Small-Signal Model of Grid-forming PEM Hydrogen Electrolyzer in Weak Grids -Supplementary Materials-

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***Abstract*—Green hydrogen production enhances energy storage capabilities and improves grid reliability. Proton Exchange Membrane Electrolyzers (PEMELs) are increasingly integrated with renewable energy sources in low-inertia systems to support sustainability goals. However, analyzing the stability and dynamics of such systems is complex, requiring comprehensive modeling for robust design and management. This paper develops a detailed small-signal model (SSM) derived from a full multiphysics computational model of a PEMEL. The system includes two power electronic interfaces (PEIs): a unidirectional DC/DC boost converter and a bidirectional DC/AC inverter, forming a two-stage energy conversion process. This configuration enables grid-forming capabilities for weak AC grids dominated by renewables, utilizing PEMEL flexibility for grid synchronization while renewable generation operates at maximum power. The proposed SSM is validated against a time-domain nonlinear model. Additionally, sensitivity analysis is explored to assess the impact of operational conditions on system dynamics. Furthermore, insights on model reduction are delivered.**

***Index Terms*—Green Hydrogen, multi-physic modeling, PEM electrolyzer, reduced order model, and small signal linearization.**

I. PEMEL LINEARIZED MATRICES DETAILS

Numerous equations in the main manuscript are presented in linearized state-space form, with the details of the linearization provided in this supplementary file as follows. The parameters with overbar represent the parameter at the operating point condition where the small-signal model has been constructed. The individual entries for each matrix in I-(12) can be defined as follows

$$a^{\mathrm{g}}_{e11} = z_1 z_8 + z_2 + z_3 z_5 z_8, \quad a^{\mathrm{g}}_{e21} = \frac{z_8}{2F\tau_e}, \tag{1a}$$

$$b^{\mathrm{pm}}_{e11} = (z_3(z_4 + z_5) + z_1)(z_7(z_{10} + z_{11}) + z_9 z_{19} z_{16}), \tag{1b}$$

$$b^{\mathrm{pm}}_{e21} = \frac{z_7(z_{10} + z_{11}) + z_9 z_{19} z_{16}}{2F\tau_e}, \quad a^{\mathrm{g}}_{e22} = -\frac{1}{\tau_e}, \tag{1c}$$

$$b^{\mathrm{pm}}_{e16} = z_3 z_5 z_6, \quad b^{\mathrm{pm}}_{e26} = \frac{z_6}{2F\tau_e}, \tag{1d}$$

$$b^{\mathrm{pm}}_{e12} = z_3 z_5 z_7 z_{12} + z_1 z_7 z_{12}, \tag{1e}$$

$$b^{\mathrm{pm}}_{e13} = z_3 z_5 z_7 z_{13} + z_1 z_7 z_{13}, \tag{1f}$$

$$b^{\mathrm{pm}}_{e14} = (z_3 z_5 + z_1)\left(z_7 z_{14} + z_9 z_{19} z_{15} z_{17}\right), \tag{1g}$$

$$b^{\mathrm{pm}}_{e15} = (z_3 z_5 + z_1)\left(z_9 z_{19} z_{15} z_{18}\right), \tag{1h}$$

$$b^{\mathrm{pm}}_{e22} = \frac{z_7 z_{12}}{2F\tau_e}, \quad b^{\mathrm{pm}}_{e23} = \frac{z_7 z_{13}}{2F\tau_e}, \tag{1i}$$

$$b^{\mathrm{pm}}_{e24} = \frac{z_7 z_{14} + z_9 z_{19} z_{15} z_{17}}{2F\tau_e}, \quad b^{\mathrm{pm}}_{e25} = \frac{z_9 z_{19} z_{15} z_{18}}{2F\tau_e}. \tag{1j}$$

The individual entries for each matrix in I-(13) can be defined as follows

$$c^{\mathrm{pm}}_{e21} = z_8, \; d^{\mathrm{pm}}_{e2*} = b^{\mathrm{pm}}_{e2*}(2F\tau_e). \tag{2}$$

The elements z_i are detailed as follows

$$z_1 = \frac{1}{C_{dl}}\left(1 - \frac{\overline{E_{\mathrm{act}}}}{\overline{E_{\mathrm{act,ss}}}}\right), \quad z_2 = -\frac{\overline{I_{\mathrm{cel}}}}{C_{\mathrm{dl}}\overline{E_{\mathrm{act,ss}}}}, \tag{3a}$$

$$z_3 = \frac{\overline{I_{\mathrm{cel}}E_{\mathrm{act}}}}{C_{\mathrm{dl}}\overline{E_{\mathrm{act,ss}}}}, \quad z_4 = \frac{R}{\alpha F}\ln\left(\frac{\overline{I_{\mathrm{cel}}}}{\overline{I_{\mathrm{cel,0}}}}\right), \tag{3b}$$

$$z_5 = \frac{\frac{R}{\alpha F}\overline{T_{\mathrm{stk}}}}{\overline{I_{\mathrm{cel}}}}, \quad z_6 = \frac{1}{\overline{R_{\mathrm{ohm}}}} = -z_7 = -z_8, \tag{3c}$$

$$z_9 = -\frac{\overline{E_{\mathrm{cel}}} - \overline{E_{\mathrm{rev}}} - \overline{E_{\mathrm{act}}}}{\overline{R_{\mathrm{ohm}}}^2}, \quad z_{19} = \frac{-L_{\mathrm{g}}}{A_{\mathrm{g}}\overline{\sigma_{\mathrm{g}}}^2}, \tag{3d}$$

$$z_{10} = \begin{pmatrix} 1.5421 \times 10^{-3} + 2 \cdot 9.84 \times 10^{-8}\overline{T_{\mathrm{stk}}} \\ +9.523 \times 10^{-5}(\log(\overline{T_{\mathrm{stk}}}) + 1) \end{pmatrix}, \tag{3e}$$

$$z_{11} = \frac{R}{2F}\log\left(\frac{\overline{P^{\mathrm{sh}}_{H_2}}\sqrt{\overline{P^{\mathrm{so}}_{O_2}}}}{(p^{\mathrm{std}})^{1.5}\overline{a^{\mathrm{so}}_{\mathrm{W}}}}\right), \quad z_{12} = \frac{R\overline{T_{\mathrm{stk}}}}{2F\overline{P^{\mathrm{sh}}_{H_2}}}, \tag{3f}$$

$$z_{13} = \frac{R\overline{T_{\mathrm{stk}}}}{4F\overline{P^{\mathrm{so}}_{O_2}}}, \quad z_{14} = \frac{-R\overline{T_{\mathrm{stk}}}}{2F\overline{p^{\mathrm{so}}_{\mathrm{W}}}}, \tag{3g}$$

$$z_{15} = 0.005139\exp\left(\frac{1268}{303} - \frac{1268}{\overline{T_{\mathrm{stk}}}}\right), \tag{3h}$$

$$z_{16} = \frac{1268(0.005139\overline{\lambda} - 0.00326)}{\overline{T_{\mathrm{stk}}}^2}\exp\left(\frac{1268}{303} - \frac{1268}{\overline{T_{\mathrm{stk}}}}\right), \tag{3i}$$

$$z_{17(18)} = 0.5\left(\frac{17.81}{p^{\mathrm{sat}}_{\mathrm{W}}} - \frac{239.85\overline{p^{\mathrm{so(sh)}}_{\mathrm{W}}}}{(p^{\mathrm{sat}}_{\mathrm{W}})^2} + \frac{336\overline{p^{\mathrm{so(sh)}}_{\mathrm{W}}}^2}{(p^{\mathrm{sat}}_{\mathrm{W}})^3}\right). \tag{3j}$$

The individual entries for the matrices in I-(19) can be provided as follows

$$\begin{aligned} a_{t11}^{\mathrm{pm}} = &- \frac{1}{C_{th}^{\mathrm{cel}}} C_l^{\mathrm{W}} \left(\overline{\dot{m}^{\mathrm{rh}}} - \overline{\dot{m}_{\mathrm{W}}^{\mathrm{dif}}} \right) - \frac{1}{C_{th}^{\mathrm{cel}}} C_g^{W} \overline{\dot{m}_{\mathrm{W}}^{\mathrm{eos}}} \\ &- \frac{N_{\mathrm{cs}}}{C_{th}^{\mathrm{cel}}} \overline{H_{H_2O,g}} 0.0134 M_{\mathrm{W}}^{w} \overline{\xi} - \frac{C_l^{\mathrm{W}}}{C_{th}^{\mathrm{cel}}} \left(\overline{\dot{m}^{\mathrm{ro}}} - \overline{\dot{m}_{\mathrm{W}}^{\mathrm{eos}}} \right) \\ &- \frac{1}{C_{th}^{\mathrm{cel}}} C_g^{W} \overline{\dot{m}_{\mathrm{W}}^{\mathrm{dif}}} + \frac{1}{C_{th}^{\mathrm{cel}}} \overline{T_c^{\mathrm{sk}}} C_l^{\mathrm{W}} 0.0134 N_{\mathrm{cs}} M_{\mathrm{W}}^{w} \overline{\xi} \\ &- \frac{1}{C_{th}^{\mathrm{cel}} R_{cnd}^{g}} \\ &- \frac{N_{\mathrm{cs}}}{C_{th}^{\mathrm{cel}}} \overline{\xi} \left(C_g^{H_2} M_{\mathrm{H}_2}^{w} + 0.5 C_g^{O_2} M_{\mathrm{O}_2}^{w} - C_l^{\mathrm{W}} M_{\mathrm{W}}^{w} \right), \end{aligned} \tag{4a}$$

$$b_{t18}^{\mathrm{pm}} = \frac{C_l^{\mathrm{W}} \overline{\dot{m}^{\mathrm{rh}}}}{C_{th}^{\mathrm{cel}}}, \; b_{t19}^{\mathrm{pm}} = \frac{1}{C_{th}^{\mathrm{cel}}} C_l^{\mathrm{W}} \overline{\dot{m}^{\mathrm{ro}}}, \; b_{t110}^{\mathrm{pm}} = \frac{1}{C_{th}^{\mathrm{cel}} R_{cnd}^{g}}, \tag{4b}$$

$$b_{t11}^{\mathrm{pm}} = \frac{C_l^{\mathrm{W}}}{C_{th}^{\mathrm{cel}}} \left(\overline{T^{\mathrm{rh}}} - \overline{T^{\mathrm{sk}}} \right), \quad b_{t12}^{\mathrm{pm}} = \frac{C_l^{\mathrm{W}}}{C_{th}^{\mathrm{cel}}} \left(\overline{T^{\mathrm{ro}}} - \overline{T^{\mathrm{sk}}} \right), \tag{4c}$$

$$b_{t13}^{\mathrm{pm}} = \frac{1}{C_{th}^{\mathrm{cel}}} C_l^{\mathrm{W}} \overline{T_c^{\mathrm{sk}}} - \frac{1}{C_{th}^{\mathrm{cel}}} \overline{H_{H_2O,g}}, \tag{4d}$$

$$\begin{aligned} b_{t15}^{\mathrm{pm}} = &- \frac{N_{\mathrm{cs}}}{C_{th}^{\mathrm{cel}}} \Big(- C_l^{\mathrm{W}} \overline{T_c^{\mathrm{sk}}} M_{\mathrm{W}}^{w} v_{\mathrm{W}} + v_{\mathrm{H}_2} M_{\mathrm{H}_2}^{w} \overline{H_{\mathrm{H}_2}} \\ &+ v_{\mathrm{O}_2} M_{\mathrm{O}_2}^{w} \overline{H_{\mathrm{O}_2}} \\ &- M_{\mathrm{W}}^{w} (0.0134 \overline{T^{\mathrm{sk}}} + 0.03) \left(C_l^{\mathrm{W}} \overline{T_c^{\mathrm{sk}}} - \overline{H_{\mathrm{W}}} \right) \Big), \end{aligned} \tag{4e}$$

$$b_{t16}^{\mathrm{pm}} = \frac{N_{\mathrm{cs}}}{C_{th}^{\mathrm{cel}}} \overline{E_{\mathrm{cel}}}, \quad b_{t17}^{\mathrm{pm}} = \frac{N_{\mathrm{cs}}}{C_{th}^{\mathrm{cel}}} \overline{I_{\mathrm{cel}}}. \tag{4f}$$

The element of the matrices in I-(23)

$$d_{g1(2)}^{\mathrm{pm}} = -(+) \frac{N_{\mathrm{cs}} \overline{D_w} A^{\mathrm{g}} M_{\mathrm{W}}^{w} \rho_{MEM}}{{}_{\mathrm{mem}} M_{\mathrm{MEM}}} 2 z_{17(18)}, \tag{5a}$$

$$d_{g3}^{\mathrm{pm}} = \begin{pmatrix} \frac{N_{\mathrm{cs}} A^{\mathrm{g}} M_{\mathrm{W}}^{w}}{\mathrm{mem}} (\overline{C_{\mathrm{H_2O}}^{\mathrm{chh}}} - \overline{C_{\mathrm{H_2O}}^{\mathrm{cho}}}) \cdot \\ 1.25 \times 10^{-10} \cdot \exp(\frac{2416}{303.15} - \frac{2416}{\overline{T^{\mathrm{stk}}}}) \frac{2416}{\overline{T^{\mathrm{stk}}}^2} \end{pmatrix}. \tag{5b}$$

The individual entries for each matrix in I-(28) can be found as follows

$$a_{g11}^{\mathrm{so}} = - \frac{\overline{m_{O_2}^{\mathrm{so}}}}{(\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}})^2} \overline{\dot{m}^{\mathrm{ox}}}, \; a_{g12}^{\mathrm{so}} = \frac{\overline{m_{\mathrm{W}}^{\mathrm{so}}}}{(\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}})^2} \overline{\dot{m}^{\mathrm{ox}}}, \tag{6a}$$

$$a_{g21}^{\mathrm{so}} = \frac{\overline{m_{O_2}^{\mathrm{so}}}}{(\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{W}^{\mathrm{so}}})^2} \overline{\dot{m}^{\mathrm{ox}}}, \; a_{g22}^{\mathrm{so}} = - \frac{\overline{m_{\mathrm{W}}^{\mathrm{so}}}}{(\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{W}^{\mathrm{so}}})^2} \overline{\dot{m}^{\mathrm{ox}}}, \tag{6b}$$

$$b_{g13}^{\mathrm{so}} = - \frac{\overline{m_{\mathrm{W}}^{\mathrm{so}}}}{\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}}}, \; b_{g23}^{\mathrm{so}} = - \frac{\overline{m_{O_2}^{\mathrm{so}}}}{\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}}}, \tag{6c}$$

$$b_{g34}^{\mathrm{so}} = N_{\mathrm{cs}} M_{O_2}^{w} v_{O_2}, \tag{6d}$$

The individual elements of each matrix in the algebraic output equation in I-(29) are defined as follows

$$c_{g11}^{\mathrm{so}} = \frac{R \overline{T^{\mathrm{so}}}}{M_{\mathrm{W}}^{w} \left(V_{\mathrm{so}} - A_{\mathrm{so}} \overline{L^{\mathrm{so}}} \right)}, c_{g22}^{\mathrm{so}} = \frac{R \overline{T^{\mathrm{so}}}}{M_{\mathrm{O}_2}^{w} \left(V_{\mathrm{so}} - A_{\mathrm{so}} \overline{L^{\mathrm{so}}} \right)}, \tag{7a}$$

$$c_{g33}^{\mathrm{so}} = -\omega_{ev}, \tag{7b}$$

$$d_{g31}^{\mathrm{so}} = -\omega_{ev} p_{\mathrm{W}}^{\mathrm{sat}} \frac{M_{\mathrm{W}}^{w}}{R \overline{T^{\mathrm{so}}} \rho_{\mathrm{W}}}, \tag{7c}$$

$$d_{g32}^{\mathrm{so}} = -\omega_{ev} p_{\mathrm{W}}^{\mathrm{sat}} \frac{M_{\mathrm{W}}^{w} (V_{\mathrm{so}} - A_{\mathrm{so}} \overline{L^{\mathrm{so}}})}{R \overline{T^{\mathrm{so}}}^2}, \tag{7d}$$

$$d_{g11}^{\mathrm{so}} = \frac{R \overline{T^{\mathrm{so}}} \overline{m_{\mathrm{W}}^{\mathrm{so}}}}{M_{\mathrm{W}}^{w} \left(V_{\mathrm{so}} - A_{\mathrm{so}} \overline{L^{\mathrm{so}}} \right)^2 \rho_{\mathrm{W}}}, \tag{7e}$$

$$d_{g21}^{\mathrm{so}} = \frac{R \overline{T^{\mathrm{so}}} \overline{m_{\mathrm{O}_2}^{\mathrm{so}}}}{M_{\mathrm{O}_2}^{w} \left(V_{\mathrm{so}} - A_{\mathrm{so}} \overline{L^{\mathrm{so}}} \right)^2 \rho_{\mathrm{W}}}, \tag{7f}$$

$$d_{g12}^{\mathrm{so}} = \frac{R \overline{m_{\mathrm{W}}^{\mathrm{so}}}}{M_{\mathrm{W}}^{w} \left(V_{\mathrm{so}} - A_{\mathrm{so}} \overline{L^{\mathrm{so}}} \right)}, d_{g22}^{\mathrm{so}} = \frac{R \overline{m_{\mathrm{O}_2}^{\mathrm{so}}}}{M_{\mathrm{O}_2}^{w} \left(V_{\mathrm{so}} - A_{\mathrm{so}} \overline{L^{\mathrm{so}}} \right)} \tag{7g}$$

The individual elements of the matrices in the state equation I-(31) are given as follows

$$\begin{aligned} b_{t11}^{\mathrm{so}} = &\frac{C_l^{\mathrm{W}}}{\overline{C_{th}^{\mathrm{so}}}} \left((\overline{\dot{m}^{\mathrm{ro}}} - \overline{\dot{m}^{eos}}) - M_{\mathrm{W}}^{w} N_{\mathrm{cs}} \overline{\xi} 0.0134 \overline{T_c^{\mathrm{sk}}} \right) \\ &+ \frac{C_g^{W} \overline{\dot{m}_{\mathrm{W}}^{\mathrm{dif}}}}{\overline{C_{th}^{\mathrm{so}}}} + \frac{N_{\mathrm{cs}}}{\overline{C_{th}^{\mathrm{so}}}} \overline{\xi} \left(0.5 C_g^{O_2} M_{\mathrm{O}_2}^{w} - C_l^{\mathrm{W}} M_{\mathrm{W}}^{w} \right), \end{aligned} \tag{8a}$$

$$\begin{aligned} a_{t11}^{\mathrm{so}} = &- \frac{1}{\overline{C_{th}^{\mathrm{so}}}} \Bigg(\overline{\dot{m}^{\mathrm{ro}}} C_l^{\mathrm{W}} + \frac{1}{R_{cnd}^{\mathrm{so}}} \\ &+ \overline{\dot{m}^{\mathrm{ox}}} \frac{\overline{m_{O_2}^{\mathrm{so}}} C_g^{O_2} + \overline{m_{\mathrm{W}}^{\mathrm{so}}} C_g^{W}}{\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}}} + \overline{\dot{m}_{\mathrm{W}}^{\mathrm{eo}}} (C_g^{W} - C_l^{\mathrm{W}}) \Bigg), \end{aligned} \tag{8b}$$

$$b_{t12}^{\mathrm{so}} = \frac{1}{\overline{C_{th}^{\mathrm{so}}}} \frac{1}{R_{cnd}^{\mathrm{so}}}, \quad b_{t13}^{\mathrm{so}} = \frac{1}{\overline{C_{th}^{\mathrm{so}}}} \overline{T^{\mathrm{sk}}} C_l^{\mathrm{W}} - \frac{1}{\overline{C_{th}^{\mathrm{so}}}} \overline{T^{\mathrm{so}}} C_l^{\mathrm{W}}, \tag{8c}$$

$$b_{t14}^{\mathrm{so}} = \frac{1}{\overline{C_{th}^{\mathrm{so}}}} (\overline{T^{\mathrm{rso}}}_c) C_l^{\mathrm{W}}, \tag{8d}$$

$$b_{t15}^{\mathrm{so}} = \frac{-(\overline{m_{O_2}^{\mathrm{so}}} C_g^{O_2} + \overline{m_{\mathrm{W}}^{\mathrm{so}}} C_g^{W}) \overline{T_c^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}} H_{\mathrm{W}}^{0} + \overline{m_{O_2}^{\mathrm{so}}} H_{\mathrm{O}_2}^{0}}{\overline{C_{th}^{\mathrm{so}}} (\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}})}, \tag{8e}$$

$$\begin{aligned} b_{t18}^{\mathrm{so}} = &- \frac{1}{\overline{C_{th}^{\mathrm{so}}}} C_l^{\mathrm{W}} \overline{T_c^{\mathrm{sk}}} \begin{pmatrix} N_{\mathrm{cs}} M_{\mathrm{W}}^{w} (0.0134 \overline{T^{\mathrm{sk}}} + 0.03) \\ + N_{\mathrm{cs}} M_{\mathrm{W}}^{w} v_{\mathrm{W}} \end{pmatrix} \\ &+ \frac{1}{\overline{C_{th}^{\mathrm{so}}}} \left(\overline{T_c^{\mathrm{sk}}} C_g^{O_2} + H_{\mathrm{O}_2}^{0} \right) N_{\mathrm{cs}} M_{O_2}^{w} v_{O_2}, \end{aligned} \tag{8f}$$

$$b_{t16}^{\mathrm{so}} = \frac{1}{\overline{C_{th}^{\mathrm{so}}}} \left(\overline{T_c^{\mathrm{sk}}} C_g^{W} + H_{\mathrm{W}}^{0} \right), \tag{8g}$$

$$b_{t17}^{\mathrm{so}} = - \frac{1}{\overline{C_{th}^{\mathrm{so}}}} \left(- \overline{T_c^{\mathrm{so}}} C_l^{\mathrm{W}} + \overline{T_c^{\mathrm{so}}} C_g^{W} + H_{\mathrm{W}}^{0} \right), \tag{8h}$$

$$\begin{aligned} b_{t19}^{\mathrm{so}} = &- \frac{\overline{H_{tot}^{\mathrm{so}}}}{(\overline{C_{th}^{\mathrm{so}}})^2} C_g^{W} + \frac{1}{\overline{C_{th}^{\mathrm{so}}}} \frac{\overline{\dot{m}^{\mathrm{ox}}} \overline{m_{\mathrm{O}_2}^{\mathrm{so}}}}{(\overline{m_{O_2}^{\mathrm{so}}} + \overline{m_{\mathrm{W}}^{\mathrm{so}}})^2} \\ &\cdot \left(\overline{T_c^{\mathrm{so}}} C_g^{O_2} + H_{\mathrm{O}_2}^{0} - \overline{T_c^{\mathrm{so}}} C_g^{W} - H_{\mathrm{W}}^{0} \right), \end{aligned} \tag{8i}$$

$$b^{\rm so}_{t110} = -\frac{\overline{H^{\rm so}_{tot}}}{(\overline{C^{\rm so}_{th}})^2}C^{O_2}_g + \frac{1}{\overline{C^{\rm so}_{th}}}\frac{\overline{\dot m^{\rm ox}}\overline{m^{\rm so}_{\rm W}}}{(\overline{m^{\rm so}_{\rm O_2}}+\overline{m^{\rm so}_{\rm W}})^2} \cdot \left(\overline{T^{\rm so}_c}C^W_g + H^0_{\rm W} - \overline{T^{\rm so}_c}C^{O_2}_g - H^0_{\rm O_2}\right), \tag{8j}$$

$$b^{\rm so}_{t111} = -\frac{\overline{H^{\rm so}_{tot}}}{(\overline{C^{\rm so}_{th}})^2}C^{\rm W}_l. \tag{8k}$$

The individual elements of the matrices in the state equation in I-(34) can be given as follows

$$a^{\rm so}_{f21} = -\frac{w^{\rm rso}_c\dot m^{\rm rso}_{max}K^{\rm rso}_p}{A^{\rm so}\rho_{\rm W}},\ a^{\rm so}_{f22} = -w^{\rm rso}_c, \tag{9a}$$

$$a^{\rm so}_{f23} = w^{\rm rso}_c\dot m^{\rm rso}_{max}K^{\rm rso}_i,\ a^{\rm so}_{f31} = -\frac{1}{A^{\rm so}\rho_{\rm W}}, \tag{9b}$$

$$b^{\rm so}_{f11} = -M^w_{\rm W}\bar\xi 0.0134N_{\rm cs}, \tag{9c}$$

$$b^{\rm so}_{f12} = -N_{\rm cs}M^w_{\rm W}\left(0.03+0.0134\overline{T^{\rm sk}}+v_{\rm W}\right). \tag{9d}$$

The elements in the algebraic equation in I-(35b) are detailed as follows

$$d^{\rm so}_{f11} = \frac{1}{2R^{\rm so}_{\rm vlv}\sqrt{\overline{P^{\rm so}}-\overline{P^{ox}}}} = d^{\rm so}_{f12} = d^{\rm so}_{f13}. \tag{10}$$

The elements of the matrices in the state equation I-(37) are explained as follows

$$a^{\rm sh}_{g11} = -\frac{\overline{m^{\rm sh}_{\rm W}}\overline{\dot m^{\rm sv}}}{(\overline{m^{\rm sh}_{\rm W}}+\overline{m^{\rm sh}_{H_2}})^2},\ a^{\rm sh}_{g12} = \frac{\overline{m^{\rm sh}_{H_2}}\overline{\dot m^{\rm sv}}}{(\overline{m^{\rm sh}_{\rm W}}+\overline{m^{\rm sh}_{H_2}})^2}, \tag{11a}$$

$$a^{\rm sh}_{g21} = \frac{\overline{m^{\rm sh}_{\rm W}}\overline{\dot m^{\rm sv}}}{(\overline{m^{\rm sh}_{\rm W}}+\overline{m^{\rm sh}_{H_2}})^2},\ a^{\rm sh}_{g22} = -\frac{\overline{m^{\rm sh}_{H_2}}\overline{\dot m^{\rm sv}}}{(\overline{m^{\rm sh}_{\rm W}}+\overline{m^{\rm sh}_{H_2}})^2}, \tag{11b}$$

$$b^{\rm sh}_{g11} = -\frac{\overline{m^{\rm sh}_{H_2}}}{(\overline{m^{\rm sh}_{\rm W}}+\overline{m^{\rm sh}_{H_2}})},\ b^{\rm sh}_{g21} = -\frac{\overline{m^{\rm sh}_{\rm W}}}{(\overline{m^{\rm sh}_{\rm W}}+\overline{m^{\rm sh}_{H_2}})}, \tag{11c}$$

$$b^{\rm sh}_{g14} = N_{\rm cs}M^w_{H_2}v_{H_2},\ b^{\rm sh}_{g24} = N_{\rm cs}M^w_{\rm W}(0.03+0.0134\overline{T^{\rm sk}}), \tag{11d}$$

$$b^{\rm sh}_{g23} = N_{\rm cs}M^w_{\rm W}\bar\xi 0.0134. \tag{11e}$$

The elements of algebraic equation I-(38) can be given as follows

$$c^{\rm sh}_{g11} = \frac{R\overline{T^{\rm sh}}}{M^w_{H_2}(V^{\rm sh}-A^{\rm sh}\overline{L^{\rm sh}})}, \tag{12a}$$

$$c^{\rm sh}_{g22} = \frac{R\overline{T^{\rm sh}}}{M^w_{\rm W}(V^{\rm sh}-A^{\rm sh}\overline{L^{\rm sh}})}, \tag{12b}$$

$$d^{\rm sh}_{g11} = \frac{R\overline{T^{\rm sh}}\overline{m^{\rm sh}_{\rm H_2}}}{M^w_{H_2}(V^{\rm sh}-A^{\rm sh}\overline{L^{\rm sh}})^2\rho_W}, \tag{12c}$$

$$d^{\rm sh}_{g21} = \frac{R\overline{T^{\rm sh}}\overline{m^{\rm sh}_{\rm W}}}{M^w_{\rm W}(V^{\rm sh}-A^{\rm sh}\overline{L^{\rm sh}})^2\rho_W}, \tag{12d}$$

$$d^{\rm sh}_{g12} = \frac{R\overline{m^{\rm sh}_{\rm H_2}}}{M^w_{H_2}(V^{\rm sh}-A^{\rm sh}\overline{L^{\rm sh}})}, \tag{12e}$$

$$d^{\rm sh}_{g22} = \frac{R\overline{m^{\rm sh}_{\rm W}}}{M^w_{\rm W}(V^{\rm sh}-A^{\rm sh}\overline{L^{\rm sh}})}, \tag{12f}$$

$$c^{\rm sh}_{g33} = -\omega_{ev},\ d^{\rm sh}_{g31} = -\omega_{ev}p^{\rm sat}_{\rm W}\frac{M^w_{\rm W}}{R\overline{T^{\rm sh}}\rho_{\rm W}}, \tag{12g}$$

$$d^{\rm sh}_{g32} = -\omega_{ev}p^{\rm sat}_{\rm W}\frac{M^w_{\rm W}(V_{\rm sh}-A_{\rm sh}\overline{L^{\rm sh}})}{R\overline{T^{\rm sh}}^2}. \tag{12h}$$

The elements of the matrices in the state equation in I-(40) can be found as follows

$$a^{\rm sh}_{t11} = \frac{-1}{\overline{C^{\rm sh}_{\rm th}}}\left(\begin{matrix}\overline{\dot m^{\rm rh}}C^{\rm W}_l + \frac{1}{R^{\rm sh}_{cnd}} + \\ \frac{\overline{\dot m^{\rm sv}}\left(\overline{m^{\rm sh}_{\rm W}}C^W_g+\overline{m^{\rm sh}_{\rm H_2}}C^{H_2}_g\right)}{\overline{m^{\rm sh}_{\rm W}}+\overline{m^{\rm sh}_{\rm H_2}}}\end{matrix}\right) - \frac{\overline{\dot m^{\rm eh}_{\rm W}}}{\overline{C^{\rm sh}_{\rm th}}}(C^W_g - C^{\rm W}_l), \tag{13a}$$

$$b^{\rm sh}_{t11} = \frac{1}{\overline{C^{\rm sh}_{\rm th}}}(\overline{\dot m^{\rm rh}}-\overline{\dot m^{\rm dif}})C^{\rm W}_l + \frac{1}{\overline{C^{\rm sh}_{\rm th}}}C^W_g\overline{\dot m^{\rm eos}} + \frac{N_{\rm cs}}{\overline{C^{\rm sh}_{\rm th}}}C^{H_2}_gM^w_{H_2}v_{H_2}\bar\xi + \frac{\overline{H_{H_2O,g}}}{\overline{C^{\rm sh}_{\rm th}}}N_{\rm cs}M^w_{\rm W}\bar\xi 0.0134, \tag{13b}$$

$$b^{\rm sh}_{t12} = \frac{1}{\overline{C^{\rm sh}_{\rm th}}R^{\rm sh}_{cnd}},\ b^{\rm sh}_{t13} = \frac{1}{\overline{C^{\rm sh}_{\rm th}}}\overline{T^{\rm sk}_c}C^{\rm W}_l - \frac{1}{\overline{C^{\rm sh}_{\rm th}}}\overline{T^{\rm sh}_c}C^{\rm W}_l, \tag{13c}$$

$$b^{\rm sh}_{t14} = \frac{1}{\overline{C^{\rm sh}_{th}}}(\overline{T^{\rm rsh}}_c)C^{\rm W}_l, \tag{13d}$$

$$b^{\rm sh}_{t15} = \frac{(\overline{m^{\rm sh}_{H_2}}C^{H_2}_g + \overline{m^{\rm sh}_{\rm W}}C^W_g)\overline{T^{\rm sh}_c} + \overline{m^{\rm sh}_{\rm W}}H^0_{\rm W} + \overline{m^{\rm sh}_{\rm H_2}}H^0_{\rm H_2}}{-(\overline{m^{\rm sh}_{H_2}}+\overline{m^{\rm sh}_{\rm W}})\overline{C^{\rm sh}_{th}}}, \tag{13e}$$

$$b^{\rm sh}_{t16} = -\frac{1}{\overline{C^{\rm sh}_{th}}}\overline{T^{\rm sk}_c}C^{\rm W}_l, \tag{13f}$$

$$b^{\rm sh}_{t17} = -\frac{1}{\overline{C^{\rm sh}_{th}}}(-\overline{T^{\rm sh}_c}C^{\rm W}_l + (\overline{T^{\rm sh}_c}C^W_g + H^0_{\rm W})), \tag{13g}$$

$$b^{\rm sh}_{t18} = \frac{1}{\overline{C^{\rm sh}_{\rm th}}}(\overline{T^{\rm sk}_c}C^W_g + H^0_{\rm W})(M^w_{\rm W}N_{\rm cs}(0.0134\overline{T^{\rm sk}}+0.03)) + \frac{1}{\overline{C^{\rm sh}_{\rm th}}}(\overline{T^{\rm sk}_c}C^{H_2}_g + H^0_{\rm H_2})N_{\rm cs}M^w_{H_2}v_{H_2}, \tag{13h}$$

$$b^{\rm sh}_{t110} = -\frac{\overline{H^{\rm sh}_{tot}}}{(\overline{C^{\rm sh}_{th}})^2}C^W_g + \frac{1}{\overline{C^{\rm sh}_{th}}}\frac{\overline{\dot m^{\rm sv}}\overline{m^{\rm sh}_{\rm H_2}}}{(\overline{m^{\rm sh}_{H_2}}+\overline{m^{\rm sh}_{\rm W}})^2}(\overline{T^{\rm sh}_c}(C^{H_2}_g - C^W_g) + H^0_{\rm H_2} - H^0_{\rm W}) \tag{13i}$$

$$b_{t19}^{\mathrm{sh}} = -\frac{\overline{H_{tot}^{\mathrm{sh}}}}{(\overline{C_{th}^{\mathrm{sh}}})^2} C_g^{H_2} + \frac{1}{\overline{C_{th}^{\mathrm{sh}}}} \frac{\overline{\dot{m}^{\mathrm{sv}}}\,\overline{m_{\mathrm{W}}^{\mathrm{sh}}}}{(\overline{m_{H_2}^{\mathrm{sh}}} + \overline{m_{\mathrm{W}}^{\mathrm{sh}}})^2} (\overline{T_c^{\mathrm{sh}}}(C_g^W - C_g^{H_2}) + H_{\mathrm{W}}^0 - H_{\mathrm{H_2}}^0) \tag{13j}$$

$$b_{t111}^{\mathrm{sh}} = -\frac{\overline{H_{tot}^{\mathrm{sh}}}}{(\overline{C_{\mathrm{th}}^{\mathrm{sh}}})^2} C_l^{\mathrm{W}}. \tag{13k}$$

The individual elements of the matrices in the state equation I-(42) can be given as follows

$$a_{f21}^{\mathrm{sh}} = -\frac{w_c^{\mathrm{rsh}} \dot{m}_{max}^{\mathrm{rsh}} K_p^{\mathrm{rsh}}}{A^{\mathrm{sh}} \rho_{\mathrm{W}}},\ a_{f22}^{\mathrm{sh}} = -w_c^{\mathrm{rsh}}, \tag{14a}$$

$$a_{f23}^{\mathrm{sh}} = w_c^{\mathrm{rsh}} \dot{m}_{max}^{\mathrm{rsh}} K_i^{\mathrm{rsh}},\ a_{f31}^{\mathrm{sh}} = -\frac{1}{A^{\mathrm{sh}} \rho_{\mathrm{W}}}, \tag{14b}$$

The elements of the algebraic equation in I-(43b) can be given as follows

$$d_{f11}^{\mathrm{sh}} = \frac{1}{2\sqrt{|\overline{p_{H_2}^{\mathrm{sh}}} + \overline{p_{\mathrm{W}}^{\mathrm{sh}}} - \overline{p_{\mathrm{W}}^{\mathrm{sv}}} - \overline{p_{H_2}^{\mathrm{sv}}}| R_{\mathrm{vlv}}^{\mathrm{sep}}}} = d_{f12}^{\mathrm{sh}}, \tag{15a}$$

$$d_{f13}^{\mathrm{sh}} = d_{f14}^{\mathrm{sh}} = -d_{f11}^{\mathrm{sh}}. \tag{15b}$$

The elements of the state equation in I-(48) can be given as follows

$$a_{t11}^{\mathrm{ro}} = -\frac{1}{C_{th}^{ro}} \left(C_l^{\mathrm{W}} \overline{\dot{m}^{\mathrm{ro}}} + \frac{1}{R_{cnd}^{\mathrm{ro}}} + \frac{1}{R_{ech}^{\mathrm{ro}}} \right), \tag{16a}$$

$$a_{t12}^{\mathrm{ro}} = \frac{1}{C_{th}^{ro}} \frac{1}{R_{ech}^{\mathrm{ro}}},\ a_{t21}^{\mathrm{ro}} = \frac{1}{C_{th}^{col}} \frac{1}{R_{ech}^{\mathrm{ro}}}, \tag{16b}$$

$$a_{t22}^{\mathrm{ro}} = -\frac{1}{C_{th}^{col}} \left(C_l^{\mathrm{W}} \overline{\dot{m}^{\mathrm{col}}} + \frac{1}{R_{ech}^{\mathrm{ro}}} \right), \tag{16c}$$

$$a_{t23}^{\mathrm{ro}} = \frac{1}{C_{th}^{col}} C_l^{\mathrm{W}} \overline{\dot{m}^{\mathrm{col}}},\ a_{t32}^{\mathrm{ro}} = \frac{1}{C_{cld}^{th}} C_l^{\mathrm{W}} \overline{\dot{m}^{\mathrm{col}}}, \tag{16d}$$

$$a_{t33}^{\mathrm{ro}} = -\frac{1}{C_{cld}^{th}} C_l^{\mathrm{W}} \overline{\dot{m}^{\mathrm{col}}},\ b_{t11}^{\mathrm{ro}} = \frac{1}{C_{th}^{ro}} C_l^{\mathrm{W}} \overline{\dot{m}^{\mathrm{ro}}}, \tag{16e}$$

$$b_{t12}^{\mathrm{ro}} = \frac{1}{C_{th}^{ro}} \frac{1}{R_{cnd}^{\mathrm{ro}}},\ b_{t13}^{\mathrm{ro}} = \frac{1}{C_{th}^{ro}} C_l^{\mathrm{W}} (\overline{T^{\mathrm{so}}} - \overline{T^{\mathrm{ro}}}), \tag{16f}$$

The elements of the matrices in I-(50) can be given as follows:

$$a_{f11}^{\mathrm{ro}} = -\omega_c^{\mathrm{ro}},\ a_{f12}^{\mathrm{ro}} = \omega_c^{\mathrm{ro}} \dot{m}^{\mathrm{ro}} K_i^{\mathrm{ro}},\ b_{f11}^{\mathrm{ro}} = \omega_c^{\mathrm{ro}} \dot{m}^{\mathrm{ro}} K_p^{\mathrm{ro}}. \tag{17}$$

The elements of the matrices in the state equation in I-(52) are given as follows

$$a_{t11}^{\mathrm{rh}} = -\frac{1}{\overline{C_{th}^{\mathrm{rh}}}} \left(\overline{C_l^{\mathrm{W}}}\,\overline{\dot{m}^{\mathrm{rh}}} + \frac{1}{R_{cnd}^{\mathrm{rh}}} \right),\ b_{t11}^{\mathrm{rh}} = \frac{1}{\overline{C_{th}^{\mathrm{rh}}}} \overline{C_l^{\mathrm{W}}}\,\overline{\dot{m}^{\mathrm{rh}}}, \tag{18a}$$

$$b_{t12}^{\mathrm{rh}} = \frac{1}{\overline{C_{th}^{\mathrm{rh}}}} \frac{1}{R_{cnd}^{\mathrm{rh}}},\ b_{t13}^{\mathrm{rh}} = \frac{1}{\overline{C_{th}^{\mathrm{rh}}}} \overline{C_l^{\mathrm{W}}} (\overline{T^{\mathrm{sh}}} - \overline{T^{\mathrm{rh}}}), \tag{18b}$$

The coefficients of the matrices in the state equation I-(54) are given as follows

$$a_{g11}^{\mathrm{dy}} = -\frac{(\overline{m_{\mathrm{W}}^{\mathrm{dy}}})\overline{\dot{m}^{\mathrm{pr}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}})^2},\ a_{g12}^{\mathrm{dy}} = \frac{\overline{m_{H_2}^{\mathrm{dy}}}\,\overline{\dot{m}^{\mathrm{pr}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}})^2}, \tag{19a}$$

$$a_{g21}^{\mathrm{dy}} = \frac{\overline{m_{\mathrm{W}}^{\mathrm{dy}}}\,\overline{\dot{m}^{\mathrm{pr}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}})^2},\ a_{g22}^{\mathrm{dy}} = -\frac{(\overline{m_{H_2}^{\mathrm{dy}}})\overline{\dot{m}^{\mathrm{pr}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}})^2}, \tag{19b}$$

$$b_{g11}^{\mathrm{dy}} = \frac{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}})\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}})^2},\ b_{g12}^{\mathrm{dy}} = -\frac{\overline{m_{H_2}^{\mathrm{sh}}}\,\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}})^2}, \tag{19c}$$

$$b_{g21}^{\mathrm{dy}} = -\frac{\overline{m_{\mathrm{W}}^{\mathrm{sh}}}\,\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}})^2},\ b_{g22}^{\mathrm{dy}} = \frac{(\overline{m_{H_2}^{\mathrm{sh}}})\overline{\dot{m}^{\mathrm{sv}}}}{(\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}})^2}, \tag{19d}$$

$$b_{g13}^{\mathrm{dy}} = \frac{\overline{m_{H_2}^{\mathrm{sh}}}}{\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}}},\ b_{g23}^{\mathrm{dy}} = \frac{\overline{m_{\mathrm{W}}^{\mathrm{sh}}}}{\overline{m_{\mathrm{W}}^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}}}, \tag{19e}$$

$$b_{g14}^{\mathrm{dy}} = -\frac{\overline{m_{H_2}^{\mathrm{dy}}}}{\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}}},\ b_{g24}^{\mathrm{dy}} = -\frac{\overline{m_{\mathrm{W}}^{\mathrm{dy}}}}{\overline{m_{\mathrm{W}}^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}}}. \tag{19f}$$

The coefficients of the matrices in the state equation I-(58) are elaborated as follows

$$a_{t11}^{\mathrm{dy}} = \frac{1}{\overline{C_{th}^{\mathrm{dy}}}} \left(\begin{array}{c} \left(\overline{C_g^W} - 2\overline{C_l^{\mathrm{W}}}\right) \overline{\dot{m}^{\mathrm{ads}}} \\ -\frac{\overline{\dot{m}^{\mathrm{pr}}}\left(\overline{m_W^{\mathrm{dy}}}\,\overline{C_g^W} + \overline{m_{H_2}^{\mathrm{dy}}}\,\overline{C_g^{H_2}}\right)}{\overline{m_W^{\mathrm{dy}}} + \overline{m_{H_2}^{\mathrm{dy}}}} - \frac{1}{R_{cnd}^{\mathrm{dy}}} \end{array} \right), \tag{20a}$$

$$b_{t11}^{\mathrm{dy}} = \frac{1}{\overline{C_{th}^{\mathrm{dy}}}} \overline{\dot{m}^{\mathrm{sv}}} \frac{\left(\overline{m_W^{\mathrm{sh}}}\,\overline{C_g^W} + \overline{m_{H_2}^{\mathrm{sh}}}\,\overline{C_g^{H_2}}\right)}{\overline{m_W^{\mathrm{sh}}} + \overline{m_{H_2}^{\mathrm{sh}}}}, \tag{20b}$$

$$b_{t12}^{\mathrm{dy}} = \frac{1}{\overline{C_{th}^{\mathrm{dy}}} R_{cnd}^{\mathrm{dy}}}, \tag{20c}$$

$$b_{t13}^{\mathrm{dy}} = \frac{1}{\overline{C_{th}^{\mathrm{dy}}}} \frac{\overline{\dot{m}^{\mathrm{pr}}}\,\overline{m_{\mathrm{W}}^{\mathrm{dy}}}}{(\overline{m_{H_2}^{\mathrm{dy}}} + \overline{m_{\mathrm{W}}^{\mathrm{dy}}})^2} (\overline{T_c^{\mathrm{dy}}}(C_g^W - C_g^{H_2}) + H_{\mathrm{W}}^0 - H_{\mathrm{H_2}}^0), \tag{20d}$$

$$b_{t14}^{\mathrm{dy}} = \frac{1}{\overline{C_{th}^{\mathrm{dy}}}} \frac{\overline{\dot{m}^{\mathrm{pr}}}\,\overline{m_{\mathrm{H_2}}^{\mathrm{dy}}}}{(\overline{m_{H_2}^{\mathrm{dy}}} + \overline{m_{\mathrm{W}}^{\mathrm{dy}}})^2} (\overline{T_c^{\mathrm{dy}}}(C_g^{H_2} - C_g^W) + H_{\mathrm{H_2}}^0 - H_{\mathrm{W}}^0), \tag{20e}$$

$$b_{t15}^{\mathrm{dy}} = \frac{-1}{\overline{C_{th}^{\mathrm{dy}}}} \frac{\overline{\dot{m}^{\mathrm{sv}}}\,\overline{m_{\mathrm{W}}^{\mathrm{sh}}}}{(\overline{m_{H_2}^{\mathrm{sh}}} + \overline{m_{\mathrm{W}}^{\mathrm{sh}}})^2} (\overline{T_c^{\mathrm{sh}}}(C_g^W - C_g^{H_2}) + H_{\mathrm{W}}^0 - H_{\mathrm{H_2}}^0), \tag{20f}$$

$$b_{t16}^{\mathrm{dy}} = \frac{-1}{\overline{C_{th}^{\mathrm{dy}}}} \frac{\overline{\dot{m}^{\mathrm{sv}}}\,\overline{m_{\mathrm{H_2}}^{\mathrm{sh}}}}{(\overline{m_{H_2}^{\mathrm{sh}}} + \overline{m_{\mathrm{W}}^{\mathrm{sh}}})^2} (\overline{T_c^{\mathrm{sh}}}(C_g^{H_2} - C_g^W) + H_{\mathrm{H_2}}^0 - H_{\mathrm{W}}^0), \tag{20g}$$

$$b_{t17}^{\mathrm{dy}} = \frac{(\overline{m_{H_2}^{\mathrm{dy}}} C_g^{H_2} + \overline{m_{\mathrm{W}}^{\mathrm{dy}}} C_g^{W}) \overline{T_c^{\mathrm{dy}}} + \overline{m_{\mathrm{W}}^{\mathrm{dy}}} H_{\mathrm{W}}^0 + \overline{m_{\mathrm{H}_2}^{\mathrm{dy}}} H_{\mathrm{H}_2}^0}{-\overline{C_{th}^{\mathrm{dy}}} (\overline{m_{H_2}^{\mathrm{dy}}} + \overline{m_{\mathrm{W}}^{\mathrm{dy}}})}, \tag{20h}$$

$$b_{t18}^{\mathrm{dy}} = \frac{1}{\overline{C_{th}^{\mathrm{dy}}}} b_{t15}^{\mathrm{sh}} (-\overline{C_{\mathrm{th}}^{\mathrm{sh}}}), \tag{20i}$$

$$b_{t19}^{\mathrm{dy}} = \frac{1}{\overline{C_{th}^{\mathrm{dy}}}} ((\overline{C_g^W} - 2\overline{C_l^{\mathrm{W}}}) (\overline{T_c^{dy}}) + H_{\mathrm{W}}^0). \tag{20j}$$

The coefficients of the matrices in I-(60) can be given as follows

$$c_{g11}^{\mathrm{dy}} = \frac{R\overline{T}^{\mathrm{dy}}}{M_{H_2}^w V^{\mathrm{dy}}},\ c_{g22}^{\mathrm{dy}} = \frac{R\overline{T}^{\mathrm{dy}}}{M_{\mathrm{W}}^w V^{\mathrm{dy}}}, \tag{21a}$$

$$d_{g11}^{\mathrm{dy}} = \frac{R\overline{m_{H_2}^{\mathrm{dy}}}}{M_{H_2}^w V^{\mathrm{dy}}},\ d_{g21}^{\mathrm{dy}} = \frac{R\overline{m_{\mathrm{W}}^{\mathrm{dy}}}}{M_{\mathrm{W}}^w V^{\mathrm{dy}}}, \tag{21b}$$

The elements of the state equation in I-(62) are elaborated as follows

$$a_{11}^{ec} = -\frac{1}{C_{th}^{ec}} \begin{pmatrix} \frac{1}{R_{cnd}^{g}} + \frac{1}{R_{cnd}^{\mathrm{rh}}} + \frac{1}{R_{cnd}^{dy}} + \frac{1}{R_{cnd}^{\mathrm{ro}}} \\ + \frac{1}{R_{cnd}^{\mathrm{sh}}} + \frac{1}{R_{cnd}^{\mathrm{so}}} + \frac{1}{R_{cnd}^{ec}} + C_p^{air} \overline{m}_d^{fan} \end{pmatrix} \tag{22a}$$

$$b_{11}^{ec} = \frac{1}{C_{th}^{ec}} \frac{1}{R_{cnd}^{g}},\ b_{12}^{ec} = \frac{1}{C_{th}^{ec}} \frac{1}{R_{cnd}^{\mathrm{so}}},\ b_{13}^{ec} = \frac{1}{C_{th}^{ec}} \frac{1}{R_{cnd}^{\mathrm{sh}}}, \tag{22b}$$

$$b_{14}^{ec} = \frac{1}{C_{th}^{ec}} \frac{1}{R_{cnd}^{\mathrm{ro}}} b_{15}^{ec} = \frac{1}{C_{th}^{ec}} \frac{1}{R_{cnd}^{\mathrm{rh}}}, \tag{22c}$$

$$b_{16}^{ec} = \frac{1}{C_{th}^{ec}} \frac{1}{R_{cnd}^{dy}}, b_{17}^{ec} = \frac{1}{C_{th}^{ec}} \left(C_p^{air} \overline{m}_d^{fan} + \frac{1}{R_{cnd}^{ec}} \right). \tag{22d}$$

The elements of the matrices in I-(64) can be formulated as follows:

$$a_{11}^{\mathrm{stg}} = -\omega_c^{\mathrm{stg}},\ a_{12}^{\mathrm{stg}} = \omega_c^{\mathrm{stg}} K_i^{\mathrm{stg}} A_{\max}^{\mathrm{rst}}, \tag{23a}$$

$$b_{11}^{\mathrm{stg}} = \omega_c^{\mathrm{stg}} K_p^{\mathrm{stg}} A_{\max}^{\mathrm{rst}} = b_{12}^{\mathrm{stg}}. \tag{23b}$$

The elements of the algebraic equation in I-(66) can be described as follows

$$c_{g11}^{\mathrm{stg}} = \frac{\sqrt{\overline{p_{H_2}^{\mathrm{sv}}} + \overline{p_{\mathrm{W}}^{\mathrm{sv}}} - P^{\mathrm{pr}}}}{R_{\mathrm{vlv}}^{\mathrm{pr}}}, \tag{24a}$$

$$d_{g11}^{\mathrm{stg}} = \frac{\overline{A^{\mathrm{rst}}}}{2\sqrt{\overline{p_{H_2}^{\mathrm{sv}}} + \overline{p_{\mathrm{W}}^{\mathrm{sv}}} - P^{\mathrm{pr}}} R_{\mathrm{vlv}}^{\mathrm{pr}}} = d_{g12}^{\mathrm{stg}}. \tag{24b}$$

The elements of the electrolyzer state matrix in I-(68) can be given as in (25).

II. DC TO AC INVERTER AND ITS CONTROL LOOPS

The AC-side system and its control loops regulate the grid voltage amplitude and frequency, allowing the electrolyzer to absorb excess power from the green microgrid for hydrogen production. Due to space limitations and the extensive coverage of this topic in prior literature, the modeling of this section is omitted from the main manuscript and is instead detailed in this supplementary material. The AC inverter employs droop control, enabling the electrolyzer unit to actively participate in grid formation alongside other grid-forming units.

To ensure accurate control, the active and reactive power measurements are filtered to remove harmonics, resulting in clean signals that support effective power-sharing among the grid's generating units. The filtered measurements are expressed as follows

$$\dot{P} = -\omega_c P + \omega_c (v_{\mathrm{od}} i_{\mathrm{od}} + v_{oq} i_{oq}), \tag{26a}$$
$$\dot{Q} = -\omega_c Q + \omega_c (-v_{\mathrm{od}} i_{oq} + v_{oq} i_{\mathrm{od}}), \tag{26b}$$

where v_{od} and v_{oq} are the direct and quadrature components of the output voltage of the VSC, i_{od} and i_{oq} are the corresponding dq components of the output current, and ω_c is the cut-off frequency of the low-pass measurement filters. The active and reactive power dictates the frequency and the output voltage of the inverter based on droop relations that are given as follows

$$\omega = \omega^{\mathrm{nom}} - mP - m_d \dot{P}, \tag{27a}$$
$$v_{\mathrm{od}}^{\mathrm{ref}} = v^{\mathrm{nom}} - nQ - n_d \dot{Q}, \tag{27b}$$
$$v_{\mathrm{oq}}^{\mathrm{ref}} = 0, \tag{27c}$$

where ω is the VSC angular frequency and ω^{nom} is its nominal value, m is the active power droop constant, v_{od}^* is the setpoint for the direct component of the output voltage, v_{oq}^* is its quadrature component, v^{nom} is the nominal output voltage, n is the reactive power droop constant, and m^d and n^d are the derivative controllers constants for the active and reactive power laws, respectively. The voltage droop in (27b) provides the voltage reference for the outer PI controller of the inverter to regulate the voltage across the filter capacitor on the AC side. The equations controlling the outer PI are given as follows

$$\frac{d\phi_{\mathrm{d}}}{dt} = v_{\mathrm{od}}^{\mathrm{ref}} - v_{\mathrm{od}}, \tag{28a}$$
$$\frac{d\phi_{\mathrm{q}}}{dt} = v_{\mathrm{oq}}^{\mathrm{ref}} - v_{\mathrm{oq}}. \tag{28b}$$

The AC voltage PI controller generates the voltage components to the AC current PI controllers. The output of the voltage PI controller is given as follows

$$i_{\mathrm{ld}}^* = H i_{\mathrm{od}} - \omega^{\mathrm{nom}} C_f v_{\mathrm{oq}} + k_{\mathrm{pv}}^{ac} (v_{\mathrm{od}}^{\mathrm{ref}} - v_{\mathrm{od}}) + k_{\mathrm{iv}}^{ac} \phi_{\mathrm{d}}, \tag{29a}$$
$$i_{\mathrm{lq}}^* = H i_{\mathrm{oq}} + \omega^{\mathrm{nom}} C_f v_{\mathrm{od}} + k_{\mathrm{pv}}^{ac} (v_{\mathrm{oq}}^{\mathrm{ref}} - v_{\mathrm{oq}}) + k_{\mathrm{iv}}^{ac} \phi_{\mathrm{q}}, \tag{29b}$$

where k_{ii}^{ac} denotes the PI controller gains, H a feedforward constant, and C_f in the capacitance of the Lc filter interfacing the unit to the weak grids. The current PI controller regulates the current passing through the filter on the AC side according to the following relations

$$\frac{d\gamma_{\mathrm{d}}}{dt} = i_{\mathrm{ld}}^{\mathrm{ref}} - i_{\mathrm{ld}}, \tag{30a}$$
$$\frac{d\gamma_{\mathrm{q}}}{dt} = i_{\mathrm{lq}}^{\mathrm{ref}} - i_{\mathrm{lq}}. \tag{30b}$$

$$a_1 = b_{e13}^{\mathrm{pm}} d_{g22}^{\mathrm{so}} + b_{e14}^{\mathrm{pm}} d_{g12}^{\mathrm{so}},\ a_2 = b_{e12}^{\mathrm{pm}} d_{g12}^{\mathrm{sh}} + b_{e15}^{\mathrm{pm}} d_{g22}^{\mathrm{sh}},\ a_3 = b_{e14}^{\mathrm{pm}} c_{g11}^{\mathrm{so}},\ a_4 = b_{e13}^{\mathrm{pm}} c_{g22}^{\mathrm{so}}\ a_5 = b_{e12}^{\mathrm{pm}} c_{g11}^{\mathrm{sh}},\ a_6 = b_{e15}^{\mathrm{pm}} c_{g22}^{\mathrm{sh}}, \tag{25a}$$

$$a_7 = b_{e13}^{\mathrm{pm}} d_{g21}^{\mathrm{so}} + b_{e14}^{\mathrm{pm}} d_{g11}^{\mathrm{so}},\ a_8 = b_{e12}^{\mathrm{pm}} d_{g11}^{\mathrm{sh}} + b_{e15}^{\mathrm{pm}} d_{g21}^{\mathrm{sh}},\ a_9 = b_{e23}^{\mathrm{pm}} d_{g22}^{\mathrm{so}} + b_{e24}^{\mathrm{pm}} d_{g12}^{\mathrm{so}},\ a_{10} = b_{e22}^{\mathrm{pm}} d_{g12}^{\mathrm{sh}} + b_{e25}^{\mathrm{pm}} d_{g22}^{\mathrm{sh}},\ a_{11} = b_{e24}^{\mathrm{pm}} c_{g11}^{\mathrm{so}}, \tag{25b}$$

$$a_{12} = b_{e23}^{\mathrm{pm}} c_{g22}^{\mathrm{so}}, a_{13} = b_{e22}^{\mathrm{pm}} c_{g11}^{\mathrm{sh}}, a_{14} = b_{e25}^{\mathrm{pm}} c_{g22}^{\mathrm{sh}},\ a_{15} = b_{e23}^{\mathrm{pm}} d_{g21}^{\mathrm{so}} + b_{e24}^{\mathrm{pm}} d_{g11}^{\mathrm{so}},\ a_{16} = b_{e22}^{\mathrm{pm}} d_{g11}^{\mathrm{sh}} + b_{e25}^{\mathrm{pm}} d_{g21}^{\mathrm{sh}},\ a_{17} = b_{t16}^{\mathrm{pm}} c_{e21}^{\mathrm{pm}}, \tag{25c}$$

$$a_{18} = a_{t11}^{\mathrm{pm}} + b_{t13}^{\mathrm{pm}} d_{g3}^{\mathrm{pm}} + b_{t16}^{\mathrm{pm}} d_{e21}^{\mathrm{pm}},\ a_{19} = b_{t13}^{\mathrm{pm}} d_{g1}^{\mathrm{pm}} d_{g12}^{\mathrm{so}} + b_{t16}^{\mathrm{pm}} d_{e23}^{\mathrm{pm}} d_{g22}^{\mathrm{so}} + b_{t16}^{\mathrm{pm}} d_{e24}^{\mathrm{pm}} d_{g12}^{\mathrm{so}}, \tag{25d}$$

$$a_{20} = b_{t13}^{\mathrm{pm}} d_{g2}^{\mathrm{pm}} d_{g22}^{\mathrm{sh}} + b_{t16}^{\mathrm{pm}} d_{e22}^{\mathrm{pm}} d_{g12}^{\mathrm{sh}} + b_{t16}^{\mathrm{pm}} d_{e25}^{\mathrm{pm}} d_{g22}^{\mathrm{sh}},\ a_{21} = b_{t13}^{\mathrm{pm}} d_{g1}^{\mathrm{pm}} c_{g11}^{\mathrm{so}} + b_{t16}^{\mathrm{pm}} d_{e24}^{\mathrm{pm}} c_{g11}^{\mathrm{so}},\ a_{22} = b_{t16}^{\mathrm{pm}} d_{e23}^{\mathrm{pm}} c_{g22}^{\mathrm{so}}, \tag{25e}$$

$$a_{23} = b_{t16}^{\mathrm{pm}} d_{e22}^{\mathrm{pm}} c_{g11}^{\mathrm{sh}},\ a_{24} = b_{t13}^{\mathrm{pm}} d_{g2}^{\mathrm{pm}} c_{g22}^{\mathrm{sh}} + b_{t16}^{\mathrm{pm}} d_{e25}^{\mathrm{pm}} c_{g22}^{\mathrm{sh}},\ a_{25} = b_{t13}^{\mathrm{pm}} d_{g1}^{\mathrm{pm}} d_{g11}^{\mathrm{so}} + b_{t16}^{\mathrm{pm}} d_{e23}^{\mathrm{pm}} d_{g21}^{\mathrm{so}} + b_{t16}^{\mathrm{pm}} d_{e24}^{\mathrm{pm}} d_{g11}^{\mathrm{so}}, \tag{25f}$$

$$a_{26} = b_{t13}^{\mathrm{pm}} d_{g2}^{\mathrm{pm}} d_{g21}^{\mathrm{sh}} + b_{t16}^{\mathrm{pm}} d_{e22}^{\mathrm{pm}} d_{g11}^{\mathrm{sh}} + b_{t16}^{\mathrm{pm}} d_{e25}^{\mathrm{pm}} d_{g21}^{\mathrm{sh}} a_{27} = b_{t12}^{\mathrm{pm}} + b_{t11}^{\mathrm{pm}} a_{28} = b_{t11}^{\mathrm{so}} + b_{t16}^{\mathrm{so}} d_{g3}^{\mathrm{pm}}, \tag{25g}$$

$$a_{29} = a_{t11}^{\mathrm{so}} + b_{t15}^{\mathrm{so}} d_{f11}^{\mathrm{so}} d_{g12}^{\mathrm{so}} + b_{t15}^{\mathrm{so}} d_{f12}^{\mathrm{so}} d_{g22}^{\mathrm{so}} + b_{t16}^{\mathrm{so}} d_{g1}^{\mathrm{pm}} d_{g12}^{\mathrm{so}} + b_{t17}^{\mathrm{so}} d_{g32}^{\mathrm{so}},\ a_{30} = b_{t16}^{\mathrm{so}} d_{g2}^{\mathrm{pm}} d_{g22}^{\mathrm{sh}}, \tag{25h}$$

$$a_{31} = b_{t19}^{\mathrm{so}} + b_{t15}^{\mathrm{so}} d_{f11}^{\mathrm{so}} c_{g11}^{\mathrm{so}} + b_{t16}^{\mathrm{so}} d_{g1}^{\mathrm{pm}} c_{g11}^{\mathrm{so}} + b_{t17}^{\mathrm{so}} c_{g33}^{\mathrm{so}},\ a_{32} = b_{t110}^{\mathrm{so}} + b_{t15}^{\mathrm{so}} d_{f12}^{\mathrm{so}} c_{g22}^{\mathrm{so}},\ a_{33} = b_{t16}^{\mathrm{so}} d_{g2}^{\mathrm{pm}} c_{g22}^{\mathrm{sh}}, \tag{25i}$$

$$a_{34} = b_{t111}^{\mathrm{so}} + b_{t15}^{\mathrm{so}} d_{f11}^{\mathrm{so}} d_{g11}^{\mathrm{so}} + b_{t15}^{\mathrm{so}} d_{f12}^{\mathrm{so}} d_{g21}^{\mathrm{so}} + b_{t16}^{\mathrm{so}} d_{g1}^{\mathrm{pm}} d_{g11}^{\mathrm{so}} + b_{t17}^{\mathrm{so}} d_{g31}^{\mathrm{so}},\ a_{35} = b_{t16}^{\mathrm{so}} d_{g2}^{\mathrm{pm}} d_{g21}^{\mathrm{sh}}, \tag{25j}$$

$$a_{36} = b_{t11}^{\mathrm{sh}} + b_{t16}^{\mathrm{sh}} d_{g3}^{\mathrm{pm}},\ a_{37} = b_{t16}^{\mathrm{sh}} d_{g1}^{\mathrm{pm}} d_{g12}^{\mathrm{so}},\ a_{38} = a_{t11}^{\mathrm{sh}} + b_{t15}^{\mathrm{sh}} d_{f11}^{\mathrm{sh}} (d_{g12}^{\mathrm{sh}} + d_{g22}^{\mathrm{sh}}) + b_{t16}^{\mathrm{sh}} d_{g2}^{\mathrm{pm}} d_{g22}^{\mathrm{sh}} + b_{t17}^{\mathrm{sh}} d_{g32}^{\mathrm{sh}}, \tag{25k}$$

$$a_{39} = b_{t15}^{\mathrm{sh}} d_{f14}^{\mathrm{sh}} (d_{g11}^{\mathrm{dy}} + d_{g21}^{\mathrm{dy}}),\ a_{40} = b_{t16}^{\mathrm{sh}} d_{g1}^{\mathrm{pm}} c_{g11}^{\mathrm{so}},\ a_{41} = b_{t19}^{\mathrm{sh}} + b_{t15}^{\mathrm{sh}} d_{f11}^{\mathrm{sh}} c_{g11}^{\mathrm{sh}},\ a_{43} = b_{t15}^{\mathrm{sh}} d_{f14}^{\mathrm{sh}} c_{g11}^{\mathrm{dy}} \tag{25l}$$

$$a_{42} = b_{t110}^{\mathrm{sh}} + b_{t15}^{\mathrm{sh}} d_{f12}^{\mathrm{sh}} c_{g22}^{\mathrm{sh}} + b_{t16}^{\mathrm{sh}} d_{g2}^{\mathrm{pm}} c_{g22}^{\mathrm{sh}} + b_{t17}^{\mathrm{sh}} c_{g33}^{\mathrm{sh}},\ a_{44} = b_{t15}^{\mathrm{sh}} d_{f14}^{\mathrm{sh}} c_{g22}^{\mathrm{dy}},\ a_{45} = b_{t16}^{\mathrm{sh}} d_{g1}^{\mathrm{pem}} d_{g11}^{\mathrm{so}}, \tag{25m}$$

$$a_{46} = b_{t111}^{\mathrm{sh}} + b_{t15}^{\mathrm{sh}} d_{f11}^{\mathrm{sh}} (d_{g11}^{\mathrm{sh}} + d_{g21}^{\mathrm{sh}}) + b_{t16}^{\mathrm{sh}} d_{g2}^{\mathrm{pem}} d_{g21}^{\mathrm{sh}} + b_{t17}^{\mathrm{sh}} d_{g31}^{\mathrm{sh}},\ a_{47} = b_{t11}^{\mathrm{dy}} + b_{t18}^{\mathrm{dy}} d_{f11}^{\mathrm{sh}} (d_{g12}^{\mathrm{sh}} + d_{g22}^{\mathrm{sh}}), \tag{25n}$$

$$a_{48} = a_{t11}^{\mathrm{dy}} + b_{t17}^{\mathrm{dy}} d_{g11}^{\mathrm{sg}} d_{g11}^{\mathrm{dy}} + b_{t17}^{\mathrm{dy}} d_{g12}^{\mathrm{sg}} d_{g21}^{\mathrm{dy}} + b_{t18}^{\mathrm{dy}} d_{f14}^{\mathrm{sh}} (d_{g11}^{\mathrm{dy}} + d_{g21}^{\mathrm{dy}}),\ a_{49} = b_{t15}^{\mathrm{dy}} + b_{t18}^{\mathrm{dy}} d_{f11}^{\mathrm{sh}} c_{g11}^{\mathrm{sh}}, \tag{25o}$$

$$a_{50} = b_{t16}^{\mathrm{dy}} + b_{t18}^{\mathrm{dy}} d_{f12}^{\mathrm{sh}} c_{g22}^{\mathrm{sh}},\ a_{51} = b_{t13}^{\mathrm{dy}} + b_{t17}^{\mathrm{dy}} d_{g11}^{\mathrm{sg}} c_{g11}^{\mathrm{dy}} + b_{t18}^{\mathrm{dy}} d_{f14}^{\mathrm{sh}} c_{g11}^{\mathrm{dy}},\ a_{52} = b_{t14}^{\mathrm{dy}} + b_{t17}^{\mathrm{dy}} d_{g12}^{\mathrm{sg}} c_{g22}^{\mathrm{dy}} + b_{t18}^{\mathrm{dy}} d_{f14}^{\mathrm{sh}} c_{g22}^{\mathrm{dy}} + b_{t19}^{\mathrm{dy}} w^{\mathrm{ads}}, \tag{25p}$$

$$a_{53} = b_{t18}^{\mathrm{dy}} d_{f11}^{\mathrm{sh}} (d_{g11}^{\mathrm{sh}} + d_{g21}^{\mathrm{sh}}),\ a_{54} = b_{t17}^{\mathrm{dy}} c_{g11}^{\mathrm{sg}},\ a_{55} = d_{g1}^{\mathrm{pem}} d_{g12}^{\mathrm{so}} + d_{g32}^{\mathrm{so}} + b_{g13}^{\mathrm{so}} d_{f11}^{\mathrm{so}} d_{g12}^{\mathrm{so}} + b_{g13}^{\mathrm{so}} d_{f12}^{\mathrm{so}} d_{g22}^{\mathrm{so}}, \tag{25q}$$

$$a_{56} = d_{g2}^{\mathrm{pem}} d_{g22}^{\mathrm{sh}},\ a_{57} = a_{g11}^{\mathrm{so}} + d_{g1}^{\mathrm{pem}} c_{g11}^{\mathrm{so}} + c_{g33}^{\mathrm{so}} + b_{g13}^{\mathrm{so}} d_{f11}^{\mathrm{so}} c_{g11}^{\mathrm{so}},\ a_{58} = a_{g12}^{\mathrm{so}} + b_{g13}^{\mathrm{so}} d_{f12}^{\mathrm{so}} c_{g22}^{\mathrm{so}},\ a_{59} = d_{g2}^{\mathrm{pem}} c_{g22}^{\mathrm{sh}}, \tag{25r}$$

$$a_{60} = d_{g1}^{\mathrm{pem}} d_{g11}^{\mathrm{so}} + d_{g31}^{\mathrm{so}} + b_{g13}^{\mathrm{so}} d_{f11}^{\mathrm{so}} d_{g11}^{\mathrm{so}} + b_{g13}^{\mathrm{so}} d_{f12}^{\mathrm{so}} d_{g21}^{\mathrm{so}},\ a_{61} = d_{g2}^{\mathrm{pem}} d_{g21}^{\mathrm{sh}} a_{62} = b_{g23}^{\mathrm{so}} d_{f11}^{\mathrm{so}} d_{g12}^{\mathrm{so}} + b_{g23}^{\mathrm{so}} d_{f12}^{\mathrm{so}} d_{g22}^{\mathrm{so}}, \tag{25s}$$

$$a_{63} = a_{g21}^{\mathrm{so}} + b_{g23}^{\mathrm{so}} d_{f11}^{\mathrm{so}} c_{g11}^{\mathrm{so}},\ a_{64} = a_{g22}^{\mathrm{so}} + b_{g23}^{\mathrm{so}} d_{f12}^{\mathrm{so}} c_{g22}^{\mathrm{so}},\ a_{65} = b_{g23}^{\mathrm{so}} d_{f11}^{\mathrm{so}} d_{g11}^{\mathrm{so}} + b_{g23}^{\mathrm{so}} d_{f12}^{\mathrm{so}} d_{g21}^{\mathrm{so}},\ a_{66} = b_{g11}^{\mathrm{sh}} d_{f11}^{\mathrm{sh}} (d_{g12}^{\mathrm{sh}} + d_{g22}^{\mathrm{sh}}), \tag{25t}$$

$$a_{67} = b_{g11}^{\mathrm{sh}} d_{f14}^{\mathrm{sh}} (d_{g11}^{\mathrm{dy}} + d_{g21}^{\mathrm{dy}}),\ a_{68} = a_{g11}^{\mathrm{sh}} + b_{g11}^{\mathrm{sh}} d_{f11}^{\mathrm{sh}} c_{g11}^{\mathrm{sh}},\ a_{69} = a_{g12}^{\mathrm{sh}} + b_{g11}^{\mathrm{sh}} d_{f12}^{\mathrm{sh}} c_{g22}^{\mathrm{sh}},\ a_{70} = b_{g11}^{\mathrm{sh}} d_{f14}^{\mathrm{sh}} c_{g11}^{\mathrm{dy}}, \tag{25u}$$

$$a_{71} = b_{g11}^{\mathrm{sh}} d_{f14}^{\mathrm{sh}} c_{g22}^{\mathrm{dy}},\ a_{72} = b_{g11}^{\mathrm{sh}} d_{f11}^{\mathrm{sh}} (d_{g11}^{\mathrm{sh}} + d_{g21}^{\mathrm{sh}}),\ a_{73} = b_{g21}^{\mathrm{sh}} d_{f11}^{\mathrm{sh}} (d_{g12}^{\mathrm{sh}} + d_{g22}^{\mathrm{sh}}) + d_{g32}^{\mathrm{sh}},\ a_{74} = b_{g21}^{\mathrm{sh}} d_{f14}^{\mathrm{sh}} (d_{g11}^{\mathrm{dy}} + d_{g21}^{\mathrm{dy}}), \tag{25v}$$

$$a_{75} = a_{g21}^{\mathrm{sh}} + b_{g21}^{\mathrm{sh}} d_{f11}^{\mathrm{sh}} c_{g11}^{\mathrm{sh}},\ a_{76} = a_{g22}^{\mathrm{sh}} + b_{g21}^{\mathrm{sh}} d_{f12}^{\mathrm{sh}} c_{g22}^{\mathrm{sh}} + c_{g33}^{\mathrm{sh}},\ a_{77} = b_{g21}^{\mathrm{sh}} d_{f14}^{\mathrm{sh}} c_{g11}^{\mathrm{dy}},\ a_{78} = b_{g21}^{\mathrm{sh}} d_{f14}^{\mathrm{sh}} c_{g22}^{\mathrm{dy}},\ a_{79} = b_{g21}^{\mathrm{sh}} d_{f11}^{\mathrm{sh}} (d_{g11}^{\mathrm{sh}} + d_{g21}^{\mathrm{sh}}) + d_{g31}^{\mathrm{sh}}, \tag{25w}$$

$$a_{80} = b_{g13}^{\mathrm{dy}} d_{f11}^{\mathrm{sh}} (d_{g12}^{\mathrm{sh}} + d_{g22}^{\mathrm{sh}}),\ a_{81} = b_{g13}^{\mathrm{dy}} d_{f14}^{\mathrm{sh}} (d_{g11}^{\mathrm{dy}} + d_{g21}^{\mathrm{dy}}) + b_{g14}^{\mathrm{dy}} d_{g11}^{\mathrm{sg}} d_{g11}^{\mathrm{dy}} + b_{g14}^{\mathrm{dy}} d_{g12}^{\mathrm{sg}} d_{g21}^{\mathrm{dy}},\ a_{82} = b_{g11}^{\mathrm{dy}} + b_{g13}^{\mathrm{dy}} d_{f11}^{\mathrm{sh}} c_{g11}^{\mathrm{sh}}, \tag{25x}$$

$$a_{83} = b_{g12}^{\mathrm{dy}} + b_{g13}^{\mathrm{dy}} d_{f12}^{\mathrm{sh}} c_{g22}^{\mathrm{sh}},\ a_{84} = a_{g11}^{\mathrm{dy}} + b_{g13}^{\mathrm{dy}} d_{f14}^{\mathrm{sh}} c_{g11}^{\mathrm{dy}} + b_{g14}^{\mathrm{dy}} d_{g11}^{\mathrm{sg}} c_{g11}^{\mathrm{dy}}, a_{85} = a_{g12}^{\mathrm{dy}} + b_{g13}^{\mathrm{dy}} d_{f14}^{\mathrm{sh}} c_{g22}^{\mathrm{dy}} + b_{g14}^{\mathrm{dy}} d_{g12}^{\mathrm{sg}} c_{g22}^{\mathrm{dy}}, a_{86} = b_{g13}^{\mathrm{dy}} d_{f11}^{\mathrm{sh}} (d_{g11}^{\mathrm{sh}} + d_{g21}^{\mathrm{sh}}), \tag{25y}$$

$$a_{87} = b_{g14}^{\mathrm{dy}} c_{g11}^{\mathrm{sg}},\ a_{88} = b_{g23}^{\mathrm{dy}} d_{f11}^{\mathrm{sh}} (d_{g12}^{\mathrm{sh}} + d_{g22}^{\mathrm{sh}}),\ a_{89} = b_{g23}^{\mathrm{dy}} d_{f14}^{\mathrm{sh}} (d_{g11}^{\mathrm{dy}} + d_{g21}^{\mathrm{dy}}) + b_{g24}^{\mathrm{dy}} d_{g11}^{\mathrm{sg}} d_{g11}^{\mathrm{dy}} + b_{g24}^{\mathrm{dy}} d_{g12}^{\mathrm{sg}} d_{g21}^{\mathrm{dy}},\ a_{a3} = b_{g11}^{\mathrm{sg}} (d_{g11}^{\mathrm{dy}} + d_{g21}^{\mathrm{dy}}), \tag{25z}$$

$$a_{90} = b_{g21}^{\mathrm{dy}} + b_{g23}^{\mathrm{dy}} d_{f11}^{\mathrm{sh}} c_{g11}^{\mathrm{sh}},\ a_{91} = b_{g22}^{\mathrm{dy}} + b_{g23}^{\mathrm{dy}} d_{f12}^{\mathrm{sh}} c_{g22}^{\mathrm{sh}},\ a_{92} = a_{g21}^{\mathrm{dy}} + b_{g23}^{\mathrm{dy}} d_{f14}^{\mathrm{sh}} c_{g11}^{\mathrm{dy}} + b_{g24}^{\mathrm{dy}} d_{g11}^{\mathrm{sg}} c_{g11}^{\mathrm{dy}},\ a_{a4} = b_{g11}^{\mathrm{sg}} c_{g11}^{\mathrm{dy}}, \tag{25aa}$$

$$a_{93} = a_{g22}^{\mathrm{dy}} + b_{g23}^{\mathrm{dy}} d_{f14}^{\mathrm{sh}} c_{g22}^{\mathrm{dy}} + b_{g24}^{\mathrm{dy}} d_{g12}^{\mathrm{sg}} c_{g22}^{\mathrm{dy}} - w_{\mathrm{ads}},\ a_{94} = b_{g23}^{\mathrm{dy}} d_{f11}^{\mathrm{sh}} \left(d_{g11}^{\mathrm{sh}} + d_{g21}^{\mathrm{sh}} \right),\ a_{95} = b_{g24}^{\mathrm{dy}} c_{g11}^{\mathrm{sg}},\ a_{96} = -d_{g1}^{\mathrm{pem}} d_{g12}^{\mathrm{so}},\ a_{a5} = b_{g12}^{\mathrm{sg}} c_{g22}^{\mathrm{dy}}, \tag{25ab}$$

$$a_{97} = -d_{g2}^{\mathrm{pem}} d_{g22}^{\mathrm{sh}} - d_{g32}^{\mathrm{sh}},\ a_{98} = d_{g1}^{\mathrm{pem}} c_{g11}^{\mathrm{so}},\ a_{99} = -d_{g2}^{\mathrm{pem}} c_{g22}^{\mathrm{sh}} - c_{g33}^{\mathrm{sh}},\ a_{a1} = -d_{g1}^{\mathrm{pem}} d_{g11}^{\mathrm{so}},\ a_{a2} = -d_{g2}^{\mathrm{pem}} d_{g21}^{\mathrm{sh}} - d_{g31}^{\mathrm{sh}},\ a_{a6} = (d_{g11}^{\mathrm{dy}} + d_{g21}^{\mathrm{dy}}) \tag{25ac}$$

The output of the of the current controller is the voltage appearing on the inverter terminals directly which is given as follows neglecting the impact of the switching

$$v_{\mathrm{id}}^{*} = -\omega^{\mathrm{nom}} L_f i_{\mathrm{lq}} + k_{\mathrm{pc}}^{ac} (i_{\mathrm{ld}}^{\mathrm{ref}} - i_{\mathrm{ld}}) + k_{\mathrm{ic}}^{ac} \gamma_{\mathrm{d}}, \tag{31a}$$

$$v_{\mathrm{iq}}^{*} = \omega^{\mathrm{nom}} L_f i_{\mathrm{ld}} + k_{\mathrm{pc}}^{ac} (i_{\mathrm{lq}}^{\mathrm{ref}} - i_{\mathrm{lq}}) + k_{\mathrm{ic}}^{ac} \gamma_{\mathrm{q}}, \tag{31b}$$

where L_f is the LC filter inductance interfacing the PEMEL with the weak grid. The correlation between the reference signal v_{idq}^{*} and the physical output v_{idq}^{*} is described in the main manuscript and interconnects the previously explained controller with the physical dynamics of the LC filter interfacing the unit to the weak grid. Lastly, the LC filter dynamics along with the coupling transformer between the hydrogen generation unit and the point of common coupling are described as follows

$$\frac{di_{\mathrm{ld}}}{dt} = -\frac{R_f}{L_f} i_{\mathrm{ld}} + \omega i_{\mathrm{lq}} + \frac{1}{L_f} v_{\mathrm{id}} - \frac{1}{L_f} v_{\mathrm{od}}, \tag{32a}$$

$$\frac{di_{\mathrm{lq}}}{dt} = -\frac{R_f}{L_f} i_{\mathrm{lq}} - \omega i_{\mathrm{ld}} + \frac{1}{L_f} v_{\mathrm{iq}} - \frac{1}{L_f} v_{\mathrm{oq}}, \tag{32b}$$

$$\frac{dv_{\mathrm{od}}}{dt} = \omega v_{\mathrm{oq}} + \frac{1}{C_f} i_{\mathrm{ld}} - \frac{1}{C_f} i_{\mathrm{od}}, \tag{32c}$$

$$\frac{dv_{\mathrm{oq}}}{dt} = -\omega v_{\mathrm{od}} + \frac{1}{C_f} i_{\mathrm{lq}} - \frac{1}{C_f} i_{\mathrm{oq}}, \tag{32d}$$

$$\frac{di_{\mathrm{od}}}{dt} = -\frac{R_c}{L_c} i_{\mathrm{od}} + \omega i_{\mathrm{oq}} + \frac{1}{L_c} v_{\mathrm{od}} - \frac{1}{L_c} v_{\mathrm{bd}}, \tag{32e}$$

$$\frac{di_{\mathrm{oq}}}{dt} = -\frac{R_c}{L_c} i_{\mathrm{oq}} - \omega i_{\mathrm{od}} + \frac{1}{L_c} v_{\mathrm{oq}} - \frac{1}{L_c} v_{\mathrm{bq}}, \tag{32f}$$

where R_f, R_c, and L_c are the filter series resistance, coupling line resistance, and coupling line inductance, respectively. v_{bd} and v_{bq} are the point of common coupling voltage components in the DG individual reference frame. A virtual resistance R_n is assumed at the point of common coupling to absorb the net current at the node to facilitate the establishment of SSM. The virtual resistance net current includes the current drawn by the inverter connected to the electrolyzer unit, the current from the microgrid equivalent source, and the current consumed by the weak grid load. Selecting a sufficiently large value for this virtual resistance enhances modeling accuracy. The corresponding formulation is given as follows

$$v_{bd} = R_n (i_{sd} + i_{od} - i_{gd}), \tag{33a}$$

$$v_{bq} = R_n (i_{sq} + i_{oq} - i_{gq}). \tag{33b}$$

III. LINEARIZATION OF DC/AC INVERTER AND ITS CONTROL LOOPS

The linearization of the power controller at the AC side in equations (26) and (27) can be given as follows

$$\begin{bmatrix}\dot{P}\\ \dot{Q}\end{bmatrix} = A_P \begin{bmatrix}P\\ Q\end{bmatrix} + B_P \begin{bmatrix}i_{\mathrm{ldq}}\\ v_{\mathrm{odq}}\\ i_{\mathrm{odq}}\end{bmatrix}, \tag{34a}$$

$$\begin{bmatrix}\omega\\ v^*_{\mathrm{odq}}\end{bmatrix} = \begin{bmatrix}C_{\mathrm{P}\omega}\\ C_{\mathrm{Pvi}}\end{bmatrix}\begin{bmatrix}P\\ Q\end{bmatrix} + \begin{bmatrix}D_{\mathrm{P}\omega}\\ D_{\mathrm{Pv}}\end{bmatrix}\begin{bmatrix}i_{\mathrm{ldq}}\\ v_{\mathrm{odq}}\\ i_{\mathrm{odq}}\end{bmatrix}, \tag{34b}$$

$$A_{\mathrm{P}} = \begin{bmatrix}-\omega_c & 0\\ 0 & -\omega_c\end{bmatrix}, B_{\mathrm{P}} = \omega_c \begin{bmatrix}0\,0 & \overline{I}_{\mathrm{od}} & \overline{I}_{\mathrm{oq}} & \overline{V}_{\mathrm{od}} & \overline{V}_{\mathrm{oq}}\\ 0\,0 & -\overline{I}_{\mathrm{oq}} & \overline{I}_{\mathrm{od}} & \overline{V}_{\mathrm{oq}} & -\overline{V}_{\mathrm{od}}\end{bmatrix}, \tag{34c}$$

$$C_{\mathrm{P}\omega} = \begin{bmatrix}-m + m_d\omega_c\, 0\end{bmatrix}, C_{\mathrm{Pv}} = \begin{bmatrix}0 & -n + n_d\omega_c\\ 0 & 0\end{bmatrix}, \tag{34d}$$

$$D_{\mathrm{P}\omega} = -\omega_c m_d \begin{bmatrix}0\,0\,\overline{I}_{\mathrm{od}}\,\overline{I}_{\mathrm{oq}}\,\overline{V}_{\mathrm{od}}\,\overline{V}_{\mathrm{oq}}\end{bmatrix}, \tag{34e}$$

$$D_{\mathrm{Pv}} = \omega_c n_d \begin{bmatrix}0\,0 & \overline{I}_{\mathrm{oq}} & -\overline{I}_{\mathrm{od}} & -\overline{V}_{\mathrm{oq}} & \overline{V}_{\mathrm{od}}\\ 0\,0 & 0 & 0 & 0 & 0\end{bmatrix}. \tag{34f}$$

The linearization of the voltage controller in equations (28) and (29) can be given as follows

$$\begin{bmatrix}\dot{\phi}_{\mathrm{d}}\\ \dot{\phi}_{\mathrm{q}}\end{bmatrix} = \begin{bmatrix}0\,0\\ 0\,0\end{bmatrix}\begin{bmatrix}\phi_{\mathrm{d}}\\ \phi_{\mathrm{q}}\end{bmatrix} + B_{V1}\begin{bmatrix}v^*_{\mathrm{od}}\\ v^*_{\mathrm{oq}}\end{bmatrix} + B_{V2}\begin{bmatrix}i_{\mathrm{ldq}}\\ v_{\mathrm{odq}}\\ i_{\mathrm{odq}}\end{bmatrix}, \tag{35a}$$

$$\begin{bmatrix}i^*_{\mathrm{ld}}\\ i^*_{\mathrm{lq}}\end{bmatrix} = C_V\begin{bmatrix}\phi_{\mathrm{d}}\\ \phi_{\mathrm{q}}\end{bmatrix} + D_{V1}\begin{bmatrix}v^*_{\mathrm{od}}\\ v^*_{\mathrm{oq}}\end{bmatrix} + D_{V2}\begin{bmatrix}i_{\mathrm{ldq}}\\ v_{\mathrm{odq}}\\ i_{\mathrm{odq}}\end{bmatrix}, \tag{35b}$$

$$B_{V1} = \begin{bmatrix}1\,0\\ 0\,1\end{bmatrix}, B_{V2} = \begin{bmatrix}0\,0 & -1 & 0 & 0\,0\\ 0\,0 & 0 & -1 & 0\,0\end{bmatrix}, C_V = \begin{bmatrix}k^{ac}_{\mathrm{iv}} & 0\\ 0 & k^{ac}_{\mathrm{iv}}\end{bmatrix}, \tag{35c}$$

$$D_{V1} = \begin{bmatrix}k^{ac}_{\mathrm{pv}} & 0\\ 0 & k^{ac}_{\mathrm{pv}}\end{bmatrix}, D_{V2} = \begin{bmatrix}0\,0 & -k^{ac}_{\mathrm{pv}} & -\omega_n C_f & H & 0\\ 0\,0 & \omega_n C_f & -k^{ac}_{\mathrm{pv}} & 0 & H\end{bmatrix} \tag{35d}$$

The linearization of the current controller in equations (30) and (31) can be given as follows

$$\begin{bmatrix}\dot{\gamma}_{\mathrm{d}}\\ \dot{\gamma}_{\mathrm{q}}\end{bmatrix} = \begin{bmatrix}0\,0\\ 0\,0\end{bmatrix}\begin{bmatrix}\gamma_{\mathrm{d}}\\ \gamma_{\mathrm{q}}\end{bmatrix} + B_{C1}\begin{bmatrix}i^*_{\mathrm{ld}}\\ i^*_{\mathrm{lq}}\end{bmatrix} + B_{C2}\begin{bmatrix}i_{\mathrm{ldq}}\\ v_{\mathrm{odq}}\\ i_{\mathrm{odq}}\end{bmatrix}, \tag{36a}$$

$$\begin{bmatrix}v^*_{\mathrm{id}}\\ v^*_{\mathrm{iq}}\end{bmatrix} = C_C\begin{bmatrix}\gamma_{\mathrm{d}}\\ \gamma_{\mathrm{q}}\end{bmatrix} + D_{C1}\begin{bmatrix}i^*_{\mathrm{ld}}\\ i^*_{\mathrm{lq}}\end{bmatrix} + D_{C2}\begin{bmatrix}i_{\mathrm{ldq}}\\ v_{\mathrm{odq}}\\ i_{\mathrm{odq}}\end{bmatrix}, \tag{36b}$$

$$B_{C1} = \begin{bmatrix}1\,0\\ 0\,1\end{bmatrix}, B_{C2} = \begin{bmatrix}-1 & 0 & 0000\\ 0 & -1 & 0000\end{bmatrix}, C_C = \begin{bmatrix}k^{ac}_{\mathrm{ic}} & 0\\ 0 & k^{ac}_{\mathrm{ic}}\end{bmatrix}, \tag{36c}$$

$$D_{C1} = \begin{bmatrix}k^{ac}_{\mathrm{pc}} & 0\\ 0 & k^{ac}_{\mathrm{pc}}\end{bmatrix}, D_{C2} = \begin{bmatrix}-k^{ac}_{\mathrm{pc}} & -\omega_n L_f & 0000\\ \omega_n L_f & -k^{ac}_{\mathrm{pc}} & 0000\end{bmatrix} \tag{36d}$$

The linearization of the LCL subsystem in equations (32a)-(32f), (33), and I-(79) can be given as follows

$$\begin{bmatrix}\dot{i}_{\mathrm{ldq}}\\ \dot{v}_{\mathrm{odq}}\\ \dot{i}_{\mathrm{odq}}\end{bmatrix} = A_{LCL}\begin{bmatrix}i_{\mathrm{ldq}}\\ v_{\mathrm{odq}}\\ i_{\mathrm{odq}}\end{bmatrix} + B_{LCL1}\begin{bmatrix}v^*_{\mathrm{id}}\\ v^*_{\mathrm{iq}}\end{bmatrix} + B_{LCL2}\begin{bmatrix}i_{\mathrm{sd}}\\ i_{\mathrm{sq}}\end{bmatrix}, \tag{37a}$$
$$+B_{LCL3}\omega + B_{LCL4}\begin{bmatrix}E^{cel}\\ E_{DC}\end{bmatrix} + B_{LCL5}\begin{bmatrix}i^{gd}\\ i^{gq}\end{bmatrix}$$

$$A_{LCL} = \begin{bmatrix}-\frac{R_f}{L_f} & \omega_o & -\frac{1}{L_f} & 0 & 0 & 0\\ -\omega_o & -\frac{R_f}{L_f} & 0 & -\frac{1}{L_f} & 0 & 0\\ \frac{1}{C_f} & 0 & 0 & \omega_o & -\frac{1}{C_f} & 0\\ 0 & \frac{1}{C_f} & -\omega_o & 0 & 0 & -\frac{1}{C_f}\\ 0 & 0 & \frac{1}{L_c} & 0 & -\frac{R_c}{L_c} - \frac{r_N}{L_c} & \omega_o\\ 0 & 0 & 0 & \frac{1}{L_c} & -\omega_o & -\frac{R_c}{L_c} - \frac{r_N}{L_c}\end{bmatrix} \tag{37b}$$

$$B_{LCL1} = \begin{bmatrix}\frac{1}{L_f} & 0\\ 0 & \frac{1}{L_f}\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix} B_{LCL2} = \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ -\frac{r_N}{L_c} & 0\\ 0 & -\frac{r_N}{L_c}\end{bmatrix}, \tag{37c}$$

$$B_{LCL3} = \begin{bmatrix}\overline{I}_{\mathrm{lq}}\\ -\overline{I}_{\mathrm{ld}}\\ \overline{V}_{\mathrm{oq}}\\ -\overline{V}_{\mathrm{od}}\\ \overline{I}_{\mathrm{oq}}\\ -\overline{I}_{\mathrm{od}}\end{bmatrix}, B_{LCL4} = \begin{bmatrix}0 & \frac{1}{L_f}\frac{\bar{v}_{\mathrm{id}}}{E_{DC}}\\ 0 & \frac{1}{L_f}\frac{\bar{v}_{\mathrm{id}}}{E_{DC}}\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}, \tag{37d}$$

$$B_{LCL5} = \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ \frac{r_N}{L_c} & 0\\ 0 & \frac{r_N}{L_c}\end{bmatrix}, \tag{37e}$$

Lastly, the linearization of the weak grid dynamics from equations (33) and I-(81) can be given as follows

$$\begin{bmatrix}\dot{i}^{gd}\\ \dot{i}^{gq}\end{bmatrix} = A_{Load}\begin{bmatrix}i^{gd}\\ i^{gq}\end{bmatrix} + B_{Load1}\begin{bmatrix}i_{\mathrm{ldq}}\\ v_{\mathrm{odq}}\\ i_{\mathrm{odq}}\end{bmatrix}, \tag{38a}$$
$$+B_{Load2}\omega + B_{Load3}\begin{bmatrix}i_{\mathrm{sd}}\\ i_{\mathrm{sq}}\end{bmatrix}$$

$$A_{Load} = \begin{bmatrix}-\frac{R_g}{L_g} - \frac{r_N}{L_g} & \omega_o\\ -\omega_o & -\frac{R_g}{L_g} - \frac{r_N}{L_g}\end{bmatrix} \tag{38b}$$

$$B_{Load} = \begin{bmatrix} 0\,0\,0\,0\,\frac{r_N}{L_g} & 0 \\ 0\,0\,0\,0\;\;0 & \frac{r_N}{L_g} \end{bmatrix} \tag{38c}$$

$$B_{Load} = \begin{bmatrix} \overline{I}_{\mathrm{gq}} \\ -\overline{I}_{\mathrm{gd}} \end{bmatrix}, B_{Load} = \begin{bmatrix} \frac{r_N}{L_g} & 0 \\ 0 & \frac{r_N}{L_g} \end{bmatrix}, \tag{38d}$$

The details of the equation I-(83b) can be given as follows

$$C_p^{il} = \left[\overline{I}_{ld}\,\overline{I}_{lq}\right] \tag{39a}$$

$$C_p^{ed} = \begin{bmatrix} \frac{\overline{V}_{id}}{\overline{E}_{DC}} \\ \frac{\overline{V}_{iq}}{\overline{E}_{DC}} \end{bmatrix} \tag{39b}$$

$$C_p^{c} = \left[\overline{V}_{id}\,\overline{V}_{iq}\,0\,0\,0\,0\right] \tag{39c}$$