ECE 253 Homework 4

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Contents

Problem 1 2D Sampling and Aliasing

Part(a)

```
clear; close all; clc;
[x y] = meshgrid(0:256,0:256);
```

- We are given the sampling period T = 1, thus the sampling frequency F_s = 1.
- x and y are the spatial coordinates of the original image given by the mesh grid in matlab
- u and v are the spatial frequency coordinates of the Fourier transform of the image.

```
z1 = cos ( 2 * pi * 1/32 .* x - 2 * pi * 1/128 .* y);
figure, imshow(z1);
```

Cutoff Frequencies for z1.

- In x-direction F_{xc} = 1/32
- In y-direction F_{yc} = 1/128

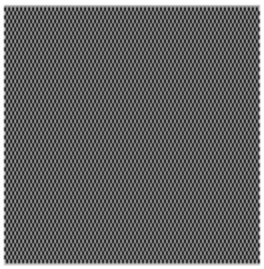
Image can be reconstructed exactly from it's samples if Cutoff freq ≤ 1/2 sampling freq F_s

- If equality holds, then the sampling is at the Nyquist rate
- If Δx and Δy are smaller than required, the image is called oversampled
- If they are larger than required, the image is undersampled

In terms of spatial frequency and the sampling frequency,

- 2 * F_{xc} = 1/16 < F_s = 1 ==> undersampled in x-direction
- 2 * F_{yc} = 1/64 < F_s = 1 ==> undersampled in y-direction

```
z2 = cos ( 2 * pi * 1/4 .* x - 2 * pi * 7/8 .* y);
figure, imshow(z2), truesize;
```



Cutoff Frequencies for z2.

- In x-direction F_{xc} = 1/4
- In y-direction F_{yc} = 7/8

In terms of spatial frequency and the sampling frequency,

- 2 * F_{xc} = 1/2 < F_s = 1 ==> undersamppled in x-direction
- 2 * F_{yc} = 7/4 > F_s = 1 ==> oversampled in y-direction

```
z3 = cos ( 2 * pi * 1/2 .* x - 2 * pi * 1/2 .* y);
figure, imshow(z3);
```

Cutoff Frequencies for z3.

- In x-direction F_{xc} = 1/2
- In y-direction F_{yc} = 1/2

In terms of spatial frequency and the sampling frequency,

- 2 * F_{xc} = 1 = F_s = 1 ==> critically sampled in x-direction or sampling is at the Nyquist rate
- 2 * F_{yc} = 1 > F_s = 1 ==> critically sampled in y-direction or sampling is at the Nyquist rate

Part(b)

Since Cosine function is periodic with period 2π . we can use any frequencies which is generated by adding any multiple of 2 π to each of the xy frequency components in z1 to obtain a identical sampled function to that of the sampled function z1. Therefore let us consider a = F_{xc} + 2π and b = F_{yc} + 2π, taking the 2π in common we get a = 33/32 b = 129/128 Any a = 1/32 + k 2π where k $\in$ z, will give us the identical sampled function.

However, we have to note that the we want the sampled function to be aliased, this means we have to consider the inequality a > 1/2 and b>1/2

```
z4 = cos ( 2 * pi * 33/32 .* x - 2 * pi * 129/128 .* y);
figure, imshow(z4);
```

In the figure above it came be seen that z4 produces the same image as sampled in z1.

Problem 2 Pre-filtering to Reduce Aliasing before Sampling

```
close all; clc;
barbara = read(Tiff('barbara.tiff', 'r'));
baby = read(Tiff('baby.tiff', 'r'));
figure, imshow(barbara), title('barbara original');
figure, imshow(baby), title('baby original');
```

baby original

```
barbBig = barbara(1:512,37:548);
babyBig = baby(201:712,201:712);
figure, imshow(barbBig), title('barbBig 512 x 512 section');
figure, imshow(babyBig), title('babyBig 512 x 512 section');
```

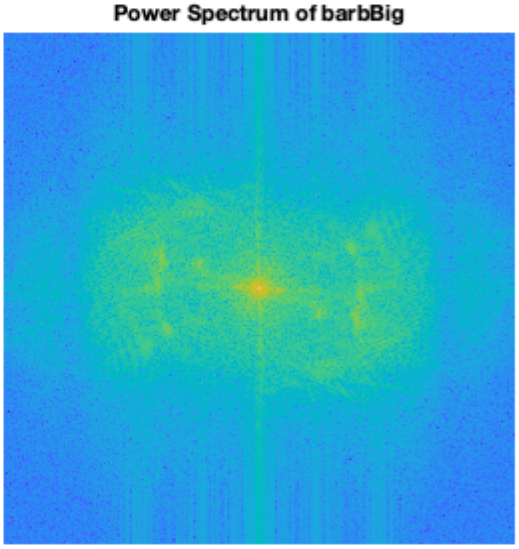
barbBig 512 x 512 section

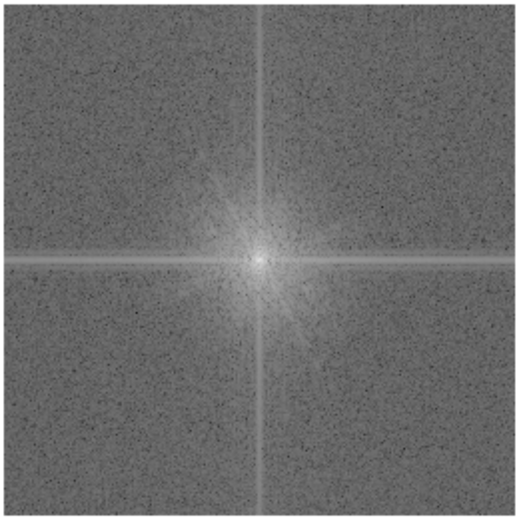
figure, plot(abs(fft2(barbBig)).^2) title('power spectrum before fftshift');

```
barbfft_1 = fftshift(abs(fft2(barbBig)).^2);
barbspectrum = log(barbfft_1 + 1);
figure, imshow(barbspectrum / max(max(barbspectrum)))
title('Power Spectrum of barbBig');

figure, imshow(barbspectrum / max(max(barbspectrum))),colormap("default")
title('Power Spectrum of barbBig');

babyfft_1 = fftshift(abs(fft2(babyBig)).^2);
babyspectrum = log(babyfft_1 + 1);
figure, imshow(babyspectrum / max(max(babyspectrum))),
title('Power Spectrum of babyBig');
% figure,
% imhist(barbBig,256);
% figure,
% imhist(babyBig,256);
```

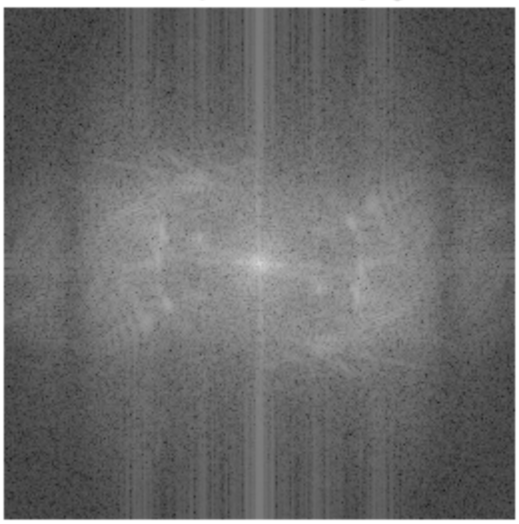
Power Spectrum of barbBig

Power Spectrum of barbBig





Power Spectrum of babyBig



Part(a)

As can be seen from the above Power Spectrum the DC coefficient is located right at the center. Note that it would initially be in the corner but the fftshift brings it to the center. Before the fftshift, the DC coefficient is located in the (1,1) position, and the low frequencies are out in the four corner areas. The fftshift swaps the quadrants. The way we can check this is through the matlab documentation on how the fftshift function shifts the DC coefficent. which would tells us that it just swaps the quadrants.

Part(b)

By just doing the visual comparision, we can say that barbbig has a higher frequency content because of the vertical and horizontal lines that we see in the image. Looking at the babyBig and it's power spectra, there are high frequency components, however not as high as barbbig. we just see the white circle in the middle which is a low frequency component. Barbara will have more of the aliasing problem because of the high frequency components in the image.

Part(c)

```
close all;
baby64_1 = imresize(babyBig,0.125,'nearest');
figure, imshow(baby64_1), truesize, title("baby64_1");

babylow = filter2(ones(8)/64,babyBig);
baby64_2 = imresize(babylow,0.125,'nearest');
figure, imshow(baby64_2, [0 255]), truesize, title("baby64_2");
```

$baby64_1$

$baby64_2$

Making it bigger to see properly.

```
baby64_1 = imresize(babyBig,0.125,'nearest');
baby64_1 = imresize(baby64_1, [500, 500]);
figure, imshow(baby64_1), truesize, title("baby64_1");

babylow = filter2(ones(8)/64,babyBig);
baby64_2 = imresize(babylow,0.125,'nearest');
baby64_2 = imresize(baby64_2, [500, 500]);
figure, imshow(baby64_2, [0 255]), truesize, title("baby64_2");
```

baby64_1

baby64_2

```
barb64_1 = imresize(barbBig,0.125,'nearest');
figure, imshow(barb64_1), truesize, title("barb64_1")
barblow = filter2(ones(8)/64,barbBig);
barb64_2 = imresize(barblow,0.125,'nearest');
figure, imshow(barb64_2, [0 255]), truesize, title("barb64_2");
```

barb64_1

barb64_2

Making it bigger to see properly

```
barb64_1 = imresize(barbBig,0.125,'nearest');
barb64_1 = imresize(barb64_1, [500, 500]);
figure, imshow(barb64_1), truesize, title("barb64_1")

barblow = filter2(ones(8)/64,barbBig);
barb64_2 = imresize(barblow,0.125,'nearest');
barb64_2 = imresize(barb64_2, [500, 500]);
figure, imshow(barb64_2, [0 255]), truesize, title("barb64_2");
```

$barb64_1$

$barb64_2$

By visually comparting the two 64x64 versions of baby and for barbara, looks like the pre-filtering helped barbara more because of the higher frequency components and also aliasing. barb64_1 looks bad, like around the scarf area where we have vertical lines in the original image with a lot of alising because it has high freq which we supress using low freq filter and thus get a better image barb64_2 baby64_1 and baby64_2 looks identical, this is likely due to the low freq nature of the image. It didn't have aliasing which required the pre-filtering, and thus it didn't make any difference.

Part(d)

```
close all;
%
%
%
%
%
%
%
% ds = 1;
% fs = 1;
% factor = 8;
```

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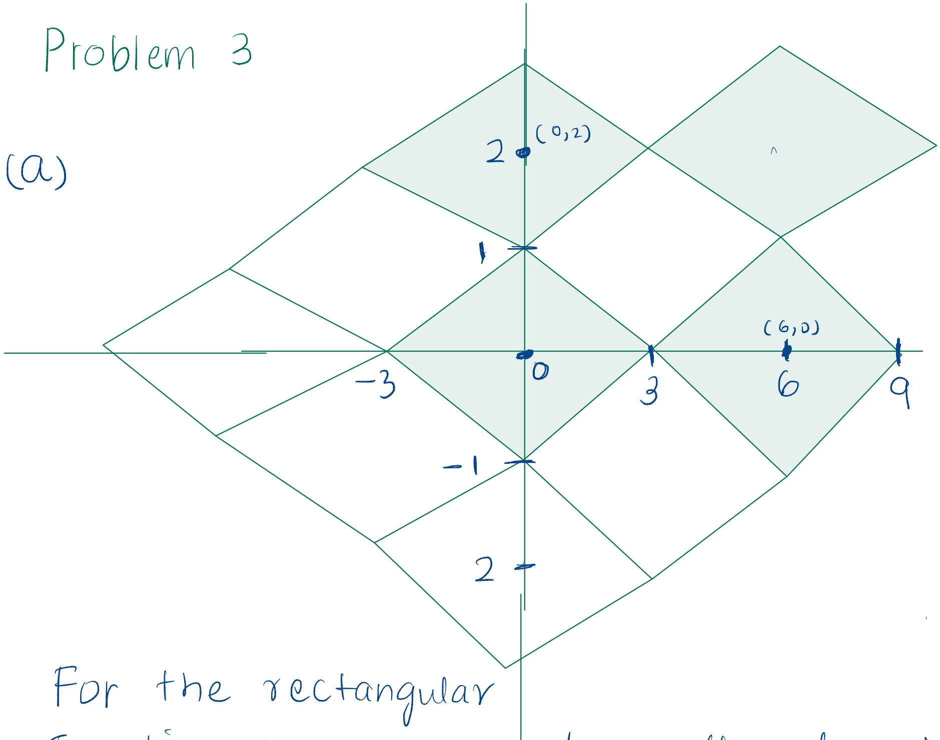
Problem 3

(a)



For the rectangular sampling lattice that has the lowest sampling density which can still ensure no aliasing (no overlap of spectral replications) has a generator matrix :

$$B = \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}$$

each column is a spectral replication location

$B = B^T$

Therefore,

$$B = (A^{-1})^T$$

$$B^T = A^{-1}$$

$$(B^T)^{-1} = A$$

$$A = \frac{1}{12}\begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1/6 & 0 \\ 0 & 1/2 \end{bmatrix}$$

Generator matrix for lattice

$\det(A) = 1/12$

(b) Generator Matrix for non-rectangular sampling lattice w/ lowest sampling density & prevents spectral replications from overlapping.

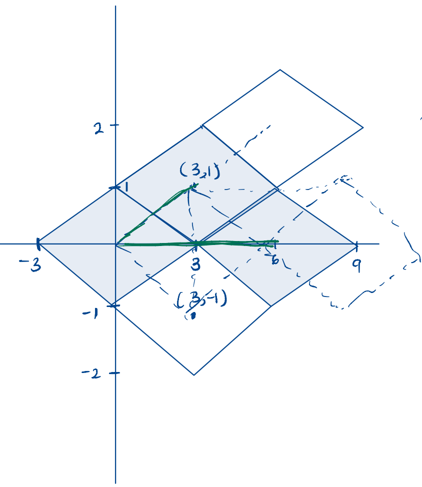
$$B = \begin{bmatrix} 6 & 3 \\ 0 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{6-0} \begin{bmatrix} 1 & -3 \\ 0 & 6 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1/6 & -1/2 \\ 0 & 1 \end{bmatrix}$$

$$(B^{-1})^T = A = \begin{bmatrix} 1/6 & 0 \\ -1/2 & 1 \end{bmatrix}$$

$$\det(A) = \frac{1}{6}$$



Hexagonal Lattice

c) Percentage reduction in samples by sampling on the non-rectangular grid as opposed to the rectangular grid.

$$\frac{\det(A)_{Rec}}{\det(A)_{NRec}} = \frac{1/12}{1/6} = 1/2 = 0.50.$$