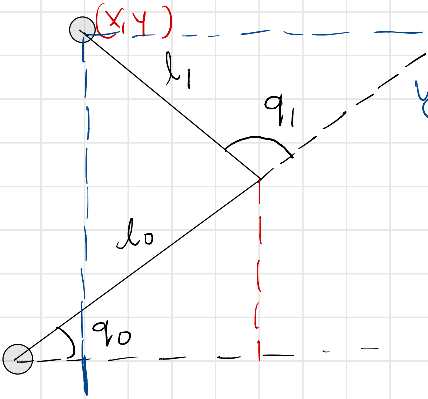
Action space $[T_{q0}, T_{q1}]$

joint torque to q_0 and q_1

$l_0 = 0.1\,m$

$l_1 = 0.11\,m$

Geometric solution

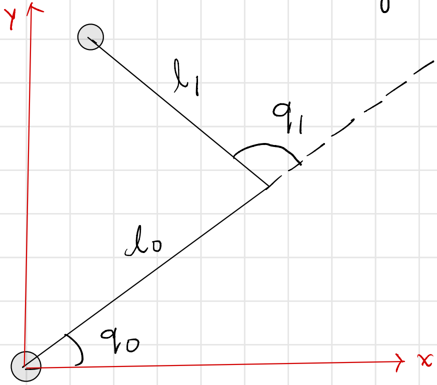


$$y = l_0 \cdot \sin(q_0) + l_1 \sin(q_0 + q_1)$$

$$x = l_0 \cdot \cos(q_0) + l_1 \cos(q_1 + q_0)$$

position: (x, y) → end poin

Establishing Coordinate Frames on Robots w/ Denavit-Hartenberg Method



D-H parameters

α_{i-1}	a_{i-1}	θ_i	d_i
0	0	q_0	0
0	l_0	q_1	0
0	l_1	0	0

we do not have rotation

D-H Matrix

$$ {}^{i-1}_{i}H = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i \cos\alpha_{i-1} & \cos\theta_i \cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1} d_i \\ \sin\theta_i \sin\alpha_{i-1} & \cos\theta_i \sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}H = \begin{bmatrix} C_0 & -S_0 & 0 & 0 \\ S_0 & C_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}H = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}H = \begin{bmatrix} C_1 & -S_1 & 0 & l_0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{3}H = {}^{0}_{1}H \, {}^{1}_{2}H \, {}^{2}_{3}H $$

using online calculater we get

$$ {}^{3}_{0}H = \begin{bmatrix} \cos q_0 \cos q_1 - \sin q_0 \sin q_1 & -\cos q_0 \sin q_1 - \cos q_1 \sin q_0 & 0 & l_1(\cos q_0 \cos q_1 - \sin q_0 \sin q_1) + L_0 \cos q_0 \\ \cos q_0 \sin q_1 + \cos q_1 \sin q_0 & \cos q_0 \cos q_1 - \sin q_0 \sin q_1 & 0 & l_1[\cos q_0 \sin q_1 + \cos q_1 \sin q_0] + L_0 \sin(q_0) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

using Trig identities we have

$$
\begin{array}{ccccc}
 & & & & \text{Position} \\
{}^{3}_{0}H = & \left[\begin{array}{cccc} \cos(q_0+q_1) & -\sin(q_0+q_1) & 0 & l_0\cos q_0 + l_1\cos(q_0+q_1) \\ \sin(q_0+q_1) & \cos(q_0+q_1) & 0 & l_0\sin q_0 + l_1\sin(q_0+q_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]
\end{array}
$$

$$f_1(q) = l_0\cos q_0 + l_1\cos(q_0+q_1)$$

$$f_2(q) = l_0 \cdot \sin q_0 + l_1\sin(q_0+q_1)$$

$$J(q) = \begin{bmatrix} \dfrac{df_1(q)}{dq_0} & \dfrac{df_1(q)}{dq_1} \\ \dfrac{df_2(q)}{dq_0} & \dfrac{df_2(q)}{dq_1} \end{bmatrix}$$

$$J(q) = \begin{bmatrix} -l_0 \cdot \sin(q_0) - l_1\sin(q_0+q_1) & -l_1\sin(q_0+q_1) \\ l_0\cos(q_0) + l_1\cos(q_0+q_1) & +l_1\cos(q_0+q_1) \end{bmatrix}$$