### Planar Graphs

#### Brendan Halstead

Outline

Definitions and Examples

Kuratowski Theorem

DMP Algorithm

Other Surfaces?

# Planar Graphs and how to find them

Brendan Halstead

July 21, 2020

#### Brendan Halstead

#### Outline

1 Definitions and Examples

2 Kuratowski's Theorem

3 DMP Algorithm

4 Other Surfaces?

and Examples
Kuratowski's

Theorem

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Other Surfaces

### What is a planar graph?

**Planar embedding**: drawing of a graph with no edges crossing **Planar graph**: has a planar embedding

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### What is a planar graph?

**Planar embedding**: drawing of a graph with no edges crossing **Planar graph**: has a planar embedding



Nonplanar embedding

DMP Algorithn

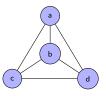
Other Surfaces

### What is a planar graph?

**Planar embedding**: drawing of a graph with no edges crossing **Planar graph**: has a planar embedding



Nonplanar embedding



Planar embedding

DMP Algorithm

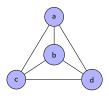
Other Surfaces?

### What is a planar graph?

**Planar embedding**: drawing of a graph with no edges crossing **Planar graph**: has a planar embedding



Nonplanar embedding



Planar embedding

 $K_4$  is planar!

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Other Surfaces? The complete bipartite graph  $K_{3,3}$ 









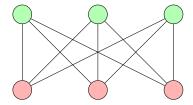




**Definitions** and Examples

### Another example

The complete bipartite graph  $K_{3,3}$ 

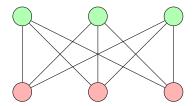


DMP Algorithn

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### Another example

The complete bipartite graph  $K_{3,3}$ 



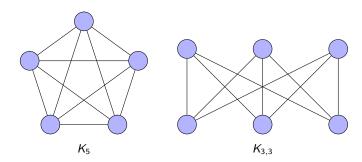
 $K_{3,3}$  is **not** planar.

DMP Algorithn

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### Kuratowski's Theorem

**Theorem:** A graph is planar if and only if it contains no subgraphs homeomorphic to  $K_5$  or  $K_{3,3}$ .



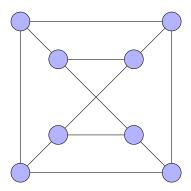
Homeomorphic to = has same basic shape as

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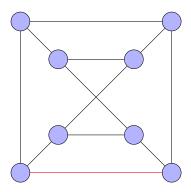


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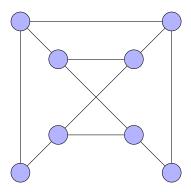


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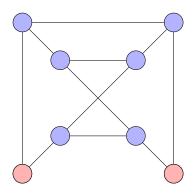


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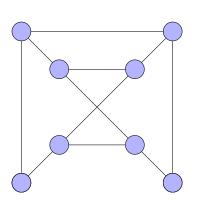
Definitions and Examples

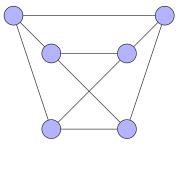
#### Kuratowski's Theorem

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### Example





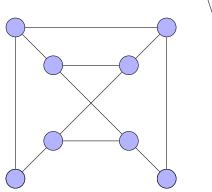
Definitions and Examples

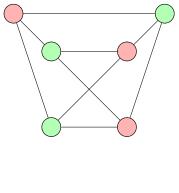
#### Kuratowski's Theorem

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# Example

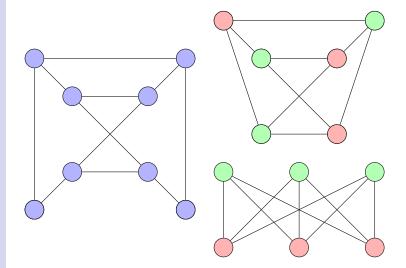




DMP Algorithn

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### Example



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Other Surfaces? Published by Demoucron, Malgrange and Pertuiset in 1964

Works by adding segments to a subgraph until graph is complete or segment is blocked

Not the most efficient, but simplest to understand

Requires pre-processing

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### DMP Planarity Algorithm

#### DMP Algorithm

Input: A 2-connected graph G

Output: A planar embedding or FALSE

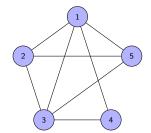
 $G_0:=\mathsf{any}\;\mathsf{cycle}\;\mathsf{in}\;\mathsf{G}$ 

Definitions and Examples

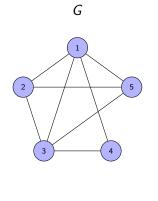
Kuratowski Theorem

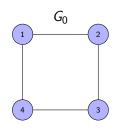
DMP Algorithm

Other



**DMP** Algorithm





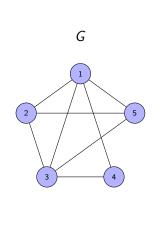
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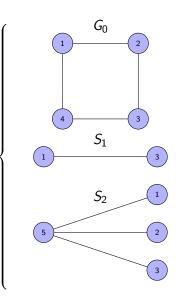
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### DMP Planarity Algorithm

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DMP Algorithm

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Input: A 2-connected graph G
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if any segment is blocked then

return FALSE

end

4□ → 4□ → 4 □ → 1 □ → 9 Q (~)

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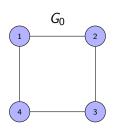
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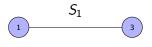
Definitions and Examples

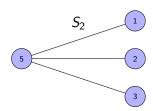
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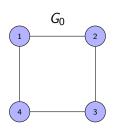
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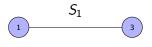
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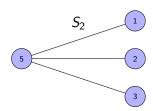
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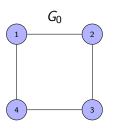
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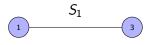
Definitions and Examples

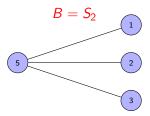
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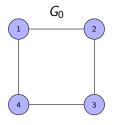
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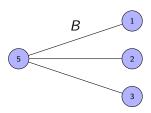
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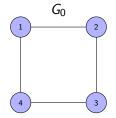
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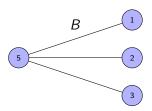
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```

Definitions and Examples

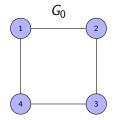
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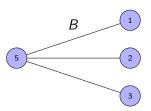
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### DMP Planarity Algorithm

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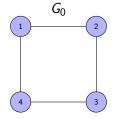
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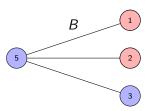
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### DMP Planarity Algorithm

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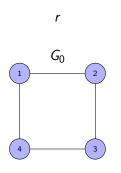
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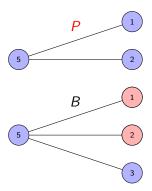
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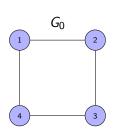
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        G_{i+1} := \text{drawing of } P \text{ in } r
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```

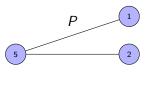
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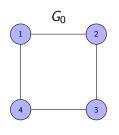
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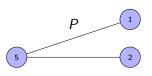
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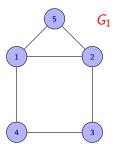
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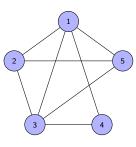


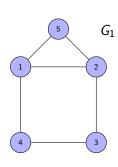




**DMP** Algorithm





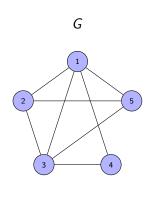


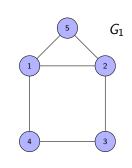
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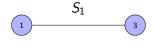
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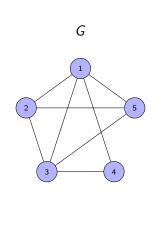


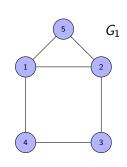


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DMP Algorithm

r := region whose boundary contacts BP := path between two contact points of B

 $G_{j+1} := \text{drawing of } P \text{ in } r$ 

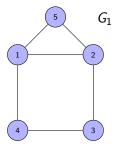
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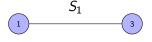
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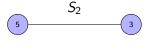
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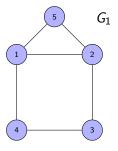


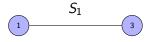




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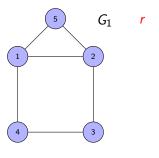
$$B = S_2$$

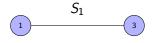
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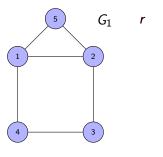


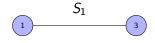
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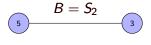
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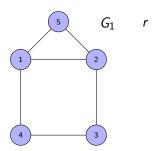


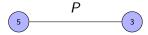
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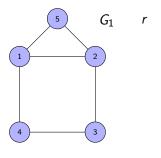


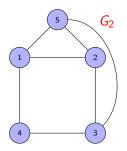
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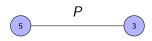
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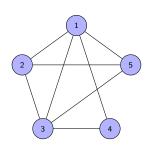


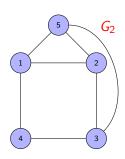
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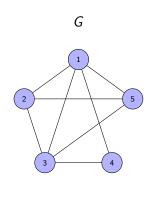
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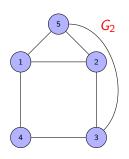
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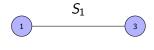
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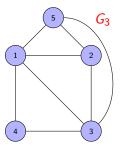
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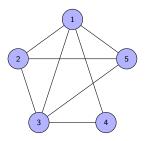


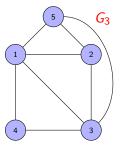
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               B := any segment
        end
       r := region whose boundary contacts B
        P := path between two contact points of B
       G_{i+1} := drawing of P in r
end
return Gi
```

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Sphere?

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Algorithn

**Sphere?** same as plane

Annulus?

Other

Surfaces?

Sphere? same as plane

Annulus? same as plane

Torus?

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Other Surfaces?

#### What about other surfaces?

**Sphere?** same as plane

Annulus? same as plane

Torus? different!