UNIT-3 OOR

Transportation Problem :-

(1) Finding initial basic feasible solution by north-west Corner Rule, Least Cost method and vogel's approx method:

Introduction :-

It is a special kind of linear programming problem (LPP) in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destinations respectively, such that the total cost of transportation is minimized.

There are 2 types of Transportation Problems

1) Balanced TP
   Supply is equal to Demand (S=D)

2) UnBalanced TP
   Supply is not equal to Demand (S≠D)

we cannot proceed Unbalanced TP into balanced TP, we need to add either Dummy Column or Dummy Row according to the situation.

Methods of TP :-

1) Finding the initial feasible solution
2) Finding optimization.

		A	B	C	Supply
Source	I	2	7	5	200
	II	3	4	2	300
	III	5	4	7	500
Demand		200	400	400	1000

Demand = Supply (i.e Balanced TP)

The techniques of Transportation Problem

(1) Northwest corner cell method
(2) Least cost cell method
(3) Vogel's approximation method (VAM)

Scanned with OKEN Scanner

(ii) North-west Corner Cell method

Destination

Source	A	B	C	D	Supply
1	3	1	7	4	300
2	2	6	5	9	400
3	8	3	3	2	500
Demand	250	350	400	200	

Sol:

It is a Balanced T.P

Demand = Supply = 1200

Source	A	B	C	D	Supply
1	$\boxed{250}$ ~~3~~	$\boxed{50}$ ~~1~~	~~7~~	~~4~~	~~300~~ ~~50~~ 0
2	~~2~~	$\boxed{300}$ ~~6~~	$\boxed{100}$ ~~5~~	~~9~~	~~400~~ ~~100~~ 0
3	~~8~~	~~3~~	$\boxed{300}$ ~~3~~	$\boxed{200}$ 2	~~500~~ ~~200~~ 0
Demand	~~250~~ 0	~~350~~ ~~300~~ 0	~~400~~ ~~300~~ 0	~~200~~ 0	1200

$(250 \times 3) + (50 \times 1) + (300 \times 6) + (100 \times 5) + (300 \times 3) + (200 \times 2) =$

$\boxed{₹\ 4400}$

⇒ ~~Compar~~ starting from north-west Corner, we need to Compare Demand & Supply ~~and~~ Values.

⇒ check which value is lesser in Demand & Supply apply that value in north-west Corner cell

⇒ As we got demand 0 in the ~~Abo~~ first column of North-west cell; so we need to delete all full column

⇒ After deleting; Again we need to select the nort-west corner from the table (B-(1))

⇒ In north-west corner Again compare demand & supply and delete the

⇒ After comparing we got the (B) row to be deleted as supply is 50 & demand is 350 (supply is Less)

⇒ Again compare to north-west column.

⇒ Apply the same Rule to all north-west columns

⇒ At last the only one cell remains i.e (2).

⇒ In that last cell we will be getting demand is Equal to supply

⇒ Therefore we got the balanced T.P (proved)

⇒ Multiply the cell & allocated cell.

$(250 \times 3) + (50 \times 1) + (300 \times 6) + (100 \times 5) +$

$(300 \times 3) + (200 \times 2) = Rs \cdot 4400$

Eg 2:

	D_1	D_2	D_3	D_4	Supply
P_1	2	3	11	7	6
P_2	1	0	6	1	1
P_3	5	8	15	9	10
Demand	7	5	3	2	

Sol:

	D_1	D_2	D_3	D_4	Supply
P_1	2	3	11	7	6
P_2	1	0	6	1	1
P_3	5	8	15	9	10
Demand	7	5	3	2	

	D_1	D_2	D_3	D_4	Supply
P_1	$\boxed{6}$ $\cancel{2}$	$\cancel{3}$	$\cancel{11}$	$\cancel{7}$	$\cancel{6}$ 0
P_2	$\boxed{0}$ $\cancel{1}$	$\cancel{0}$	$\cancel{6}$	$\cancel{1}$	$\cancel{1}$ 0
P_3	$\cancel{5}$	$\boxed{5}$ $\cancel{8}$	$\boxed{3}$ $\cancel{15}$	$\boxed{2}$ 9	$\cancel{10}$ $\cancel{7}$ $\cancel{2}$ 0
Demand	$\cancel{7}$ $\cancel{1}$	$\cancel{5}$ 0	$\cancel{3}$ 0	$\cancel{2}$ 0	

$$\text{Min } z = (6 \times 2) + (5 \times 8) + (3 \times 15) + (2 \times 9) = 115$$

$$12 + 40 + 45 + 18$$

(2) Least-Cost Method :

	A	B	C	D	Supply
1	3	1	7	4	300
2	2	6	5	9	400
3	8	3	3	2	500
Demand	250	350	400	200	

- ⇒ First we need to find which value is least among all cells (i.e. 1)
- ⇒ ~~Check~~ Compare demand & Supply to that cell. and then same process
- ⇒ Again finding least cell
- ⇒ If we're having same value at 2 different cells, we can select any one of the cell.
- ⇒ Apply Same process. Atlast we will be remaining with one cell
- ⇒ For that cell Supply & demand will become same (i.e. 50)
- ⇒ After all that multiply the ~~valo~~ cell value & allocated value.

Source	A	B	C	D	Supply
1	3	1 [300]	7	4	~~300~~ 0
2	2 [250]	6 [50]	5 [100]	9	~~400~~ ~~150~~ ~~50~~ 0
3	8	8	3 [300]	2 [200]	~~500~~ ~~300~~ 0
	~~250~~ 0	~~350~~ ~~50~~ 0	~~400~~ ~~100~~ 0	~~200~~ 0	

$\Rightarrow (300 \times 1) + (250 \times 2) + (50 \times 6) + (100 \times 5) + (300 \times 3) + (200 \times 2)$

$= 2900.$

Eg 2:- Least value = 15

supply = Demand

	D_1	D_2	D_3	D_4	Supply
P_1	2 [6]	3	11	7	~~6~~ 0
P_2	1	0 [1]	6	1	~~1~~ 0
P_3	5 [1]	8 [4]	15 [3]	9 [2]	~~10~~ ~~8~~ ~~7~~ ~~3~~ 0
Demand	~~7~~ ~~1~~ 0	~~5~~ ~~4~~ 0	~~3~~ 0	~~2~~ 0	

$\Rightarrow (6 \times 2) + (1 \times 0) + (1 \times 5) + (4 \times 8) + (3 \times 15) + (2 \times 9) =$

$12 + 5 + 32 + 45 + 18 = \underline{112}$

(3) Vogel's Approximation Method

	A	B	C	D	Supply
I	3	1	7	4	300
II	2	6	5	9	400
III	8	3	3	2	500
Demand	250	350	400	200	

⇒ Before solving the problem, Firstly we need to identify the difference between row and column.

⇒ First select the least value/cell in row and next select another least value; Find difference in b/w 2 cells

⇒ The difference values is called as Penalities and select

→ the maximum value in row.

→ select that cell and check the Demand & Supply

⇒ Again check the difference and continue with same process until we get the single cell

⇒ check Demand & Supply for that single cell, so that Demand = Supply

→ After multiply cell value & allocated value for transportation cost

	A	B	C	D	Supply	Row Diff			
I	~~3~~	$\boxed{300}$ 1	7	4	~~300~~ 0	2	3	–	–
II	$\boxed{250}$ ~~2~~	~~6~~	$\boxed{150}$ 5	~~9~~	~~400~~ ~~150~~ ~~150~~ ~~100~~ 0	3	1	1	1
III	~~8~~	$\boxed{50}$ ~~3~~	$\boxed{250}$ ~~3~~	$\boxed{200}$ ~~2~~	~~500~~ ~~250~~ 0	1	1	1	0
Demand	~~250~~ 0	~~350~~ ~~50~~ 0	~~400~~ ~~150~~ 0	~~200~~ 0					

1	2	2	2
–	3	2	2
—	3	2	7
—	3	2	–

$\rightarrow (300\times1)+(250\times2)+(150\times5)+(50\times3)+(250\times3)+(200\times2)$

$\rightarrow 300+500+750+150+750+400 = \underline{2850}$

Row column diff = Select the least cell and subtract it from another least cell

(4) Testing for Optimality of balanced transportation Problems

	M_1	M_2	M_3	M_4	
P_1	10	2	20	11	15
P_2	12	7	9	20	25
P_3	5	14	16	5	10
	5	15	15	15	

Step 1 :- ~~Least Cost Method~~

	M_1	M_2	M_3	M_4	
P_1	10	2	20	11	15
P_2	12	7	9	20	25
P_3	54	14	16	18	10
	5	15	15	15	

3	1	7	4
2	6	5	9
8	3	3	2

	A	B	C	D	Supply
	[200] 3	[50] 1	7	4	~~250~~ ~~50~~ 0
	2	[250] 6	[100] 5	9	~~350~~ ~~100~~ 0
	8	3	[250] 3	[150] 2	~~400~~ ~~150~~
Demand	~~200~~ 0	~~300~~ ~~250~~ 0	~~350~~ ~~250~~ 0	~~150~~ 0	1000 0

UV method / MODI Method

← Step - 1

$\Rightarrow (200 \times 3) + (50 \times 1) + (250 \times 6) + (100 \times 5) + (250 \times 3) + (150 \times 2)$

$\Rightarrow$ Total Cost of transpoltation = 3700

Now we need to check the optimality

Appilication of u-v method to optimize the solution

[200] 3	[50] 1	7	4
2	[250] 6	[100] 5	9
8	3	[250] 3	[150] 2

→ obtained solution from north-west Corner Cell method

$\Rightarrow$ For optimization First we need first u & v values

Step-2: Find u & v values

u values for rows

v values for columns

For finding u & v values $\boxed{u_i + v_j = C_{ij}}$

Before finding u&v we need to check allocations

	$V_1=3$	$V_2=1$	$V_3=0$	$V_4=-1$
$u_1=0$	[200] 3	[50] 1	7	4
$u_2=5$	2	[250] 6	[100] 5	9
$u_3=3$	8	3	[250] 3	[150] 2

$$\boxed{u_i+v_j=c_{ij}}$$

∴ No. of allocations = 6

$m+n-1=6$

m = rows

n = columns

$\Rightarrow 3+4-1=6$

$6=6$ (proved)

Always $u_1=0$

We need to find u & v values only for allocated cells.

$c_{ij}=3$

$c_{ij}=u_i+v_j$

$3=0+V_1$

$\boxed{V_1=3}$

$c_{ij}=1$

$1=u_1+V_2$

$1=0+V_2$

$\boxed{V_2=1}$

~~$c_{ij}=7$~~

~~$7=u_1+V_3$~~

~~$7=$~~

$c_{ij}=6$

$6=u_2+V_2$

$6=u_2+1$

$\boxed{u_2=5}$

$c_{ij}=5$

$c_{ij}=u_2+V_3$

$5=5+V_3$

$V_3=0$

$c_{ij}=3$

$3=u_3+V_3$

$3=u_3+0$

$\boxed{u_3=3}$

$c_{ij}=2$

$c_{ij}\ 2=u_3+V_4$

$2=3+V_4$

$\boxed{V_4=-1}$

Step 3 :- we need to find penalities

using $\boxed{P_{ij}=u_i+V_j-c_{ij}}$

→ we need to find penalities for non allocated cells.

$u_i + v_j - c_{ij}$

$C_{13} = 0 + 0 - 7 = -7$

$C_{14} = 0 - 1 - 4 = -5$

$C_{21} = 5 + 3 - 2 = +6$

$C_{24} = 5 + (-1) - 9 = 5 - 1 - 9 = -5$

$C_{31} = 3 + 3 - 8 = -2$

$C_{32} = 3 + 1 - 3 = 1$

<u>Rule</u>: If we get '0' or less than 0 means optimality Reached.

If we get any positive value or greater than 0, we have further steps

⇒ We need to find maximum positive value ($i.e = 6 = C_{21}$)

$\boxed{C_{21} = 2}$ ⇒ This cell is called New Basic Cell (NBC). we need to form a loop with NBC along with allocated cells

	$v_1 = 3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$	[200] 3 (−)	[50] 1 (+)	7	4
$u_2 = 5$	2 (+)	[250] 6 (−)	[100] 5	9
$u_3 = 3$	8	3	[250] 3	[150] 2

New Basic cell we have to allocate positive cell sign and after (−) & after (+) (Continue).

⇒ We have to observe the negative signs: allocated values (i.e 200 & 250)

⇒ select the minimum value and add with the positive value and abstract with negative value

⇒ After forming a new table, we need to find u & v values for allocated cells

	$v_1 = 3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$	3	$\boxed{250}$ 1	7	4
$u_2 = 5$	$\boxed{200}$ 2	$\boxed{50}$ 6	$\boxed{100}$ 5	9
$u_3 = 3$	8	3	$\boxed{250}$ 3	$\boxed{150}$ 2

$$\boxed{u_{ij} + v_j = c_{ij}}$$

$u_1 = 0$

$c_{ij} = 1$

$0 + v_2 = 1$

$\boxed{v_2 = 1}$

$c_{ij} = 6$

$u_2 + 1 = 6$

$\boxed{u_2 = 5}$

$c_{ij} = 5$

$u_2 + v_3 = 5$

$5 + v_3 = 5$

$\boxed{v_3 = 0}$

$c_{ij} = 2$

$u_2 + v_1 = 2$

$5 + v_1 = 2$

$\boxed{v_1 = -3}$

$c_{ij} = 3$

$u_3 + v_3 = 3$

$u_3 + 0 = 3$

$\boxed{u_3 = 3}$

$c_{ij} = 2$

$u_3 + v_4 = 2$

$3 + v_4 = 2$

$\boxed{v_4 = -1}$

Now, we need to find penalties for non-allocated cells using $\boxed{p_{ij} = u_i + v_j - c_{ij}}$

$u_i + v_j - c_{ij}$

$C_{11} = 0-3-3 = -6,$

$C_{13} = 0+0-7 = -7$

$C_{14} = 0-1-4 = -5$

$C_{24} = 5-1-9 = -5$

$C_{31} = 3-3-8 = -8$

$C_{32} = 3+1-3 = 1$

If we get all -ve negative or less than 0 then it is optimized

	$V_1=3$	$V_2=1$	$V_3=0$	$V_4=-1$
$u_1=0$	3	[250] 1	7	4
$u_2=5$	[200] 2	[50] (-) 6	[100] (+) 5	9
$u_3=3$	8	(+) 3	[250] (-) 3	[150] 2

After allocation the table is

	$V_1=2$	$V_2=1$	$V_3=1$	$V_4=0$
$u_1=0$	3	[250] 1	7	4
$u_2=4$	[200] 2	6	[150] 5	9
$u_3=2$	8	[50] 3	[200] 3	2

We need check whether allocation cells is 6 or not

(Allocated) Again find u & v values $\boxed{u_{ij} + v_{ij} = c_{ij}}$

(Non-Allocated) Penalties ($P_{ij} = u_{ij} + v_{ij} - c_{ij}$)

Penalities ($p_{ij} = q_{ij} + v_{ij} - c_{ij}$)

$C_{11} = 0 - 2 - 3 = -5$

$C_{13} = 0 + 1 - 7 = -6$

$C_{14} = 0 + 0 - 4 = -4$

$C_{22} = 4 + 1 - 6 = -1$

$C_{24} = 4 + 0 - 9 = -5$

$C_{31} = 2 - 2 - 8 = -8$

Final solution.

Here, we got all values ~~or~~ negative value which are less than, 0., If so, ~~the~~ it is optimized.

* We have to repeat the same ~~process~~ Procedure ~~th~~ untill the solution is optimized

Now, Final total cost of transportation is

$(250 \times 1) + (200 \times 2) + (150 \times 5) + (50 \times 3) + (200 \times 3) + (150 \times 2) = 2450$ (Optimum Answer).