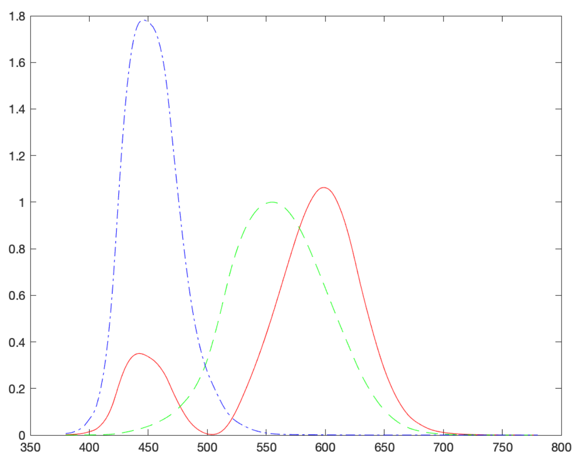
ECE 253, Solutions for Homework 3 (Fall 2022)

1) **Chromaticity Diagrams**

(a) Here is the matlab code to plot the tristimulus values:

```
>> load cie -ascii
>> l = cie(:,1);
>> X = cie(:,2);
>> Y = cie(:,3);
>> Z = cie(:,4);
>> plot(l,X,'r-',l,Y,'g--',l,Z,'b-.');
```

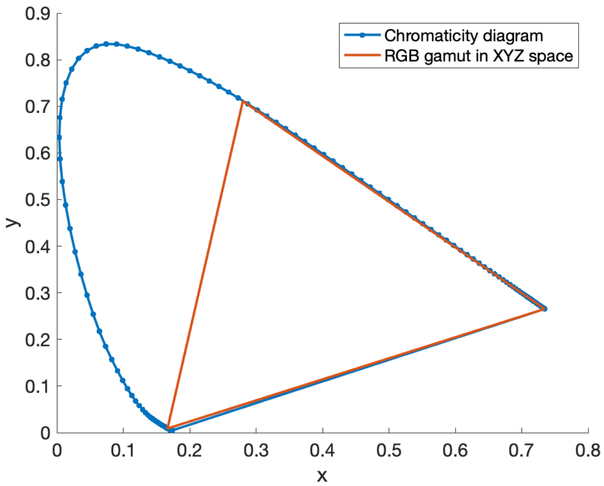


Here is the code to plot the chromaticity diagram. The connected line of purples is obtained by simply tacking the first point of each vector on to its end:

```
>> x = X ./ (X+Y+Z);
>> y = Y ./ (X+Y+Z);
>> x = [x;x(1)];
>> y = [y;y(1)];
>> plot(x,y)
```

(b) The triangle corresponding to the gamut in the RGB space is defined by the three vertices: (1, 0, 0), (0, 1, 0) and (0, 0, 1). Plug these coordinates into the given RGB-to-XYZ transformation.

```
>> CIERGB2XYZ = [0.49000 0.32000 0.2; 0.17697 0.81240 0.01063; 0 0.01
0.99000];
>> verticesRGB = CIERGB2XYZ*[1 0 0; 0 1 0; 0 0 1];
>> verticesRGB = verticesRGB./sum(verticesRGB, 1);
>> figure; hold on;
>> plot(x, y,'.-','MarkerSize',12,'LineWidth',2);
```

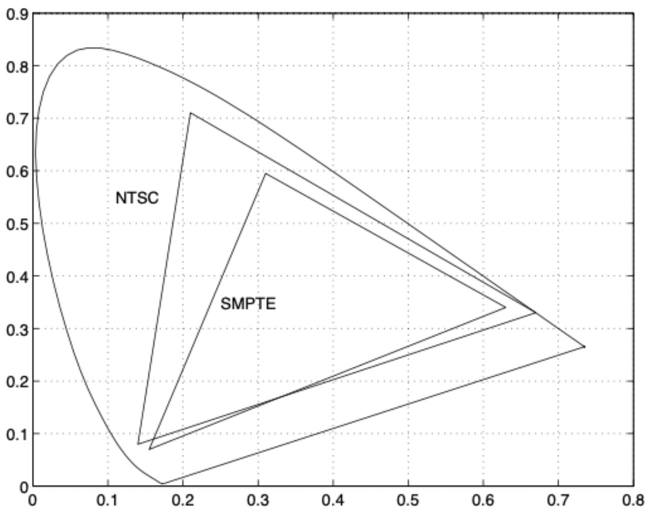



```
>> set(gca,'FontSize',15);
>> xlabel('x');ylabel('y');
>> plot([verticesRGB(1,:) verticesRGB(1, 1)], ...[verticesRGB(2,:)
verticesRGB(2, 1)],'LineWidth', 2)
>> legend('Chromaticity diagram','RGB gamut in XYZ space')
```

(c) You were given the conversions from XYZ to RGB. What we need, however, is to convert from RGB to XYZ in order to plot in the xy chromaticity diagram. Specifically, we need to convert each of the pure phosphor colors to XYZ, then to xy. For example, the vector [1 0 0]' represents the pure NTSC red phosphor. That is, it is the NTSC red primary expressed in the NTSC coordinates. By using the inverse of the given matrix, we can convert this (1 0 0) point from the NTSC space to the XYZ space. Likewise, we have to convert (0 1 0) and (0 0 1) to the XYZ space. These three XYZ tristimulus values then get converted to xy and plotted in the xy plane. Here is the code to plot the NTSC triangle inside the xy-chromaticity horse-shoe:

```
>> Nmat = [1.910 -0.532 -0.288; -0.985 2.0 -0.028; 0.058 -0.118 0.898];
>> invNmat = inv(Nmat);
>> xyzR = invNmat * [1 0 0]';
>> xR = xyzR(1) / sum(xyzR);
>> yR = xyzR(2) / sum(xyzR);
>> xyzG = invNmat * [0 1 0]';
>> xG = xyzG(1) / sum(xyzG);
>> yG = xyzG(2) / sum(xyzG);
>> xyzB = invNmat * [0 0 1]';
>> xB = xyzB(1) / sum(xyzB);
>> yB = xyzB(2) / sum(xyzB);
>> a = [xR xG xB xR];
```

```
>> b = [yR yG yB yR];
>> plot(x,y,a,b)
>> grid
```



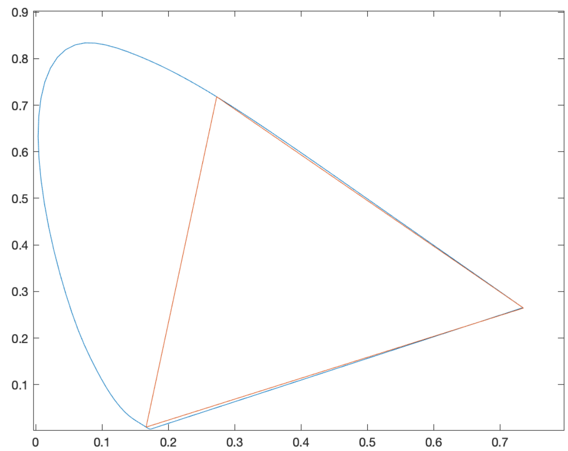
(d) We could get the CIE-RGB triangle in exactly the same way, starting with a corner point such as (1,0,0) expressed in the CIE RGB system, and then using a 3x3 transformation to convert it to XYZ. But knowing that these were monochromatic spectral primaries with values 435.8, 546.1m, and 700nm, we can just read the closest lines directly from the cie data file, since those give us the XYZ coordinates directly. Specifically, the closest data points are for wavelengths 436, 546, and 700nm:

436.0000 0.3344 0.0180 1.6564
546.0000 0.3741 0.9841 0.0123
700.0000 0.0114 0.0041 0

The Matlab code for creating the triangle is:

```
>> c1 = [0.3344 0.0180 1.6564];
>> c2 = [0.3741 0.9841 0.0123];
>> c3 = [0.0114 0.0041 0];
>> c1x = c1(1) / sum(c1);
>> c1y = c1(2) / sum(c1);
>> c2x = c2(1) / sum(c2);
>> c2y = c2(2) / sum(c2);
>> c3x = c3(1) / sum(c3);
>> c3y = c3(2) / sum(c3);
>> triX = [c1x c2x c3x c1x];
>> triY = [c1y c2y c3y c1y];
```

```
>> plot(x,y,triX,triY)
```



2) **Contrast and saturation enhancement for color images**

(a) After converting to HSI, we equalize the I plane and convert back:

```
>> newi = histeq(i);
>> [rHE, gHE, bHE] = hsi2rgb(h,s,newi);
>> imHE = reshape([rHE gHE bHE],500,1110,3);
```

After this equalization, and conversion back to RGB, there are many pixels which are out of range. The maximum values for R, G, and B are 337, 307, and 314. We can visualize the areas with out-of-range pixels, for example

```
>> figure, imshow(rHE > 255)
>> figure, imshow(bHE > 255)
```

These places where out-of-range values get truncated are the places where we would expect to see some color shifts (that is, shifts in the hue). For example, in the fields near the top, the rHE plane shows lots of out-of-range pixels. The original fields are somewhat brownish, and they show a shift towards being yellowish in the equalized image (original image on top, equalized one on bottom):

(b) After converting to HSI, we take the square root of the S plane and then convert back:

```
>> newS = sqrt(s);
>> [rSAT gSAT bSAT] = hsi2rgb(h,newS,i);
>> imSAT = reshape([rSAT gSAT bSAT], 500, 1110, 3);
>> imshow(imSAT / 255)
```

The light pink shirt is now deep pink. The white shirt turned mostly light blue. The faces are quite reddish. And the white hair seems to have streaks of pink and blue. Overall it looks weird and not natural.

(c) Other transformations give more pleasing results. For example if we simply double all values of saturation that are less than 0.5, and set to 1.0 all values that are greater than 0.5, then the values with low saturation are not being modified as strongly as with the square root operation, and we get a picture like this:

Comments on plotting the saturation transformation function:
For transformation function $y = f(x)$, we can plot it by creating a dummy variable and apply the transformation to it.

```
>> x = 0:0.01:1
>> y = sqrt(x)
```

```
>> plot(x, y)
```

If you want to plot with real data, make sure to convert the matrix to a vector before using the "plot()" function.

```
>> [h,s,i] = rgb2hsi(r,g,b)
>> snew = sqrt(s)
>> plot(s(:), snew(:))
```

3) **Change of Reference White**

Tristimulus values for a color C are the ratio between the "power knob setting" used to match the color C in the color matching experiment, and the "power knob settings" that were used to match the reference white for that coordinate system.

For the tristimulus values in a coordinate system that uses reference white W_2, we simply put, in the denominator of each ratio, the "power knob settings" that were used to match W_2

$$\hat{T}_1(C) = \frac{A_1(C)}{A_1(W_2)} = \frac{A_1(C)}{A_1(W_1)} \times \frac{A_1(W_1)}{A_1(W_2)} = \frac{T_1(C)}{T_1(W_2)}$$

$$\hat{T}_2(C) = \frac{A_2(C)}{A_2(W_2)} = \frac{A_2(C)}{A_2(W_1)} \times \frac{A_2(W_1)}{A_2(W_2)} = \frac{T_2(C)}{T_2(W_2)}$$

$$\hat{T}_3(C) = \frac{A_3(C)}{A_3(W_2)} = \frac{A_3(C)}{A_3(W_1)} \times \frac{A_3(W_1)}{A_3(W_2)} = \frac{T_3(C)}{T_3(W_2)}$$

4) **Adding Colors**

(a) Tristimulus values are additive. The tristimulus values for W are (1,1,1). So:

$$R = \frac{1}{2}(T_1 + 1)$$

$$G = \frac{1}{2}(T_2 + 1)$$

$$B = \frac{1}{2}(T_3 + 1)$$

The chromaticity coordinates are:

$$r = \frac{T_1 + 1}{T_1 + T_2 + T_3 + 3}$$

$$g = \frac{T_2 + 1}{T_1 + T_2 + T_3 + 3}$$

(b) We are told that $t_1 = t_2 = 0.1$ so we know that:

$$\frac{T_1}{T_1+T_2+T_3} = \frac{T_2}{T_1+T_2+T_3} = 0.1$$

and in particular $T_1 = T_2$. Also

$$T_1 = 0.1(T_1+T_2+T_3) = 0.1(2T_1+T_3)$$

Multiplying by 10, we get $10T_1 = 2T_1 + T_3$ and so $T_3 = 8T_1$. The tristimulus values for color C are therefore (T_1, T_1, T_8). So we know that T_1 is positive, since if it were negative then all 3 tristimulus values would be negative, which can't happen (since that would mean that all 3 of the primaries get moved over to the test light side of the box, and turned up on that side). We can substitute $T_2 = T_1$ and $T_3 = 8T_1$ into the expressions for r and g to obtain:

$$r = \frac{T_1+1}{10T_1+3}$$

$$g = \frac{T_1+1}{10T_1+3} = r$$

So, the chromaticity coordinates for A are equal to each other, so the point lies on the line $x = y$. We can say additionally that it will lie on the line segment that connects (0.1,0.1) with (1/3,1/3), that is to say, that goes from the chromaticity coordinates of C to W. To show this, we need to show:

$$0.1 < \frac{T_1+1}{10T_1+3} < \frac{1}{3}$$

Since T_1 is positive, $10T_1 + 3$ is greater than zero, so we can safely multiply by it:

$$0.1 < \frac{T_1+1}{10T_1+3} \Leftrightarrow T_1 + .3 < T_1 + 1$$

which is true. And

$$\frac{T_1+1}{10T_1+3} < \frac{1}{3} \Leftrightarrow T_1 + 1 < \frac{10}{3}T_1 + 1$$

which is also true. This is what we needed to show, so the point is on the line segment which connects C (0.1, 0.1) and W (1/3, 1/3).

Here, it is important to note that although $A = \frac{1}{2}(C+W)$ is the relation between the tristimulus values of A, C, and W, the same relation is not necessarily satisfied for their chromaticity coordinates. Therefore, simply saying that the chromaticity coordinates of A is on the line segment that connects the chromaticity coordinates of C and W because $r = g = \frac{1}{2}(0.1 + 1/3) = \frac{13}{60}$, and $0.1 < \frac{13}{60} < \frac{1}{3}$ is not acceptable.