SC-612 Discrete Mathematics
Tutorial 2

1. The n^{th} statement in a list of 100 statements is "Exactly n of the statements in this list are false."

   a) What conclusions can you draw from these statements?

   b) Answer part (a) if the n^{th} statement is "At least n of the statements in this list are false."

   c) Answer part (b) assuming that the list contains 99 statements.

2. Let p and q be the propositions

   p: You drive over 65 miles per hour.
   q: You get a speeding ticket.

   Write these propositions using p and q and logical connectives.

   a) You do not drive over 65 miles per hour.

   b) You drive over 65 miles per hour, but you do not get a speeding ticket.

   c) You will get a speeding ticket if you drive over 65 miles per hour.

   d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

   e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

   f) You get a speeding ticket, but you do not drive over 65 miles per hour.

   g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

3. Determine whether each of these conditional statements is true or false.

   a) If $1 + 1 = 2$, then $2 + 2 = 5$.

   b) If $1 + 1 = 3$, then $2 + 2 = 4$.

   c) If $1 + 1 = 3$, then $2 + 2 = 5$.

   d) If monkeys can fly, then $1 + 1 = 3$.

4. Write each of these sentences in the form "p if and only if q" in English.

   a) If it is hot outside you buy an ice cream cone, and if you buy an ice cream cone it is hot outside.

   b) For you to win the contest it is necessary and sufficient that you have the only winning ticket.

   c) You get promoted only if you have connections, and you have connections only if you get promoted.

   d) If you watch television your mind will decay, and conversely.

   e) The trains run late on exactly those days when I take it.

5. State the converse, contrapositive, and inverse of each of these conditional statements.

   a) If it snows today, I will ski tomorrow.

   b) I come to class whenever there is going to be a quiz.

   c) A positive integer is a prime only if it has no divisors other than I and itself.

6. Construct a truth table for each of these compound propositions.

   a) $p \rightarrow (\neg q \vee r)$

   b) $\neg p \rightarrow (q \rightarrow r)$

   c) $(p \rightarrow q) \vee (\neg p \rightarrow r)$

   d) $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

   e) $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$

   f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$