex Method

Simplex Algorithm

Duality in Linear Programming

Dual Simplex Method / Two-phase Simplex Method

UNIT-2 Linear Programming

Linear Programming Problem.

- Graphical Method
- Simple x Method
- Two-phase Simplex Method
  - Dual simplex method

Topic-2

Graphical Method :- (This method is only appilicable when there are 2 variables)

(Geometry)

1) Maximize $z = 12x_1 + 16x_2$

$10x_1 + 20x_2 \leq 120$

$8x_1 + 8x_2 \leq 80$

x_1 and $x_2 \geq 0$

Q. solve the following LPP using Graphical Method

2) Minimize $z = 4x_1 + 6x_2$

$x_1 + x_2 \geq 8$

$6x_1 + x_2 \geq 12$

x_1 and $x_2 \geq 0$

3) Maximize $z = 100x_1 + 60x_2$

Subject to $5x_1 + 10x_2 \leq 50$

$8x_1 + 2x_2 \geq 16$

$3x_1 - 2x_2 \geq 6$

3 Constraints (3 Equations)

x_1 and $x_2 \geq 0$

Sol:-

Maximize $Z = 12x_1 + 16x_2$ (objective function)

$10x_1 + 20x_2 \leq 120 \longrightarrow ①$

$8x_1 + 8x_2 \leq 80 \longrightarrow ②.$

x_1 and $x_2 \geq 0$.

$\Downarrow$

For Eq ①

$10x_1 + 20x_2 = 120 \longrightarrow ①$

x_1	0	12
x_2	6	0

when $x_1 = 0$.

$10(0) + 20x_2 = 120$

$20x_2 = 120$

$x_2 = \frac{\cancel{120}\ 6}{\cancel{20}}$

$\boxed{x_2 = 6}$

when $x_2 = 0$

$10x_1 + 20(0) = 120$

$10x_1 = 120$

$x_1 = \frac{120}{10} = 12$

$\boxed{x_1 = 12}$

Plot the points in the graph

(0,6) (12,0)

Eq ② $\Rightarrow$ $8x_1 + 8x_2 = 80 \longrightarrow ②$

x_1	0	10
x_2	10	0

when $x_1 = 0$

$8(0) + 8x_2 = 80$

$8x_2 = 80$

$x_2 = \frac{\cancel{80}}{\cancel{8}}\ 10$

$\boxed{x_2 = 10}$

when $x_2 = 0$

$8x_1 + 8(0) = 80$

$8x_1 = 80$

$x_1 = \cancel{80}/\cancel{8}\ 10$

$\boxed{x_1 = \cancel{8}\,10}$

Again plot the points in the graph

(0,10), (10,0).

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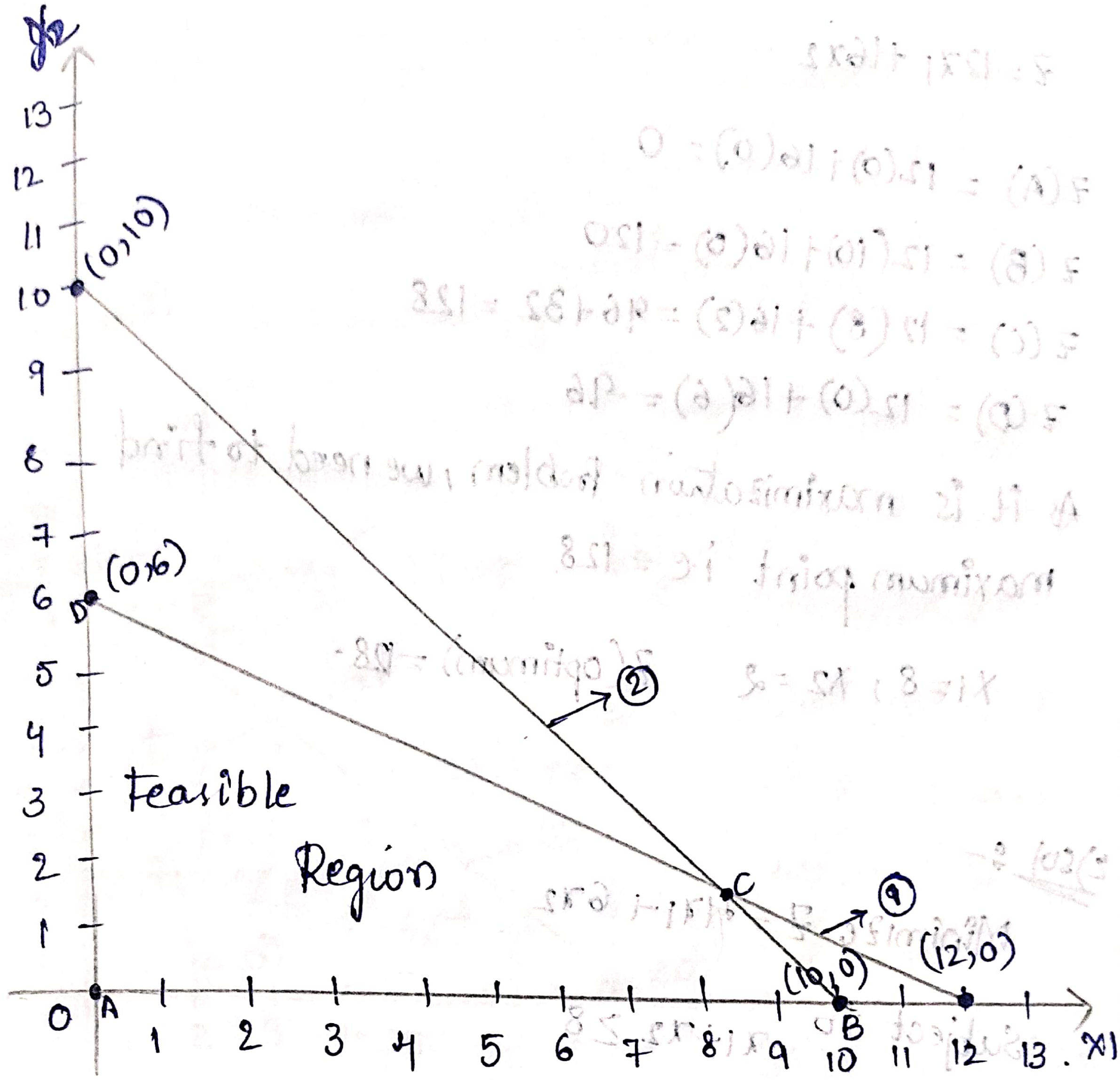


We need to find feasable region. Inorder to find feasible region observe two Constraints that they are g lesserthan Equal to or Greater than Equal to

Basedon

* Feasible Region should be below both lines.

The closed polygon (ABCD) is the Feasible Region

A (0,0)
B (10,0)
C (8,2)
D (0,6)

} substitue the values in Objective Function (Z) inorder to find the optimum point

$z = 12x_1 + 16x_2$

$z(A) = 12(0) + 16(0) = 0$

$z(B) = 12(10) + 16(0) = 120$

$z(C) = 12(8) + 16(2) = 96 + 32 = 128$

$z(D) = 12(0) + 16(6) = 96$

As it is maximization Problem, we need to find maximum point i.e = 128

$x_1 = 8,\ x_2 = 2$ $\quad z(\text{optimum}) = 128.$

2) Sol :-

Minimize $z = 4x_1 + 6x_2$

subject to $x_1 + x_2 \geq 8$

$6x_1 + x_2 \geq 12$

x_1 and $x_2 \geq 0$

⇓

$x_1 + x_2 = 8 \rightarrow (1)$

when $x_1 = 0$.

$0 + x_2 = 8$

$\boxed{x_2 = 8}$

when $x_2 = 0$.

$x_1 + 0 = 8$

$\boxed{x_1 = 8}$

x_1	0	8
x_2	8	0

points are
(0, 8)
(8, 0)

Eq (2) ⇒ $6x_1 + x_2 = 12 \rightarrow (2)$

when $x_1 = 0$

$6(0) + x_2 = 12$

$\boxed{x_2 = 12}$

when $x_2 = 0$

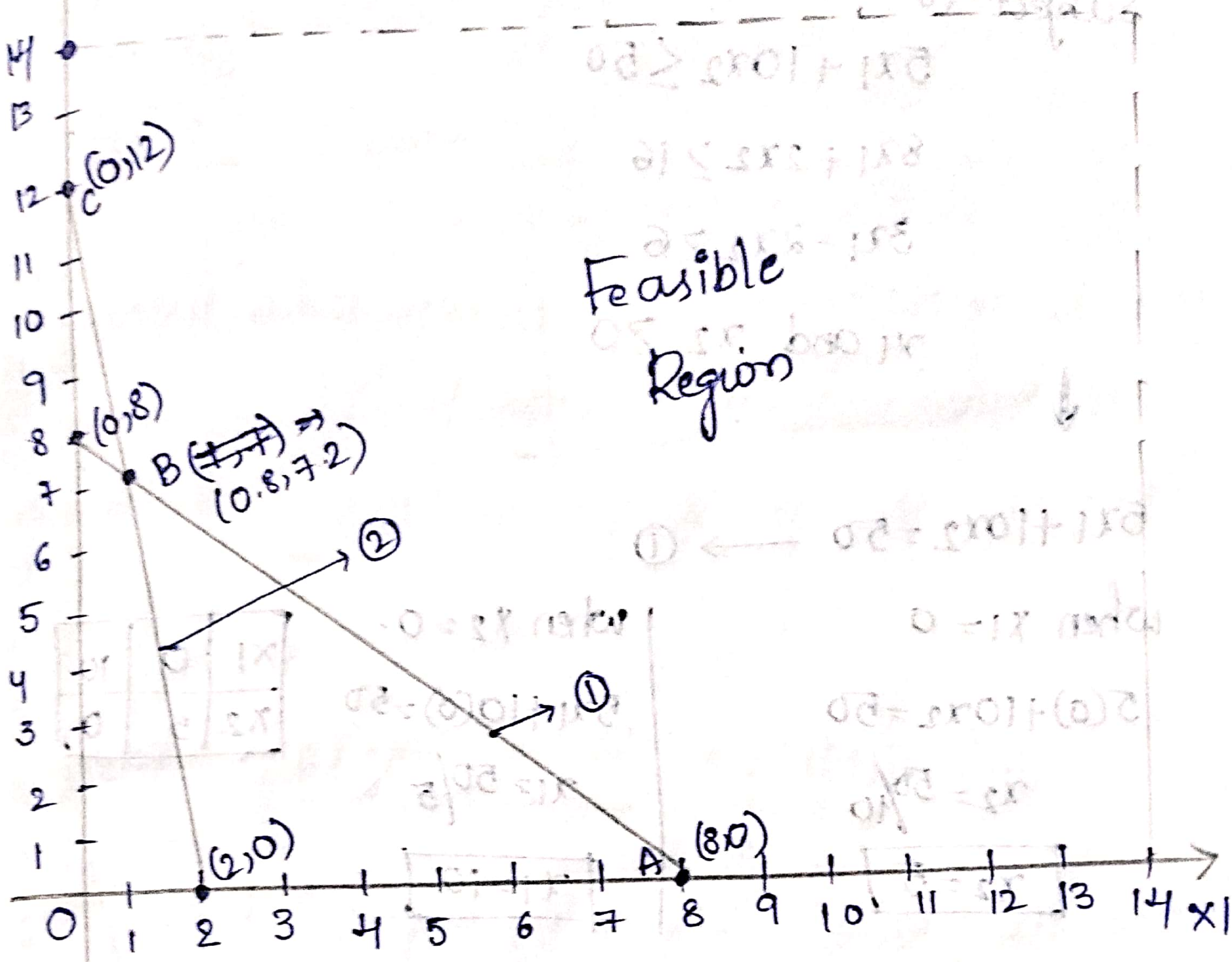
$6x_1 + (0) = 12$

$x_1 = 12/6$

$\boxed{x_1 = 2}$

x_1	0	2
x_2	12	0

points are
(0, 2) (12, 0)



For Minimum Feasible Region should be above the 2 lines

Finding optimum solution $Z = 4x_1 + 6x_2$

A $Z(A) = 4(8) + 6(0) = 32$

$Z(B) = 4(0.8) + 6(7.2) = 3.2 + 43.2 \Rightarrow 46.4$

$Z(C) = 4(0) + 6(12) = 72$

As it is minimum problem, we need to find least value

Optimum solution is A

$x_1 = 8$, $x_2 = 0$, $Z(\text{optimum}) = 32$

③ Sol :-

Maximize $= 100x_1 + 60x_2$

Subject to

$5x_1 + 10x_2 \leq 50$

$8x_1 + 2x_2 \geq 16$

$3x_1 - 2x_2 \geq 6$

x_1 and $x_2 \geq 0$

⇓

$5x_1 + 10x_2 = 50 \longrightarrow ①$

when $x_1 = 0$

$5(0) + 10x_2 = 50$

$x_2 = 50/10$

$\boxed{x_2 = 5}$

when $x_2 = 0$.

$5x_1 + 10(0) = 50$

$x_1 = 50/5$

$\boxed{x_1 = 10}$

x_1	0	10
x_2	5	0

Eq ② ⇒

$8x_1 + 2x_2 = 16 \longrightarrow ②$

When $x_1 = 0$

$8(0) + 2x_2 = 16$

$2x_2 = 16$

$x_2 = 16/2 \ 8$

$\boxed{x_2 = 8}$

when $x_2 = 0$.

$8x_1 + 2(0) = 16$

$8x_1 = 16$

$x_1 = 16/8 \ 2$

$\boxed{x_1 = 2}$

x_1	0	2
x_2	8	0

Eq ③ ⇒

$3x_1 - 2x_2 = 6$.

when $x_1 = 0$

$3(0) - 2x_2 = 6$

$-2x_2 = 6$

$x_2 = -6/2$

$\boxed{x_2 = -3}$

x_1	0	2
x_2	-3	0

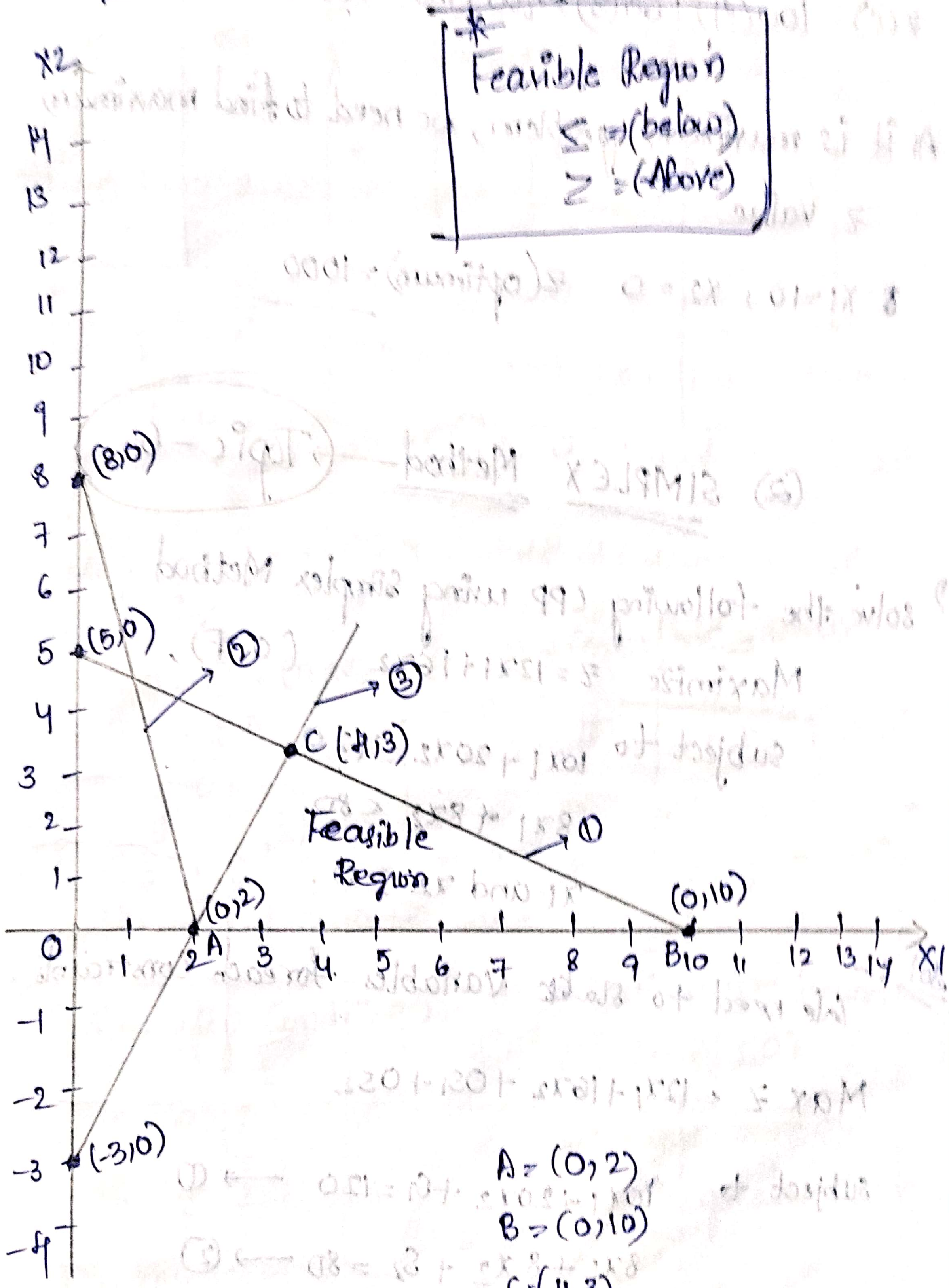
when $x_2 = 0$.

$3x_1 - 2(0) = 6$

$3x_1 = 6$

$x_2 = 6/3$

$\boxed{x_2 = 2}$



A = (0, 2)

B = (0, 10)

C = (4, 3)

Sub. A, B, C in Z

The optimum ...

$Z(A) = 100(2) + 60(0) = 200$

$Z(B) = 100(10) + 60(0) = 1000$

$Z(C) = 100(4) + 60(3) = 400 + 180 = 580$.

As it is maximum problem, we need to find maximum z value

B $x_1 = 10$, $x_2 = 0$ $Z(\text{optimum}) = 1000$

(2) SIMPLEX Method → Topic-6

1) Solve the following LPP using Simplex Method.

≤ 0

Maximize $Z = 12x_1 + 16x_2$ (O.F),

subject to $10x_1 + 20x_2 \leq 120$.

$8x_1 + 8x_2 \leq 80$.

x_1 and $x_2 \geq 0$.

Sol :- We need to slack variables for each constraints.

$\text{Max } Z = 12x_1 + 16x_2 + 0S_1 + 0S_2$

subject to $10x_1 + 20x_2 + S_1 = 120 \longrightarrow (1)$

$8x_1 + 8x_2 + S_2 = 80 \longrightarrow (2)$

x_1, x_2, S_1 and $S_2 \geq 0$

Initial Simplex Table

CBi	Cij	12	16	0	0	Solution	Ratio
	Basic variable	x1	x2	S1	S2		
0	S1	10	$\boxed{20}$	1	0	120	$120/20 = \underline{6}$
0	S2	8	8	0	1	80	$80/8 = 10$
	Zj	0	0	0	0	0	
	Cj − Zj	12	16	0	0		

$$Z_j = \sum_{i=1}^{2} (CB_i)(a_{ij})$$

$\Rightarrow (0\times10) + (0\times8)$
$= 0 + 0$
$= 0$

$(0\times20) + (0\times8)$
$= 0 + 0$
$= 0$

$0\times1 + (0\times0)$
$= 0 + 0$
$= 0$

$(0\times0) + (0\times1)$
$= 0$

$(0\times120) + (0\times80)$
$= 0 + 0 = 0$

Optimality Condition:-

For Max:-
all $C_j - Z_j \le 0$

For Min:-
all $C_j - Z_j \ge 0$

As, it is maximum
all $C_j - Z_j \le 0$;

But, here some of values is greater than 0.

For optimality, we need to do further steps.

Select the maximum value in $C_j - Z_j$, (i.e = 16)

That column is called Key column.

Then, we need to find ratio;

Ratio b/w Ratio column & key column

After finding Ratio, we need to select least value in Ratio column.

& Then select that row; That row is called key row

The Intersection value is called Key element (i.e 20)

x_2 = Entering variable | column var = Entering var.

s_1 = leaving variable | Row variable = leaving var

<u>Iteration - I</u>

C_{Bi}	C_j	12	16	0	0	Solution	Ratio
	B.V	x_1	x_2	s_1	s_2		
16	x_2	$\frac{1}{2}$ $(10/20)$	1 $(20/20)$	$1/20$	0 $(0/20)$	$(120/20)$ 6	$6/(1/2) = 12$
0	s_2	4	0	$-2/5$	1	32	$32/4 = 8$ least value
	Z_j	8	16	$4/5$	0		
$C_j - Z_j$		4	0	$-4/5$	0		

Intersection point = 4

New value = old val − $\dfrac{\text{corr. key col. val} \times \text{corr. key row value}}{\text{key element}}$

$8 - \dfrac{8 \times 10}{20}$ = New value

$8 - \dfrac{80}{20} = 4 \Rightarrow$ New value

$8 - \dfrac{8 \times 20}{20}$ = N.V

$8 - \dfrac{160}{20} = 0 \Rightarrow$ New value

$0 - \dfrac{8 \times 1}{20} = 0 - \dfrac{8}{20} = \left(-\dfrac{2}{5}\right)$ = N.V

$1 - \dfrac{8 \times 0}{20} = 1 - \dfrac{0}{20} = 1$ = New val

$80 - \dfrac{8 \times 120}{20} = 80 - \dfrac{960}{20}$

$= 80 - 48 \Rightarrow 32$ = New value

After Z_j & $C_j - Z_j$, ($C_j - Z_j$) should be 0 & less than 0 then it is optimized)

If not optimized,

Select maximum value in $C_j - Z_j$ (i.e 4)

Same process as in Initial Simplex method Table

Iteration - II

C_{Bi}	C_j	12	16	0	0	Sol.	Ratio
	B.V	X_1	X_2	S_1	S_2		
16.	X_2	0	1	1/10	−1/8	2	
12	X_1	1	0	−1/10	1/4	8	
	Z_j	12	16	2/5	1	128	
	$C_j - Z_j$	0	0	(−2/5)	(−1)		

check optimality (All $C_j - Z_j$ values should be 0 or less than '0') ∴ It is optimized

New Values

$\frac{1}{2} - \left(\frac{\frac{1}{2} \times 4}{4}\right) = 0$; $0 - \left(\frac{\frac{1}{2} \times 1}{4}\right) = -\frac{1}{8}$

$1 - \left(\frac{\frac{1}{2} \times 0}{4}\right) = 1$; $6 - \left(\frac{\frac{1}{2} \times 32}{4}\right) = 2$

$\frac{1}{20} - \left(\frac{\frac{1}{2} \times (-2/5)}{4}\right) = \frac{1}{10}$.

Finally;

$X_1 = 12$, $X_2 = 16$ $Z(\text{optimum}) = 128$

(3) Two-Phase Simplex Method → Topic - 8

⇒ Minimization type of objective Function

⇒ Constraints

For '≥' or '=' type Constraints

subtract slack variable and add artificial variable

For '≤' type Constraints

add slack variable

⇒ phase 1 and phase 2

For phase 1: (Same process) Simplex method with revised objective Function

→ At the end of phase 1 check whether the objective function value is zero in the optimal table.

If yes
Go to phase 2.

By replacing the original value of objective function

Eg: Min $z = 10x_1 + 6x_2 + 2x_3$

solve to

$$-x_1 + x_2 + x_3 \geq 1$$

$$3x_1 + x_2 - x_3 \geq 2$$

x_1, x_2 and $x_3 \geq 0$.

Sol:

Min $z = 10x_1 + 6x_2 + 2x_3$

subject to

$$-x_1 + x_2 + x_3 - S_1 + A_1 = 1$$

$$3x_1 + x_2 - x_3 - S_2 + A_2 = 2$$

$$x_1, x_2, x_3, S_1, S_2, A_1, A_2 \geq 0$$

Min $z = 10x_1 + 6x_2 + 2x_3 + 0S_1 + 0S_2 + A_1 + A_2$

Min $z = A_1 + A_2$ (Take only Artifical values)

subject to

$$-x_1 + x_2 + x_3 - S_1 + A_1 = 1$$

$$3x_1 + x_2 - x_3 - S_2 + A_2 = 2$$

$$x_1, x_2, x_3, S_1, S_2, A_1, A_2 \geq 0.$$

Initial Table

C_{Bi}	C_j	0	0	0	0	0	1	1	Solution
	B.V	x_1	x_2	x_3	S_1	S_2	A_1	A_2	
R_1 1	A_1	-1	1	1	-1	0	1	0	1
R_2 1	A_2	3	1	-1	0	-1	0	1	2
	z_j	2	2	0	-1	-1	1	1	3
$C_j - z_j$		-2	-2	0	1	1	0	0	

z_j values

$-1+3=2$ | $1-1=0$ | $1(-1)=-1$ | $1(0)+1(1)=1$

$1+1=2$ | $1(-1)+0=-1$ | $1(1)+0=1$

After finding $c_j - z_j$ check optimality

For Min z problem

$$\boxed{\text{all } C_j - z_j \text{ are } \geq 0}$$

we got 2 -ve values, so we didn't reach optimality.

Select the most -ve value, that -ve value column is key column

select the most +ve value, in solution column It is key row

x_1 is Entering variable

A_2 is Leaving variable

CBi	Cj	0	0	0	0	0	1	Solution
	B.V.	X1	X2	X3	S1	S2	A1	
R3 0	X1	1	1/3	-1/3	0	-1/3	0	~~2/3~~ 2/3
R4 1	A1	0	4/3	2/3	-1	-1/3	1	5/3
	Zj	0	4/3	2/3	-1	-1/3	1	
Cj-Zj →		0	-4/3	-2/3	1	1/3	0	

Row3:-

$\boxed{\text{New value for Entering val = Old value / key element}}$

Row4 = old val - key column val × R3 value

= -1-(-1)×1 = -1+1 = 0.

= 1-[(-1) × 1/3] = 1+1/3 = 4/3

Same formula for all Row 4 values

Cj-Zj values are not less than 0 or zero.

It didn't reach optimality.

Again find key column & Row.

X2 = Entering var. A1 = leaving var.

<u>Iteration -2</u>

CBi	Cj	0	0	0	0	0	Solution
	B.V.	X1	X2	X3	S1	S2	
R5 0	X2	0	1	1/2	-3/4	-1/4	5/4
R6 0	X1	1	0	-1/2	1/4	-1/4	1/4
	Zj	0	0	0	1/4	0	0
Cj-Zj →		0	0	0	0	0	

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R5
New Values for Entering Variable = oldvalue / Key element

$\boxed{\text{Row-6. oldvalue} - \text{key column value} \times R_5}$

finding
After z_j, $C_j - z_j$;
we need to check; If the values are 0 or less than zero
So, we have reached the optimality

Phase-2 :-

The objective function values are zero in optimal Table then phase-2

CBi	Cj	10	6	2	0	0	solution
	B.V	x1	x2	x3	s1	s2	
6	x2	0	1	1/2	-3/4	-1/4	5/4
10	x1	1	0	-1/2	1/4	-1/4	1/4
	zj	10	6	-2	-2	-4	10
	cj - zj	0	0	4	2	4	4

Before In phase 1 we took Artifical values in phase 2, we need to take all values expect aritifical Values

Min $z = 10x_1 + 6x_2 + 2x_3$

$x_1 = 1/4$ $\Rightarrow 10(1/4) + 6(5/4) + 2(0) = 10$

$x_2 = 5/4$, $= \frac{10}{4} + \frac{30}{4} + 0 = 10$

$x_3 = 0$ $= \frac{40}{4} + 0 = 10$

$z(\text{opt}) = 10$. $\boxed{10 = 10}$

way to cross checking the objective Function

Topic-5

Pivotal Reduction of general System of equations

Find all the basic solutions Corresponding to the Systems of equations

$$2x_1 + 3x_2 - 2x_3 - 7x_4 = 1$$
$$x_1 + x_2 + x_3 + 3x_4 = 6$$
$$x_1 - x_2 + x_3 + 5x_4 = 4$$

Sol:- The given statement of equations are

$$2x_1 + 3x_2 - 2x_3 - 7x_4 = 1$$
$$x_1 + x_2 + x_3 + 3x_4 = 6$$
$$x_1 - x_2 + x_3 + 5x_4 = 4$$

Case-I :- Canonical form in terms of basic

Variables x_1, x_2 and x_3;

The matrix form of the above system is.

$$\begin{bmatrix} 2 & 3 & -2 & -7 & 1 \\ 1 & 1 & 1 & 3 & 6 \\ 1 & -1 & 1 & 5 & 4 \end{bmatrix}$$

First we pivot on the element $a_{11} = 2$.

$\Rightarrow$ $R_1 \to \frac{R_1}{2}$

$$\begin{bmatrix} 1 & \frac{3}{2} & -1 & -\frac{7}{2} & \frac{1}{2} \\ 1 & 1 & 1 & 3 & 6 \\ 1 & -1 & 1 & 5 & 4 \end{bmatrix}$$

$\Rightarrow$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & \frac{3}{2} & -1 & -\frac{7}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & 2 & \frac{13}{2} & \frac{11}{2} \\ 0 & -\frac{5}{2} & 2 & \frac{17}{2} & \frac{7}{2} \end{bmatrix}$$

$\rightarrow$ then we povit on $a_{22} = -\frac{1}{2}$

$R_2 \to (-2) R_2$

$$\begin{bmatrix} 1 & \frac{3}{2} & -1 & -\frac{7}{2} & \frac{1}{2} \\ 0 & 1 & -4 & -13 & -11 \\ 0 & -\frac{5}{2} & 2 & \frac{17}{2} & \frac{7}{2} \end{bmatrix}$$

$\Rightarrow R_1 \to R_1 - \frac{}{2} R_2,\ R_3 \to R_3 + \frac{}{2} R_1)$

$$\begin{bmatrix} 1 & 0 & 5 & 16 & 17 \\ 0 & 1 & -4 & -13 & -11 \\ 0 & 0 & -8 & -24 & -24 \end{bmatrix}$$

$\Rightarrow$ Then we pivot on $a'_{33} = -8$

$$R_3 \to \frac{R_3}{(-8)}$$

$$\begin{bmatrix} 1 & 0 & 5 & 16 & 17 \\ 0 & 1 & -4 & -13 & -11 \\ 0 & 0 & 1 & 3 & 3 \end{bmatrix}$$

$\Rightarrow R_1 \to R_1 - 5R_3,\ R_2 \to R_2 + 4R_3$.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 3 & 3 \end{bmatrix}$$

From the above matrix, we get the required Cononical form as

$$x_1 + x_4 = 2$$
$$x_2 - x_4 = 1$$
$$x_3 + 3x_4 = 3$$

From this Cononical forms, we can write the solutions of x_1, x_2, x_3 in terms of x_4 as

$$x_1 = 2 - x_4 \qquad x_3 = 3 - 3x_4$$
$$x_2 = 1 + x_4$$

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The solution can be obtained by setting

independent variable is zero i.e $x_4 = 0$.

$\therefore$ The basic solution is $x_1 = 2, x_2 = 1, x_3 = 3$ (Basic variable)

and $x_4 = 0$ (Non-basic variable)

Since all $x_J \geq 0$ $(J = 1, 2, 3, 4)$

$\therefore$ It is a basic feasible solution.

Standard form of LPP :- → Topic-1

An LPP is said to be in Standard form, if the following four Conditions are satisfied:

(1) The objective function must be either maximization or minimization; (max or min)

(2) All the Constraints should hold with an equality sign.

(3) The right-hand side (RHS) of all Constraints is non-Negative (~~RHS Variable ≥ 0~~) (positive)

(4) All the Variables involved in the problem are non negative (~~decision Variable ≥ 0~~) (positive)

Eg:- write the LPP into standard form.

$$\text{Max } z = x_1 + x_2 - 2x_3 \quad \text{-OR}$$

Sub. to
$$x_1 + x_2 + x_3 \leq 15$$
$$2x_1 - x_2 + x_3 \leq -10$$
$$x_1 + 2x_3 = 10$$

Con

$x_3 = x_3' - x_3''$

$x_1, x_2 \geq 0$, x_3 - unrestricted in sign

Sol:

$$x_1, x_2 \geq 0$$
$$x_3 = x_3' - x_3'' \; ; \; x_3' \geq 0, \; x_3'' \geq 0$$

LPP : $\text{Max } z = x_1 + x_2 - 2(x_3' - x_3'')$

Subject to $x_1 + x_2 + x_3' - x_3'' \leq 15$

$2x_1 - x_2 + x_3' - x_3'' \leq -10$

$x_1 + 2(x_3' - x_3'') = 10$ (All decision variables are already non-negative)

In LPP; $x_1 \geq 0$; $x_2 \geq 0$; $x_3' \geq 0$; $x_3'' \geq 0$

Constraints into Equality (Constraints are in the

LPP: $\text{Max } z = x_1 + x_2 - 2x_3' + 2x_3''$

Subject to $x_1 + x_2 + x_3' - x_3'' \leq 15$

$-(2x_1 + x_2 - x_3' + x_3'') \geq 10$

$x_1 + 2x_3' - 2x_3'' = 10$

All decision variables are ≥ 0, and also Right hand side is non - Negative.

Introduce Slack / surplus Variable :- (Constraints are equal)

Standard form of LPP:

$\leq \rightarrow +S_1$

$\geq \rightarrow S_1$

$\text{Max } z = x_1 + x_2 - 2x_3' + 2x_3''$

Subject to

$$\left.\begin{aligned} x_1 + x_2 + x_3' - x_3'' + S_1 &= 15 \\ -2x_1 + x_2 - x_3' + x_3'' - S_2 &= 10 \\ x_1 + 2x_3' - 2x_3'' &= 10 \end{aligned}\right\}$$

$x_1, x_2, x_3', x_3'', S_1, S_2 \geq 0$

Eg 2:-

write the LPP into Standard form

Maximize $Z = 4x_1 - 2x_2 + 6x_3$

Subject to $2x_1 - x_2 \leq 6$

$x_1 + 4x_2 + 7x_3 \geq 7$

$x_1 - 6x_3 \leq -8$

$x_1 \geq 0$, $x_2 \geq 0$; x_3 - unrestricted in sign.

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Sol :-

1) The objective function must be either maximization or minimization

Satisfy the Condition

2) All the Variables involved in the problem are non-negative. $x_2 \geq 2 \Rightarrow x_2 - 2 \geq 0$

$$x_2' = x_2 - 2 \geq 0$$

$$x_2' \geq 0 \ ; \ x_2' = x_2 - 2$$

$$\text{(or)} \ x_2 = x_2' + 2$$

$x_3 \rightarrow$ unrestricted sign

$x_3 = x_3' - x_3''$; $x_3' \geq 0$; $x'' \geq 0$

x_3 — unrestricted

$$x_3' = \begin{cases} x_3 \ ; \ x_3 \geq 0 \\ 0 \ ; \ x_3 < 0 \end{cases} \quad = \begin{cases} 0 & x_3 \geq 0 \\ -x_3 & x_3 < 0 \end{cases}$$

$$x_3' - x_3'' = \begin{cases} x_3 & x_3 \geq 0 \\ x_3 & x_3 < 0 \end{cases}$$

Eg :-

$$x_3 = 7 = 7 - 0$$

$$x_3' = 7 \geq 0$$

$$x_3'' = 0 \geq 0$$

$$x_3 = -7 = 0 - 7$$

$$\downarrow$$

$$x_3' = 0 \geq 0$$

$$x_3'' = 7 \geq 0$$

Min $z = 4x_1 - 2(x_2' + 2) + 6(x_3' - x_3'')$

Subject to $2x_1 - (x_2' + 2) \leq 6$

Scanned with OKEN Scanner

$$x_1 + 4(x_2' + 2) + 7(x_3' - x_3'') \geq 0$$

$$x_1 - 6(x_3' - x_3'') \leq -8$$

$$x_1, x_2', x_3', x_3'' \geq 0$$

Satisfied

(3) RHS Constraints non-Negatives

$$\text{Min } z = 4x_1 - 2x_2' - 4 + 6x_3' - 6x_3''$$

$$\text{s.t } 2x_1 - x_2' \leq 8$$

$$\left.\begin{array}{l} x_1 + 4x_2' + 7x_3' - 7x_3'' \geq -1 \\ x_1 - 6x_3' + 6x_3'' \geq -8 \end{array}\right\} \times$$

$$x_1, x_2', x_3', x_3'' \geq 0$$

$$\text{min } z = 4x_1 - 2x_2' - 4 + 6x_3' - 6x_3''$$

$$\text{s.t } \left.\begin{array}{l} 2x_1 - x_2' \leq 8 \\ -x_1 - 4x_2' - 7x_3' + 7x_3'' \leq 1 \\ -x_1 + 6x_3' - 6x_3'' \geq 8 \end{array}\right\} \checkmark$$

$$x_1, x_2', x_3', x_3'' \geq 0$$

Satisfied

(4) All Constraints with Equal sign

$$\text{min } z = 4x_1 - 2x_2' - 4 + 6x_3' - 6x_3''$$

$$\text{Sub. to } 2x_1 - x_2' + S_1 = 8$$

$$-x_1 - 4x_2' - 7x_3' + 7x_3'' + S_2 = 1$$

$$-x_1 + 6x_3' - 6x_3'' - S_3 = 8$$

$$x_1, x_2', x_3', x_3'', S_1, S_2, S_3 \geq 0$$

Satisfied

Eg (3):

Write LPP into standard-form

$\max z = x_1+x_2$

s.t $|4x_1+x_2| \leq 6$

$7x_1-x_2 \geq 5$

$x_1 \geq 0; x_2 \geq 0$

Sol:-

$\max z = x_1+x_2$

s.t $-6 \leq 4x_1+x_2 \leq 6$

$7x_1-x_2 \geq 5$

$x_1 \geq 0; x_2 \geq 0$

$\text{Max } z = x_1+x_2$

$4x_1+x_2 \geq -6$

$4x_1+x_2 \leq 6$

$7x_1-x_2 \geq 5$

$x_1 \geq 0; x_2 \geq 0$

RHS should be positive

$-4x_1-x_2 \leq 6$

$4x_1+x_2 \leq 6$

$7x_1-x_2 \geq 5$

$x_1 \geq 0; x_2 \geq 0$

Constraints Equal

$-4x_1-x_2+s_1=6$

$4x_1+x_2+s_2=6$

$7x_1-x_2-s_3=5$

$x_1 \geq 0; x_2 \geq 0; s_1, s_2, s_3 \geq 0$

Eg(4):- Write LPP into standard form

$$\text{Max } z = x_1 + 2x_2 - x_3$$

$$\text{s.t } x_1 + x_2 - x_3 \leq 5$$

$$-x_1 + 2x_2 + 3x_3 \geq -4$$

$$2x_1 + 3x_2 - 4x_3 \geq 3$$

$$x_1 + x_2 + x_3 = 2$$

$x_1 \geq 0$; $x_2 \geq P$; x_3 unrestricted in sign

Sol:- $x_1 \geq 0$

$x_2 \geq P \Rightarrow x_2 - P \geq 0$

$\therefore x_2' = x_2 - P$; $x_2 = x_2' + P$; $x_2' \geq 0$

$$\text{Max } z = x_1 + 2(x_2' + P) - (x_3' - x_3'')$$

$$\text{s.t } x_1 + (x_2' + P) - (x_3' - x_3'') \leq 5$$

$$-x_1 + 2(x_2' + P) + 3(x_3' - x_3'') \geq -4$$

$$2x_1 + 3(x_2' + P) - 4(x_3' - x_3'') \geq 3$$

$$x_1 + (x_2' + P) + (x_3' - x_3'') = 2$$

$$x_1, x_2', x_3', x_3'' \geq 0$$

After introducing Slack/surplus Variable

$$\text{Max } z = x_1 + 2(x_2' + P) - x_3' + x_3''$$

s.t

$$x_1 + x_2' - x_3' + x_3'' + S_1 = 5 - P$$

$$x_1 - 2x_2' - 3x_3' + 3x_3'' + S_2 = 4 + 2P$$

$$2x_1 + 3x_2' - 4x_3' - 4x_3'' - S_3 = 3 - 3P$$

$$x_1 + x_2' + x_3' - x_3'' = 2 - P$$

$$x_1, x_2', x_3', x_3'' \geq 0.$$

RHS :-

$5-P\geq 0$; $4+2P\geq 0$; $3-3P\geq 0$, $2-P\geq 0$

$P\leq 5$; $2P\geq -4$; $-3P\geq -3$; $P\leq 2$

$P\geq -2$ $\quad$ $3P\leq 3$

$P\leq 1$

$P\geq -2$; $P\leq 1$

$P\in[-2,1]$

Solution of a System of Linear Simultaneous Equations

Pb :-

$$x-2y+3z=7$$
$$2x+y+z=4$$
$$-3x+2y-2z=-10$$

$\longrightarrow$ ③

Steps :-

(1) Matrix form

(2) Getting two zeros in a column

(3) Solving Equations

Sol :-

Step-1 :-

$$\begin{array}{c}\\ \underbrace{\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 2 & 1 & 1 & 4 \\ -3 & 2 & -2 & -10 \end{array}\right]}_{\text{Variables}}\end{array}\begin{array}{l}\rightarrow R_1\\ \rightarrow R_2\\ \rightarrow R_3\end{array}$$

Constants

Numbers in R_1 & R_2

Step-2 $R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & -2 & 3 & 7 \\ 2 & 1 & 1 & -4 \\ -2 & 0 & 1 & -3 \end{bmatrix}$$

$R_2 \to 2 * R_2$

$$\begin{bmatrix} 1 & -2 & 3 & 7 \\ 4 & 2 & 2 & -8 \\ -2 & 0 & 1 & -3 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & -2 & 3 & 7 \\ 5 & 0 & 5 & 15 \\ -2 & 0 & 1 & -3 \end{bmatrix}$$

Here we got two zeros in R_2 and R_3 selecting that two rows to form a new equation

Step-3 :-

$5x + 5z = 15 \longrightarrow ①$

$-2x + z = -3 \longrightarrow ②$

$$\begin{array}{r} 5x + \cancel{5z} = 15 \\ -10x + \cancel{5z} = -15 \\ \hline 15x = 30 \end{array}$$

$\boxed{x = 2}$

$2x + y + z = 4 \longrightarrow ③$

(From step-1 matrix 2 row)

Sub x in Eq ②

$-2(2) + z = -3$

$-4 + z = -3$

$\boxed{z = 1}$

Sub x & z in Eq ③

$2x + y + z = 4$

$2(2) + y + 1 = 4$

$y = -1$