Small-Signal Modeling and Analysis of a Grid-Forming PEM Hydrogen Electrolyzer -Supplementary Materials-

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***Abstract*—Green hydrogen production using Proton Exchange Membrane Electrolyzers (PEMELs) is increasingly integrated with renewable energy sources to support sustainability goals and enhance energy storage capabilities and grid resilience. However, analyzing the stability and dynamics of such systems is intricate, requiring comprehensive modeling for robust design and management. This paper develops a detailed small-signal multiphysics model (SSM) of a grid-forming PEMEL (GFPEMEL). The GFPEMEL adopts a unidirectional dc/dc buck converter and a bidirectional dc/ac inverter for grid integration. The developed model captures the electrical, electrochemical, thermal, and fluidic dynamics of the GFPEMEL system. The proposed SSM is validated against a time-domain multiphysics nonlinear model. Additionally, sensitivity and participation factor analyses are used to assess the impact of the operational conditions on the system dynamics. Stack temperature, current consumption, and ac network strength have been shown to influence the low-frequency modes, highlighting the importance of detailed PEMEL modeling in grid dynamic studies.**

***Index Terms*—Green Hydrogen, grid-forming, multiphysics modeling, PEM electrolyzer, and small signal modeling.**

I. OUTLINE

In this supplementary document, the linearization details and the state-space matrix construction of the PEMEL, the dc-dc PEI, and the dc-ac PEI are thoroughly explained. Accompanying this document is a MATLAB '.m' code file that enables the generation of the matrices and the complete construction of the GFPEMEL SSM, allowing researchers to build upon the outcomes of this work efficiently.

This supplementary document is structured as follows: In Section II, the linearization details of the PEMEL unit (excluding its PEI) are provided. In Section III, the linearization of the dc-dc buck converter is explained in detail. In Section IV, the ac grid interfacing control loops and LCL filter dynamics are described, where in Section V, their linearization is provided.

II. PEMEL LINEARIZED MATRICES DETAILS

Numerous equations in the main manuscript are presented in linearized state-space form for the electrolysis unit and its ancillaries. The details of the linearization are provided in this supplementary file as follows. The parameter with an overbar represent its value at the operating point condition where the small-signal model has been constructed. Equations referenced from the main manuscript as referenced as I-(equation number) in this supplementary document.

The individual entries for each matrix in I-(12) can be defined as follows:

$$a_{e11}^{\mathrm{pm}} = z_1 z_8 + z_2 + z_3 z_5 z_8, \quad a_{e21}^{\mathrm{pm}} = \frac{z_8}{2F\tau_e}, \tag{1a}$$

$$b_{e11}^{\mathrm{pm}} = (z_3(z_4+z_5)+z_1)(z_7(z_{10}+z_{11})+z_9 z_{19} z_{16}), \tag{1b}$$

$$b_{e21}^{\mathrm{pm}} = \frac{z_7(z_{10}+z_{11})+z_9 z_{19} z_{16}}{2F\tau_e}, \quad a_{e22}^{\mathrm{pm}} = -\frac{1}{\tau_e}, \tag{1c}$$

$$b_{e16}^{\mathrm{pm}} = z_3 z_5 z_6, \quad b_{e26}^{\mathrm{pm}} = \frac{z_6}{2F\tau_e}, \tag{1d}$$

$$b_{e12}^{\mathrm{pm}} = z_3 z_5 z_7 z_{12} + z_1 z_7 z_{12}, \tag{1e}$$

$$b_{e13}^{\mathrm{pm}} = z_3 z_5 z_7 z_{13} + z_1 z_7 z_{13}, \tag{1f}$$

$$b_{e14}^{\mathrm{pm}} = (z_3 z_5 + z_1)(z_7 z_{14} + z_9 z_{19} z_{15} z_{17}), \tag{1g}$$

$$b_{e15}^{\mathrm{pm}} = (z_3 z_5 + z_1)(z_9 z_{19} z_{15} z_{18}), \tag{1h}$$

$$b_{e22}^{\mathrm{pm}} = \frac{z_7 z_{12}}{2F\tau_e}, \quad b_{e23}^{\mathrm{pm}} = \frac{z_7 z_{13}}{2F\tau_e}, \tag{1i}$$

$$b_{e24}^{\mathrm{pm}} = \frac{z_7 z_{14} + z_9 z_{19} z_{15} z_{17}}{2F\tau_e}, \quad b_{e25}^{\mathrm{pm}} = \frac{z_9 z_{19} z_{15} z_{18}}{2F\tau_e}. \tag{1j}$$

The individual entries for each matrix in I-(13) can be defined as follows:

$$c_{e1}^{\mathrm{pm}} = z_8, \; d_{e*}^{\mathrm{pm}} = b_{e2*}^{\mathrm{pm}}(2F\tau_e). \tag{2}$$

The elements z_i are detailed as follows:

$$z_1 = \frac{1}{C_{dl}}\left(1 - \frac{\overline{E_{\mathrm{act}}}}{\overline{E_{\mathrm{ass}}}}\right), \quad z_2 = -\frac{\overline{i_{\mathrm{cl}}}}{C_{\mathrm{dl}}\overline{E_{\mathrm{ass}}}}, \tag{3a}$$

$$z_3 = \frac{\overline{i_{\mathrm{cl}}}\,\overline{E_{\mathrm{act}}}}{C_{\mathrm{dl}}\overline{E_{\mathrm{ass}}}}, \quad z_4 = \frac{R}{\alpha F}\ln\left(\frac{\overline{i_{\mathrm{cl}}}}{\overline{j_{\mathrm{cel},0}A^{\mathrm{pm}}}}\right), \tag{3b}$$

$$z_5 = \frac{\frac{R}{\alpha F}\overline{T_{\mathrm{sk}}}}{\overline{i_{\mathrm{cl}}}}, \quad z_6 = \frac{1}{\overline{R_{\mathrm{ohm}}}} = -z_7 = -z_8, \tag{3c}$$

$$z_9 = -\frac{\overline{E_{\mathrm{cl}}} - \overline{E_{\mathrm{oc}}} - \overline{E_{\mathrm{act}}}}{\overline{R_{\mathrm{ohm}}}^2}, \tag{3d}$$

$$z_{10} = \begin{pmatrix} 1.5421\times10^{-3} + 2\cdot 9.84\times10^{-8}\overline{T_{\mathrm{sk}}} \\ +9.523\times10^{-5}(\log(\overline{T_{\mathrm{sk}}})+1) \end{pmatrix}, \tag{3e}$$

$$z_{11} = \frac{R}{2F}\log\left(\frac{\overline{p_{H_2}^{\mathrm{sh}}}\sqrt{\overline{p_{O_2}^{\mathrm{so}}}}}{(p^{\mathrm{std}})^{1.5}\overline{R_A}}\right), \quad z_{12} = \frac{R\overline{T_{\mathrm{sk}}}}{2F\overline{p_{H_2}^{\mathrm{sh}}}}, \tag{3f}$$

$$z_{13} = \frac{R\overline{T_{\mathrm{sk}}}}{4F\overline{p_{O_2}^{\mathrm{so}}}}, \quad z_{14} = \frac{-R\overline{T_{\mathrm{sk}}}}{2F\overline{p_{\mathrm{W}}^{\mathrm{so}}}}, \tag{3g}$$

$$z_{15} = 0.005139\exp\left(\frac{1268}{303} - \frac{1268}{\overline{T_{\mathrm{sk}}}}\right), \tag{3h}$$

$$z_{16} = \frac{1268(0.005139\overline{\lambda} - 0.00326)}{\overline{T_{\mathrm{sk}}}^2}\exp\left(\frac{1268}{303} - \frac{1268}{\overline{T_{\mathrm{sk}}}}\right), \tag{3i}$$

$$z_{17(18)} = 0.5\left(\frac{17.81}{p_{\mathrm{W}}^{\mathrm{sat}}} - \frac{239.85\overline{p_{\mathrm{W}}^{\mathrm{so(sh)}}}}{(p_{\mathrm{W}}^{\mathrm{sat}})^2} + \frac{336\overline{p_{\mathrm{W}}^{\mathrm{so(sh)}}}^2}{(p_{\mathrm{W}}^{\mathrm{sat}})^3}\right), \tag{3j}$$

$$z_{19} = \frac{-L^{\mathrm{pm}}}{A^{\mathrm{pm}}\overline{\sigma_{\mathrm{pm}}}^2}. \tag{3k}$$

The individual entries for the matrices in I-(19) can be provided as follows:

$$\begin{aligned}
a_{t11}^{\mathrm{pm}} =& -\frac{1}{C_{\mathrm{th}}^{\mathrm{cl}}}C_l^{\mathrm{W}}\left(\overline{\dot{m}^{\mathrm{rh}}} - \overline{\dot{m}_{\mathrm{W}}^{\mathrm{dif}}}\right) - \frac{1}{C_{\mathrm{th}}^{\mathrm{cl}}}C_g^{\mathrm{W}}\overline{\dot{m}_{\mathrm{W}}^{\mathrm{eos}}} \\
&-\frac{N_{\mathrm{cs}}}{C_{\mathrm{th}}^{\mathrm{cl}}}(\overline{T_c^{\mathrm{sk}}}C_g^{\mathrm{W}} + H_W^0)0.0134M_{\mathrm{W}}^w\overline{\xi} \\
&-\frac{C_l^{\mathrm{W}}}{C_{\mathrm{th}}^{\mathrm{cl}}}\left(\overline{\dot{m}^{\mathrm{ro}}} - \overline{\dot{m}_{\mathrm{W}}^{\mathrm{eos}}}\right) \\
&-\frac{1}{C_{\mathrm{th}}^{\mathrm{cl}}}C_g^{\mathrm{W}}\overline{\dot{m}_{\mathrm{W}}^{\mathrm{dif}}} + \frac{1}{C_{\mathrm{th}}^{\mathrm{cl}}}\overline{T_c^{\mathrm{sk}}}C_l^{\mathrm{W}}0.0134N_{\mathrm{cs}}M_{\mathrm{W}}^w\overline{\xi} \\
&\quad - \frac{1}{C_{\mathrm{th}}^{\mathrm{cl}}R_{\mathrm{cnd}}^{\mathrm{pm}}} \\
&-\frac{N_{\mathrm{cs}}}{C_{\mathrm{th}}^{\mathrm{cl}}}\overline{\xi}\left(C_g^{H_2}M_{\mathrm{H_2}}^w + 0.5C_g^{O_2}M_{\mathrm{O_2}}^w - C_l^{\mathrm{W}}M_{\mathrm{W}}^w\right),
\end{aligned} \tag{4a}$$

$$b_{t11}^{\mathrm{pm}} = \frac{C_l^{\mathrm{W}}}{C_{\mathrm{th}}^{\mathrm{cl}}}\left(\overline{T^{\mathrm{rh}}} - \overline{T^{\mathrm{sk}}}\right),\ b_{t12}^{\mathrm{pm}} = \frac{C_l^{\mathrm{W}}}{C_{\mathrm{th}}^{\mathrm{cl}}}\left(\overline{T^{\mathrm{ro}}} - \overline{T^{\mathrm{sk}}}\right), \tag{4b}$$

$$b_{t13}^{\mathrm{pm}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{cl}}}C_l^{\mathrm{W}}\overline{T_c^{\mathrm{sk}}} - \frac{1}{C_{\mathrm{th}}^{\mathrm{cl}}}(\overline{T_c^{\mathrm{sk}}}C_g^{\mathrm{W}} + H_W^0), \tag{4c}$$

$$\begin{aligned}
b_{t15}^{\mathrm{pm}} =& -\frac{N_{\mathrm{cs}}}{C_{\mathrm{th}}^{\mathrm{cl}}}\Big(- C_l^{\mathrm{W}}\overline{T_c^{\mathrm{sk}}}M_{\mathrm{W}}^w v_{\mathrm{W}} \\
&+v_{\mathrm{H_2}}M_{\mathrm{H_2}}^w(\overline{T_c^{\mathrm{sk}}}C_g^{H_2} + H_{H_2}^0) + v_{\mathrm{O_2}}M_{\mathrm{O_2}}^w(\overline{T_c^{\mathrm{sk}}}C_g^{O_2} + H_{O_2}^0) \\
&-M_{\mathrm{W}}^w(0.0134\overline{T^{\mathrm{sk}}} + 0.03)\left(C_l^{\mathrm{W}}\overline{T_c^{\mathrm{sk}}} - \overline{T_c^{\mathrm{sk}}}C_g^{\mathrm{W}} - H_W^0\right)\Big),
\end{aligned} \tag{4d}$$

$$b_{t16}^{\mathrm{pm}} = \frac{N_{\mathrm{cs}}}{C_{\mathrm{th}}^{\mathrm{cl}}}\overline{E_{\mathrm{cl}}}, \quad b_{t17}^{\mathrm{pm}} = \frac{N_{\mathrm{cs}}}{C_{\mathrm{th}}^{\mathrm{cl}}}\overline{i_{\mathrm{cl}}}, \tag{4e}$$

$$b_{t18}^{\mathrm{pm}} = \frac{C_l^{\mathrm{W}}\overline{\dot{m}^{\mathrm{rh}}}}{C_{\mathrm{th}}^{\mathrm{cl}}},\ b_{t19}^{\mathrm{pm}} = \frac{C_l^{\mathrm{W}}}{C_{\mathrm{th}}^{\mathrm{cl}}}\overline{\dot{m}^{\mathrm{ro}}},\ b_{t110}^{\mathrm{pm}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{cl}}R_{\mathrm{cnd}}^{\mathrm{pm}}}. \tag{4f}$$

The element of the matrices in I-(23) are detailed as follows:

$$d_{g1(2)}^{\mathrm{pm}} = -(+)\frac{N_{\mathrm{cs}}\overline{D_w}A^{\mathrm{pm}}M_{\mathrm{W}}^w\rho_{me}}{\delta_{\mathrm{me}}M_{\mathrm{me}}}2z_{17(18)}, \tag{5a}$$

$$d_{g3}^{\mathrm{pm}} = \begin{pmatrix} \frac{N_{\mathrm{cs}}A^{\mathrm{pm}}M_{\mathrm{W}}^w}{\delta_{\mathrm{me}}}(\overline{C_{\mathrm{H_2O}}^{\mathrm{chh}}} - \overline{C_{\mathrm{H_2O}}^{\mathrm{cho}}})\cdot \\ 1.25\times 10^{-10}\cdot\exp(\frac{2416}{303.15} - \frac{2416}{\overline{T^{\mathrm{sk}}}})\frac{2416}{\overline{T^{\mathrm{sk}}}^2} \end{pmatrix}. \tag{5b}$$

The individual entries for each matrix in I-(28) can be found as follows:

$$a_{g11}^{\mathrm{so}} = -\frac{\overline{m_{O_2}^{\mathrm{so}}}\,\overline{\dot{m}^{\mathrm{ox}}}}{(\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}})^2},\ a_{g12}^{\mathrm{so}} = \frac{\overline{m_{\mathrm{W}}^{\mathrm{so}}}\,\overline{\dot{m}^{\mathrm{ox}}}}{(\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}})^2}, \tag{6a}$$

$$a_{g21}^{\mathrm{so}} = \frac{\overline{m_{O_2}^{\mathrm{so}}}\,\overline{\dot{m}^{\mathrm{ox}}}}{(\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{W}^{\mathrm{so}}})^2},\ a_{g22}^{\mathrm{so}} = -\frac{\overline{m_{\mathrm{W}}^{\mathrm{so}}}\,\overline{\dot{m}^{\mathrm{ox}}}}{(\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{W}^{\mathrm{so}}})^2}, \tag{6b}$$

$$b_{g13}^{\mathrm{so}} = -\frac{\overline{m_{\mathrm{W}}^{\mathrm{so}}}}{\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}}},\ b_{g23}^{\mathrm{so}} = -\frac{\overline{m_{O_2}^{\mathrm{so}}}}{\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}}}, \tag{6c}$$

$$b_{g24}^{\mathrm{so}} = N_{\mathrm{cs}}M_{O_2}^w v_{O_2}. \tag{6d}$$

The individual elements of each matrix in the algebraic output equation in I-(29) are defined as follows:

$$c_{g11}^{\mathrm{so}} = \frac{R\overline{T^{\mathrm{so}}}}{M_{\mathrm{W}}^w\left(V^{\mathrm{so}} - A^{\mathrm{so}}\overline{L^{\mathrm{so}}}\right)}, c_{g22}^{\mathrm{so}} = \frac{R\overline{T^{\mathrm{so}}}}{M_{\mathrm{O_2}}^w\left(V^{\mathrm{so}} - A^{\mathrm{so}}\overline{L^{\mathrm{so}}}\right)}, \tag{7a}$$

$$c_{g33}^{\mathrm{so}} = -\omega_{ev}, \tag{7b}$$

$$d_{g11}^{\mathrm{so}} = \frac{R\overline{T^{\mathrm{so}}}\,\overline{m_{\mathrm{W}}^{\mathrm{so}}}}{M_{\mathrm{W}}^w\left(V^{\mathrm{so}} - A^{\mathrm{so}}\overline{L^{\mathrm{so}}}\right)^2\rho_{\mathrm{W}}}, \tag{7c}$$

$$d_{g21}^{\mathrm{so}} = \frac{R\overline{T^{\mathrm{so}}}\,\overline{m_{\mathrm{O_2}}^{\mathrm{so}}}}{M_{\mathrm{O_2}}^w\left(V^{\mathrm{so}} - A^{\mathrm{so}}\overline{L^{\mathrm{so}}}\right)^2\rho_{\mathrm{W}}}, \tag{7d}$$

$$d_{g31}^{\mathrm{so}} = -\omega_{ev}p_{\mathrm{W}}^{\mathrm{sat}}\frac{M_{\mathrm{W}}^w}{R\overline{T^{\mathrm{so}}}\rho_{\mathrm{W}}}, \tag{7e}$$

$$d_{g12}^{\mathrm{so}} = \frac{R\overline{m_{\mathrm{W}}^{\mathrm{so}}}}{M_{\mathrm{W}}^w\left(V^{\mathrm{so}} - A^{\mathrm{so}}\overline{L^{\mathrm{so}}}\right)}, d_{g22}^{\mathrm{so}} = \frac{R\overline{m_{\mathrm{O_2}}^{\mathrm{so}}}}{M_{\mathrm{O_2}}^w\left(V^{\mathrm{so}} - A^{\mathrm{so}}\overline{L^{\mathrm{so}}}\right)}, \tag{7f}$$

$$d_{g32}^{\mathrm{so}} = -\omega_{ev}p_{\mathrm{W}}^{\mathrm{sat}}\frac{M_{\mathrm{W}}^w(V^{\mathrm{so}} - A^{\mathrm{so}}\overline{L^{\mathrm{so}}})}{R\overline{T^{\mathrm{so}}}^2}. \tag{7g}$$

The individual elements of the matrices in the state equation I-(31) are given as follows:

$$\begin{aligned}
a_{t11}^{\mathrm{so}} =& -\frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}\Bigg(\overline{\dot{m}^{\mathrm{ro}}}C_l^{\mathrm{W}} + \frac{1}{R_{\mathrm{cnd}}^{\mathrm{so}}} + \\
&\overline{\dot{m}^{\mathrm{ox}}}\frac{\overline{m_{O_2}^{\mathrm{so}}}C_g^{O_2} + \overline{m_{\mathrm{W}}^{\mathrm{so}}}C_g^{\mathrm{W}}}{\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}}} + \overline{\dot{m}_{\mathrm{W}}^{\mathrm{eo}}}(C_g^{\mathrm{W}} - C_l^{\mathrm{W}})\Bigg),
\end{aligned} \tag{8a}$$

$$\begin{aligned}
b_{t11}^{\mathrm{so}} =& \frac{C_l^{\mathrm{W}}}{C_{\mathrm{th}}^{\mathrm{so}}}\left((\overline{\dot{m}^{\mathrm{ro}}} - \overline{\dot{m}^{eos}}) - M_{\mathrm{W}}^w N_{\mathrm{cs}}\overline{\xi}0.0134\overline{T_c^{\mathrm{sk}}}\right) \\
&+\frac{C_g^{\mathrm{W}}\overline{\dot{m}_{\mathrm{W}}^{\mathrm{dif}}}}{C_{\mathrm{th}}^{\mathrm{so}}} + \frac{N_{\mathrm{cs}}}{C_{\mathrm{th}}^{\mathrm{so}}}\overline{\xi}\left(0.5C_g^{O_2}M_{\mathrm{O_2}}^w - C_l^{\mathrm{W}}M_{\mathrm{W}}^w\right),
\end{aligned} \tag{8b}$$

$$b_{t12}^{\mathrm{so}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}\frac{1}{R_{\mathrm{cnd}}^{\mathrm{so}}}, \quad b_{t13}^{\mathrm{so}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}\overline{T^{\mathrm{sk}}}C_l^{\mathrm{W}} - \frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}\overline{T^{\mathrm{so}}}C_l^{\mathrm{W}}, \tag{8c}$$

$$b_{t14}^{\mathrm{so}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}(\overline{T_c^{\mathrm{rso}}})C_l^{\mathrm{W}}, \tag{8d}$$

$$b_{t15}^{\mathrm{so}} = \frac{-(\overline{m_{O_2}^{\mathrm{so}}}C_g^{O_2} + \overline{m_{\mathrm{W}}^{\mathrm{so}}}C_g^{\mathrm{W}})\overline{T_c^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}}H_{\mathrm{W}}^0 + \overline{m_{O_2}^{\mathrm{so}}}H_{\mathrm{O_2}}^0}{C_{\mathrm{th}}^{\mathrm{so}}(\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}})}, \tag{8e}$$

$$b_{t16}^{\mathrm{so}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}\left(\overline{T_c^{\mathrm{sk}}}C_g^{\mathrm{W}} + H_{\mathrm{W}}^0\right), \tag{8f}$$

$$b_{t17}^{\mathrm{so}} = -\frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}\left(-\overline{T_c^{\mathrm{so}}}C_l^{\mathrm{W}} + \overline{T_c^{\mathrm{so}}}C_g^{\mathrm{W}} + H_{\mathrm{W}}^0\right), \tag{8g}$$

$$\begin{aligned}
b_{t18}^{\mathrm{so}} =& -\frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}C_l^{\mathrm{W}}\overline{T_c^{\mathrm{sk}}}\begin{pmatrix} N_{\mathrm{cs}}M_{\mathrm{W}}^w(0.0134\overline{T^{\mathrm{sk}}} + 0.03) \\ +N_{\mathrm{cs}}M_{\mathrm{W}}^w v_{\mathrm{W}} \end{pmatrix} \\
&+ \frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}\left(\overline{T_c^{\mathrm{sk}}}C_g^{O_2} + H_{\mathrm{O_2}}^0\right)N_{\mathrm{cs}}M_{O_2}^w v_{O_2},
\end{aligned} \tag{8h}$$

$$b_{t19}^{\mathrm{so}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}\frac{\overline{\dot{m}^{\mathrm{ox}}}\,\overline{m_{\mathrm{O_2}}^{\mathrm{so}}}}{(\overline{m_{\mathrm{O_2}}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}})^2}\begin{pmatrix} \overline{T_c^{\mathrm{so}}}C_g^{O_2} + H_{\mathrm{O_2}}^0 \\ -\overline{T_c^{\mathrm{so}}}C_g^{\mathrm{W}} - H_{\mathrm{W}}^0 \end{pmatrix}, \tag{8i}$$

$$b_{t110}^{\mathrm{so}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{so}}}\frac{\overline{\dot{m}^{\mathrm{ox}}}\,\overline{m_{\mathrm{W}}^{\mathrm{so}}}}{(\overline{m_{\mathrm{O_2}}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}})^2}\begin{pmatrix} \overline{T_c^{\mathrm{so}}}C_g^{\mathrm{W}} + H_{\mathrm{W}}^0 \\ -\overline{T_c^{\mathrm{so}}}C_g^{O_2} - H_{\mathrm{O_2}}^0 \end{pmatrix}. \tag{8j}$$

The individual elements of the matrices in the state equation in I-(34) can be given as follows:

$$a_{f21}^{\mathrm{so}} = -\frac{\omega_c^{\mathrm{rso}}\dot{m}_{\mathrm{mx}}^{\mathrm{rso}}K_p^{\mathrm{rso}}}{A^{\mathrm{so}}\rho_{\mathrm{W}}},\ a_{f22}^{\mathrm{so}} = -\omega_c^{\mathrm{rso}}, \tag{9a}$$

$$a_{f23}^{\mathrm{so}} = \omega_c^{\mathrm{rso}}\dot{m}_{\mathrm{mx}}^{\mathrm{rso}}K_i^{\mathrm{rso}},\ a_{f31}^{\mathrm{so}} = -\frac{1}{A^{\mathrm{so}}\rho_{\mathrm{W}}}, \tag{9b}$$

$$b_{f11}^{\mathrm{so}} = -M_{\mathrm{W}}^{w}\overline{\xi}0.0134N_{\mathrm{cs}}, \tag{9c}$$

$$b_{f12}^{\mathrm{so}} = -N_{\mathrm{cs}}M_{\mathrm{W}}^{w}\left(0.03+0.0134\overline{T^{\mathrm{sk}}}+v_{\mathrm{W}}\right). \tag{9d}$$

The elements in the algebraic equation in I-(35b) are detailed as follows:

$$d_{f11}^{\mathrm{so}} = \frac{1}{2R_{\mathrm{vlv}}^{\mathrm{so}}\sqrt{\overline{p_{\mathrm{O}_2}^{\mathrm{so}}}+\overline{p_{\mathrm{W}}^{\mathrm{so}}}-P^{atm}}} = d_{f12}^{\mathrm{so}}. \tag{10}$$

The elements of the matrices in the state equation I-(37) are explained as follows:

$$a_{g11}^{\mathrm{sh}} = -\frac{\overline{m_{\mathrm{W}}^{\mathrm{sh}}}\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}}+\overline{m_{H_2}^{\mathrm{sh}}})^2},\ a_{g12}^{\mathrm{sh}} = \frac{\overline{m_{H_2}^{\mathrm{sh}}}\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}}+\overline{m_{H_2}^{\mathrm{sh}}})^2}, \tag{11a}$$

$$a_{g21}^{\mathrm{sh}} = \frac{\overline{m_{\mathrm{W}}^{\mathrm{sh}}}\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}}+\overline{m_{H_2}^{\mathrm{sh}}})^2},\ a_{g22}^{\mathrm{sh}} = -\frac{\overline{m_{H_2}^{\mathrm{sh}}}\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}}+\overline{m_{H_2}^{\mathrm{sh}}})^2}, \tag{11b}$$

$$b_{g11}^{\mathrm{sh}} = -\frac{\overline{m_{H_2}^{\mathrm{sh}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}}+\overline{m_{H_2}^{\mathrm{sh}}})},\ b_{g21}^{\mathrm{sh}} = -\frac{\overline{m_{\mathrm{W}}^{\mathrm{sh}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}}+\overline{m_{H_2}^{\mathrm{sh}}})}, \tag{11c}$$

$$b_{g23}^{\mathrm{sh}} = N_{\mathrm{cs}}M_{\mathrm{W}}^{w}\overline{\xi}0.0134, \tag{11d}$$

$$b_{g14}^{\mathrm{sh}} = N_{\mathrm{cs}}M_{H_2}^{w}v_{H_2},\ b_{g24}^{\mathrm{sh}} = N_{\mathrm{cs}}M_{\mathrm{W}}^{w}(0.03+0.0134\overline{T^{\mathrm{sk}}}). \tag{11e}$$

The elements of algebraic equation I-(38) can be given as follows:

$$c_{g11}^{\mathrm{sh}} = \frac{R\overline{T^{\mathrm{sh}}}}{M_{H_2}^{w}(V^{\mathrm{sh}}-A^{\mathrm{sh}}\overline{L^{\mathrm{sh}}})}, \tag{12a}$$

$$c_{g22}^{\mathrm{sh}} = \frac{R\overline{T^{\mathrm{sh}}}}{M_{\mathrm{W}}^{w}(V^{\mathrm{sh}}-A^{\mathrm{sh}}\overline{L^{\mathrm{sh}}})},\ c_{g33}^{\mathrm{sh}} = -\omega_{ev}, \tag{12b}$$

$$d_{g11}^{\mathrm{sh}} = \frac{R\overline{T^{\mathrm{sh}}}\overline{m_{\mathrm{H}_2}^{\mathrm{sh}}}}{M_{H_2}^{w}(V^{\mathrm{sh}}-A^{\mathrm{sh}}\overline{L^{\mathrm{sh}}})^2\rho_W}, \tag{12c}$$

$$d_{g21}^{\mathrm{sh}} = \frac{R\overline{T^{\mathrm{sh}}}\overline{m_{\mathrm{W}}^{\mathrm{sh}}}}{M_{\mathrm{W}}^{w}(V^{\mathrm{sh}}-A^{\mathrm{sh}}\overline{L^{\mathrm{sh}}})^2\rho_W}, \tag{12d}$$

$$d_{g31}^{\mathrm{sh}} = -\omega_{ev}p_{\mathrm{W}}^{\mathrm{sat}}\frac{M_{\mathrm{W}}^{w}}{R\overline{T^{\mathrm{sh}}}\rho_{\mathrm{W}}}, \tag{12e}$$

$$d_{g12}^{\mathrm{sh}} = \frac{R\overline{m_{\mathrm{H}_2}^{\mathrm{sh}}}}{M_{H_2}^{w}(V^{\mathrm{sh}}-A^{\mathrm{sh}}\overline{L^{\mathrm{sh}}})}, \tag{12f}$$

$$d_{g22}^{\mathrm{sh}} = \frac{R\overline{m_{\mathrm{W}}^{\mathrm{sh}}}}{M_{\mathrm{W}}^{w}(V^{\mathrm{sh}}-A^{\mathrm{sh}}\overline{L^{\mathrm{sh}}})}, \tag{12g}$$

$$d_{g32}^{\mathrm{sh}} = -\omega_{ev}p_{\mathrm{W}}^{\mathrm{sat}}\frac{M_{\mathrm{W}}^{w}(V^{\mathrm{sh}}-A^{\mathrm{sh}}\overline{L^{\mathrm{sh}}})}{R\overline{T^{\mathrm{sh}}}^2}. \tag{12h}$$

The elements of the matrices in the state equation in I-(40) can be found as follows:

$$a_{t11}^{\mathrm{sh}} = \frac{-1}{C_{\mathrm{th}}^{\mathrm{sh}}}\left(\begin{array}{c}\overline{\dot{m}^{\mathrm{rh}}}C_l^{\mathrm{W}}+\frac{1}{R_{\mathrm{cnd}}^{\mathrm{sh}}}+\\ \frac{\overline{\dot{m}^{\mathrm{sv}}}\left(\overline{m_{\mathrm{W}}^{\mathrm{sh}}}C_g^{\mathrm{W}}+\overline{m_{\mathrm{H}_2}^{\mathrm{sh}}}C_g^{H_2}\right)}{\overline{m_{\mathrm{W}}^{\mathrm{sh}}}+\overline{m_{\mathrm{H}_2}^{\mathrm{sh}}}}\end{array}\right) - \frac{\overline{\dot{m}_{\mathrm{W}}^{\mathrm{eh}}}}{C_{\mathrm{th}}^{\mathrm{sh}}}(C_g^{\mathrm{W}}-C_l^{\mathrm{W}}), \tag{13a}$$

$$\begin{aligned}b_{t11}^{\mathrm{sh}} =& \frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}}(\overline{\dot{m}^{\mathrm{rh}}}-\overline{\dot{m}^{\mathrm{dif}}})C_l^{\mathrm{W}}+\frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}}C_g^{\mathrm{W}}\overline{\dot{m}^{\mathrm{eos}}}\\ &+\frac{N_{\mathrm{cs}}}{C_{\mathrm{th}}^{\mathrm{sh}}}C_g^{H_2}M_{H_2}^{w}v_{H_2}\overline{\xi}+\frac{(\overline{T_c^{\mathrm{sk}}}C_g^{\mathrm{W}}+H_W^0)}{C_{\mathrm{th}}^{\mathrm{sh}}}N_{\mathrm{cs}}M_{\mathrm{W}}^{w}\overline{\xi}0.0134,\end{aligned} \tag{13b}$$

$$b_{t12}^{\mathrm{sh}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}R_{\mathrm{cnd}}^{\mathrm{sh}}},\ b_{t13}^{\mathrm{sh}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}}\overline{T_c^{\mathrm{sk}}}C_l^{\mathrm{W}}-\frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}}\overline{T_c^{\mathrm{sh}}}C_l^{\mathrm{W}}, \tag{13c}$$

$$b_{t14}^{\mathrm{sh}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}}(\overline{T_c^{\mathrm{rsh}}})C_l^{\mathrm{W}}, \tag{13d}$$

$$b_{t15}^{\mathrm{sh}} = \frac{(\overline{m_{H_2}^{\mathrm{sh}}}C_g^{H_2}+\overline{m_{\mathrm{W}}^{\mathrm{sh}}}C_g^{\mathrm{W}})\overline{T_c^{\mathrm{sh}}}+\overline{m_{\mathrm{W}}^{\mathrm{sh}}}H_{\mathrm{W}}^0+\overline{m_{\mathrm{H}_2}^{\mathrm{sh}}}H_{\mathrm{H}_2}^0}{-(\overline{m_{H_2}^{\mathrm{sh}}}+\overline{m_{\mathrm{W}}^{\mathrm{sh}}})C_{\mathrm{th}}^{\mathrm{sh}}}, \tag{13e}$$

$$b_{t16}^{\mathrm{sh}} = -\frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}}\overline{T_c^{\mathrm{sk}}}C_l^{\mathrm{W}}, \tag{13f}$$

$$b_{t17}^{\mathrm{sh}} = -\frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}}(-\overline{T_c^{\mathrm{sh}}}C_l^{\mathrm{W}}+(\overline{T_c^{\mathrm{sh}}}C_g^{\mathrm{W}}+H_{\mathrm{W}}^0)), \tag{13g}$$

$$\begin{aligned}b_{t18}^{\mathrm{sh}} =& \frac{M_{\mathrm{W}}^{w}N_{\mathrm{cs}}}{C_{\mathrm{th}}^{\mathrm{sh}}}(\overline{T_c^{\mathrm{sk}}}C_g^{\mathrm{W}}+H_{\mathrm{W}}^0)((0.0134\overline{T^{\mathrm{sk}}}+0.03))\\ &+\frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}}(\overline{T_c^{\mathrm{sk}}}C_g^{H_2}+H_{\mathrm{H}_2}^0)N_{\mathrm{cs}}M_{H_2}^{w}v_{H_2},\end{aligned} \tag{13h}$$

$$b_{t110}^{\mathrm{sh}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}}\frac{\overline{\dot{m}^{\mathrm{sv}}}\overline{m_{\mathrm{H}_2}^{\mathrm{sh}}}}{(\overline{m_{H_2}^{\mathrm{sh}}}+\overline{m_{\mathrm{W}}^{\mathrm{sh}}})^2}\left(\begin{array}{c}\overline{T_c^{\mathrm{sh}}}(C_g^{H_2}-C_g^{\mathrm{W}})\\ +H_{\mathrm{H}_2}^0-H_{\mathrm{W}}^0\end{array}\right), \tag{13i}$$

$$b_{t19}^{\mathrm{sh}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{sh}}}\frac{\overline{\dot{m}^{\mathrm{sv}}}\overline{m_{\mathrm{W}}^{\mathrm{sh}}}}{(\overline{m_{H_2}^{\mathrm{sh}}}+\overline{m_{\mathrm{W}}^{\mathrm{sh}}})^2}\left(\begin{array}{c}\overline{T_c^{\mathrm{sh}}}(C_g^{\mathrm{W}}-C_g^{H_2})\\ +H_{\mathrm{W}}^0-H_{\mathrm{H}_2}^0\end{array}\right). \tag{13j}$$

The individual elements of the matrices in the state equation I-(42) can be given as follows:

$$a_{f21}^{\mathrm{sh}} = -\frac{\omega_c^{\mathrm{rsh}}\dot{m}_{\mathrm{mx}}^{\mathrm{rsh}}K_p^{\mathrm{rsh}}}{A^{\mathrm{sh}}\rho_{\mathrm{W}}},\ a_{f22}^{\mathrm{sh}} = -\omega_c^{\mathrm{rsh}}, \tag{14a}$$

$$a_{f23}^{\mathrm{sh}} = \omega_c^{\mathrm{rsh}}\dot{m}_{\mathrm{mx}}^{\mathrm{rsh}}K_i^{\mathrm{rsh}},\ a_{f31}^{\mathrm{sh}} = -\frac{1}{A^{\mathrm{sh}}\rho_{\mathrm{W}}}. \tag{14b}$$

The elements of the algebraic equation in I-(43b) can be given as follows:

$$d_{f11}^{\mathrm{sh}} = \frac{1}{2\sqrt{\overline{p_{H_2}^{\mathrm{sh}}}+\overline{p_{\mathrm{W}}^{\mathrm{sh}}}-\overline{p_{\mathrm{W}}^{\mathrm{sv}}}-\overline{p_{H_2}^{\mathrm{sv}}}}R_{\mathrm{vlv}}^{\mathrm{sep}}} = d_{f12}^{\mathrm{sh}}, \tag{15a}$$

$$d_{f13}^{\mathrm{sh}} = d_{f14}^{\mathrm{sh}} = -d_{f11}^{\mathrm{sh}}. \tag{15b}$$

The elements of the state equation in I-(49) can be given as follows:

$$a_{t11}^{\mathrm{ro}} = -\frac{1}{C_{\mathrm{th}}^{\mathrm{ro}}}\left(C_l^{\mathrm{W}}\overline{\dot{m}^{\mathrm{ro}}}+\frac{1}{R_{\mathrm{cnd}}^{\mathrm{ro}}}+\frac{1}{R_{ech}^{\mathrm{ro}}}\right), \tag{16a}$$

$$a_{t12}^{\mathrm{ro}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{ro}}}\frac{1}{R_{ech}^{\mathrm{ro}}},\ a_{t21}^{\mathrm{ro}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{col}}}\frac{1}{R_{ech}^{\mathrm{ro}}}, \tag{16b}$$

$$a_{t22}^{\mathrm{ro}} = -\frac{1}{C_{\mathrm{th}}^{\mathrm{col}}}\left(C_l^{\mathrm{W}}\overline{\dot{m}^{\mathrm{col}}}+\frac{1}{R_{ech}^{\mathrm{ro}}}\right), \tag{16c}$$

$$a_{t23}^{\mathrm{ro}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{col}}}C_l^{\mathrm{W}}\overline{\dot{m}^{\mathrm{col}}},\ a_{t32}^{\mathrm{ro}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{cld}}}C_l^{\mathrm{W}}\overline{\dot{m}^{\mathrm{col}}}, \tag{16d}$$

$$a_{t33}^{\mathrm{ro}} = -\frac{1}{C_{\mathrm{th}}^{\mathrm{cld}}}C_l^{\mathrm{W}}\overline{\dot{m}^{\mathrm{col}}},\ b_{t11}^{\mathrm{ro}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{ro}}}C_l^{\mathrm{W}}\overline{\dot{m}^{\mathrm{ro}}}, \tag{16e}$$

$$b_{t12}^{\mathrm{ro}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{ro}}}\frac{1}{R_{\mathrm{cnd}}^{\mathrm{ro}}},\ b_{t13}^{\mathrm{ro}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{ro}}}C_l^{\mathrm{W}}(\overline{T^{\mathrm{so}}}-\overline{T^{\mathrm{ro}}}), \tag{16f}$$

The elements of the matrices in I-(51) can be given as follows:

$$a_{f11}^{\mathrm{ro}} = -\omega_c^{\mathrm{ro}},\ a_{f12}^{\mathrm{ro}} = \omega_c^{\mathrm{ro}} \dot{m}_{\mathrm{mx}}^{\mathrm{ro}} K_i^{\mathrm{ro}},\ b_{f11}^{\mathrm{ro}} = \omega_c^{\mathrm{ro}} \dot{m}_{\mathrm{mx}}^{\mathrm{ro}} K_p^{\mathrm{ro}}. \tag{17}$$

The elements of the matrices in the state equation in I-(53) are given as follows:

$$a_{t11}^{\mathrm{rh}} = -\frac{1}{C_{\mathrm{th}}^{\mathrm{rh}}}\left(C_l^{\mathrm{W}}\overline{\dot{m}^{\mathrm{ro}}} + \frac{1}{R_{\mathrm{cnd}}^{\mathrm{rh}}}\right),\ b_{t11}^{\mathrm{rh}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{rh}}}C_l^{\mathrm{W}}\overline{\dot{m}^{\mathrm{ro}}}, \tag{18a}$$

$$b_{t12}^{\mathrm{rh}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{rh}}}\frac{1}{R_{\mathrm{cnd}}^{\mathrm{rh}}},\ b_{t13}^{\mathrm{rh}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{rh}}}C_l^{\mathrm{W}}(\overline{T^{\mathrm{sh}}} - \overline{T^{\mathrm{rh}}}), \tag{18b}$$

The coefficients of the matrices in the state equation I-(55) are given as follows:

$$a_{g11}^{\mathrm{dy}} = -\frac{(\overline{m_{\mathrm{W}}^{\mathrm{dy}}})\overline{\dot{m}^{\mathrm{pr}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}})^2},\ a_{g12}^{\mathrm{dy}} = \frac{\overline{m_{H_2}^{\mathrm{dy}}}\,\overline{\dot{m}^{\mathrm{pr}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}})^2}, \tag{19a}$$

$$a_{g21}^{\mathrm{dy}} = \frac{\overline{m_{\mathrm{W}}^{\mathrm{dy}}}\,\overline{\dot{m}^{\mathrm{pr}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}})^2},\ a_{g22}^{\mathrm{dy}} = -\frac{(\overline{m_{H_2}^{\mathrm{dy}}})\overline{\dot{m}^{\mathrm{pr}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}})^2}, \tag{19b}$$

$$b_{g11}^{\mathrm{dy}} = \frac{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}})\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}})^2},\ b_{g12}^{\mathrm{dy}} = -\frac{\overline{m_{H_2}^{\mathrm{sh}}}\,\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}})^2}, \tag{19c}$$

$$b_{g21}^{\mathrm{dy}} = -\frac{\overline{m_{\mathrm{W}}^{\mathrm{sh}}}\,\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}})^2},\ b_{g22}^{\mathrm{dy}} = \frac{(\overline{m_{H_2}^{\mathrm{sh}}})\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}})^2}, \tag{19d}$$

$$b_{g13}^{\mathrm{dy}} = \frac{\overline{m_{H_2}^{\mathrm{sh}}}}{\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}}},\ b_{g23}^{\mathrm{dy}} = \frac{\overline{m_{\mathrm{W}}^{\mathrm{sh}}}}{\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}}}, \tag{19e}$$

$$b_{g14}^{\mathrm{dy}} = -\frac{\overline{m_{H_2}^{\mathrm{dy}}}}{\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}}},\ b_{g24}^{\mathrm{dy}} = -\frac{\overline{m_{\mathrm{W}}^{\mathrm{dy}}}}{\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}}}. \tag{19f}$$

The coefficients of the matrices in the state equation I-(59) are elaborated as follows:

$$a_{t11}^{\mathrm{dy}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{dy}}}\begin{pmatrix} \left(C_g^{\mathrm{W}} - 2C_l^{\mathrm{W}}\right)\overline{\dot{m}^{\mathrm{ads}}} \\ -\frac{\overline{\dot{m}^{\mathrm{pr}}}\left(\overline{m_W^{\mathrm{dy}}}C_g^{\mathrm{W}} + \overline{m_{H_2}^{\mathrm{dy}}}\,\overline{C_g^{H_2}}\right)}{\overline{m_W^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}}} \\ -\frac{1}{R_{\mathrm{cnd}}^{\mathrm{dy}}} \end{pmatrix}, \tag{20a}$$

$$b_{t11}^{\mathrm{dy}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{dy}}}\overline{\dot{m}^{\mathrm{sv}}}\frac{\left(\overline{m_W^{\mathrm{sh}}}C_g^{\mathrm{W}} + \overline{m_{H_2}^{\mathrm{sh}}}\,\overline{C_g^{H_2}}\right)}{\overline{m_W^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}}}, \tag{20b}$$

$$b_{t12}^{\mathrm{dy}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{dy}}R_{\mathrm{cnd}}^{\mathrm{dy}}}, \tag{20c}$$

$$b_{t13}^{\mathrm{dy}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{dy}}}\frac{\overline{\dot{m}^{\mathrm{pr}}}\,\overline{m_{\mathrm{W}}^{\mathrm{dy}}}}{(\overline{m_{H_2}^{\mathrm{dy}}} + \overline{m_{\mathrm{W}}^{\mathrm{dy}}})^2}\begin{pmatrix} \overline{T_c^{\mathrm{dy}}}(C_g^{\mathrm{W}} - C_g^{H_2}) \\ +H_{\mathrm{W}}^0 - H_{\mathrm{H}_2}^0 \end{pmatrix}, \tag{20d}$$

$$b_{t14}^{\mathrm{dy}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{dy}}}\frac{\overline{\dot{m}^{\mathrm{pr}}}\,\overline{m_{\mathrm{H}_2}^{\mathrm{dy}}}}{(\overline{m_{H_2}^{\mathrm{dy}}} + \overline{m_{\mathrm{W}}^{\mathrm{dy}}})^2}\begin{pmatrix} \overline{T_c^{\mathrm{dy}}}(C_g^{H_2} - C_g^{\mathrm{W}}) \\ +H_{\mathrm{H}_2}^0 - H_{\mathrm{W}}^0 \end{pmatrix}, \tag{20e}$$

$$b_{t15}^{\mathrm{dy}} = \frac{-1}{C_{\mathrm{th}}^{\mathrm{dy}}}\frac{\overline{\dot{m}^{\mathrm{sv}}}\,\overline{m_{\mathrm{W}}^{\mathrm{sh}}}}{(\overline{m_{H_2}^{\mathrm{sh}}} + \overline{m_{\mathrm{W}}^{\mathrm{sh}}})^2}\begin{pmatrix} \overline{T_c^{\mathrm{sh}}}(C_g^{\mathrm{W}} - C_g^{H_2}) \\ +H_{\mathrm{W}}^0 - H_{\mathrm{H}_2}^0 \end{pmatrix}, \tag{20f}$$

$$b_{t16}^{\mathrm{dy}} = \frac{-1}{C_{\mathrm{th}}^{\mathrm{dy}}}\frac{\overline{\dot{m}^{\mathrm{sv}}}\,\overline{m_{\mathrm{H}_2}^{\mathrm{sh}}}}{(\overline{m_{H_2}^{\mathrm{sh}}} + \overline{m_{\mathrm{W}}^{\mathrm{sh}}})^2}\begin{pmatrix} \overline{T_c^{\mathrm{sh}}}(C_g^{H_2} - C_g^{\mathrm{W}}) \\ +H_{\mathrm{H}_2}^0 - H_{\mathrm{W}}^0 \end{pmatrix}, \tag{20g}$$

$$b_{t17}^{\mathrm{dy}} = \frac{(\overline{m_{H_2}^{\mathrm{dy}}}C_g^{H_2} + \overline{m_{\mathrm{W}}^{\mathrm{dy}}}C_g^{\mathrm{W}})\overline{T_c^{\mathrm{dy}}} + \overline{m_{\mathrm{W}}^{\mathrm{dy}}}H_{\mathrm{W}}^0 + \overline{m_{\mathrm{H}_2}^{\mathrm{dy}}}H_{\mathrm{H}_2}^0}{-C_{\mathrm{th}}^{\mathrm{dy}}(\overline{m_{H_2}^{\mathrm{dy}}} + \overline{m_{\mathrm{W}}^{\mathrm{dy}}})}, \tag{20h}$$

$$b_{t18}^{\mathrm{dy}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{dy}}}b_{t15}^{\mathrm{sh}}(-C_{\mathrm{th}}^{\mathrm{sh}}), \tag{20i}$$

$$b_{t19}^{\mathrm{dy}} = \frac{1}{C_{\mathrm{th}}^{\mathrm{dy}}}\left(\left(C_g^{\mathrm{W}} - 2C_l^{\mathrm{W}}\right)\left(\overline{T_c^{dy}}\right) + H_{\mathrm{W}}^0\right). \tag{20j}$$

The coefficients of the matrices in I-(61) can be given as follows:

$$c_{g11}^{\mathrm{dy}} = \frac{R\overline{T^{\mathrm{dy}}}}{M_{H_2}^{w}V^{\mathrm{dy}}},\ c_{g22}^{\mathrm{dy}} = \frac{R\overline{T^{\mathrm{dy}}}}{M_{\mathrm{W}}^{w}V^{\mathrm{dy}}}, \tag{21a}$$

$$d_{g11}^{\mathrm{dy}} = \frac{R\overline{m_{H_2}^{\mathrm{dy}}}}{M_{H_2}^{w}V^{\mathrm{dy}}},\ d_{g21}^{\mathrm{dy}} = \frac{R\overline{m_{\mathrm{W}}^{\mathrm{dy}}}}{M_{\mathrm{W}}^{w}V^{\mathrm{dy}}}, \tag{21b}$$

The elements of the state equation in I-(63) are elaborated as follows:

$$a_{11}^{ec} = -\frac{1}{C_{\mathrm{th}}^{ec}}\begin{pmatrix} \frac{1}{R_{\mathrm{cnd}}^{\mathrm{pm}}} + \frac{1}{R_{\mathrm{cnd}}^{\mathrm{rh}}} + \frac{1}{R_{\mathrm{cnd}}^{dy}} + \frac{1}{R_{\mathrm{cnd}}^{\mathrm{ro}}} \\ +\frac{1}{R_{\mathrm{cnd}}^{\mathrm{sh}}} + \frac{1}{R_{\mathrm{cnd}}^{\mathrm{so}}} + \frac{1}{R_{\mathrm{cnd}}^{ec}} + C_p^{air}\overline{m}_d^{fan} \end{pmatrix} \tag{22a}$$

$$b_{11}^{ec} = \frac{1}{C_{\mathrm{th}}^{ec}}\frac{1}{R_{\mathrm{cnd}}^{\mathrm{pm}}},\ b_{12}^{ec} = \frac{1}{C_{\mathrm{th}}^{ec}}\frac{1}{R_{\mathrm{cnd}}^{\mathrm{so}}},\ b_{13}^{ec} = \frac{1}{C_{\mathrm{th}}^{ec}}\frac{1}{R_{\mathrm{cnd}}^{\mathrm{sh}}}, \tag{22b}$$

$$b_{14}^{ec} = \frac{1}{C_{\mathrm{th}}^{ec}}\frac{1}{R_{\mathrm{cnd}}^{\mathrm{ro}}} b_{15}^{ec} = \frac{1}{C_{\mathrm{th}}^{ec}}\frac{1}{R_{\mathrm{cnd}}^{\mathrm{rh}}}, \tag{22c}$$

$$b_{16}^{ec} = \frac{1}{C_{\mathrm{th}}^{ec}}\frac{1}{R_{\mathrm{cnd}}^{dy}}, b_{17}^{ec} = \frac{1}{C_{\mathrm{th}}^{ec}}\left(C_p^{air}\overline{m}_d^{fan} + \frac{1}{R_{\mathrm{cnd}}^{ec}}\right). \tag{22d}$$

where C_p^{air} and $\overline{m}_d^{fan}$ are the air density and air flow produced by the enclosure fan. The elements of the matrices in I-(65) can be formulated as follows:

$$a_{11}^{\mathrm{stg}} = -\omega_c^{\mathrm{stg}},\ a_{12}^{\mathrm{stg}} = \omega_c^{\mathrm{stg}}K_i^{\mathrm{stg}}A_{\mathrm{mx}}^{\mathrm{rst}}, \tag{23a}$$

$$b_{11}^{\mathrm{stg}} = \omega_c^{\mathrm{stg}}K_p^{\mathrm{stg}}A_{\mathrm{mx}}^{\mathrm{rst}} = b_{12}^{\mathrm{stg}}. \tag{23b}$$

The elements of the algebraic equation in I-(67) can be described as follows:

$$c_{g11}^{\mathrm{stg}} = \frac{\sqrt{\overline{p_{H_2}^{\mathrm{sv}}} + \overline{p_{\mathrm{W}}^{\mathrm{sv}}} - P^{\mathrm{pr}}}}{R_{\mathrm{vlv}}^{\mathrm{pr}}}, \tag{24a}$$

$$d_{g11}^{\mathrm{stg}} = \frac{\overline{A^{\mathrm{rst}}}}{2\sqrt{\overline{p_{H_2}^{\mathrm{sv}}} + \overline{p_{\mathrm{W}}^{\mathrm{sv}}} - P^{\mathrm{pr}}}R_{\mathrm{vlv}}^{\mathrm{pr}}} = d_{g12}^{\mathrm{stg}}. \tag{24b}$$

The matrices of the accumulated SSM of the electrolyzer without its interfaces in I-(68); A_{el}, B_{el}, C_{el}, and D_{el}, are detailed in equations (25)-(27). The individual elements of the state matrix A_{el} in (25) are expanded in (28).

III. Dc-Dc PEI SSM matrices

The matrices of the SSM of the dc-dc PEI in I-(75) are detailed as follows:

$$A^{dc} = \begin{bmatrix} A_{\mathrm{lc}}^{\mathrm{dc}} + B_{\mathrm{lc},2}^{\mathrm{dc}}D_{ct,1}^{\mathrm{dc}} & B_{\mathrm{lc},2}^{\mathrm{dc}}C_{ct}^{\mathrm{dc}} \\ B_{\mathrm{ct},1}^{\mathrm{dc}} & A_{\mathrm{ct}}^{\mathrm{dc}} \end{bmatrix} \tag{29a}$$

$$B_{\mathrm{cl}}^{\mathrm{dc}} = \begin{bmatrix} B_{\mathrm{lc},1}^{\mathrm{dc}} + B_{\mathrm{lc},2}^{\mathrm{dc}}D_{\mathrm{ct},2}^{\mathrm{dc}} \\ B_{\mathrm{ct},2}^{\mathrm{dc}} \end{bmatrix} \tag{29b}$$

$$B_{\mathrm{p}}^{\mathrm{dc}} = \begin{bmatrix} B_{\mathrm{lc},3}^{\mathrm{dc}} \\ \mathbf{0}_{2\times1} \end{bmatrix},\ \begin{bmatrix} C_{ec}^{\mathrm{dc}} \\ C_{ed}^{\mathrm{dc}} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_{\mathrm{cs}}} & 0\,0\,0\,0 \\ 0 & 0\,1\,0\,0 \end{bmatrix} \tag{29c}$$

$$
A_{el} =
\begin{bmatrix}
a^{\mathrm{pm}}_{e11} & 0 & b^{\mathrm{pm}}_{e11} & a_1 & a_2 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 & a_4 & a_5 & a_6 & 0 & 0 & a_7 & a_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a^{\mathrm{pm}}_{e21} & a^{\mathrm{pm}}_{e22} & b^{\mathrm{pm}}_{e21} & a_9 & a_{10} & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} & a_{12} & a_{13} & a_{14} & 0 & 0 & a_{15} & a_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{17} & b^{\mathrm{pm}}_{t15} & a_{18} & a_{19} & a_{20} & b^{\mathrm{pm}}_{t19} & 0 & 0 & b^{\mathrm{pm}}_{t18} & 0 & b^{\mathrm{pm}}_{t110} & a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 & a_{25} & a_{26} & 0 & 0 & a_{27} & 0 & 0 & 0 & 0 & 0 \\
0 & b^{\mathrm{so}}_{t18} & a_{28} & a_{29} & a_{30} & 0 & 0 & 0 & 0 & 0 & b^{\mathrm{so}}_{t12} & a_{31} & a_{32} & 0 & a_{33} & 0 & 0 & a_{34} & a_{35} & b^{\mathrm{so}}_{t14} & 0 & b^{\mathrm{so}}_{t13} & 0 & 0 & 0 & 0 & 0 \\
0 & b^{\mathrm{sh}}_{t18} & a_{36} & a_{37} & a_{38} & 0 & 0 & 0 & 0 & a_{39} & b^{\mathrm{sh}}_{t12} & a_{40} & 0 & a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & 0 & b^{\mathrm{sh}}_{t14} & b^{\mathrm{sh}}_{t13} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b^{\mathrm{ro}}_{t11} & 0 & a^{\mathrm{ro}}_{t11} & a^{\mathrm{ro}}_{t12} & 0 & 0 & 0 & b^{\mathrm{ro}}_{t12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b^{\mathrm{ro}}_{t13} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a^{\mathrm{ro}}_{t21} & a^{\mathrm{ro}}_{t22} & a^{\mathrm{ro}}_{t23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & a^{\mathrm{ro}}_{t32} & a^{\mathrm{ro}}_{t33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & b^{\mathrm{rh}}_{t11} & 0 & 0 & 0 & a^{\mathrm{rh}}_{t11} & 0 & b^{\mathrm{rh}}_{t12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b^{\mathrm{rh}}_{t13} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_{47} & 0 & 0 & 0 & 0 & a_{48} & b^{\mathrm{dy}}_{t12} & 0 & 0 & a_{49} & a_{50} & a_{51} & a_{52} & 0 & a_{53} & 0 & 0 & 0 & a_{54} & 0 & 0 & 0 & 0 \\
0 & 0 & b^{ec}_{t11} & b^{ec}_{t12} & b^{ec}_{t13} & b^{ec}_{t14} & 0 & 0 & b^{ec}_{t15} & b^{ec}_{t16} & a^{ec}_{t11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & d^{\mathrm{pm}}_{g3} & a_{55} & a_{56} & 0 & 0 & 0 & 0 & 0 & 0 & a_{57} & a_{58} & 0 & a_{59} & 0 & 0 & a_{60} & a_{61} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & b^{\mathrm{so}}_{g24} & 0 & a_{62} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{63} & a_{64} & 0 & 0 & 0 & 0 & a_{65} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & b^{\mathrm{sh}}_{g14} & 0 & 0 & a_{66} & 0 & 0 & 0 & 0 & a_{67} & 0 & 0 & 0 & a_{68} & a_{69} & a_{70} & a_{71} & 0 & a_{72} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & b^{\mathrm{sh}}_{g24} & b^{\mathrm{sh}}_{g23} & 0 & a_{73} & 0 & 0 & 0 & 0 & a_{74} & 0 & 0 & 0 & a_{75} & a_{76} & a_{77} & a_{78} & 0 & a_{79} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_{80} & 0 & 0 & 0 & 0 & a_{81} & 0 & 0 & 0 & a_{82} & a_{83} & a_{84} & a_{85} & 0 & a_{86} & 0 & 0 & 0 & a_{87} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_{88} & 0 & 0 & 0 & 0 & a_{89} & 0 & 0 & 0 & a_{90} & a_{91} & a_{92} & a_{93} & 0 & a_{94} & 0 & 0 & 0 & a_{95} & 0 & 0 & 0 & 0 \\
0 & b^{\mathrm{so}}_{f12} & b^{\mathrm{so}}_{f11} & -d^{\mathrm{so}}_{g32} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c^{\mathrm{so}}_{g33} & 0 & 0 & 0 & 0 & 0 & -d^{\mathrm{so}}_{g31} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -d^{\mathrm{pm}}_{g3} & a_{96} & a_{97} & 0 & 0 & 0 & 0 & 0 & 0 & a_{98} & 0 & 0 & a_{99} & 0 & 0 & a_{a1} & a_{a2} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a^{\mathrm{so}}_{f21} & 0 & a^{\mathrm{so}}_{f22} & 0 & 0 & 0 & a^{\mathrm{so}}_{f23} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a^{\mathrm{sh}}_{f21} & 0 & a^{\mathrm{sh}}_{f22} & 0 & 0 & 0 & a^{\mathrm{sh}}_{f23} & 0 & 0 \\
0 & 0 & b^{\mathrm{ro}}_{f11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a^{\mathrm{ro}}_{f11} & 0 & 0 & 0 & a^{\mathrm{ro}}_{f12} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{a3} & 0 & 0 & 0 & 0 & 0 & a_{a4} & a_{a5} & 0 & 0 & 0 & 0 & 0 & a^{\mathrm{stg}}_{g11} & 0 & 0 & 0 & a^{\mathrm{stg}}_{g12} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a^{\mathrm{so}}_{f31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a^{\mathrm{sh}}_{f31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{a6} & 0 & 0 & 0 & 0 & 0 & c^{\mathrm{dy}}_{g11} & c^{\mathrm{dy}}_{g22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{25}
$$

$$
B_{el} = \begin{bmatrix} b^{\mathrm{pm}}_{e16} & b^{\mathrm{pm}}_{e26} & b^{\mathrm{pm}}_{t17} + b^{\mathrm{pm}}_{t16} d^{\mathrm{pm}}_{e6} & \mathbf{0}_{1\times 24} \end{bmatrix}^{\top}, \; D_{el} = d^{\mathrm{pm}}_{e6} \tag{26}
$$

$$
C_{el} = \left[c^{\mathrm{pm}}_{e1} \; 0 \; d^{\mathrm{pm}}_{e1} \begin{pmatrix} d^{\mathrm{pm}}_{e3} d^{\mathrm{so}}_{g22} \\ + d^{\mathrm{pm}}_{e4} d^{\mathrm{so}}_{g12} \end{pmatrix} \begin{pmatrix} d^{\mathrm{pm}}_{e2} d^{\mathrm{sh}}_{g12} \\ + d^{\mathrm{pm}}_{e5} d^{\mathrm{sh}}_{g22} \end{pmatrix} \mathbf{0}_{1\times 6} \; d^{\mathrm{pm}}_{e4} c^{\mathrm{so}}_{g11} \; d^{\mathrm{pm}}_{e3} c^{\mathrm{so}}_{g22} \; d^{\mathrm{pm}}_{e2} c^{\mathrm{sh}}_{g11} \; d^{\mathrm{pm}}_{e5} c^{\mathrm{sh}}_{g22} \mathbf{0}_{1\times 2} \begin{pmatrix} d^{\mathrm{pm}}_{e3} d^{\mathrm{so}}_{g21} \\ + d^{\mathrm{pm}}_{e4} d^{\mathrm{so}}_{g11} \end{pmatrix} d^{\mathrm{pm}}_{e2} d^{\mathrm{sh}}_{g11} + d^{\mathrm{pm}}_{e5} d^{\mathrm{sh}}_{g21} \; \mathbf{0}_{1\times 8} \right] \tag{27}
$$

where the individual matrices in (29) are detailed as follows in terms of dc-dc PEI parameters:

$$
A^{\mathrm{dc}}_{\mathrm{lc}} = \begin{bmatrix} 0 & \frac{1}{C_{lv}} & 0 \\ \frac{-1}{L_s} & 0 & \frac{\overline{D}}{L_s} \\ 0 & \frac{-\overline{D}}{C_{dc}} & \frac{-\overline{\tilde{p}}}{C_{dc}\overline{E_{dc}}^2} \end{bmatrix}, \tag{30a}
$$

$$
B^{\mathrm{dc}}_{\mathrm{lc},1} = \begin{bmatrix} \frac{-N_{\mathrm{sk}}}{C_{lv}} \\ 0 \\ 0 \end{bmatrix} B^{\mathrm{dc}}_{\mathrm{lc},3} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{C_{dc}\overline{E_{dc}}} \end{bmatrix} \tag{30b}
$$

$$
B^{\mathrm{dc}}_{\mathrm{lc},2} = \begin{bmatrix} 0 \\ \frac{\overline{E_{dc}}}{L_s} \\ \frac{-\overline{i_{lv}}}{C_{dc}} \end{bmatrix}, D^{\mathrm{dc}}_{ct,2} = \begin{bmatrix} -N_{\mathrm{sk}} k^{dc}_{pc} \end{bmatrix} \tag{30c}
$$

$$
A^{\mathrm{dc}}_{\mathrm{ct}} = \begin{bmatrix} 0 & 0 \\ k^{dc}_{iv} & 0 \end{bmatrix}, B^{\mathrm{dc}}_{\mathrm{ct},1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & k^{dc}_{pv} \end{bmatrix}, \tag{30d}
$$

$$
B^{\mathrm{dc}}_{\mathrm{ct},2} = \begin{bmatrix} 0 \\ -N_{\mathrm{sk}} \end{bmatrix}, \tag{30e}
$$

$$
C^{\mathrm{dc}}_{ct} = \begin{bmatrix} k^{dc}_{pc} k^{dc}_{iv} & k^{dc}_{ic} \end{bmatrix} D^{\mathrm{dc}}_{ct,1} = \begin{bmatrix} 0 & 0 & k^{dc}_{pc} k^{dc}_{pv} \end{bmatrix}, \tag{30f}
$$

IV. DC TO AC INVERTER AND ITS CONTROL LOOPS

The ac interfacing and its control loops regulate the grid voltage amplitude and frequency, allowing the electrolyzer to absorb excess power from the green microgrid for hydrogen production. The ac inverter employs derivative droop control, enabling the electrolyzer unit to actively participate in grid formation alongside other grid-forming units. To ensure accurate control, the active and reactive power measurements are filtered to remove the harmonics, according to the following first-order filters, which significantly influences the control performance:

$$
\dot{P} = -\omega_c P + \omega_c (v_{\mathrm{od}} i_{\mathrm{od}} + v_{oq} i_{oq}), \tag{31a}
$$

$$
\dot{Q} = -\omega_c Q + \omega_c (-v_{\mathrm{od}} i_{oq} + v_{oq} i_{\mathrm{od}}), \tag{31b}
$$

where v_{od} and v_{oq} are the direct and quadrature components of the output voltage of the voltage source converter (VSC), i_{od} and i_{oq} are the corresponding dq components of the output current, and ω_c is the cut-off frequency of the low-pass measurement filters. The active and reactive power dictate the frequency and the output voltage of the inverter, respectively, based on derivative droop relations that are restated from the

$$a_1 = b^{\mathrm{pm}}_{e13}d^{\mathrm{so}}_{g22} + b^{\mathrm{pm}}_{e14}d^{\mathrm{so}}_{g12},\ a_2 = b^{\mathrm{pm}}_{e12}d^{\mathrm{sh}}_{g12} + b^{\mathrm{pm}}_{e15}d^{\mathrm{sh}}_{g22},\ a_3 = b^{\mathrm{pm}}_{e14}c^{\mathrm{so}}_{g11},\ a_4 = b^{\mathrm{pm}}_{e13}c^{\mathrm{so}}_{g22}\ a_5 = b^{\mathrm{pm}}_{e12}c^{\mathrm{sh}}_{g11},\ a_6 = b^{\mathrm{pm}}_{e15}c^{\mathrm{sh}}_{g22}, \tag{28a}$$

$$a_7 = b^{\mathrm{pm}}_{e13}d^{\mathrm{so}}_{g21} + b^{\mathrm{pm}}_{e14}d^{\mathrm{so}}_{g11},\ a_8 = b^{\mathrm{pm}}_{e12}d^{\mathrm{sh}}_{g11} + b^{\mathrm{pm}}_{e15}d^{\mathrm{sh}}_{g21},\ a_9 = b^{\mathrm{pm}}_{e23}d^{\mathrm{so}}_{g22} + b^{\mathrm{pm}}_{e24}d^{\mathrm{so}}_{g12},\ a_{10} = b^{\mathrm{pm}}_{e22}d^{\mathrm{sh}}_{g12} + b^{\mathrm{pm}}_{e25}d^{\mathrm{sh}}_{g22},\ a_{11} = b^{\mathrm{pm}}_{e24}c^{\mathrm{so}}_{g11}, \tag{28b}$$

$$a_{12} = b^{\mathrm{pm}}_{e23}c^{\mathrm{so}}_{g22}, a_{13} = b^{\mathrm{pm}}_{e22}c^{\mathrm{sh}}_{g11}, a_{14} = b^{\mathrm{pm}}_{e25}c^{\mathrm{sh}}_{g22},\ a_{15} = b^{\mathrm{pm}}_{e23}d^{\mathrm{so}}_{g21} + b^{\mathrm{pm}}_{e24}d^{\mathrm{so}}_{g11},\ a_{16} = b^{\mathrm{pm}}_{e22}d^{\mathrm{sh}}_{g11} + b^{\mathrm{pm}}_{e25}d^{\mathrm{sh}}_{g21},\ a_{17} = b^{\mathrm{pm}}_{t16}c^{\mathrm{pm}}_{e1}, \tag{28c}$$

$$a_{18} = a^{\mathrm{pm}}_{t11} + b^{\mathrm{pm}}_{t13}d^{\mathrm{pm}}_{g3} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e1},\ a_{19} = b^{\mathrm{pm}}_{t13}d^{\mathrm{pm}}_{g1}d^{\mathrm{so}}_{g12} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e3}d^{\mathrm{so}}_{g22} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e4}d^{\mathrm{so}}_{g12}, \tag{28d}$$

$$a_{20} = b^{\mathrm{pm}}_{t13}d^{\mathrm{pm}}_{g2}d^{\mathrm{sh}}_{g22} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e2}d^{\mathrm{sh}}_{g12} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e5}d^{\mathrm{sh}}_{g22},\ a_{21} = b^{\mathrm{pm}}_{t13}d^{\mathrm{pm}}_{g1}c^{\mathrm{so}}_{g11} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e4}c^{\mathrm{so}}_{g11},\ a_{22} = b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e3}c^{\mathrm{so}}_{g22}, \tag{28e}$$

$$a_{23} = b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e2}c^{\mathrm{sh}}_{g11},\ a_{24} = b^{\mathrm{pm}}_{t13}d^{\mathrm{pm}}_{g2}c^{\mathrm{sh}}_{g22} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e5}c^{\mathrm{sh}}_{g22},\ a_{25} = b^{\mathrm{pm}}_{t13}d^{\mathrm{pm}}_{g1}d^{\mathrm{so}}_{g11} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e3}d^{\mathrm{so}}_{g21} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e4}d^{\mathrm{so}}_{g11}, \tag{28f}$$

$$a_{26} = b^{\mathrm{pm}}_{t13}d^{\mathrm{pm}}_{g2}d^{\mathrm{sh}}_{g21} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e2}d^{\mathrm{sh}}_{g11} + b^{\mathrm{pm}}_{t16}d^{\mathrm{pm}}_{e5}d^{\mathrm{sh}}_{g21}a_{27} = b^{\mathrm{pm}}_{t12} + b^{\mathrm{pm}}_{t11}a_{28} = b^{\mathrm{so}}_{t11} + b^{\mathrm{so}}_{t16}d^{\mathrm{pm}}_{g3}, \tag{28g}$$

$$a_{29} = a^{\mathrm{so}}_{t11} + b^{\mathrm{so}}_{t15}d^{\mathrm{so}}_{f11}d^{\mathrm{so}}_{g12} + b^{\mathrm{so}}_{t15}d^{\mathrm{so}}_{f12}d^{\mathrm{so}}_{g22} + b^{\mathrm{so}}_{t16}d^{\mathrm{pm}}_{g1}d^{\mathrm{so}}_{g12} + b^{\mathrm{so}}_{t17}d^{\mathrm{so}}_{g32},\ a_{30} = b^{\mathrm{so}}_{t16}d^{\mathrm{pm}}_{g2}d^{\mathrm{sh}}_{g22}, \tag{28h}$$

$$a_{31} = b^{\mathrm{so}}_{t19} + b^{\mathrm{so}}_{t15}d^{\mathrm{so}}_{f11}c^{\mathrm{so}}_{g11} + b^{\mathrm{so}}_{t16}d^{\mathrm{pm}}_{g1}c^{\mathrm{so}}_{g11} + b^{\mathrm{so}}_{t17}c^{\mathrm{so}}_{g33},\ a_{32} = b^{\mathrm{so}}_{t110} + b^{\mathrm{so}}_{t15}d^{\mathrm{so}}_{f12}c^{\mathrm{so}}_{g22},\ a_{33} = b^{\mathrm{so}}_{t16}d^{\mathrm{pm}}_{g2}c^{\mathrm{sh}}_{g22}, \tag{28i}$$

$$a_{34} = b^{\mathrm{so}}_{t15}d^{\mathrm{so}}_{f11}d^{\mathrm{so}}_{g11} + b^{\mathrm{so}}_{t15}d^{\mathrm{so}}_{f12}d^{\mathrm{so}}_{g21} + b^{\mathrm{so}}_{t16}d^{\mathrm{pm}}_{g1}d^{\mathrm{so}}_{g11} + b^{\mathrm{so}}_{t17}d^{\mathrm{so}}_{g31},\ a_{35} = b^{\mathrm{so}}_{t16}d^{\mathrm{pm}}_{g2}d^{\mathrm{sh}}_{g21}, \tag{28j}$$

$$a_{36} = b^{\mathrm{sh}}_{t11} + b^{\mathrm{sh}}_{t16}d^{\mathrm{pm}}_{g3},\ a_{37} = b^{\mathrm{sh}}_{t16}d^{\mathrm{pm}}_{g1}d^{\mathrm{so}}_{g12},\ a_{38} = a^{\mathrm{sh}}_{t11} + b^{\mathrm{sh}}_{t15}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g12} + d^{\mathrm{sh}}_{g22}) + b^{\mathrm{sh}}_{t16}d^{\mathrm{pm}}_{g2}d^{\mathrm{sh}}_{g22} + b^{\mathrm{sh}}_{t17}d^{\mathrm{sh}}_{g32}, \tag{28k}$$

$$a_{39} = b^{\mathrm{sh}}_{t15}d^{\mathrm{sh}}_{f14}(d^{\mathrm{dy}}_{g11} + d^{\mathrm{dy}}_{g21}),\ a_{40} = b^{\mathrm{sh}}_{t16}d^{\mathrm{pm}}_{g1}c^{\mathrm{so}}_{g11},\ a_{41} = b^{\mathrm{sh}}_{t19} + b^{\mathrm{sh}}_{t15}d^{\mathrm{sh}}_{f11}c^{\mathrm{sh}}_{g11},\ a_{43} = b^{\mathrm{sh}}_{t15}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g11} \tag{28l}$$

$$a_{42} = b^{\mathrm{sh}}_{t110} + b^{\mathrm{sh}}_{t15}d^{\mathrm{sh}}_{f12}c^{\mathrm{sh}}_{g22} + b^{\mathrm{sh}}_{t16}d^{\mathrm{pm}}_{g2}c^{\mathrm{sh}}_{g22} + b^{\mathrm{sh}}_{t17}c^{\mathrm{sh}}_{g33},\ a_{44} = b^{\mathrm{sh}}_{t15}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g22},\ a_{45} = b^{\mathrm{sh}}_{t16}d^{\mathrm{pem}}_{g1}d^{\mathrm{so}}_{g11}, \tag{28m}$$

$$a_{46} = b^{\mathrm{sh}}_{t15}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g11} + d^{\mathrm{sh}}_{g21}) + b^{\mathrm{sh}}_{t16}d^{\mathrm{pem}}_{g2}d^{\mathrm{sh}}_{g21} + b^{\mathrm{sh}}_{t17}d^{\mathrm{sh}}_{g31},\ a_{47} = b^{\mathrm{dy}}_{t11} + b^{\mathrm{dy}}_{t18}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g12} + d^{\mathrm{sh}}_{g22}), \tag{28n}$$

$$a_{48} = a^{\mathrm{dy}}_{t11} + b^{\mathrm{dy}}_{t17}d^{\mathrm{sg}}_{g11}d^{\mathrm{dy}}_{g11} + b^{\mathrm{dy}}_{t17}d^{\mathrm{sg}}_{g12}d^{\mathrm{dy}}_{g21} + b^{\mathrm{dy}}_{t18}d^{\mathrm{sh}}_{f14}(d^{\mathrm{dy}}_{g11} + d^{\mathrm{dy}}_{g21}),\ a_{49} = b^{\mathrm{dy}}_{t15} + b^{\mathrm{dy}}_{t18}d^{\mathrm{sh}}_{f11}c^{\mathrm{sh}}_{g11}, \tag{28o}$$

$$a_{50} = b^{\mathrm{dy}}_{t16} + b^{\mathrm{dy}}_{t18}d^{\mathrm{sh}}_{f12}c^{\mathrm{sh}}_{g22},\ a_{51} = b^{\mathrm{dy}}_{t13} + b^{\mathrm{dy}}_{t17}d^{\mathrm{sg}}_{g11}c^{\mathrm{dy}}_{g11} + b^{\mathrm{dy}}_{t18}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g11},\ a_{52} = b^{\mathrm{dy}}_{t14} + b^{\mathrm{dy}}_{t17}d^{\mathrm{sg}}_{g12}c^{\mathrm{dy}}_{g22} + b^{\mathrm{dy}}_{t18}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g22} + b^{\mathrm{dy}}_{t19}w^{\mathrm{ads}}, \tag{28p}$$

$$a_{53} = b^{\mathrm{dy}}_{t18}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g11} + d^{\mathrm{sh}}_{g21}),\ a_{54} = b^{\mathrm{dy}}_{t17}c^{\mathrm{sg}}_{g11},\ a_{55} = d^{\mathrm{pem}}_{g1}d^{\mathrm{so}}_{g12} + d^{\mathrm{so}}_{g32} + b^{\mathrm{so}}_{g13}d^{\mathrm{so}}_{f11}d^{\mathrm{so}}_{g12} + b^{\mathrm{so}}_{g13}d^{\mathrm{so}}_{f12}d^{\mathrm{so}}_{g22}, \tag{28q}$$

$$a_{56} = d^{\mathrm{pem}}_{g2}d^{\mathrm{sh}}_{g22},\ a_{57} = a^{\mathrm{so}}_{g11} + d^{\mathrm{pem}}_{g1}c^{\mathrm{so}}_{g11} + c^{\mathrm{so}}_{g33} + b^{\mathrm{so}}_{g13}d^{\mathrm{so}}_{f11}c^{\mathrm{so}}_{g11},\ a_{58} = a^{\mathrm{so}}_{g12} + b^{\mathrm{so}}_{g13}d^{\mathrm{so}}_{f12}c^{\mathrm{so}}_{g22},\ a_{59} = d^{\mathrm{pem}}_{g2}c^{\mathrm{sh}}_{g22}, \tag{28r}$$

$$a_{60} = d^{\mathrm{pem}}_{g1}d^{\mathrm{so}}_{g11} + d^{\mathrm{so}}_{g31} + b^{\mathrm{so}}_{g13}d^{\mathrm{so}}_{f11}d^{\mathrm{so}}_{g11} + b^{\mathrm{so}}_{g13}d^{\mathrm{so}}_{f12}d^{\mathrm{so}}_{g21},\ a_{61} = d^{\mathrm{pem}}_{g2}d^{\mathrm{sh}}_{g21}a_{62} = b^{\mathrm{so}}_{g23}d^{\mathrm{so}}_{f11}d^{\mathrm{so}}_{g12} + b^{\mathrm{so}}_{g23}d^{\mathrm{so}}_{f12}d^{\mathrm{so}}_{g22}, \tag{28s}$$

$$a_{63} = a^{\mathrm{so}}_{g21} + b^{\mathrm{so}}_{g23}d^{\mathrm{so}}_{f11}c^{\mathrm{so}}_{g11},\ a_{64} = a^{\mathrm{so}}_{g22} + b^{\mathrm{so}}_{g23}d^{\mathrm{so}}_{f12}c^{\mathrm{so}}_{g22},\ a_{65} = b^{\mathrm{so}}_{g23}d^{\mathrm{so}}_{f11}d^{\mathrm{so}}_{g11} + b^{\mathrm{so}}_{g23}d^{\mathrm{so}}_{f12}d^{\mathrm{so}}_{g21},\ a_{66} = b^{\mathrm{sh}}_{g11}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g12} + d^{\mathrm{sh}}_{g22}), \tag{28t}$$

$$a_{67} = b^{\mathrm{sh}}_{g11}d^{\mathrm{sh}}_{f14}(d^{\mathrm{dy}}_{g11} + d^{\mathrm{dy}}_{g21}),\ a_{68} = a^{\mathrm{sh}}_{g11} + b^{\mathrm{sh}}_{g11}d^{\mathrm{sh}}_{f11}c^{\mathrm{sh}}_{g11},\ a_{69} = a^{\mathrm{sh}}_{g12} + b^{\mathrm{sh}}_{g11}d^{\mathrm{sh}}_{f12}c^{\mathrm{sh}}_{g22},\ a_{70} = b^{\mathrm{sh}}_{g11}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g11}, \tag{28u}$$

$$a_{71} = b^{\mathrm{sh}}_{g11}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g22},\ a_{72} = b^{\mathrm{sh}}_{g11}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g11} + d^{\mathrm{sh}}_{g21}),\ a_{73} = b^{\mathrm{sh}}_{g21}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g12} + d^{\mathrm{sh}}_{g22}) + d^{\mathrm{sh}}_{g32},\ a_{74} = b^{\mathrm{sh}}_{g21}d^{\mathrm{sh}}_{f14}(d^{\mathrm{dy}}_{g11} + d^{\mathrm{dy}}_{g21}), \tag{28v}$$

$$a_{75} = a^{\mathrm{sh}}_{g21} + b^{\mathrm{sh}}_{g21}d^{\mathrm{sh}}_{f11}c^{\mathrm{sh}}_{g11},\ a_{76} = \begin{pmatrix} b^{\mathrm{sh}}_{g21}d^{\mathrm{sh}}_{f12}c^{\mathrm{sh}}_{g22} \\ +a^{\mathrm{sh}}_{g22} + c^{\mathrm{sh}}_{g33} \end{pmatrix}, a_{77} = b^{\mathrm{sh}}_{g21}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g11},\ a_{78} = b^{\mathrm{sh}}_{g21}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g22},\ a_{79} = b^{\mathrm{sh}}_{g21}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g11} + d^{\mathrm{sh}}_{g21}) + d^{\mathrm{sh}}_{g31}, \tag{28w}$$

$$a_{80} = b^{\mathrm{dy}}_{g13}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g12} + d^{\mathrm{sh}}_{g22}),\ a_{81} = b^{\mathrm{dy}}_{g13}d^{\mathrm{sh}}_{f14}(d^{\mathrm{dy}}_{g11} + d^{\mathrm{dy}}_{g21}) + b^{\mathrm{dy}}_{g14}d^{\mathrm{sg}}_{g11}d^{\mathrm{dy}}_{g11} + b^{\mathrm{dy}}_{g14}d^{\mathrm{sg}}_{g12}d^{\mathrm{dy}}_{g21},\ a_{82} = b^{\mathrm{dy}}_{g11} + b^{\mathrm{dy}}_{g13}d^{\mathrm{sh}}_{f11}c^{\mathrm{sh}}_{g11}, \tag{28x}$$

$$a_{83} = b^{\mathrm{dy}}_{g12} + b^{\mathrm{dy}}_{g13}d^{\mathrm{sh}}_{f12}c^{\mathrm{sh}}_{g22},\ a_{84} = \begin{pmatrix} a^{\mathrm{dy}}_{g11} + b^{\mathrm{dy}}_{g13}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g11} \\ +b^{\mathrm{dy}}_{g14}d^{\mathrm{sg}}_{g11}c^{\mathrm{dy}}_{g11} \end{pmatrix}, a_{85} = a^{\mathrm{dy}}_{g12} + b^{\mathrm{dy}}_{g13}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g22} + b^{\mathrm{dy}}_{g14}d^{\mathrm{sg}}_{g12}c^{\mathrm{dy}}_{g22}, a_{86} = b^{\mathrm{dy}}_{g13}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g11} + d^{\mathrm{sh}}_{g21}), \tag{28y}$$

$$a_{87} = b^{\mathrm{dy}}_{g14}c^{\mathrm{sg}}_{g11},\ a_{88} = b^{\mathrm{dy}}_{g23}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g12} + d^{\mathrm{sh}}_{g22}),\ a_{89} = \begin{pmatrix} b^{\mathrm{dy}}_{g23}d^{\mathrm{sh}}_{f14}(d^{\mathrm{dy}}_{g11} + d^{\mathrm{dy}}_{g21}) \\ +b^{\mathrm{dy}}_{g24}d^{\mathrm{sg}}_{g11}d^{\mathrm{dy}}_{g11} + b^{\mathrm{dy}}_{g24}d^{\mathrm{sg}}_{g12}d^{\mathrm{dy}}_{g21}, \end{pmatrix}\ a_{a3} = b^{\mathrm{sg}}_{g11}(d^{\mathrm{dy}}_{g11} + d^{\mathrm{dy}}_{g21}), \tag{28z}$$

$$a_{90} = b^{\mathrm{dy}}_{g21} + b^{\mathrm{dy}}_{g23}d^{\mathrm{sh}}_{f11}c^{\mathrm{sh}}_{g11},\ a_{91} = b^{\mathrm{dy}}_{g22} + b^{\mathrm{dy}}_{g23}d^{\mathrm{sh}}_{f12}c^{\mathrm{sh}}_{g22},\ a_{92} = a^{\mathrm{dy}}_{g21} + b^{\mathrm{dy}}_{g23}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g11} + b^{\mathrm{dy}}_{g24}d^{\mathrm{sg}}_{g11}c^{\mathrm{dy}}_{g11},\ a_{a4} = b^{\mathrm{sg}}_{g11}c^{\mathrm{dy}}_{g11}, \tag{28aa}$$

$$a_{93} = \begin{pmatrix} a^{\mathrm{dy}}_{g22} + b^{\mathrm{dy}}_{g23}d^{\mathrm{sh}}_{f14}c^{\mathrm{dy}}_{g22} \\ +b^{\mathrm{dy}}_{g24}d^{\mathrm{sg}}_{g12}c^{\mathrm{dy}}_{g22} - w_{\mathrm{ads}} \end{pmatrix},\ a_{94} = b^{\mathrm{dy}}_{g23}d^{\mathrm{sh}}_{f11}(d^{\mathrm{sh}}_{g11} + d^{\mathrm{sh}}_{g21}),\ a_{95} = b^{\mathrm{dy}}_{g24}c^{\mathrm{sg}}_{g11},\ a_{96} = -d^{\mathrm{pem}}_{g1}d^{\mathrm{so}}_{g12},\ a_{a5} = b^{\mathrm{sg}}_{g12}c^{\mathrm{dy}}_{g22}, \tag{28ab}$$

$$a_{97} = -d^{\mathrm{pem}}_{g2}d^{\mathrm{sh}}_{g22} - d^{\mathrm{sh}}_{g32},\ a_{98} = d^{\mathrm{pem}}_{g1}c^{\mathrm{so}}_{g11},\ a_{99} = -d^{\mathrm{pem}}_{g2}c^{\mathrm{sh}}_{g22} - c^{\mathrm{sh}}_{g33},\ a_{a1} = -d^{\mathrm{pem}}_{g1}d^{\mathrm{so}}_{g11},\ a_{a2} = -d^{\mathrm{pem}}_{g2}d^{\mathrm{sh}}_{g21} - d^{\mathrm{sh}}_{g31},\ a_{a6} = (d^{\mathrm{dy}}_{g11} + d^{\mathrm{dy}}_{g21}) \tag{28ac}$$

main manuscript as follows:

$$\omega = \omega^{\mathrm{nom}} - mP - m_d\dot{P}, \tag{32a}$$

$$v^{\mathrm{ref}}_{\mathrm{od}} = v^{\mathrm{nom}} - nQ - n_d\dot{Q}, \tag{32b}$$

$$v^{\mathrm{ref}}_{\mathrm{oq}} = 0, \tag{32c}$$

The voltage droop in (32b) provides the voltage reference for the outer PI controller of the inverter to regulate the voltage across the filter capacitor on the ac side. The PI state equations eliminating the voltage tracking error are given as follows:

$$\frac{d\phi_{\mathrm{d}}}{dt} = v^{\mathrm{ref}}_{\mathrm{od}} - v_{\mathrm{od}}, \tag{33a}$$

$$\frac{d\phi_{\mathrm{q}}}{dt} = v^{\mathrm{ref}}_{\mathrm{oq}} - v_{\mathrm{oq}}. \tag{33b}$$

The ac voltage PI controller, based on its integral state ϕ_d & ϕ_q and the voltage tracking error, generates the inductor reference current components fed to the ac current PI controllers. The output of the voltage PI controller is given as follows:

$$i^*_{\mathrm{ld}} = Hi_{\mathrm{od}} - \omega^{\mathrm{nom}}C_f v_{\mathrm{oq}} + k^{ac}_{\mathrm{pv}}(v^{\mathrm{ref}}_{\mathrm{od}} - v_{\mathrm{od}}) + k^{ac}_{\mathrm{iv}}\phi_{\mathrm{d}}, \tag{34a}$$

$$i^*_{\mathrm{lq}} = Hi_{\mathrm{oq}} + \omega^{\mathrm{nom}}C_f v_{\mathrm{od}} + k^{ac}_{\mathrm{pv}}(v^{\mathrm{ref}}_{\mathrm{oq}} - v_{\mathrm{oq}}) + k^{ac}_{\mathrm{iv}}\phi_{\mathrm{q}}, \tag{34b}$$

where k^{ac}_{pv} and k^{ac}_{iv} denote the PI controller gains, H denotes a feedforward constant, and C_f denotes the capacitance of the LC filter interfacing the unit to the weak grids. The current PI controller regulates the current passing through the filter on the ac side according to the following integral state equations

$$\frac{d\gamma_{\mathrm{d}}}{dt} = i_{\mathrm{ld}}^{\mathrm{ref}} - i_{\mathrm{ld}}, \tag{35a}$$
$$\frac{d\gamma_{\mathrm{q}}}{dt} = i_{\mathrm{lq}}^{\mathrm{ref}} - i_{\mathrm{lq}}. \tag{35b}$$

The output of the current controller is the reference voltage appearing directly on the inverter terminals and fed to the LC filter, which is given as follows:

$$v_{\mathrm{id}}^{*} = -\omega^{\mathrm{nom}} L_f i_{\mathrm{lq}} + k_{\mathrm{pc}}^{ac}(i_{\mathrm{ld}}^{\mathrm{ref}} - i_{\mathrm{ld}}) + k_{\mathrm{ic}}^{ac}\gamma_{\mathrm{d}}, \tag{36a}$$
$$v_{\mathrm{iq}}^{*} = \omega^{\mathrm{nom}} L_f i_{\mathrm{ld}} + k_{\mathrm{pc}}^{ac}(i_{\mathrm{lq}}^{\mathrm{ref}} - i_{\mathrm{lq}}) + k_{\mathrm{ic}}^{ac}\gamma_{\mathrm{q}}, \tag{36b}$$

where L_f is the LC filter inductance interfacing the PEMEL with the weak grid, k_{pc}^{ac} and k_{ic}^{ac} are the PI controller gains. The correlation between the reference signal v_{idq}^{*} and the physical output v_{idq} is described in the main manuscript and interconnects the previously explained controller with the physical dynamics of the LC filter interfacing the unit to the weak grid along with the dc link dynamics, restated as follows:

$$v_{\mathrm{id(q)}} = v_{\mathrm{id(q)}}^{*}\frac{E_{dc}}{E_{dc}^{\mathrm{ref}}}, \tag{37}$$

Lastly, the LC filter dynamics along with the coupling line between the hydrogen generation unit and the point of common coupling are described as follows:

$$\frac{di_{\mathrm{ld}}}{dt} = -\frac{R_f}{L_f} i_{\mathrm{ld}} + \omega i_{\mathrm{lq}} + \frac{1}{L_f} v_{\mathrm{id}} - \frac{1}{L_f} v_{\mathrm{od}}, \tag{38a}$$
$$\frac{di_{\mathrm{lq}}}{dt} = -\frac{R_f}{L_f} i_{\mathrm{lq}} - \omega i_{\mathrm{ld}} + \frac{1}{L_f} v_{\mathrm{iq}} - \frac{1}{L_f} v_{\mathrm{oq}}, \tag{38b}$$
$$\frac{dv_{\mathrm{od}}}{dt} = \omega v_{\mathrm{oq}} + \frac{1}{C_f} i_{\mathrm{ld}} - \frac{1}{C_f} i_{\mathrm{od}}, \tag{38c}$$
$$\frac{dv_{\mathrm{oq}}}{dt} = -\omega v_{\mathrm{od}} + \frac{1}{C_f} i_{\mathrm{lq}} - \frac{1}{C_f} i_{\mathrm{oq}}, \tag{38d}$$
$$\frac{di_{\mathrm{od}}}{dt} = -\frac{R_c}{L_c} i_{\mathrm{od}} + \omega i_{\mathrm{oq}} + \frac{1}{L_c} v_{\mathrm{od}} - \frac{1}{L_c} v_{\mathrm{bd}}, \tag{38e}$$
$$\frac{di_{\mathrm{oq}}}{dt} = -\frac{R_c}{L_c} i_{\mathrm{oq}} - \omega i_{\mathrm{od}} + \frac{1}{L_c} v_{\mathrm{oq}} - \frac{1}{L_c} v_{\mathrm{bq}}, \tag{38f}$$

where R_f, R_c, and L_c are the filter series resistance, coupling line resistance, and coupling line inductance, respectively. v_{bd} and v_{bq} are the point of common coupling voltage components in the GFPEMEL individual reference frame. A virtual resistance R_n is assumed at the point of common coupling to absorb the net current at the node to facilitate the establishment of SSM. The virtual resistance net current includes the current drawn by the inverter connected to the electrolyzer unit i_{od} & i_{oq}, the current from the microgrid equivalent source i_{sd} & i_{sq}, and the current consumed by the weak grid load i_{gd} & i_{gq}. Selecting a sufficiently large value for this virtual resistance enhances modeling accuracy. The corresponding formulation for the virtual resistance is given as follows:

$$v_{bd} = R_n(i_{sd} + i_{od} - i_{gd}), \tag{39a}$$
$$v_{bq} = R_n(i_{sq} + i_{oq} - i_{gq}). \tag{39b}$$

V. LINEARIZATION OF DC/AC INVERTER AND ITS CONTROL LOOPS

The linearization of the power controller at the ac side in equations (31) and (32) can be given as follows:

$$\begin{bmatrix}\dot{P}\\ \dot{Q}\end{bmatrix} = A_P \begin{bmatrix}P\\ Q\end{bmatrix} + B_P \begin{bmatrix} i_{\mathrm{ldq}} \\ v_{\mathrm{odq}} \\ i_{\mathrm{odq}}\end{bmatrix}, \tag{40a}$$

$$\begin{bmatrix}\omega\\ v_{\mathrm{odq}}^{*}\end{bmatrix} = \begin{bmatrix}C_{\mathrm{P}\omega}\\ C_{\mathrm{Pvi}}\end{bmatrix}\begin{bmatrix}P\\ Q\end{bmatrix} + \begin{bmatrix}D_{\mathrm{P}\omega}\\ D_{\mathrm{Pv}}\end{bmatrix}\begin{bmatrix} i_{\mathrm{ldq}} \\ v_{\mathrm{odq}} \\ i_{\mathrm{odq}}\end{bmatrix}, \tag{40b}$$

$$A_{\mathrm{P}} = \begin{bmatrix}-\omega_c & 0\\ 0 & -\omega_c\end{bmatrix}, B_{\mathrm{P}} = \omega_c\begin{bmatrix}0 & 0 & \bar{i}_{\mathrm{od}} & \bar{i}_{\mathrm{oq}} & \bar{v}_{\mathrm{od}} & \bar{v}_{\mathrm{oq}}\\ 0 & 0 & -\bar{i}_{\mathrm{oq}} & \bar{i}_{\mathrm{od}} & \bar{v}_{\mathrm{oq}} & -\bar{v}_{\mathrm{od}}\end{bmatrix}, \tag{40c}$$

$$C_{\mathrm{P}\omega} = \begin{bmatrix}-m + m_d\omega_c & 0\end{bmatrix}, C_{\mathrm{Pv}} = \begin{bmatrix}0 & -n + n_d\omega_c\\ 0 & 0\end{bmatrix}, \tag{40d}$$

$$D_{\mathrm{P}\omega} = -\omega_c m_d \begin{bmatrix}0 & 0 & \bar{i}_{\mathrm{od}} & \bar{i}_{\mathrm{oq}} & \bar{v}_{\mathrm{od}} & \bar{v}_{\mathrm{oq}}\end{bmatrix}, \tag{40e}$$

$$D_{\mathrm{Pv}} = \omega_c n_d \begin{bmatrix}0 & 0 & \bar{i}_{\mathrm{oq}} & -\bar{i}_{\mathrm{od}} & -\bar{v}_{\mathrm{oq}} & \bar{v}_{\mathrm{od}}\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}. \tag{40f}$$

The linearization of the voltage controller in equations (33) and (34) can be given as follows:

$$\begin{bmatrix}\dot{\phi}_{\mathrm{d}}\\ \dot{\phi}_{\mathrm{q}}\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\phi_{\mathrm{d}}\\ \phi_{\mathrm{q}}\end{bmatrix} + B_{V1}\begin{bmatrix}v_{\mathrm{od}}^{*}\\ v_{\mathrm{oq}}^{*}\end{bmatrix} + B_{V2}\begin{bmatrix} i_{\mathrm{ldq}} \\ v_{\mathrm{odq}} \\ i_{\mathrm{odq}}\end{bmatrix}, \tag{41a}$$

$$\begin{bmatrix}i_{\mathrm{ld}}^{*}\\ i_{\mathrm{lq}}^{*}\end{bmatrix} = C_V\begin{bmatrix}\phi_{\mathrm{d}}\\ \phi_{\mathrm{q}}\end{bmatrix} + D_{V1}\begin{bmatrix}v_{\mathrm{od}}^{*}\\ v_{\mathrm{oq}}^{*}\end{bmatrix} + D_{V2}\begin{bmatrix} i_{\mathrm{ldq}} \\ v_{\mathrm{odq}} \\ i_{\mathrm{odq}}\end{bmatrix}, \tag{41b}$$

$$B_{V1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, B_{V2} = \begin{bmatrix}0 & 0 & -1 & 0 & 0 & 0\\ 0 & 0 & 0 & -1 & 0 & 0\end{bmatrix}, C_V = \begin{bmatrix}k_{\mathrm{iv}}^{ac} & 0\\ 0 & k_{\mathrm{iv}}^{ac}\end{bmatrix}, \tag{41c}$$

$$D_{V1} = \begin{bmatrix}k_{\mathrm{pv}}^{ac} & 0\\ 0 & k_{\mathrm{pv}}^{ac}\end{bmatrix}, D_{V2} = \begin{bmatrix}0 & 0 & -k_{\mathrm{pv}}^{ac} & -\omega_n C_f & H & 0\\ 0 & 0 & \omega_n C_f & -k_{\mathrm{pv}}^{ac} & 0 & H\end{bmatrix} \tag{41d}$$

The linearization of the current controller in equations (35) and (36) can be given as follows:

$$\begin{bmatrix}\dot{\gamma}_{\mathrm{d}}\\ \dot{\gamma}_{\mathrm{q}}\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\gamma_{\mathrm{d}}\\ \gamma_{\mathrm{q}}\end{bmatrix} + B_{C1}\begin{bmatrix}i_{\mathrm{ld}}^{*}\\ i_{\mathrm{lq}}^{*}\end{bmatrix} + B_{C2}\begin{bmatrix} i_{\mathrm{ldq}} \\ v_{\mathrm{odq}} \\ i_{\mathrm{odq}}\end{bmatrix}, \tag{42a}$$

$$\begin{bmatrix}v_{\mathrm{id}}^{*}\\ v_{\mathrm{iq}}^{*}\end{bmatrix} = C_C\begin{bmatrix}\gamma_{\mathrm{d}}\\ \gamma_{\mathrm{q}}\end{bmatrix} + D_{C1}\begin{bmatrix}i_{\mathrm{ld}}^{*}\\ i_{\mathrm{lq}}^{*}\end{bmatrix} + D_{C2}\begin{bmatrix} i_{\mathrm{ldq}} \\ v_{\mathrm{odq}} \\ i_{\mathrm{odq}}\end{bmatrix}, \tag{42b}$$

$$B_{C1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, B_{C2} = \begin{bmatrix}-1 & 0 & 0 & 0 & 0 & 0\\ 0 & -1 & 0 & 0 & 0 & 0\end{bmatrix}, C_C = \begin{bmatrix}k_{\mathrm{ic}}^{ac} & 0\\ 0 & k_{\mathrm{ic}}^{ac}\end{bmatrix}, \tag{42c}$$

$$D_{C1} = \begin{bmatrix}k_{\mathrm{pc}}^{ac} & 0\\ 0 & k_{\mathrm{pc}}^{ac}\end{bmatrix}, D_{C2} = \begin{bmatrix}-k_{\mathrm{pc}}^{ac} & -\omega_n L_f & 0 & 0 & 0 & 0\\ \omega_n L_f & -k_{\mathrm{pc}}^{ac} & 0 & 0 & 0 & 0\end{bmatrix} \tag{42d}$$

The linearization of the LCL subsystem in equations (38a)-(38f), (37), and (39) can be given as follows:

$$A_{inv}=\begin{bmatrix} A_P & 0 & 0 & B_P & 0\\ B_{V1}C_{Pv} & 0 & 0 & B_{V2}+B_{V1}D_{\mathrm{Pv}} & 0\\ B_{C1}D_{V1}C_{Pv} & B_{C1}C_V & 0 & \begin{pmatrix} B_{c1}D_{V2}+B_{C2}\\ +B_{C1}D_{V1}D_{\mathrm{Pv}}\end{pmatrix} & 0\\ \begin{pmatrix} B_{LCL1}D_{C1}D_{V1}C_{Pv}+\\ +B_{LCL3}C_{P\omega}\end{pmatrix} & B_{LCL1}D_{C1}C_V & B_{LCL1}C_C & \begin{pmatrix} A_{LCL}+\\ B_{LCL1}(D_{C1}D_{V2}+D_{C2})\\ +B_{LCL3}D_{Pw}\\ +B_{LCL1}D_{C1}D_{v1}D_{Pv}\end{pmatrix} & 0\\ B_{load2}C_{P\omega} & 0 & 0 & B_{load1} & A_{Load}\end{bmatrix}, B_{is}^{inv}=\begin{bmatrix}0\\ B_{LCL2}\\ B_{load3}\end{bmatrix} B_{ed}^{inv}=\begin{bmatrix}0\\0\\0\\B_{LCL4}\\0\end{bmatrix} \tag{40aa}$$

$$C_p^{\mathrm{inv}}=\left[C_p^{il}D_{C1}D_{V1}C_{Pv}\;\; C_p^{il}D_{C1}C_V\;\; C_p^{il}C_c\;\; C_p^{il}(D_{C1}D_{V2}+D_{C2}+D_{C1}D_{V1}D_{pv})+C_p^{c}\;\; \mathbf{0}_{2\times 1})\right],\;\; D_p^{\mathrm{inv}}=C_p^{il}C_p^{ed} \tag{40ab}$$

$$\begin{bmatrix}\dot{i}_{\mathrm{ldq}}\\ \dot{v}_{\mathrm{odq}}\\ \dot{i}_{\mathrm{odq}}\end{bmatrix}=A_{LCL}\begin{bmatrix}i_{\mathrm{ldq}}\\ v_{\mathrm{odq}}\\ i_{\mathrm{odq}}\end{bmatrix}+B_{LCL1}\begin{bmatrix}v_{\mathrm{id}}^*\\ v_{\mathrm{iq}}^*\end{bmatrix}+B_{LCL2}\begin{bmatrix}i_{\mathrm{sd}}\\ i_{\mathrm{sq}}\end{bmatrix},$$
$$+B_{LCL3}\;\omega+B_{LCL4}\begin{bmatrix}E_{cl}\\ E_{dc}\end{bmatrix}+B_{LCL5}\begin{bmatrix}i_{gd}\\ i_{gq}\end{bmatrix} \tag{43a}$$

$$A_{LCL}=\begin{bmatrix}-\frac{R_f}{L_f} & \overline{\omega} & -\frac{1}{L_f} & 0 & 0 & 0\\ -\overline{\omega} & -\frac{R_f}{L_f} & 0 & -\frac{1}{L_f} & 0 & 0\\ \frac{1}{C_f} & 0 & 0 & \overline{\omega} & -\frac{1}{C_f} & 0\\ 0 & \frac{1}{C_f} & -\overline{\omega} & 0 & 0 & -\frac{1}{C_f}\\ 0 & 0 & \frac{1}{L_c} & 0 & -\frac{R_c}{L_c}-\frac{r_N}{L_c} & \overline{\omega}\\ 0 & 0 & 0 & \frac{1}{L_c} & -\overline{\omega} & -\frac{R_c}{L_c}-\frac{r_N}{L_c}\end{bmatrix} \tag{43b}$$

$$B_{LCL1}=\begin{bmatrix}\frac{1}{L_f} & 0\\ 0 & \frac{1}{L_f}\\ 0&0\\0&0\\0&0\\0&0\end{bmatrix} B_{LCL2}=\begin{bmatrix}0&0\\0&0\\0&0\\0&0\\ -\frac{r_N}{L_c} & 0\\ 0 & -\frac{r_N}{L_c}\end{bmatrix}, \tag{43c}$$

$$B_{LCL3}=\begin{bmatrix}\overline{i_{\mathrm{lq}}}\\ -\overline{i_{\mathrm{ld}}}\\ \overline{v_{\mathrm{oq}}}\\ -\overline{v_{\mathrm{od}}}\\ \overline{i_{\mathrm{oq}}}\\ -\overline{i_{\mathrm{od}}}\end{bmatrix}, B_{LCL4}=\begin{bmatrix}0 & \frac{1}{L_f}\frac{\overline{v_{\mathrm{id}}}}{E_{dc}}\\ 0 & \frac{1}{L_f}\frac{\overline{v_{\mathrm{id}}}}{E_{dc}}\\ 0&0\\0&0\\0&0\\0&0\end{bmatrix}, \tag{43d}$$

$$B_{LCL5}=\begin{bmatrix}0&0\\0&0\\0&0\\0&0\\ \frac{r_N}{L_c} & 0\\ 0 & \frac{r_N}{L_c}\end{bmatrix}, \tag{43e}$$

Lastly, the linearization of the grid load dynamics from equations (39) and I-(79) can be given as follows:

$$\begin{bmatrix}\dot{i}_{gd}\\ \dot{i}_{gq}\end{bmatrix}=A_{Load}\begin{bmatrix}i_{gd}\\ i_{gq}\end{bmatrix}+B_{Load1}\begin{bmatrix}i_{\mathrm{ldq}}\\ v_{\mathrm{odq}}\\ i_{\mathrm{odq}}\end{bmatrix},$$
$$+B_{Load2}\;\omega+B_{Load3}\begin{bmatrix}i_{\mathrm{sd}}\\ i_{\mathrm{sq}}\end{bmatrix} \tag{44a}$$

$$A_{Load}=\begin{bmatrix}-\frac{R_g}{L_g}-\frac{r_N}{L_g} & \overline{\omega}\\ -\overline{\omega} & -\frac{R_g}{L_g}-\frac{r_N}{L_g}\end{bmatrix} \tag{44b}$$

$$B_{Load}=\begin{bmatrix}0\,0\,0\,0\,\frac{r_N}{L_g} & 0\\ 0\,0\,0\,0 & 0\,\frac{r_N}{L_g}\end{bmatrix} \tag{44c}$$

$$B_{Load}=\begin{bmatrix}\bar{i}_{\mathrm{gq}}\\ -\bar{i}_{\mathrm{gd}}\end{bmatrix}, B_{Load}=\begin{bmatrix}\frac{r_N}{L_g} & 0\\ 0 & \frac{r_N}{L_g}\end{bmatrix}, \tag{44d}$$

The matrices of the SSM of the dc-ac PEI in I-(80), which results as accumlation of subsystems (40a-f), (41a-d), (42a-d), and (43a-e), are detailed in equation (40aa) and (40ab). The details of matrices C_p^{il}, C_p^{ed}, and C_p^{c} in equation (40ab) are obtained through the linearization of I-(78) can be given as follows:

$$C_p^{il}=\left[\bar{i}_{ld}\,\bar{i}_{lq}\right] \tag{45a}$$

$$C_p^{ed}=\begin{bmatrix}\frac{\overline{v}_{id}}{\overline{E}_{dc}}\\ \frac{\overline{v}_{iq}}{\overline{E}_{dc}}\end{bmatrix} \tag{45b}$$

$$C_p^{c}=\left[\overline{v}_{id}\,\overline{v}_{iq}\,0\,0\,0\,0\right] \tag{45c}$$

All details regarding the construction of the entire GF-PEMEL unit have been provided in .m Matalb code in the Github link referenced in the main manuscript. The nonlinear time-domain model is confidential due to funding restrictions.