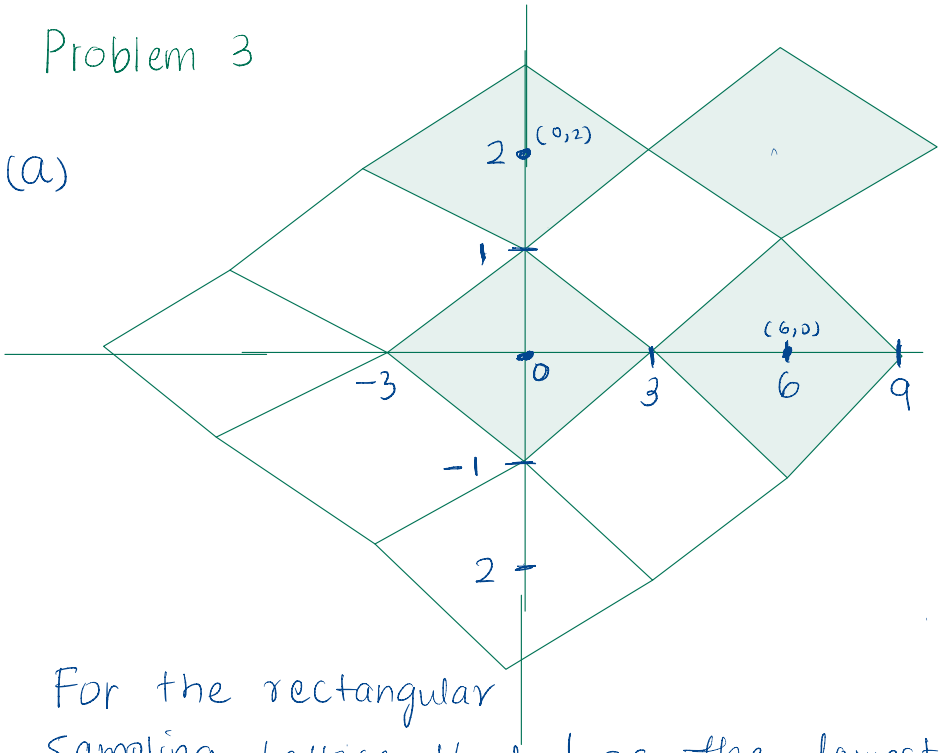
Problem 3

(a)



For the rectangular sampling lattice that has the lowest sampling density which can still ensure no aliasing (no overlap of spectral replications) has a generator matrix :

$$B = \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix} \qquad B = B^T$$

↑ each column is a spectral replication location

Therefore,

$$B = (A^{-1})^T$$

$$B^T = A^{-1}$$

$$(B^T)^{-1} = A$$

$$A = \frac{1}{12}\begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$$

$$\hookrightarrow = \begin{bmatrix} 1/6 & 0 \\ 0 & 1/2 \end{bmatrix} \leftarrow \text{Generator matrix for lattice}$$

$$\det(A) = 1/12 \leftarrow$$

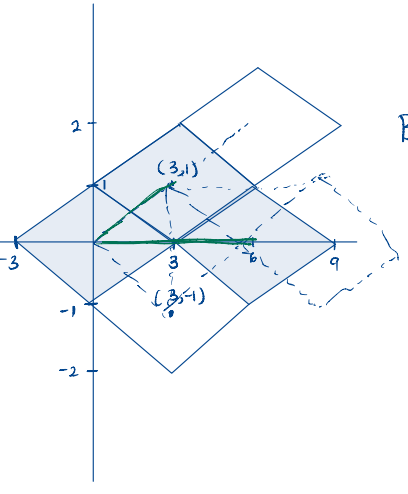
(b) Generator Matrix for non-rectangular sampling lattice w/ lowest sampling density & Prevents spectral replications from overlapping.

$$B^{-1} = \frac{1}{6-0}\begin{bmatrix} 1 & -3 \\ 0 & 6 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1/6 & -1/2 \\ 0 & 1 \end{bmatrix}$$

$$(B^{-1})^T = A = \begin{bmatrix} 1/6 & 0 \\ -1/2 & 1 \end{bmatrix}$$

$$\det(A) = \frac{1}{6}$$



$$B = \begin{bmatrix} 6 & 3 \\ 0 & 1 \end{bmatrix}$$

Hexagonal Lattice

(c) Percentage reduction in samples by Sampling on the non-rectangular grid as opposed to the rectangular grid.

$$\frac{\det(A)_{Rec}}{\det(A)_{NRec}} = \frac{1/12}{1/6} = \frac{1}{2} = 0.50.$$