UNIT-1

Part-1 :-

Introduction and classical Optimization Techniques :-

Statements of an optimization problem, Design Vectors, Design Constraints, Constraint Surface, Objective function, objective function Surfaces, classification of optimization problems.

Classical Optimization Techniques :-

Single Variable Optimization, Multi-variable optimization without Constraints, Necessary and Sufficient Conditions for minimum/maximum, multi variable optimization with equality Constraints, solution by method of Lagrange's multiplers, multi-variable optimization with Inequality Constraints, Kuhn-Tucker Conditions

Part-2

Classical Optimization Techniques :-

(1) Single Variable Optimization :-

(a) Different types of optimal points

(i) Relative (or) Local minimum $\rightarrow f(x^*) \leq f(x+h)$

(ii) Relative (or) local maximum $\rightarrow f(x^*) \geq f(x+h)$

(iii) Global (or) Absolute maximum $\rightarrow f(x^*) \leq f(x)$

(iv) Global (or) Absolute minimum $\rightarrow f(x^*) \geq f(x)$

Examples

1) Find the optimal points and the optimality as optimal points of the following functions

(i) $f(x) = 3x^4 - 4x^3 - 24x^2 + 48x + 15$

(ii) $f(x) = 12x^5 - 45x^4 + 40x^3 + 5$

Optimality as optimal points means maximum and minimum (or) Point of inflexes

Sol :- (i) $f(x) = 3x^4 - 4x^3 - 24x^2 + 48x + 15$

In this we have 2 conditions necessary condition and sufficient condition

Necessary Condition $\Rightarrow f'(x) = 0$.

Sufficient Condition $\Rightarrow f''(x) = 0$.

$f'(x) = 3(4)x^3 - 4(3)x^2 - 24(2)x + 48 = 0$

$12x^3 - 12x^2 - 48x + 48 = 0$

$\Rightarrow 12\{x^3 - x^2 - 4x + 4\} = 0$

$\Rightarrow 12\{\cancel{x^2 - 4x} - (x^3 - 4x) + (4 - x^2)\} = 0$

$\Rightarrow 12\{x(x^2 - 4)(x - 1)\} = 0$

$\Rightarrow x^2 - 4 = 0; \quad x - 1 = 0$

$x = 2 \text{ (or) } -2 \quad x = 1$

Sufficient Condition $f''(x) = 0$

$f(x) = 3x^4 - 4x^3 - 24x^2 + 48x + 15$

$f'(x) = 12x^3 - 12x^2 - 48x + 48 = 0$

$f''(x) = 12(3)x^2 - 12(2)x - 48$

$$\boxed{f''(x) = 36x^2 - 24x - 48 = 0}$$

Substitute x values in $f''(x)$.

$x = 1, x = 2, x = -2$

If $x = 1$

$f''(1) = 36(1)^2 - 24(1) - 48$

$= 36 - 72$

$= -36 < 0$

$x = 1$ is relative maximum

At $x=2$;

$36x^2-24x-48 = 36(2)^2-24(2)-48$

$= 36(4)-48-48$

$= 144-96$

$= 48>0$

$x=2$ is relative minimum

At $x=-2$;

$36x^2-24x-48 = 36(-2)^2-24(-2)-48$

$= 36(4)+\cancel{48}-\cancel{48}$

$= 144>0$

$x=-2$ is relative minimum

(ii) $f(x) = 12x^5-45x^4+40x^3+5$

$f'(x)=0$

$12(5)x^4-45(4)x^3+40(3)x^2 = 0$

$60x^4-180x^3+120x^2=0$

$60x^2(x^2-3x+2)=0$

$\cancel{60x^2(x(x-3)+2)=0}$

$60x^2(x^2-2x-x+2)=0$

$60x^2(x(x-2)-1(x-2))=0$

$60x^2((x-1)(x-2))=0$

$x^2=0 \Rightarrow x=0$

$x-1=0$; $x=1$

$x-2=0$; $x=2$

Sufficient Condition $f''(x)=0$

$f'(x) = \cancel{240612}\,(\cancel{5})\,x^4 - \cancel{45(4)x^3 + 40(3)x}$

$= \cancel{240x^4} -$

$f''(x) = 60(4)x^3 - 180(3)x^2 + 120(2)x = 0$

$f''(x) = 240x^3 - 540x^2 + 240x$

$f''(x) = 0$

$240x^3 - 540x^2 + 240x = 0$

If $x=2$

$f''(2) = 240(2)^3 - 540(2)^2 + 240(2) = 0$

$= 240(8) - 540(4) + 480$

$= 1920 - 2160 + 480 \Rightarrow 2400 - 2160.$

$= 240 > 0$

$x=2$ is relative minimum

If $x=0$

$f''(0) = 0$

$= \cancel{240} \cdot f''(x) = 240x^3 - 540x^2 + 240 \cdot x$

$= 0$

If $f''(0)$ becomes 0, then $f'''(0)$

$f'''(x) = 240(3)x^2 - 540(2)x^2 + 240$

$f'''(0) = \cancel{240}\ 0 - 0 + 240$

$f'''(0) = 240.$

$x=0$ will be point out Inflection

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(2) Multi-Variable Optimization without Constraints

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \qquad |n - \lambda I| = 0$$

Positive definite - matrix is positive then all its ~~negative definite~~ eign values are positive

Negative definite - matrix is negative then all its eigen values are negative

$A_1 = a_{11}$

$$A_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \qquad A_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

* A matrix will be positive definite if all the values of $A_1, A_2, A_3 \ldots A_n$ are positive
* A matrix will be negative definite if and only if sign of A_j is (-1) for $j = 1, 2, 3, \ldots n$
* If some of the A_j are positive and remaining are A_j then matrix A will be semi-definite.
* Similarly Negative Semi-definite
* If none of the above was satisficed, then it is INDEFINITE Matrix.

Examples

(a) $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

$A_1 = |5| = 5$ $\quad A_2 = \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix} = 5 - 4 = 1 > 0$

Positive definite

(b) $B = \begin{bmatrix} -5 & 2 \\ 2 & +3 \end{bmatrix}$

$B_1 = -5 < 0$ $\quad B_2 = \begin{vmatrix} -5 & 2 \\ 2 & +3 \end{vmatrix} = (-5)(+3) - (2)(2)$

$= -15 - 4 = -19 < 0$

Negative definite

(c) $C = \begin{bmatrix} 8 & 4 \\ 4 & 2 \end{bmatrix}$

$C_1 = 8$ $\quad C_2 = \begin{vmatrix} 8 & 4 \\ 4 & 2 \end{vmatrix} = 16 - 16 = 0.$

Positive Semi-Definite

(D) $D = \begin{bmatrix} -8 & -4 \\ 4 & 2 \end{bmatrix}$

$D_1 = -8$ $\quad D_2 = \begin{vmatrix} -8 & -4 \\ 4 & 2 \end{vmatrix} = -16 + 16 = 0$

Negative Semi-definite

(E) $E = \begin{bmatrix} 6 & 2 \\ 8 & 1 \end{bmatrix}$

$E_1 = 6$ $\quad E_2 = \begin{vmatrix} 6 & 2 \\ 8 & 1 \end{vmatrix} = 6 - 16$

$= -10$

Indefinite matrix

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Example :- Determine the nature (positive, definite, negative, indefinite, semi definites) of following matrix

(a) $A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix} = 36 \rightarrow$ Positive Definite.

(b) $A = \begin{pmatrix} 4 & -3 & 0 \\ -3 & 0 & 4 \\ 0 & 4 & 2 \end{pmatrix}$

$A_1 = |4| = 4$ $A_2 = \begin{vmatrix} 4 & -3 \\ -3 & 0 \end{vmatrix} = 0 - 9 = -9$

$A_3 = \begin{vmatrix} 4 & -3 & 0 \\ -3 & 0 & 4 \\ 0 & 4 & 2 \end{vmatrix} = \cancel{-23}\ -82$

$\Rightarrow$ Indefinite Matrix.

(c) $A = \begin{pmatrix} -1 & -1 & -1 \\ -1 & -2 & -2 \\ -1 & -2 & -3 \end{pmatrix}$

$A_1 = [-1] = -1$; $A_2 = \begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix} = 2 - 1 = 1$.

$A_3 = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -2 & -2 \\ -1 & -2 & -3 \end{bmatrix} =$ Negative Indefinite

Necessary Conditions :- (Topic-3)

If $f(x)$ has an extreme point (max or min) at $X = X^*$ and if the partial derivatives of $f(x)$ exist at X^* then $\frac{\partial f}{\partial x_1}(x^*) = \frac{\partial f}{\partial x_2}(x^*) \ldots \frac{\partial f}{\partial x_n}(x^*) = 0$

Sufficient Conditions :-

A Sufficient Condition for a Stationary point X^* to be an extreme point is that the matrix of Second partial derivatives (Hessian matrix) of $f(x)$ evaluated at X^* is

(i) Positive definite when X^* is a relative minimum point

(ii) Negative definite when X^* is a relative maximum point

Optimization :-

A collective word for maximization and minimization is known as optimization

Finding the maximum and Minimum :-

$f(x_1, x_2, x_3, \ldots x_n) = 0$

Step-1 :- $\frac{\partial f}{\partial x_1} = 0$; $\frac{\partial f}{\partial x_2} = 0 \ldots \ldots \frac{\partial f}{\partial x_n} = 0$

Critical points $(a_1, a_2, a_3 \ldots \ldots a_n)$

Step-2 :- $f_{11} = \frac{\partial^2 f}{\partial x_1^2}$ $f_{12} = \frac{\partial^2 f}{\partial x_1 \cdot \partial x_2}$ $f_{13} = \frac{\partial^2 f}{\partial x_1 \cdot \partial x_3}$

Hessian matrix = $\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$

$|H| < 0$ Sadle point

$|H| > 0$ $f_{11} > 0 \Rightarrow$ Local minimum

$|H| > 0$ $f_{1} < 0 \Rightarrow$ Local maximum

$|H| = 0$ Test fails.

Example :-

Examine $f(x) = x_1^2 + 4x_2^2 + 4x_3^2 + 4x_1x_2 + 4x_1x_3 + 16x_2x_3$ for relative extrema.

Sol :- From the Necessary Condition

Given Equation

$f(x) = x_1^2 + 4x_2^2 + 4x_3^2 + 4x_1x_2 + 4x_1x_3 + 16x_2x_3$.

$\frac{\partial f_0}{\partial x_1} = 0$; $\frac{\partial f_1}{\partial x_2} = 0$; $\frac{\partial f}{\partial x_3} = 0$

$\frac{\partial f}{\partial x_1} = 2x_1 + 4x_2 + 4x_3 = 0$

$\frac{\partial f}{\partial x_2} = 8x_2 + 4x_1 + 16x_3 = 0$

$\frac{\partial f}{\partial x_3} = 8x_3 + 4x_1 + 16x_2 = 0$

We can have only one solution is the point $(0,0,0)$.

Now, Considering the hessian matrix (J) at $(0,0,0)$

Hessian matrix $(J)\ (0,0,0)$ =

$$\begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \delta x_2} & \frac{\delta^2 f}{\delta x_1 . \delta x_3} \\ \frac{\delta^2 f}{\delta x_2 \cdot \delta x_1} & \frac{\delta^2 f}{\delta x_2^2} & \frac{\delta^2 f}{\delta x_2 \cdot \delta x_3} \\ \frac{\delta^2 f}{\delta x_3 \delta x_1} & \frac{\delta^2 f}{\delta x_3 \cdot \delta x_2} & \frac{\delta^2 f}{\delta x_3^2} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 4 \\ 4 & 8 & 16 \\ 4 & 16 & 8 \end{bmatrix}$$

$J_1 = |2| = 2 > 0$ positive definite

$J_2 = \begin{vmatrix} 2 & 4 \\ 4 & 8 \end{vmatrix} = 16 - 16 = 0$, Indefinite

$J_3 = \begin{vmatrix} 2 & 4 & 4 \\ 4 & 8 & 16 \\ 4 & 16 & 8 \end{vmatrix}$ $= 2(64-256) - 4(4(8) - 16(4)) + 4(4(16) - 8(4))$

$= 2(-192) - 4(32 - 64) + 4(64-32)$

$= -384 - 4(-32) + 4(32)$

$= -384 + 128 + 128$

$= -384 + 256$

$= -128 < 0$

Negative definite

$\therefore$ J is Indefinite

$\therefore (0,0,0)$ is Sadle point of $f(x)$

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Multi-variable Optimization with Equality Constraints (Direct substitution Method)

Topic-4

Statement

$\max/\min f(x)$

Subject to $g_j(x) = b_j$

$j = 1, 2, \dots n$

$X = (x_1, x_2 \dots x_n)^T$

Solution :-

Equality Constraints $f(x), g_j(x)$

No Constraints $\cancel{F(x)}$ $F(x)$

Two methods $\Rightarrow$ Direct substitution Method

$\Rightarrow$ Lagrange Multiplier Method.

Direct Substitution Method :-

$f(x), g_j(x) = b_j$

$g_1(x) = b_1$

$g_2(x) = b_2$

$\vdots$

$g_m(x) = b_m$

$m < n$ (It gives solution

$m > n$ (It doesn't solution

$f(x)$

$\downarrow$

$F(X) \to$ No Constraints.

<u>Example</u>: Find the minimum values of $x^2+y^2+z^2$ subject to $x+y+2z=12$

<u>sol</u>: $\min f(x) = x^2+y^2+z^2$

Subject to $x+y+2z=12$

$\cancel{x}\cdot X = (x,y,z)^T$

<u>Direct substitution Method</u>:-

$Z =$ From sub $x+y+2z=12$

$$z = \frac{12-x-y}{2}$$

Substitute z in min values $x^2+y^2+z^2$.

$$\Rightarrow x^2+y^2+\left(\frac{12-x-y}{2}\right)^2$$

$$= x^2+y^2+\frac{1}{4}\left(12-(x+y)\right)^2 \longrightarrow ①$$

<u>using necessary Conditions</u>:-

$$\frac{\partial F}{\partial x} = \frac{\delta F}{\delta y} = 0$$

$$\frac{\delta F}{\delta x} = 2x + \frac{\cancel{2}}{\cancel{4}\,2}(12-x-y)(-1)$$

$$= 2x - \frac{1}{2}(12-x-y) \longrightarrow ①$$

$$\frac{\delta F}{\delta y} = 2y + \frac{1}{\cancel{4}\,2}(12-x-y)(2)(-1)$$

$$= 2y - \frac{1}{2}(12-x-y) \longrightarrow ②$$

$\frac{\delta F}{\delta x} = 0 \Rightarrow 2x$

$2x - \frac{1}{2}(12 - x - y) = 0$

$2x = \frac{1}{2}(12 - x - y) \Rightarrow 2x - \frac{1}{2}(12 - x - y) = 0 \longrightarrow (2)$

$\frac{\delta F}{\delta y} = 0$

$2y - \frac{1}{2}(12 - x - y) = 0 \longrightarrow (3).$

$2y = \frac{1}{2}(12 - x - y) = 0$

$\frac{\delta F}{\delta x} = \frac{\delta F}{\delta y}$

$2x - \cancel{\frac{1}{2}(12 - x - y)} = 2y - \cancel{\frac{1}{2}(12 - x - y)}$

$\cancel{2}x = \cancel{2}y$

$\boxed{x = y}$

Substitute $x = y$ in eq(2)

$2x - \frac{1}{2}(12 - x - x) = 0$

$2x = \frac{1}{2}(12 - 2x)$

$2x = \frac{\cancel{2}}{\cancel{2}}(6 - x)$

$2x = 6 - x$

$3x = 6$

$\boxed{x = 2}$

Similarly;

$2y - \frac{1}{2}(12 - y - y) = 0$

$2y = \frac{1}{2}(12 - 2y)$

$2y = \frac{\cancel{2}}{\cancel{2}}(6 - y)$

$3y = 6$

$\boxed{y = 2}$

$z = \frac{12 - x - y}{2} \Rightarrow \frac{12 - 2 - 2}{2} = \frac{12 - 4}{2} = \frac{\cancel{8}}{\cancel{2}} = 4$

Extreme point $(2, 2, 4)$

Sufficient Condition (Hessian Matrix)

$$H = \begin{bmatrix} \frac{\delta^2 f}{\delta x^2} & \frac{\delta^2 f}{\delta x \cdot \delta y} \\ \frac{\delta^2 f}{\delta y \cdot \delta x} & \frac{\delta^2 f}{\delta y^2} \end{bmatrix}$$

$\frac{\delta^2 f}{\delta x^2} = 2 + \frac{1}{2} = \frac{5}{2}$; $\frac{\delta^2 f}{\delta y^2} = 2 + \frac{1}{2} = \frac{5}{2}$

$\frac{\delta^2 f}{(\delta x)(\delta y)} = \frac{1}{2}$ $\qquad$ $\frac{\delta^2 y}{(\delta y)(\delta x)} = \frac{1}{2}$

Hessian Matrix $= \begin{bmatrix} \frac{5}{2}, & \frac{1}{2} \\ \frac{1}{2}, & \frac{5}{2} \end{bmatrix}$

$H_1 = \frac{5}{2} > 0$, $H_2 = \frac{25}{4} - \frac{1}{4} = \frac{24}{4} = 6 > 0$

$\therefore$ H is positive definite

(2,2,4) be the min point

$f(x) = 2^2 + 2^2 + 4^2$

$= 4 + 4 + 16$

$= 8 + 16$

$f_{min} = 24 > 0$

Topic 5 → Solution by method of Lagrange Multiplers..

Topic-6

Kuhn-Tucker Conditions (Inequality Constraint)

2 Variables & one Constraint	2 variables & 2 Constraint
$\frac{\delta L}{\delta x_1} = 0 \rightarrow ①$	$\frac{\delta L}{\delta x_1} = 0 \rightarrow ①$
$\frac{\delta L}{\delta x_2} = 0 \rightarrow ②$	$\frac{\delta L}{\delta x_2} = 0 \rightarrow ②$
$\lambda(h) = 0 \rightarrow ③$	$\lambda_1(h_1) = 0 \rightarrow ③$
$h \leq 0 \rightarrow ④$	$\lambda_2(h_2) = 0 \rightarrow ④$
$x_1, x_2 \geq 0 \rightarrow ⑤$ $\lambda \geq 0$ maximum	$h_1 \leq 0 \rightarrow ⑤, h_2 \leq 0 \rightarrow ⑥$
$\lambda \leq 0$ minimum	$x_1, x_2 \geq 0$, $\lambda_1, \lambda_2 \geq 0 \rightarrow$ max
	$\lambda_1, \lambda_2 \leq 0 \rightarrow$ min

Eq ①, ②, ③ will be used for Calculation & ④ & ⑤ for checking

Kuhn-Tucker [Non linear programming with 2 Variables & 1 Inequality Constraints]

Pb MAX $z = 2x_1^2 - 7x_2^2 + 12x_1x_2$

Sub to $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geq 0$

Sol:- Kuhn-Tucker Conditions

Kuhn-Tucker Cond...

$\frac{\delta L}{\delta x_1} = 0 \rightarrow$ ①

$\frac{\delta L}{\delta x_2} = 0 \rightarrow$ ②

& $\lambda(h) = 0 \rightarrow$ ③

$h \leq 0 \rightarrow$ ④

$x_1 x_2 \geq 0 \rightarrow$ ⑤

$\lambda \geq 0$ { If it is min then $\lambda \leq 0$ (max (a) min given in sum)

$$L(x_1, x_2, \lambda) = (2x_1^2 - 7x_2^2 + 12x_1x_2) - \lambda(2x_1 + 5x_2 - 98)$$

$h = 2x_1 + 5x_2 - 98$

$\frac{\delta L}{\delta x_1} = 0 \Rightarrow 4x_1 + 12x_2 - 2\lambda = 0 \rightarrow$ ①

$\frac{\delta L}{\delta x_2} = 0 \Rightarrow -14x_2 + 12x_1 - 5\lambda = 0 \rightarrow$ ②

$\lambda(2x_1 + 5x_2 - 98) = 0 \rightarrow$ ③

$2x_1 + 5x_2 - 98 \leq 0 \rightarrow$ ④

$x_1, x_2 \geq 0$; $\lambda \geq 0 \rightarrow$ ⑤

Note :- 1,2,3 used for calculations 4 & 5 for checking

Case (i) $\lambda = 0$ $\Rightarrow \boxed{x_1 = 0; x_2 = 0}$

put in ① & ②. put in z ; then z = 0

$4x_1 + 12x_2 = 0$ It is invalid

$-14x_2 + 12x_1 = 0$ z will be a value.

Case - (ii) $\lambda \neq 0$

from ⑤

$2x_1 + 5x_2 - 98 = 0 \longrightarrow$ ④

Solve ①, ② & ④

$x_1 = 44$, $x_2 = 2$, $\lambda = 100$.

Calculate in calc

For checking put x_1, x_2 in eq ④

$2(44) + 5(2) - 98$

$88 + 10 - 98$

$\cancel{98} - \cancel{98} = 0$

Eq ④ is satisfied

Eq ⑤ also satisfied.

$\boxed{Z\ Max = 4900}$

Kuhn-Tucker (NLPP with 2 Variables and 2 Inequality Constraints :-

Pb :- Using Kuh-Tucker Conditions Solve the following

$$Max\ Z = 10x_1 + 10x_2 - x_1^2 - x_2^2$$

Sub to $x_1 + x_2 \leq 8$

$-x_1 + x_2 \leq 5$

Sol :-

$$\frac{\delta L}{\delta x_1} = 0 \longrightarrow ①$$

$$\frac{\delta L}{\delta x_2} = 0 \longrightarrow ②$$

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$\lambda_1(h_1)=0 \rightarrow (3)$

$\lambda_2(h_2)=0 \rightarrow (4)$

$h_1 \le 0 \rightarrow (5)$

$h_2 \le 0 \rightarrow (6)$

$x_1, x_2 \ge 0 \;, \lambda_1, \lambda_2 \ge 0 \rightarrow (7)$

$\Rightarrow L(x_1, x_2, \lambda_1, \lambda_2) = (10x_1 + 10x_2 - x_1^2 - x_2^2) - \lambda_1(x_1+x_2-8) - \lambda_2(-x+x_2-5)$

$h_1 = x_1+x_2-8, \; h_2 = -x_1+x_2-5$

$\frac{\delta L}{\delta x_1} = 0 \Rightarrow 10-2x_1-\lambda_1+\lambda_2 = 0 \rightarrow (1)$

$\frac{\delta L}{\delta x_2} = 0 \Rightarrow 10-2x_2-\lambda_1-\lambda_2 = 0 \rightarrow (2)$

$\lambda_1(x_1+x_2-8)=0 \rightarrow (3) \;, \lambda_2(-x_1+x_2-5)=0 \rightarrow (4)$

$x_1+x_2-8 \le 0 \rightarrow (5) \;, -x_1+x_2-5 \le 0$

$x_1 x_2 \ge 0.$

$\lambda_1 \lambda_2 \ge 0$ (In problem given max so, $\lambda_1 \lambda_2 \ge 0$).

We have 2λ ie, λ_1 & λ_2, So we are having 4 Cases.

Case-(i) :- $\lambda_1=0$ and $\lambda_2=0$

Put in (1) & (2)

$10-2x_1=0$ & $10-2x_2=0$

$x_1=5$ & $x_2=5$

eq (5) is not satisfied

$\therefore$ Rejected.

Case-(ii) :- $\lambda_1=0$ & $\lambda_2 \ne 0$.

Put in (1) & (2)

$\left.\begin{array}{l} 10-2x_1+\lambda_2=0 \\ 10-2x_2-\lambda_2=0 \end{array}\right\}$ solve

From (4) $-x_1+x_2-5=0$

$x_1=2.5 \quad x_2=7.5$

eq (5) is not satisfied

$\therefore$ Rejected.

Case-iii $\lambda_1 \neq 0$ and $\lambda_2 = 0$

From ③ $\Rightarrow x_1 + x_2 - 8 = 0$

Put in eq ① & ②

$10 - 2x_1 - \lambda_1 = 0$

$10 - 2x_2 - \lambda_1 = 0$.

$\Longrightarrow$

$10 - 2x_1 = \cancel{\lambda_1}$
$10 - 2x_2 = \cancel{\lambda_1}$

$20 - 4x_2 = 0$
$4x_2 = 20$
20 ...

$x_1 = 4$, $x_2 = 4$, $\lambda_1 = 10$

⑤ & ⑥ & ⑦ satisfied

Put in ⑦ $\boxed{Z_{max} = 48}$

Case - iv :-

$\lambda_1 \neq 0$ & $\lambda_2 \neq 0$

From ③ $\Rightarrow x_1 + x_2 - 8 = 0$
From ④ $\Rightarrow -x_1 + x_2 - 5 = 0$
solve

$x_1 = 1.5$, $x_2 = 6.5$

⑤ & ⑥ satisfied

Put x_1 & x_2 in ① & ②

$-\lambda_1 + \lambda_2 = -7$ | $-\lambda_1 - \lambda_2 = 3$

$\lambda_1 = 2$; $\lambda_2 = -5$

Eq ⑦ Not satisfied

Rejected.

Topic-5 :- Lagrange Multiplex Method

Statement :-

Min/MAX $f(x)$

subject to

$g_j(x) = b_j$

$X = (x_1, x_2, x_3, \ldots x_m)^T$

$j = 1, 2, 3, \ldots m$

1) Convert the problem into no constraints problem