Problem 1

Prove Claim 3.3 from class: let S be a non-empy subset of $\mathbb{R}$. Then $\max(S)$ exists (that is, S has a maximal element) $\iff$ S is bounded above and $\sup(S) \in S$.

Solution

Problem 2

Let S be a non-empty subset of $\mathbb{R}$. Let $UB(S)$ be thee set of all upper bounds of S (note that this set may be empty) and $LB(S)$ be the set of all lower bounds of S. Also let $-S = \{-s : s \in S\}$

i. Let $M \in \mathbb{R}$. Prove that $M = \sup(S)$ if and only if $M = \min(UB(S))$ (the minimal element of $UB(S)$). Also prove that $M = \inf(S)$ if and only if $M = \max(LB(S))$ (the maximal element of $LB(S)$). This is essentially a reeformulation of the definition of sup and inf.

Solution

Problem 3

Solution

Problem 4

Solution

Problem 5

Solution

Problem 6

Solution

Problem 7

Solution

Problem 8

Solution