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Homework 5

1. pdf: $f_X(x) = \frac{\lambda}{2} e^{-\lambda|x|}$ for random var X.

three-level quantizer q for X :

$$q(x) = \begin{cases} +b & \text{for } x > a \\ 0 & \text{for } -a \leq x \leq +a \\ -b & \text{for } x < -a \end{cases}$$

For the centroid condition to be met we know that

$$b = r_k = \frac{\int_{t_k}^{t_{k+1}} u\, p_u(u)\, du}{\int_{t_k}^{t_{k+1}} p_u(u)\, du} = \mathbb{E}[u \mid u \in k^{th} \text{ interval}]$$

$k = 1, 2, 3, \cdots, L$

We only look at positive x. $t_k \to a$ $t_{k+1} \to \infty$

$$b = \frac{\int_a^\infty x \frac{\lambda}{2} e^{-\lambda x}\, dx}{\int_a^\infty \frac{\lambda}{2} e^{-\lambda x}\, dx}$$

using $\int_a^\infty x e^{-\lambda x}\, dx = \frac{a e^{-\lambda a}}{\lambda} + \frac{e^{-\lambda a}}{\lambda^2}$

we get

$$b = \frac{\frac{\cancel{e^{-\lambda a}}}{\lambda^{\cancel{2}}}(a\cancel{\lambda}+1)}{-\frac{e^{-\lambda x}}{\lambda}\Big|_a^\infty \approx +\frac{\cancel{e^{-\lambda a}}}{\cancel{\lambda}}}$$

$$b = \frac{a\lambda+1}{\lambda} = a + \frac{1}{\lambda}$$

b The nearest neighbor condition for optimality says that the decision boundary to be midpoint between the reconstruction levels, i.e.

$$t_k = \frac{r_k + r_{k+1}}{2}$$

$$a = \frac{b+0}{2} = \frac{b}{2}$$

since $b = a + \frac{1}{\lambda}$ we can use it to also satisfy centroid condition w/ nn-condition.

$$\therefore 2a = a + \frac{1}{\lambda} \quad \therefore a = \frac{1}{\lambda}$$

$$\therefore b = \frac{2}{\lambda}$$

2 Lloyd Algorithm for quantizer Design.

$$T = \{1, 2, 3, 4, 8, 9, 12\} = \{t_1, t_2 \cdots t_K\}$$

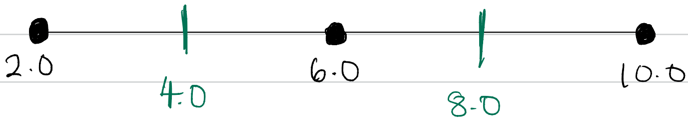
$K=7$

$$C_0 = \{2.0, 6.0, 10.0\} = \{r_1, r_2, r_3\}$$

$=$ codebook

Round 1:

Given $\{r_i\}$, set the $\{t_i\}$ halfway in between.



$$D_1 = \{4.0, 8.0\}$$

Set the $\{r_i\}$ to be the center of mass

$$C_1 = \left\{ \frac{1+2+3+4}{4}, 8, \frac{9+12}{2} \right\}$$

$$C_1 = \{2.5, 8.0, 10.5\}$$

Choose left

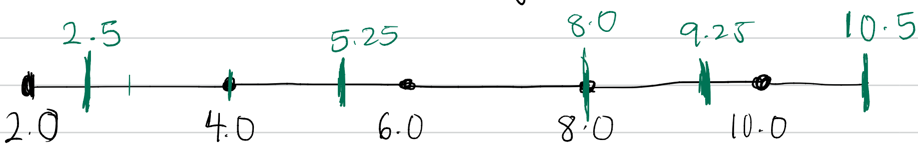
$$C_1 = \left\{ \frac{1+2+3}{3}, 4.0, \frac{8+9+12}{3} \right\}$$

$$= \{2.0, 4.0, 9.67\}$$

Chose right

Choose left

Round 2: Set $\{t_i\}$ halfway in between



$$D_2 = \left\{ \frac{2.5+8}{2} = 5.25 \, , \; \frac{8+10.5}{2} = 9.25 \right\}$$

$$C_2 = \left\{ \frac{1+2+3+4}{4} \, , \; \frac{8+9}{2} \, , \; 12 \right\}$$

$$= \{ 2.5 \, , 8.5 \, , 12.0 \}$$

Round 3: Set $\{t_i\}$ halfway

$$D_3 = \left\{ \frac{2.5+8.5}{2} = 5.5 \, , \; \frac{8.5+12.0}{2} = 10.25 \right\}$$

$$C_3 = \left\{ \frac{1+2+3+4}{4} \, , \; \frac{8+9}{2} \, , \; 12 \right\}$$

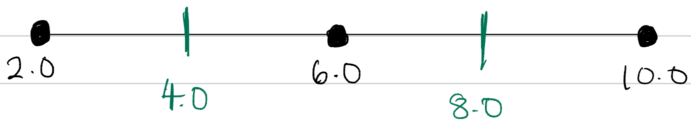
$$= \{ 2.5 \, , 8.5 \, , 12.0 \}$$

Since $C_2 = C_3 \Rightarrow$ converged.

Choose right

Round 1:

Given $\{r_i\}$, set the $\{t_i\}$ halfway in between.



$$D_1 = \{4.0, 8.0\}$$

Set the $\{r_i\}$ to be the center of mass

$$C_1 = \left\{ \frac{1+2+3}{3}, 4.0, \frac{8+9+12}{3} \right\}$$

$$= \{2.0, 4.0, 9.67\}$$

Choose right

Round 2:

$$D_1 = \left\{ \frac{2.0+4.0}{2}, \frac{4.0+9.67}{2} \right\}$$

$$= \{3.0, 6.833\}$$

$$C_2 = \left\{ \frac{1+2}{2}, \frac{3+4}{2}, \frac{8+9+12}{3} \right\}$$

$$C_2 = \{1.5, 3.5, 9.67\}$$

choose right

Round 3: set $\{t_i\}$ halfway in between

$$D_3 = \left\{ \frac{1.5+3.5}{2} = 2.5, \quad \frac{3.5+9.67}{2} = 6.58 \right\}$$
$$= \{2.5, 6.58\}$$

$$C_3 = \left\{ \frac{1+2}{2}, \frac{3+4}{2}, \frac{8+9+12}{3} \right\}$$

$$C_3 = \{1.5, 3.5, 9.67\}$$

Since $C_2 = C_3 \Rightarrow$ converged.

The tie-breaking rule (Right or left) converges in same number of iterations.