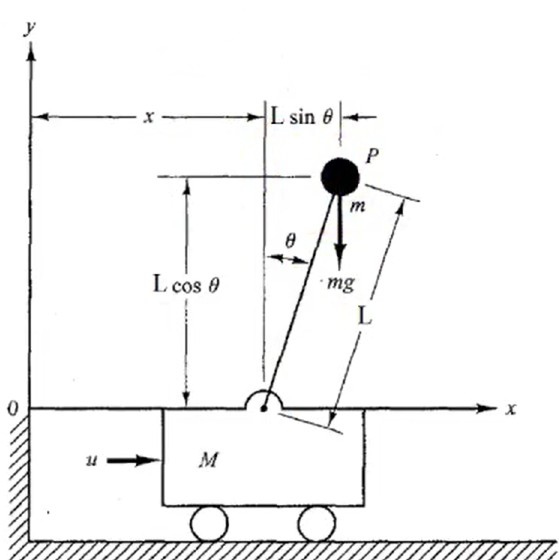
**Lagrangian Calculation:**

****

**Part 1: Kinetic Energy(T)**

1. **Pendulum:**

And we have:

1. **Cart:** The cart has kinetic energy due to its horizontal motion

**Thus, the total kinetic energy is equal to:**

**Part 2: Potential Energy(V)**

1. **Pendulum:** The pendulum has gravitational potential energy.
2. **Cart:** the cart has no potential energy

**Thus, the total potential energy is equal to:**

**Now the Lagrangian can be computed as follows:**

**As we know, the Euler Lagrange Equation is as follows:**

Where, generally represents external forces.

Here, since we have two degrees of freedom, we have to solve the Euler-Lagrange equations twice.

1. **For (cart’s horizontal position)**
2. **Computing :**
3. **Computing :**
4. **Final Equation:**
5. **For (pendulum angle)**
6. **Computing :**
7. **Computing :**
8. **Final Equation:**

**Finally, as the Euler-Lagrange model we have:**

**Solving for and we get:**

**To turn it into the state space form, we define the state variables as:**

**Having:**

And as said in the exercise instructions, the model parameters are:

s

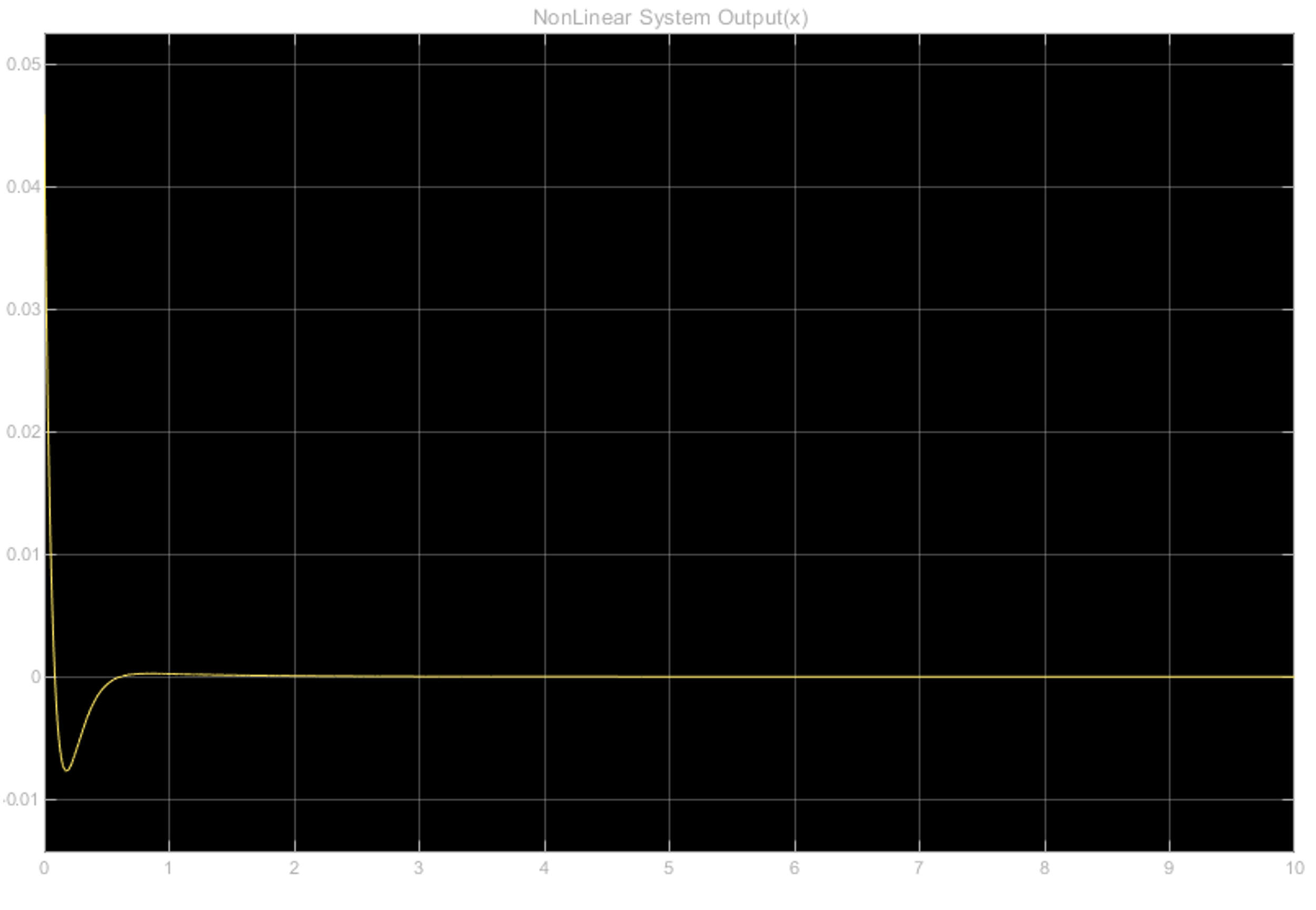
**After this, the system is linearized by calculating its Jacobian and substituting the equilibrium point (which is 90 degrees because the pendulum axis is assumed to be upwards).**

**Then a PID Controller is designed using MATLAB PIDTuner and applied as a regulator to keep the system at zero.**

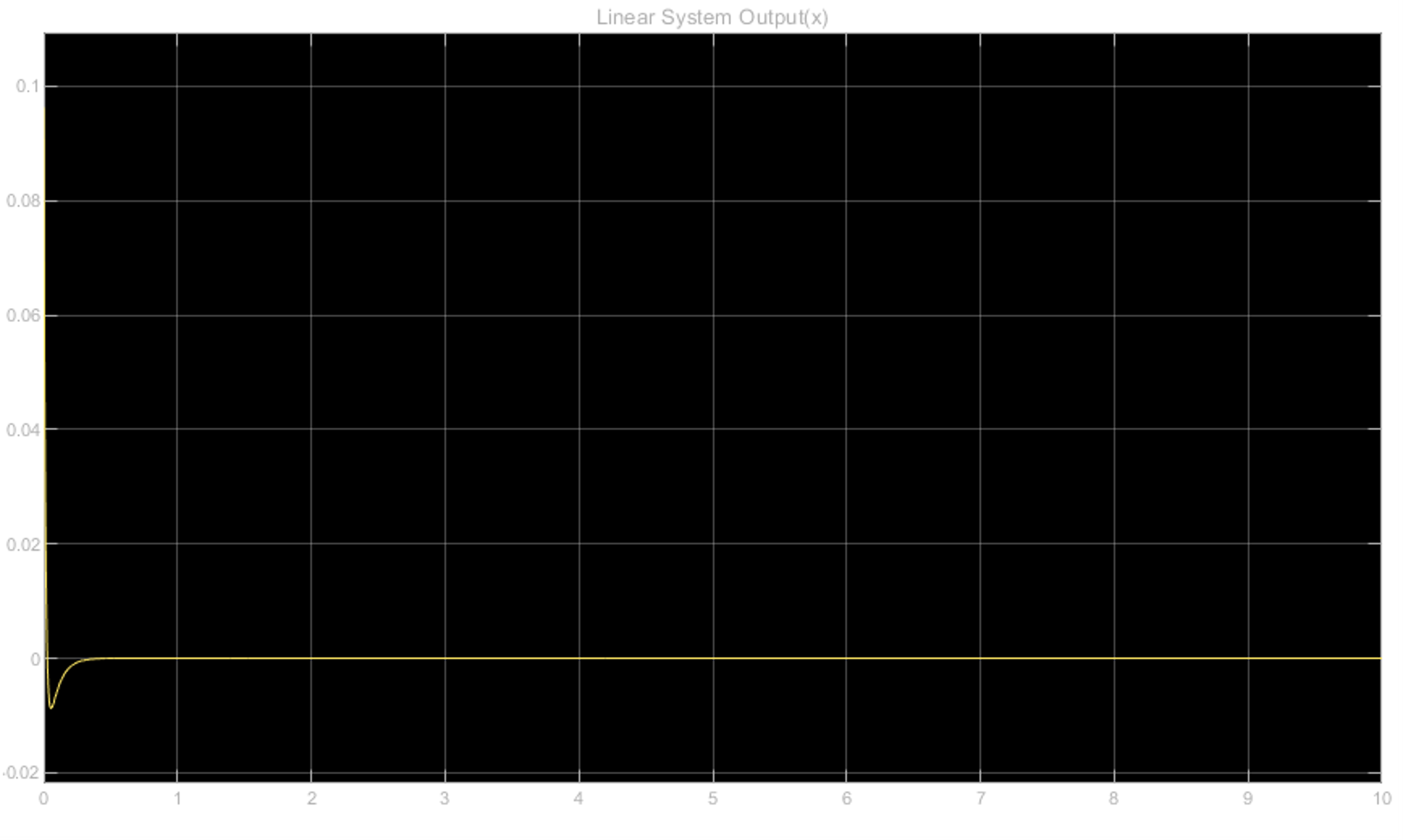
**Everything is done in MATLAB using Symbolic Math Toolbox and the system in simulated in Simulink.**

**And the results are as follows:**

**Non-Linear System Model:**

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**Linearized System:**

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