Problem 3) Change of Reference white

T_1, T_2, T_3 - tristimulus values for color C, relative to reference white (W_1)

$$T_1(C) = \frac{A_1(C)}{A_1(W_1)}, \quad T_2(C) = \frac{A_2(C)}{A_2(W_1)}, \quad T_3(C) = \frac{A_3(C)}{A_3(C)}$$

to get $\hat{T}_1, \hat{T}_2$ & $\hat{T}_3$ with reference W_2 for color C.

Tristimulus value of color C with reference W_2 can be written as

$$\hat{T}_1(C) = \frac{A_1(C)}{A_1(W_2)}, \quad \hat{T}_2(C) = \frac{A_2(C)}{A_2(W_2)}, \quad \hat{T}_3(C) = \frac{A_3(C)}{A_3(W_2)}$$

— eq(1)

Now,
consider color C = ref white W_2, then we get

$$T_1(W_2) = \frac{A_1(W_2)}{A_1(W_1)}, \quad T_2(W_2) = \frac{A_2(W_2)}{A_2(W_1)},$$

$$T_3(W_2) = \frac{A_3(W_2)}{A_3(W_1)}$$

— eq(2)

Now let us substitude All $A_i(W_2)$ in eq(1) set with $A_i(W_2)$ from equation set in eq(2).

This gives us,

$$\hat{T}_1(c) = \frac{A_1(c)}{T_1(w_2)\,A_1(w_1)}$$

$$= \frac{T_1(c)}{T_1(w_2)}$$

$$\hat{T}_2(c) = \frac{A_2(c)}{T_2(w_2)\cdot A_2(w_1)}$$

$$= \frac{T_2(c)}{T_2(w_2)}$$

$$\hat{T}_3(c) = \frac{A_3(c)}{T_3(w_2)\,A_3(w_1)}$$

$$= \frac{T_3(c)}{T_3(w_2)}$$

Problem 4 Adding colors

color C has tristimulus values T_1, T_2, T_3
Chromaticity coordinates are $t_1 = 0.1$ & $t_2 = 0.1$
the reference white of system : W
Additive mixture of colors : $A = \frac{1}{2}(C+W)$

a) let R, G, B denote the tristimulus values of color A.

By superposition tristimulus values can be writen as

$$R = \frac{1}{2}(T_1 + \hat{T}_1), \quad G = \frac{1}{2}(T_2 + \hat{T}_2), \quad B = \frac{1}{2}(T_3 + \hat{T}_3)$$

where $\hat{T}_1, \hat{T}_2, \hat{T}_3$ are tristimulus valus of W, which is basically $(1,1,1)$. $\hat{T}_i(w) = \frac{N_i(w)}{N_i(w)}$ .... where N_i is knob posi

$$\therefore R = \frac{1}{2}(T_1+1), \quad G = \frac{1}{2}(T_2+1), \quad B = \frac{1}{2}(T_3+1)$$

Chromaticity coordinates r and g are

$$r = \frac{R}{R+G+B} \qquad g = \frac{G}{R+G+B}$$

$$= \frac{\cancel{\frac{1}{2}}(T_1+1)}{\cancel{\frac{1}{2}}(T_1+T_2+T_3+3)} \qquad = \frac{T_2+1}{T_1+T_2+T_3+3}$$

$$= \frac{T_1+1}{T_1+T_2+T_3+3}$$

Now, since we are given $t_1 = t_2 = 0.1$

$$t_1 = \frac{T_1}{T_1 + T_2 + T_3} = t_2 = \frac{T_2}{T_1 + T_2 + T_3} = 0.1$$

$\therefore T_1 = T_2$

$\therefore T_1 = (0.1)(T_1 + T_2 + T_3)$

$10T_1 = 2T_1 + T_3$

$\boxed{8T_1 = T_3}$

Basically

(T_1, T_2, T_3)

$\Downarrow$

$(T_1, T_1, 8T_1)$

Therefore $T_1 > 0$

T_1 = positive

(all lights cannot move other side)

Now, let's say

$T_1 = 1$ then

$T_2 = 1$

$T_3 = 8$

$T_1 = T_2 = \frac{1}{8} T_3$

Note: this system of equations can have more than one sol'n. we will consider this, (infinite solutions)

Now, from earlier

$$R = \frac{1}{2}(T_1 + 1) \qquad G = \frac{1}{2}(T_1 + 1) \qquad B = \frac{1}{2}(8T_1 + 1)$$

$$R = \frac{1}{2}(1+1) \qquad G = \frac{1}{\cancel{2}}(\cancel{2}) \qquad B = \frac{9}{2}$$

$$\boxed{R = 1} \qquad \boxed{G = 1} \qquad \boxed{B = 4.5}$$

$$r = \frac{T_1 + 1}{10T_1 + 3} \qquad g = \frac{T_1 + 1}{10T_1 + 3} = r$$

$$\therefore \quad r = g = \frac{2}{13} \approx \boxed{0.154}$$

(b) let chromaticity coordinates of W be w_1, w_2, w_3.

Tristimulus values of W are $T_i^*(W) = \frac{\cancel{K_i(W)}}{\cancel{K_i(W)}} = 1$

$\therefore w_1 = \frac{1}{1+1+1} = \frac{1}{3}$ $\quad w_2 = w_3 = \frac{1}{3}$

Thus, we can consider the line segment that connects between two endpoints and they are $(0.1, 0.1)$ and $(\frac{1}{3}, \frac{1}{3})$

Thus the equation of the line that passes through both this points is

and since $r = g = \frac{2}{13}$ we know all three point lies along the line $y = x$

Now, we just need to check

$(0.1, 0.1) < (r, g) < (\frac{1}{3}, \frac{1}{3})$

$0.1 < \frac{2}{13} < \frac{1}{3} = 0.33$

≈ 0.15385

It lies on the line segment

Proved!

certainly greater than 0.1 and smaller than 0.33

Above ~~Proff~~ is just for one sol'n, since we have many sol'ns for T_1, T_2, & T_3.
we can have different points representing colour A.

general solu'n check:
For this we have to prove

$$(0.1, 0.1) < (r, g) < \left(\frac{1}{3}, \frac{1}{3}\right)$$

where $r = g = \dfrac{T_1 + 1}{10T_1 + 3}$

$$0.1 < \frac{T_1 + 1}{10T_1 + 3} < \frac{1}{3}$$

$$\frac{1}{10}(10T_1 + 3) < T_1 + 1 < \frac{10T_1 + 3}{3}$$

$$\underbrace{T_1 + 0.3 < T_1 + 1}_{\text{condition 1}} < \underbrace{\frac{10T_1}{3} + 1}_{\text{condition 2}}$$

Let's first look at condition 1

$$T_1 + 0.3 < T_1 + 1$$

This is true $\because$ $0.3 < 1$
since $T_1 > 0$
was established earlier.

Now, looking at conditional 2

$$T_1 \cancel{+1} < \frac{10}{3} T_1 \cancel{+1}$$

This is also true since

$$\frac{10}{3} \approx 3.33 > 1.$$

Thus, the point (r, g) is on the line segment which connects chromaticity coordinates of $C \to (0.1, 0.1)$ and $W \to (\frac{1}{3}, \frac{1}{3})$