ECE 253 Homework 4

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Contents

Problem 1 2D Sampling and Aliasing

Part(a)

```
clear; close all; clc;
[x y] = meshgrid(0:256,0:256);
figure, subplot(1,2,1), imshow(x, [0, 255])
subplot(1,2,2), imshow(y, [0, 255]), title('x, y ')
```

- We are given the sampling period T = 1, thus the sampling frequency F_s = 1.
- x and y are the spatial coordinates of the original image given by the mesh grid in matlab
- u and v are the spatial frequency coordinates of the Fourier transform of the image.

```
z1 = cos ( 2 * pi * 1/32 .* x - 2 * pi * 1/128 .* y);
figure, imshow(z1), title('z1 matrix/image');
```

Spatial Frequencies for z1.

- In x-direction = 1/32
- In y-direction = 1/128

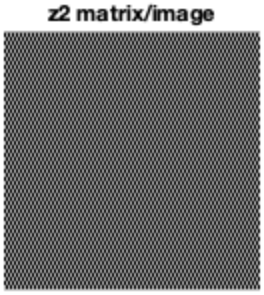
Image can be reconstructed exactly from it's samples if Cutoff freq ≤ 1/2 sampling freq F_s

- If equality holds, then the sampling is at the Nyquist rate
- If Δx and Δy are smaller than required, the image is called oversampled
- If they are larger than required, the image is undersampled

In terms of spatial frequency and the sampling frequency,

- 2 * 1/32 = 1/16 < F_s = 1 ==> oversampled in x-direction
- 2 * 1/128 = 1/64 < F_s = 1 ==> oversampled in y-direction

```
z2 = cos ( 2 * pi * 1/4 .* x - 2 * pi * 7/8 .* y);
figure, imshow(z2), truesize, title('z2 matrix/image');
```



Spatial Frequencies for z2.

- In x-direction = 1/4
- In y-direction = 7/8

In terms of spatial frequency and the sampling frequency,

z3 matrix/image

- 2 * F_{xc} = 1/2 < F_s = 1 ==> oversamppled in x-direction
- 2 * F_{yc} = 7/4 > F_s = 1 ==> undersampled in y-direction

```
z3 = cos ( 2 * pi * 1/2 .* x - 2 * pi * 1/2 .* y);
figure, imshow(z3), title('z3 matrix/image');
```

Spatial Frequencies for z3.

- In x-direction = 1/2
- In y-direction = 1/2

In terms of spatial frequency and the sampling frequency,

- 2 * F_{xc} = 1 = F_s = 1 ==> critically sampled in x-direction or sampling is at the Nyquist rate
- 2 * F_{yc} = 1 > F_s = 1 ==> critically sampled in y-direction or sampling is at the Nyquist rate

Part(b)

Since Cosine function is periodic with period 2π . we can use any frequencies which is generated by adding any multiple of 2 π to each of the xy frequency components in z1 to obtain a identical sampled function to that of the sampled function z1. Therefore let us consider a = F_{xc} + 2π and b = F_{yc} + 2π, taking the 2π in common we get a = 33/32 b = 129/128 Any a = 1/32 + k 2π where k $\in$ z, will give us the identical sampled function.

However, we have to note that the we want the sampled function to be aliased, this means we have to consider the inequality a>1/2 and b>1/2

```
z4 = cos ( 2 * pi * 33/32 .* x - 2 * pi * 129/128 .* y);
figure, imshow(z4), title('z3 matrix/image');
```

In the figure above it came be seen that z4 produces the same image as sampled in z1.

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