Income Inequality & Technology: What can Agent-Based Models Tell Us?

Becky R. Haney[1], Loren Haarsma[2], Anthony J. Ditta

Abstract

Several empirical studies have noted that when looking within and across countries, there appears to be an inverted U-shaped relationship between income and inequality. This empirical regularity was first noted by Simon Kuznets (1955), and thus is often called the Kuznets Curve. A long line of empirical research has attempted to determine if and how inequality relates to income, particularly because of its importance to understanding economic development in poor countries. Barro (2000) theorizes that a third variable, technology, may be an underlying force behind both income and inequality, and that the Kuznets curve is the result of a technological impulse in the economy. This paper explores this theory using an agent-based models of technology in economies: technical innovations are available to only a few agents at first but eventually become widely used; old technologies get adapted into new innovations; and the economy becomes more interdependent as trade in technological devices and their components increases. In this paper, agents are self-interested utility-maximizers who extract, consume, and invent. Through trade and technology the economy grows from a group of isolated, self-sustaining, resource-poor agents to a heavily interdependent society with a high degree of specialization, technological sophistication, and income. This controlled environment demonstrates that inequality and income can rise together immediately following a significant innovation, but as the new technology diffuses throughout the economy, income continues to rise as inequality falls. Importantly, the steady state level of inequality grows higher as technology and interdependence rise. These results support Barro's theory and suggest that observed Kuznets curves in real economies may be responses to technological impulses. The software is available as open source under the GPL license and is available for download, replication, and extension by the research community.

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• Introduction

A persistent empirical fact first noticed by Simon Kuznets (1955) is that within and across countries, rises in inequality precede periods of economic growth. The inequality eventually dies back down while growth continues, thus creating an inverted U-shaped curve in the relationship between inequality and income, the so-called Kuznets Curve.[3] Several decades of research have endeavored to determine if this is a true relationship, and if so, what form the relationship between income and inequality takes. At stake are implications for policies related to economic development. (Deininger and Squire 1998; García-Peñalosa and Turnovsky 2007) Although the data needed to examine this question have greatly improved and empirical methods to examine this relationship have grown increasingly more sophisticated, ultimately the empirical analysis has produced conflicting results. (Deininger & Squire, 1996; Barro, 2000; Banerjee & Duflo, 2003; Atolia, Chatterjee, & Turnovsky, 2012)

One problem with empirical analysis of such a complex set of relationships is that even if inequality always precedes growth, it could be the case that both of these are actually caused by a third preceding event that is not easily modeled, a form of omitted variable bias. Barro (2000) suggests three potential omitted variables that might result in a Kuznets-type effect on inequality: demographic shifts, financial innovations, technological innovations. Demographic shifts can result in an empirical Kuznets curve as an economy begins to shift people and resources from agricultural to industrial sectors. Inequality initially rises, but then dies down after the majority of the shift has taken place. Similarly, as a society moves from simple to more complex financial instruments inequality initially rises, but again dies down once that shift becomes widespread.

His third theory relates to technological innovation. The invention of new technologies, such as the assembly line, electricity, and computerization, greatly increase output and incomes for the select few who are first able to move into those sectors. This causes income and inequality to rise. As those technologies become more commonplace, income continues to rise, but inequality falls. He concludes that his and others' conflicting empirical results may be a result of "a dynamic effect whereby the adoption of each type of new technology

[3] Similarly, the argument has been made that the relationship between economic growth and environmental quality is an inverted U as well. As incomes grow, demand for higher air quality and preservation of environmental goods and services emerge. This relationship has been called the "environmental Kuznet's curve."(See, for example, Dasgupta & Maler, 1994)

has a transitory, Kuznets-type effect on the distribution of income. This perspective explains not only the statistical significance of the standard Kuznets relation but also its relatively poor fit." (2000, 29) Unfortunately, an empirical examination of the relationship between income and inequality in the presence of technological innovation is difficult at best, and there exists little empirical research in the last decade to examine this hypothesis.

A recent theoretical study, however, bears out Barro's theory. Atolia, Chatterjee, and Turnovsky (2012) use a dynamic general equilibrium approach and produce numerical simulations that explore the relationship between income, inequality, and productivity. This approach bypasses data issues by positing analytical relationships between a few key macroeconomic variables and running simulations to trace out growth paths. Following standard dynamic modeling techniques, they consider a decentralized economy with households and a single representative firm. Households are assumed to start with different endowments of capital but are identical in every other way. Households choose how much to work and consume in a way to maximize utility over their lifetime, subject to their initial capital endowment.

Productivity enters into the framework through a standard production function for the representative firm. Productivity and population grow at prescribed rates and the economy settles into a dynamic equilibrium. That is, the growth rates of capital, labor and productivity converge to a steady state. The steady-state equilibrium of their model does not depend on the speed of technological change and can be solved mathematically. However, to examine the intermediate paths that economies take on their way to the steady-state requires numerical simulations. (See also Nelson and Winter 1985, 209) They show how under the simple conditions of their model, a Kuznets curve can be seen as a response in inequality to an impulse in productivity. They use their numerical simulations to demonstrate that even though two economies end up in the dynamic equilibrium, the amount of inequality an economy must pass through on its way there is sensitive to whether the productivity shock is introduced gradually or instantly. Thus, numerical simulations are particularly helpful for shedding light on the Kuznets curve. Following standard exogenous growth models, Atolia et al., have one representative firm and model productivity, or technological innovation, as an exogenous parameter in the production function. Inequality results from differences in initial endowments of capital, not from differences in technology. Technology enters as an improvement in productivity that has a steady growth rate and is instantly diffused across all agents.

While these analytical models serve well to analyze aggregate growth and substitutability between capital and labor, they provide less insight into the process of technological change. (Nelson and Winter 1985, 206) They stress equilibrium conditions, and ignore the time paths of technological innovation or treat them *ad hoc*. (Nelson and Winter 1985, 209) In actual economies, technological progress is uneven, across agents and over time. It is partly exogenous (the source of inspiration for a new invention is not always clear) and partly endogenous (previous investments in human capital, such as learning-by-doing, create a more fertile ground for inspirations to come about). Thus, in order to explore these effects which are not included in the model of Atolia et al., this paper uses an agent-based model to examine how impulses in technology and both endogenous and exogenous impulses in productivity, rather than steady growth, affect income and inequality.

Agent-based models have been shown to be particularly powerful tools for exploring technological change. (Dawid 2006) Using an agent-based model, we explore technological change and its relationship to interdependence, wealth and inequality. Barro's theory is born out in the results of our agent-based simulation. We observe a rise in both income and inequality upon realization of technological invention. After the technology has had time to diffuse, income continues to rise, but inequality (as measured by the Gini coefficient) dies down. The initial gains are concentrated in the lucky few who initially discover the technology. However, as more agents trade with those technologically advanced agents and the technological improvements become commonplace, inequality falls.

Our model also uncovers an additional feature of the relationship between inequality and technology. As technology becomes more sophisticated, inequality may decline after the initial impulse, but it settles down to a higher level. In other words, it does not fully self-correct. This paper will argue that this phenomenon can be understood as a result of the increasing interdependence between agents that develops as technological sophistication increases. Modern economies are highly interdependent; each resource is produced by the use of several other resources, which are then used in the production of others, and so on. An entire economy can crash with the removal of one key resource, such as oil. Our agent-based model includes this interdependence.

Interdependence creates feedback loops; when an exogenous shock hits an economy, it can lead to unexpected and non-self-correcting behaviors (Lengnick 2013). In 2008 the financial crisis provided a dramatic illustration of the level of interdependence and presence of feedback loops present in the global economy. Nearly all of the major banks in the world teetered on the verge of collapse. Before this, few understood how deeply interdependent the global financial institutions were; even fewer predicted the far-reaching extent of the damage. The difficulty of predicting these events may stem from the increasingly interdependent nature of the global economy. Agent-based computational economic models such as ours may provide helpful insights into the both the benefits and vulnerabilities of such an interdependent economy. (Lengnick 2013; Lengnick and Wohltmann 2012; Rajiv 2010; "Economics Focus: Agents of Change" *The Economist* 2010)

Interdependence creates reinforcing feedback loops in an economy. This results in non-linear economic growth paths. This type of economic growth model is outside the predictive capacity of the standard dynamic stochastic general equilibrium models, and this is precisely where agent-based computational economic models may provide useful insights. (Lengnick 2013) Describing how technological innovation produces interdependence like that of modern economies, and how this relates to income and inequality, is one of the main contributions of our model.

Rather than building high levels of technology and interdependence in our model from the start, our model starts with a simple society and then endows agents with the capacity to invent and trade technology. Initially, each agent is self-sufficient and extracts all the resources it needs without trade or tools (even simpler than a hunter-gather society). Over time, the society in our model grows into a technologically sophisticated, interdependent economy in which agents specialize, invent increasingly higher order devices (devices made of other devices, which are made of other devices, and so on) to speed up resource extraction, and spend most of their time producing and trading these devices. Simpler devices are gradually repurposed from resource extraction devices into becoming components of more complex devices, mimicking the combinatorial evolution and "exaptation" processes of real-world technological innovation. (Nelson and Winter 1985; Dew, Sarasvathy, and Venkataraman 2004; Arthur 2009) Each agent in this model is in some ways analogous to a single human. But because each agent is infinitely long-lived, does not reproduce, and initially is completely self-sufficient, each agent might also represent a village, nation, or firm. In the following sections, we describe the model and examine the emergence of technology, interdependence, income and inequality. Interdependence turns out to be a key variable in determining income and inequality, but few empirical measure of the concept of "interdependence" are available in the literature. Thus, a second contribution of this paper is to describe what economic interdependence is, as well as explore possible measures of interdependence.

- ## Description of the Model

Our model consists of four elements: agents, resources, devices and trade.

Agents. The initial conditions of the model set agents to be self-interested, rational actors in the following ways. They are perfectly healthy, non-altruistic, diligent in working, and able to switch between tasks with no transaction costs. They calculate costs and benefits to decide which resource to extract or which device to make. All decisions are based on which use of time gives them the largest gain. They are all honest and equally good bargainers so that they have no strategic or information advantage over each other in trade.

However, their rationality is bounded. They have limited memory and only a certain amount of foresight. They do not anticipate gains from trade when deciding which resource to extract. They can only communicate pair-wise during a potential trading transaction. The only information they can use for trading resources is the value they put on each resource. The only additional information they use for purchasing or selling devices is the gain-per-minute (GPM) they achieved from extracting resources at the end of the previous day, and the prices they remember from the last few days of trades. Agents are not able to anticipate gains from future trades when calculating the benefit of producing a device; they only consider the benefit it provides them in extracting resources for themselves. All of these parameters act as "dials" that can be adjusted to create alternative societies for comparison.

Resources. Resources in our model are abstractions for things such as food, water, shelter, leather, dye, etc. Resources are infinitely obtainable in this paper. Each resource is assigned a utility curve which reflects its value to the agents. Agents consume a constant proportion of each of their resources each day, thus agents' consumption grows at the same rate as their income.

Devices. Devices are inventions which speed the extraction of resources or speed the production of simpler devices. Initially, agents don't know how to make any devices. As agents gain experience in extracting a resource, they become increasingly likely to invent a "tool" for it: a combination of units of several different resources which speeds the extraction of the initial resource. Similarly, as agents gain experience with tools they become increasingly likely to invent a "machine": a combination of tools. Factories and industries are similarly invented out of the next lower-order device. Tool-making machines and machine-making factories are invented in highly evolved societies to speed production of lower order devices which are then combined to produce the higher order devices.

Trade. Trade happens daily in decentralized markets as agents meet each other in pairs and barter to determine if they can make mutually beneficial exchanges. The intuition behind simple, bi-lateral trade is described by Kreps (1990, 196):

> we can imagine consumers wandering around a large market square, with all their possessions on their backs. They have chance meetings with each other, and when two consumers meet, they examine what each has to offer, to see if they can arrange a mutually agreeable trade…. If an exchange is made, the two swap goods and wander around in search of more advantageous trades made at chance meetings.

Bi-lateral trade is the norm in agent-based computational economics. It allows for macroeconomic phenomenon to emerge as a result of individual agents' procurement decisions rather than as a result of top-down control. (Tesfatsion 2006) Thus, this model provides a plausible explanation for how the macro patterns of inequality and wealth currently observed could have emerged in the presence of technology and interdependence without centralized planning or control. This is an example of the "generative" contributions agent-based models make to scientific questions. (Epstein 2006) Gintis (2006) demonstrates that prices in de-centralized, bi-lateral trade agent-based models under a broad set of plausible, general conditions can converge to those expected in a Walrasian market.

Although our model allows us to make each resource and each agent individually unique, for this paper each resource is identical and each agent starts with identical capabilities and preferences. In addition, there are no geographic restrictions on our agents regarding resource extraction or trading partners. All of these choices initially stack the deck against the emergence of interdependence, specialization, and income inequality. When they occur in our model, they emerge not from the initial conditions of the agents and resources, but from the particular history of contingent decisions of agents regarding when to specialize and the contingent history of technological innovations discovered by the agents. (Miller and Page 2007, 70; Safarzyńska and Bergh 2009, 344)

The following sections describe the decisions and actions performed by the agents each "day." A day in the life of each agent is broken into six phases: 1. resource extraction, 2. resource trading, 3. device invention, 4. device trading, 5. device production, and 6. consumption of resources and depreciation of devices.

Phase 1: Resource Extraction

The environment contains a certain number of resources (in this paper, 24 or 48) which agents can extract. Agents have a certain number of minutes each "day" to extract resources (in this paper, 600 minutes.) Agents gain one unit of experience for a resource by extracting one unit of it, but lose three units of experience per day if they go a day without extracting it. The resource effort curve translates cumulative experience into number of minutes to extract the next unit (Fig. 1). Each agent's experience with each resource must be tracked separately. There is a maximum experience beyond which extracting additional units does not further increase experience with that resource (in this paper, 600). In principle, each agent could have a different effort curve for each resource, reflecting that each agent could have different "talents" or different "opportunities" than other agents. In this paper, we keep the effort curve the same for all agents and all resources.

Agents are rewarded for specializing because increased experience with a resource decreases the effort required to extract it.

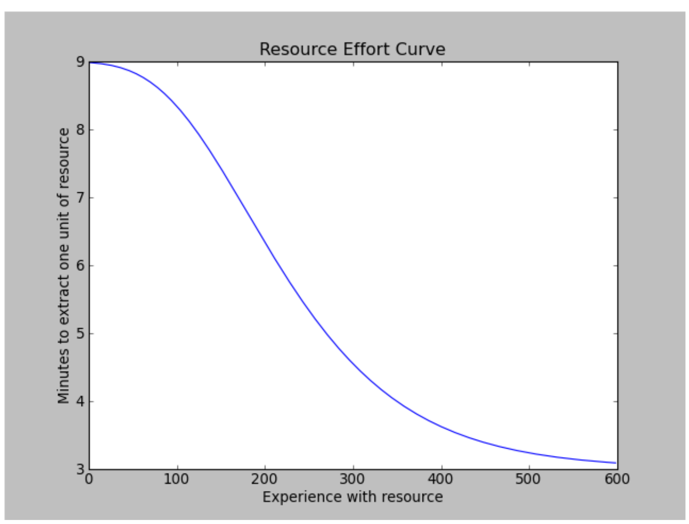

Figure 1. Resource Effort Curve

Agents value resources for the use they provide them, measured as "utility." At the end of each day, agents consume a proportion of all their holdings. Utility curves incorporate all of the knowledge agents have about the resources, including how much of it they anticipate using per day. N^{th}-root utility curves are defined as $U(x)=Dx^{1/n}$, where D indicates the utility provided by the first unit and x is the number of units owned. We restrict resources to discrete units; thus the marginal utility of the k^{th} unit is simply $U(k)-U(k-1)$. For large x, marginal utilities are approximated by $MU(x) = C * x^{(1/n)-1}$, *where* $C = D/n$. Although we do not assign resources to specific real-world analogs, we indicate that some resources are more preferred by starting them at high levels of utility (large D) and giving them slow diminishing marginal utility (small n). Figures 2 and 3 contain representative utility and marginal utility curves for six types of resources.

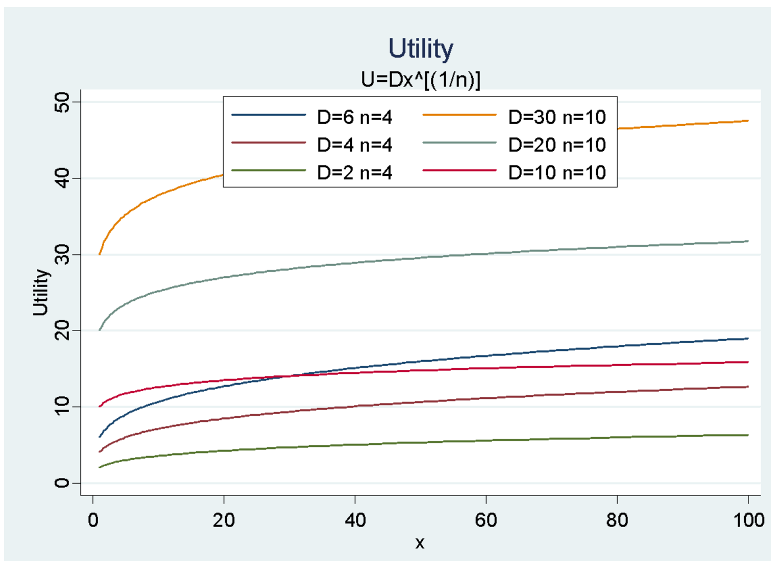

Figure 2. Representative Utility Curves

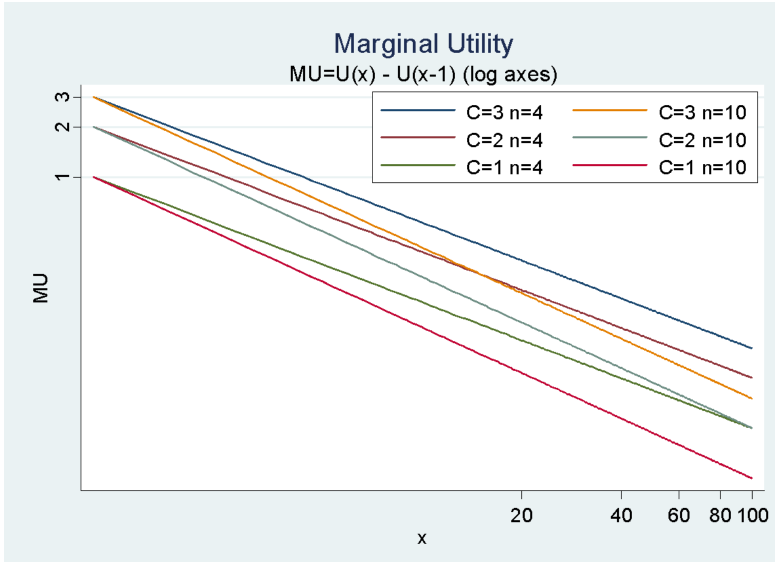

Figure 3.Representative MU

So that least preferred resources will not be ignored by agents in the model, the *MU*(*x*) of more preferred resources must fall below the *MU* of the first unit of least preferred resources at a small enough quantity *x*. This guarantees that agents will start extracting least preferred resources before they specialize too much in most preferred resources. Utility curves could vary across agents, across resources, or both. In this paper, we keep them all identical.

Agents extract resources in quantities that provide the greatest *gain per minute* (GPM), where the gain is measured in terms of utility. If an agent owns x_r units of resource r, the GPM of obtaining one more unit is:

$$\text{Gain per minute (GPM)}_r = \text{MU}(x_r+1)/e_r \qquad (1)$$

where e_r is the agent's current effort required to extract resource r. Agents calculate GPM_r for each resource at the start of the resource extraction phase of the day. Each time they extract a unit of a resource they recalculate GPM for that resource because their marginal utility and effort have changed. Agents extract the resource which gives the largest GPM. They may switch between resources frequently throughout the day to maximize the gain. Having the appropriate device for a resource reduces effort. In this paper, tools reduce effort by a factor of 3, machines by 9, factories by 27, and industries by 81.

During resource extraction, agents do not anticipate gains that they might make later during resource trading. They decide which resource to extract based only on which gives them the greatest GPM at that moment.

Phase 2: Resource Trading

Agents are randomly paired and trade multiple resources with each other before switching to trade with another agent. Strategic behavior in bartering (bluffing, low-balling, etc.) is not explored in this paper. Agents are assumed to be truthful and equally good bargainers, similar to how trade is implemented initially in similar agent-based computational economic models. When a pair of agents "meet," each agent prioritizes their resources from least valuable, the ones they are willing to trade away, to most valuable, the ones they would most like to receive. The value of a resource is simply the MU of the last unit of each resource they hold. The benefit to an agent of receiving a trade is simply the sum of the MU of the next x units of the inbound resource. Similarly, the cost of a trade is the sum of the MU of the last y units of the outbound resource. An agent agrees to exchange x units of the inbound resource for y units of the outbound resource if the benefit exceeds the cost.

When two agents are paired, each presents a portfolio of their five least valuable resources to the other. One agent is randomly assigned "first-mover" (agent A). Agent A chooses the pair of resources with the highest internal valuation, which equals the ratio of MU(inbound) / MU(outbound). Agent B then calculates its internal valuation for that pair of resources. If A and B's internal valuations indicate that there could be a mutually beneficial trade, the bargaining price is calculated as the geometric mean of A's and B's internal valuations, following Epstein and Axtell (1996, 104). Because resources are exchanged in integer units, the integer ratio closest to the bargaining price is used.

Agent A then chooses the quantities of the resources at that price ratio which will give it the largest surplus, and makes an offer. For example, if the ratio is 2:1, A will calculate its surplus for 2:1, 4:2, and so on, until it finds the maximum. This maximum is unique because of diminishing marginal utility for all resources. If B will benefit from this trade, B accepts. Otherwise B counters with quantities that will give it the largest surplus at that price. If A will benefit from this trade, the trade occurs. If not, they stop the process for that pair of resources. Then they switch roles, with B becoming the first mover, and attempt to make a trade again. A and B are each given a predetermined number of turns at being the "first mover," currently set at five. They can trade or not trade for all five attempts, after which another pair of agents is considered.

When agents calculate price ratios for proposed trades, they consider only the utilities of the resources. They do not "look ahead" to the next day's extraction phase to consider whether they are more efficient at extracting one resource or the other. Although allowing our agents such foresight would speed specialization and economic growth, we leave it out of this version of the model to show that it is not necessary for emergence of specialization and interdependence.

Phase 3: Device Invention.

Because the ability of our agents to invent new technologies depends on their experience, our agents are somewhat analogous to the firms in the models developed by Haag and Liedl (2001), Squazzoni and Boero (2002), and Nelson and Winter (1985). Those models are particularly interested in the impact of investments on innovation, so they explicitly model investment of resources into R&D but keep the inventions themselves abstract. Conversely, we are interested in tracing out the time paths of income, inequality, and interdependence that emerge as each new technology is introduced into the economy. Thus, we keep investment in R&D as simple as learning-by-doing, but we explicitly model the inventions themselves.

Resources in our model are kept abstract rather than identifying them with particular real-world resources such as shaped wood, chipped stone, or leather. Therefore, the tools and higher order devices constructed from resources in our model are also kept abstract rather than identifying them with particular real-world tools (e.g. hand-axes). Each agent is given one opportunity per day to invent a tool. An agent considers a combination of six resources: the two for which it has the most experience and four more chosen randomly from resources it holds. If tools have already been invented for all six resources considered, the agent stops. If a tool has not yet been invented by any agent for at least one of the resources, the agent attempts to invent a tool which combines one unit of each of the other five considered resources in order to make a tool for the sixth. The probability of success is the sum of the agent's experiences with the considered resources multiplied by an adjustable proportionality constant. In this paper, that constant was set so that it was very improbable that agents would invent any tools after building up experience with resources for just one day, but became fairly probable (with a probability always less than one) after ten or more days.

Once an agent invents a tool for a resource, that particular combination of component resources becomes the formula. All subsequently-made tools for that resource follow that formula. Other agents can learn the formula for making that tool by purchasing one. This simulates how new technology is, at first, available only to the agent who invents it, but knowledge of how to make the devices – and knowledge of how to use such devices to develop higher order technologies – gradually diffuses through society by imitation.

Agents attempt to invent higher order devices in similar ways. A machine consists of one primary tool combined with five secondary tools. (No machine can be invented until at least six tools have been invented.) The machine speeds the extraction of the same resource as the primary tool. The probability of successfully inventing a machine depends not on the agent's experience with the resources, but on the agent's experience making the component tools. Eventually, factories are invented by combining a primary machine and five secondary machines, and industries are invented by combining a primary factory and five secondary factories. The proportionality constant for invention can be set so that higher order devices are invented relatively quickly or relatively slowly. For this paper, it was set so that the first machines would start being invented at about the same time that all the tools had been invented, the first factories would start being invented at about the same time that all the machines had been invented, and so forth.

For this paper, a tool for a resource reduces the effort for extracting that resource by a factor of 3. Tools have a lifetime of 150 minutes (1/4 of a day) of use before wearing out. A machine speeds resource extraction by a factor of 9 and has a lifetime of 300 minutes of use. A factory speeds extraction by 27 and has a lifetime of 600 minutes. An industry speeds extraction by 81 and has a lifetime of 1200 minutes.

In a similar fashion, agents can invent tool-making machines (composed of tools) which speed the production of a particular tool by a factor of 9, and machine-making factories (composed of machines) which speed the production of a machine by a factor of 9.

Phase 4: Device Trading

Similar to resource trading, in this phase, agents go through several rounds of pair-wise bartering. A buying agent selects a device which the selling agent knows how to produce. The buying agent then makes two calculations. First, the buying agent calculates the potential benefit from purchasing and using the device. The benefit is the net gain in utility from extracting the resource with the device for the lifetime of the device minus extracting the resource without the device. (Agents first calculate that they will use up all currently held higher-order devices for the resource before calculating the benefit of using the new device under consideration.)

$$\text{Benefit(device)} = [U(z) - U(x)] - [U(y) - U(x)] \tag{2}$$

where x = the starting amount of the resource after extracting it using all currently held higher order devices for that resource; z = final amount of the resource after using the purchased device over its entire lifetime; y = final amount of the resource if it was extracted for the same amount of time without the device. The convexity of the marginal utility curves causes the benefit of a device to decrease as x, the level at which the device begins to be used, increases.

Second, the buying agent calculates how much it would cost if it were to produce the device for itself. If the buying agent has zero experience, it cannot produce the device for itself. If the buying agent has experience producing the device, the production cost of the device equals the sum of the cost of the component pieces of the device, plus the opportunity cost of the time required to produce the device. The opportunity cost of the time is calculated using the agent's final gain-per-minute at the end of the previous day, GPM(t-1). The construction time for the device is taken from the device effort curve (Fig. 4) and is multiplied by GPM(t-1) to compute the opportunity cost for producing the device.

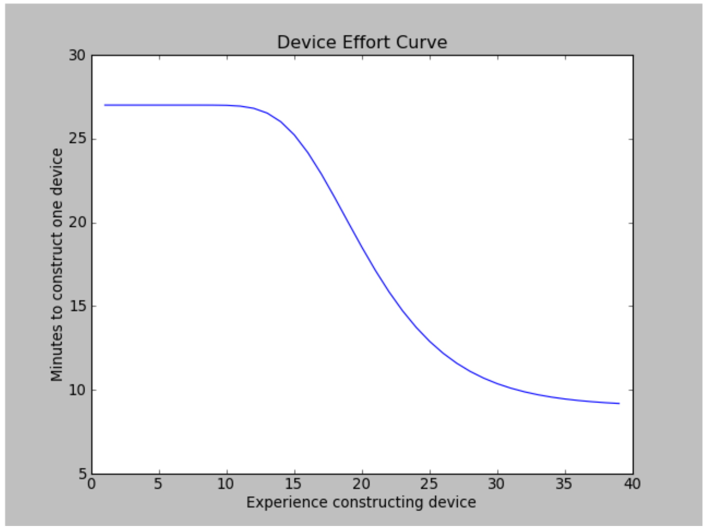

Figure 4. Device Effort Curve

The cost of the input components is calculated differently for tools than for higher order devices. For a tool, the inputs are units of resources, so the cost is the sum of the marginal utilities of the resources which comprise the tool. For higher order devices, the inputs are lower level devices which in turn may be composed of even lower level components and so on, so the cost of inputs is calculated recursively. However, if an agent has purchased one or more lower-order component devices during the last five days, it is allowed to remember those purchase prices, and assume that it can once again purchase the component device at the average of those prices. If an agent calculates that it can make a component device at a lower cost than its average purchase price, it will do so.

$$\text{ProductionCost(device)} = [\text{GPM}(t-1) * \text{DeviceConstructTime}] + \Sigma\ \text{Cost(input components)} \quad (3)$$

Each time an agent makes a tool, it gains one experience at that task, reducing its effort for making that device in the future. The experience gain for machines, factories, and industries is 2, 4, and 8 per device made. If an agent does not make a device on a given day, it loses one experience at making that device. If it has experience of less than 2 and purchases a device, its experience for making that device rises to 2, simulating its ability to "reverse engineer" a device.

The buying agent then places a buyer's value on the device, equal to whichever is smaller: the Benefit(device) or the ProductionCost(device). This is the maximum price which the buyer would pay.

The potential seller then calculates its ProductionCost(device). This is the seller's value for the device. For the trade to be successful, the seller's value must be lower than the buyer's value. This is possible when the seller's production cost is lower than the buyer's production cost because the seller has experience at making the device and/or because the seller can obtain the components more cheaply. When this happens, there is an opportunity for the buying agent to give some units of some of its resources to purchase the device.

Since devices require knowledge and time to make and have significant input costs, they are not as liquid as resources. For this reason, agents remember the benefit they get from every device traded over the last five days.

Trade "networks" develop in the device market. In each round of device trading the first agent in each pair is randomly selected, but the second agent is chosen by the first agent from the list of traders with whom it remembers having conducted successful device trades. Agents trade a bundle of resources for one device. Both the device seller and device buyer determine their preferred bundle from the device buyer's resource menu. The device seller starts with one unit of the resources it wants the most and the device buyer starts with the resources it values the least. The device seller continues to add to the bundle until it has just overcome the cost of making the device. The device buyer adds to the bundle until it just overcomes the gain of getting the device. These are their reservation prices. Because the agents are equally skilled bargainers, they then split the difference on each resource to determine the amounts traded to purchase the device.

When an agent sells a device, it "mentally reserves" the time required to make the device and all component pieces of the device and the total time for resource extraction is reduced by that amount. If it doesn't already own some of the component pieces, it makes a mental note to purchase the necessary component pieces if possible. For this reason, device trading always starts with trading industries, followed by trading factories, then machines, then tools. This allows agents to purchase lower-order component pieces whenever possible and favorable.

During this phase, agents also decide whether to make devices for themselves to use during the next day's resource extraction phase. Each agent considers all devices it knows now to make. It calculates the cost of making that device using equation 3, and the potential benefits of using that device during the next day using equation 2. Because the lifetimes of tools are ¼ of a day and the lifetimes of machines are ½ of one day, the agent calculates the costs and benefits of making up to 2 machines and up to 4 tools. If the benefits outweigh the costs, the agent plans to make the devices for its own use.

Phase 5: Device Production.

During this phase, agents produce the devices which they agreed to make for sale or decided to make for their own use, starting from the simplest devices and ending with the most complex. Producing devices costs agents both resources and time. For each device it makes, an agent subtracts the component pieces of the device from its possessions. (If the agent does not own the necessary component piece, it first has to make it.)

For each device it makes, an agent also subtracts the time required to make it from the time it can spend in the resource extraction phase of the next day.

Phase 6: Resource Consumption and Device Depreciation

Agents consume a constant proportion (λ) of their holdings on average each day, currently set at .25. The consumption of units occurs according to a binomial distribution, with the probability of consumption of each unit equal to λ. If an agent starts with x units of resource r, on average it will end the phase with y units,

$$y_r = x_r(1-\lambda) \tag{4}$$

Devices also depreciate, losing 3 minutes of use during this phase of the day whether they were used or not. With all of the elements of the model in place, the next section describes our approach to verification and testing of the model.

• Model Verification and Testing

The model was designed and programmed by the authors in Python, an open source, object-oriented programming language known for its ease of use and somewhat self-documenting syntax. The program begins by reading in the parameter values that set the context for the simulation. The typical parameters that are controlled by the researcher to define a simulation context includes: Number of Agents, Number of Resources, Number of Simulation Days, and whether there is Trade (on/off), Tool-making (on/off), and Higher-order devices (on/off). Input parameters related to trade and innovation that can be varied by the researcher to test a variety of hypotheses such as: how much experience agents need in order to be eligible to invent a tool or device; how likely it is an agent will discover a device; how long it takes for an agent to "reverse-engineer" a device and be able to make it itself (patent length); how many opportunities for agent-to-agent trading there are at the end of each day; and how many days agents remember the prices from their trades in higher-order devices. A third set of parameters relates to device technology: number of resources in a tool, and in each higher-order device; the lifetime, in minutes, of each device; and, the factor by which each device speeds extraction. Parameters related to agents and resources define the shape of the marginal utility curves and the effort curves, and the lifetime of each resource. These agent and resource parameters can be set the same or varied across agents, across resources, or across both.

The program was unit and system tested methodically. Systematic parameter sweeps were performed on input parameters and the model was found to be robust to all but the most extreme perturbations of parameters. The model produces qualitatively similar, expected results across a wide range of combinations of input parameters.

One of the areas of the program that bears particular attention is the parameter sweep involving the diminishing marginal utility exponent, because it is a significant driver of specialization and trade. Specialization begins when agents calculate that they will gain more by continuing to extract the same resource repeatedly rather than switching to extract a different resource. Thus, the level of specialization depends on shape of the GPM functions. GPM (defined in Eq. 1) is calculated as the Marginal Utility of the next unit extracted divided by the time it will take to extract it (Effort):

$$\mathbf{GPM = MU \,/\, Effort}$$

For agents to want to specialize, they need the GPM of the resources they hold to be greater than the GPM of the resources they have not extracted yet. This happens whenever the effort to extract a resource decreases faster than the marginal utility of that resource. If Marginal Utility were held constant while effort decreased, agents would tend to specialize because every unit of a resource that is extracted reduces the effort required to extract the next one, which continuously increases the GPM for that resource. However, if Effort is held constant while marginal utility dropped, agents would tend not to specialize because the value to the agent of the next unit of the same resource goes down for each unit that is extracted, which continuously reduces the GPM for that resource. Thus, specialization in a resource is more likely to occur the slower the rate that Marginal Utility declines for the resource. The shape of the resulting GPM curve that combines both opposing effects determines whether agents will be rewarded for specializing or not.

With the standard nth-root Utility function, $U(x)=Dx^{1/n}$, the steepness of the decline in MU is varied by adjusting the diminishing marginal utility exponent, **n**. Figure 5 below graphs GPM as a function of units held for various values of **n**. The x-axis represents the number of units held of a resource, the y-axis represents the Gain Per Minute.

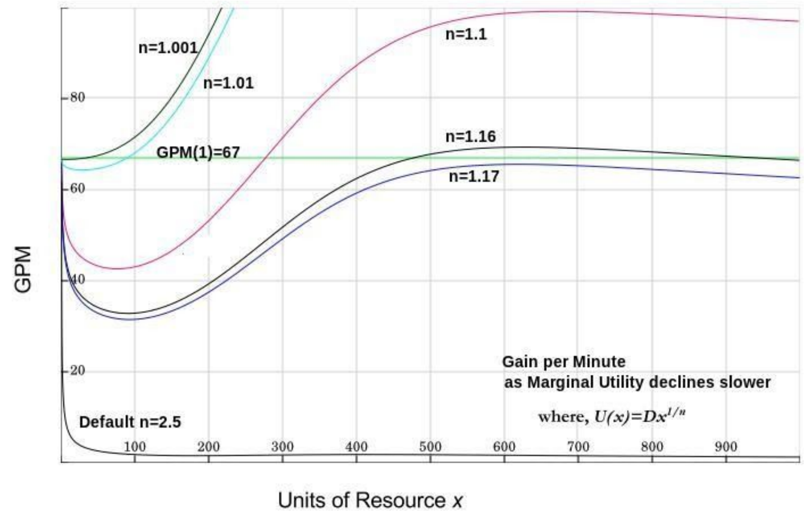

Figure 5. Parameter Sweep: Marginal Utility and Gain per Minute (GPM)

GPM, and thus level of specialization, is sensitive to the value of **n**. An agent will have little incentive to specialize if the GPM of a new resource is always higher than the GPM of a held resource. The flat, green line on the figure is drawn at GPM(1)=67, the GPM of the first unit of all resources. At one extreme, when the marginal utility exponent **n**=1.001 (the top curve in Figure 5), so that marginal utility barely decreases, agents will automatically specialize. The GPM of the first resource they randomly will always be higher than that of any other resource. Thus, no matter how many resources are in the environment, the agent will only specialize in harvesting one. At the other extreme, agents might never specialize. When the marginal utility exponent **n**=2.5 (the lowest curve in Figure 5 and the value used in the simulations for this paper), the MU curve drops precipitously after the first unit. After the first unit is extracted, the Gain Per Minute of extracting a unit of any other resource looks better.

Figure 6 displays selected results from the model from parameter sweeps over the marginal utility exponent **n**. The lines represent the average trade volume across 100 runs of the same simulation, with standard error bars included. As the marginal utility exponent increases (from 1.1 to 1.5 to 2.0), trade volume (which correlates with specialization) declines.

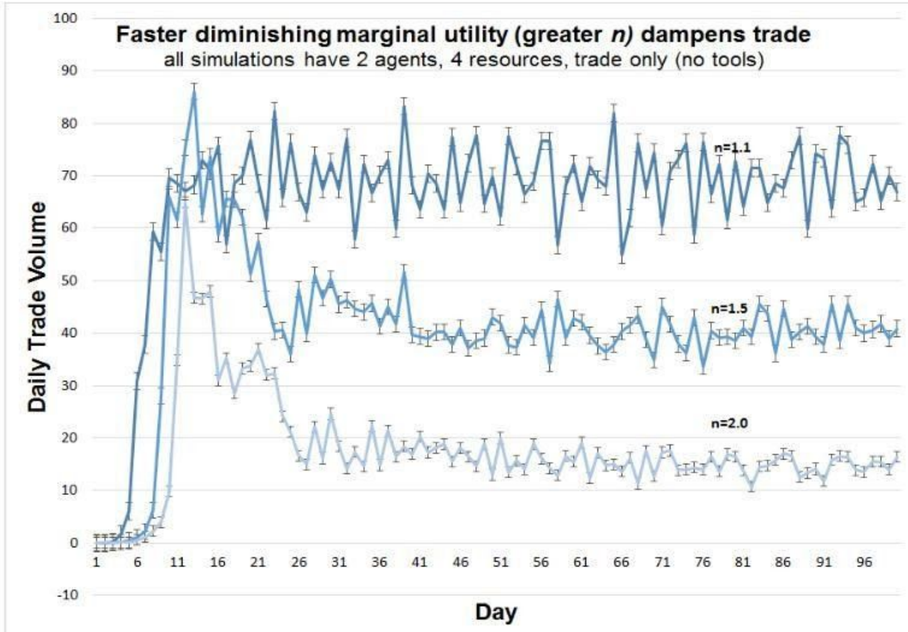

Figure 6. Marginal Utility Exponent Parameter Sweeps and Trade

The marginal utility exponent is set at **n**=2.5 in the paper to stack the deck against specialization, so that when specialization occurs it is an emergent property of the system, rather than an artifact of the program.

• Results and Analysis

In this paper, all agents have effort and marginal utility curves. Interdependence and inequality emerge, in spite of homogeneity, through random chance. During the first few days of the run, each agent employs essentially the same strategy: extract units of resources with the highest GPM. Whether agents specialize by continuing to extract units of the same few resources, or diversify by extracting a portfolio of several resources, depends on the shape of the GPM function, as illustrated in Figs. 5 and 6. As noted above, the marginal utility exponent, *n*, drives that shape of the GPM curves. In this paper it is set at high level and the agents have an incentive to diversify and work independently, rather than specialize and trade.

Thus, the agents in the simulation are not predisposed toward specialization, trade, and interdependence. However, once some specialization begins to occur, it is reinforced through trade, and then locked in with the invention of tools and higher-order devices. When agents who specialize in extracting different resources meet and trade resources, they are further rewarded for specialization. The invention of devices leads to additional specialization, as each device has one particular agent as its inventor, and that inventor immediately receives the benefit of experience in making that device. In our model, this is the primary source of initial income inequality, that is, the beginning of the Kuznets curve.

The steady-state level of inequality that the economy achieves following the Kuznets curve depends on the level of interdependence. The primary source of interdependence in this model is the combinatorial evolution of devices. (Arthur 2009) Each tool is composed of units of five resources. Each machine is composed is six tools. Factories are made of six machines. Industries are made of six factories. Once the economy has grown to the point where agents are routinely building factories and industries, the extraction of each resource depends upon the extraction of every other resource, or nearly every other resource, in order to make the necessary components.

Trade and Technology Produce Greater Income

Using the same simulation parameters in all other respects, we introduce trade, tools, and higher-order devices stepwise with four different scenarios:

(1) Agents do not trade resources and do not make tools;
(2) Agents trade resources but do not make tools;
(3) Agents trade resources and make, use, and trade tools but not higher order devices;
(4) Agents trade resources and make, use, and trade all order devices.

Each simulation includes 48 agents and 48 resources and runs for 500 model days and is run several times in order to examine variance in economic steady states and time paths.

Our model has three ways of measuring economic growth, though all convey similar information: growth in units of resources extracted per day, growth in units held by agents, growth in total utility held by agents. Agents make decisions based on utility growth. Because of the diminishing nature of marginal utility curves, it is more informative to display units of resources extracted per agent per day.

No Trade/No Tools. The lowest (blue) line in Figure 7 shows the typical growth time path of resource extraction over 500 days when agents extract resources for themselves without trade or tools. As agents gain experience, resource extraction speed increases by a factor of three, from an average of 67 to 187 units extracted per day per agent. This results directly from the maximum and minimum settings of the effort curve (Figure 1). There is no variation in starting and ending income once the economy reaches its steady state. The time path is not completely deterministic because agents randomly choose which resources to begin extracting, but there are only tiny variations in growth paths across simulations for the No trade/No tools scenario.

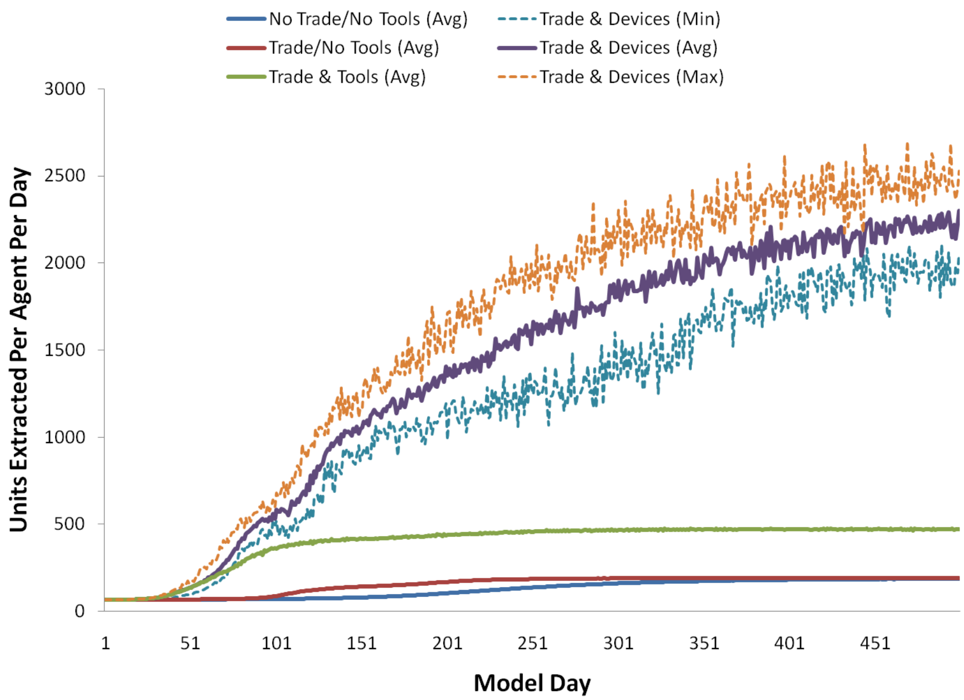

Figure 7.Agent's Income as Trade, Tools & Devices are introduced

Trade/No Tools. The addition of trade allows agents to specialize. Specialization allows agents to reach maximum experience and therefore maximum extraction speed more quickly. However, because there are only 48 resources, agents are able to gain enough experience in 500 days that they can efficiently extract each resource for themselves. By day 500, scenarios with and without trade show equivalent extraction.

Trade/Tools. Allowing invention of tools increases average extraction per day per agent from 67 units on day 1 to around 485 units by day 150: a factor of about seven. This follows from the fact that tools reduce extraction time by a factor of three and experience reduces extraction time by another factor of three. Combined, they reduce extraction time by a factor of nine, but some time must be used to produce the tools. The invention of tools also reduces the amount of time required for society to reach its steady state income by roughly one-third, from day 150 to day 100.

Trade/Tools & Higher Order Devices. Adding higher-order devices increases extraction on average by a factor of twelve after 500 days. This is the combined result of greatly reduced extraction time resulting from using higher-order devices minus the drag that is caused using time to produce the devices. After 500 days in these runs of the model, agents were producing machines, but not yet producing factories or industries. The maximum and minimum outcomes across all runs for each day (dashed bars around the top line in Figure 7) reveal the wide variety of path-dependent economic outcomes possible in societies which start identically. The time path dependences are caused by random variation in timing of inventions and variation in particularly productive trades in different runs.

Income Inequality, Technology Invention, and Technology Diffusion

It is hypothesized that the invention and use of technology in modern economies leads not only to income growth and greater interdependence, but also to wealth inequality. (Nelson and Winter 1985, 369) We calculate the Gini coefficient for each day in our model under each of the four scenarios. The Gini coefficient is a common measure of inequality where 0 represents complete equality (every agent has the same amount) and 1 represents maximum inequality (one agent has everything). The bottom two lines in Figure 8 illustrate that without devices, every agent remains equally well-off, with or without trade. When tools are introduced a modest surge in inequality appears around day 50, because only a few random agents initially discover tools. However, trade occurs and technology is diffused throughout the economy. By day 100, inequality dies down to a level only slightly greater than society without tools or trade. Random discovery of a labor-saving device by only a few agents promotes income inequality; ease of knowledge transfer ensures this disparity is short-lived. This is an example of "negative feedback" in a complex system. (Miller and Page 2007, 50) That is, in a simple economy with little interdependence, inequality that results from random good fortune can be self-correcting.

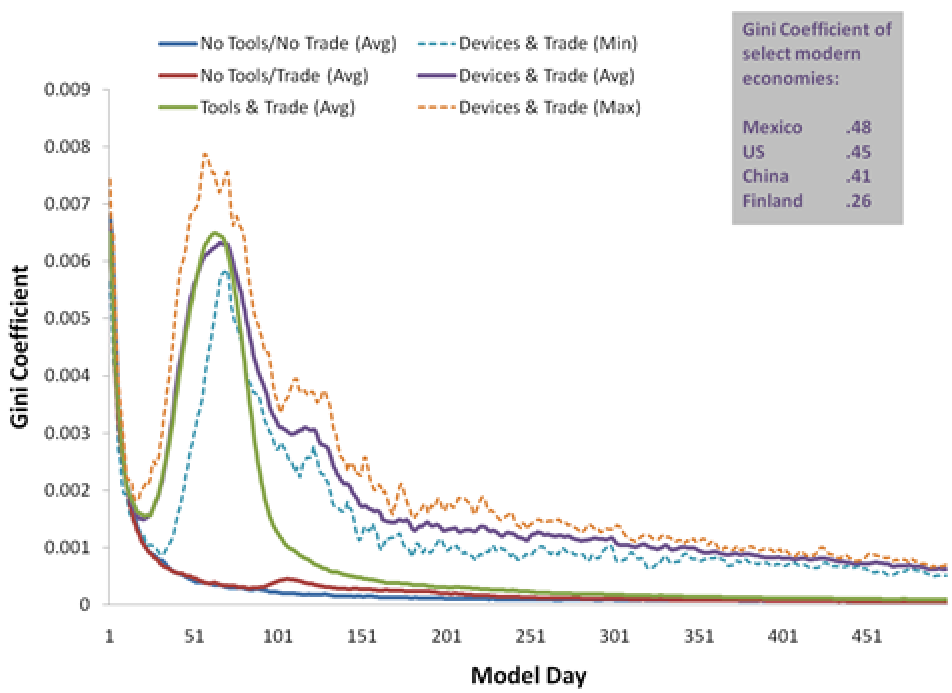

Figure 8.Inequality in Artificial and Modern Societies.

The situation is different when the economy includes higher-order devices and becomes highly interdependent (upper curves in Figure 8). Trade with preferred trading partners for higher order devices generates networks of winners that are able to maintain the disparity in income. This is an example of "positive feedback" in a complex system. (Miller and Page 2007, 56) Positive feedback occurs when agents' interactions do not cancel each other out but reinforce outcomes. Inequality in a highly interdependent complex economy can be self-reinforcing, and can remain a permanent feature, as illustrated by the top solid line in Figure 8.

Thus, our model shows the Kuznets curve can arise in an economy in which agents and resources are initially identical, purely from the innovation, diffusion, and use of technology. The rising phase of a Kuznets curve occurs when technological innovation at first benefits only a few agents. The falling phase of a Kuznets curve occurs as the technology diffuses to the rest of the agents through trade and imitation. The key result of our model is the following: the higher steady-state inequality left over once the technology has fully diffused remains high because the degree of specialization and interdependence that emerges in a technological sophisticated economy locks in some of the inequalities.

In our model, the steady-state inequality is still quite small compared to actual modern societies. We believe this is because the parameters of these simulations are set to produce quick and easy knowledge transfer, and the model overall is designed with the assumption that agents are homogenous, fair, and have equal bargaining skills and power. In actual societies, heterogeneity, power imbalance, and other contributors to inequality exist. The Gini coefficients of selected modern economies included in Figure 8 illustrate the variety of inequality in household incomes that exist today. (CIA 2008) As our model is extended to include more contextual aspects of a society, increased inequality is an expected outcome and a barometer of its ability to describe how technology relates to the Kuznets curves found in and across modern economies. The next section examines in detail the role of interdependence in economic growth and inequality.

Measuring Interdependence

In modern economies, increases in technology lead not only to increases in overall wealth and productivity, but also to increases in interdependence. This interdependence has both benefits and risks. The use of technology and high degree of specialization allows for increased productivity. However, a shock to one sector of the economy can have ripple effects to many other sectors, such as when rises in oil prices impact the price of many other commodities. Consider truck manufacturing. This industry relies on petroleum extraction and refining industries, plastics, electronics, glass, metal refining, and many other industries – each of which in turn rely on the trucking industry and other industries. The high degree of interdependence in modern economies was not pre-planned, but emerged over time as technology increased in complexity. Arthur (2009) argues that such interdependences arise and self-organize over time because of the particular way technology evolves. Technology evolves combinatorically. New inventions can be combined with existing technologies to create yet more innovations. Neither batteries nor glass nor plastics nor circuit boards nor refined metal nor refined petroleum were invented for making trucks, but all have been incorporated into modern trucks. And technology evolves through *exaptation*. Existing technologies are modified as they are incorporated into new innovations, and sometimes stop being used for the purposes for which they were originally invented as those tasks get taken over by new technologies. Because of the importance of combinatoric evolution and exaptation of technology in real world economies, our model includes all of these features in the invention of tools, machines, factories, and industries.

In order to investigate and measure this increasing wealth and interdependence, the model was run repeatedly with 24 agents and 24 resources out to 1500 "days," by which point agents were routinely making, trading, and using the highest order devices, industries, and device-making devices. The behavior of individual agents varied significantly from run to run, as agents randomly decided to specialize in one resource rather than another or invented one device rather than another. However, the overall macro-level behavior of the model was identical in every case, with small variations around the mean similar to those shown in Figure 7.

Over time, agents invent and use increasingly higher order devices (Fig. 9). Agents initially extract resources without devices (region in white on left side of figure). This is replaced by extraction with tools (blue region) typically during days 15 through 50. Machines begin to be invented, and resource extraction with machines (red region) takes over by day 100. By day 300, nearly all resource extraction is done with factories (green region), and industries (purple) are starting to be invented. Throughout this time, over-all wealth continues to rise.

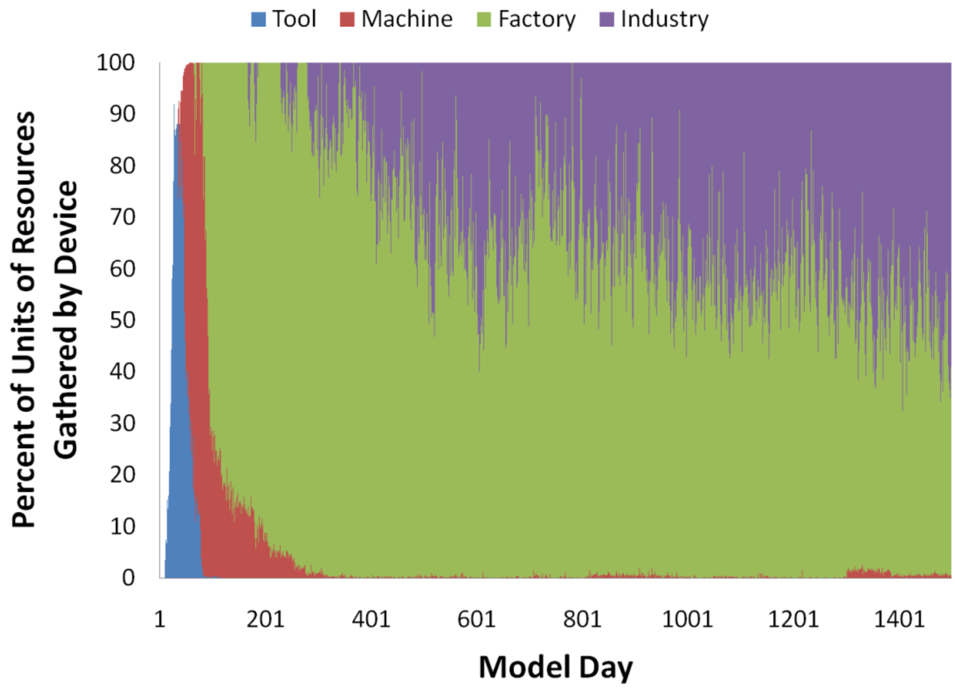

Figure 9. Agents' Use of Devices to Extract Resources.

Because each industry is made of 6 factories, each factory made of 6 machines, and each machine made from 6 tools – and making each device requires time – this high level of technological sophistication would not be possible without device-making devices. Agents can invent machines which speed the manufacture of tools and factories which speed the manufacture of machines (Figure 10). By day 300, nearly every tool is made with a tool-making machine, and nearly half of all machines are made with machine-making factories. At this point, tools are no longer used for the purpose for which they were originally invented – extracting resource. Tools are made only to serve as components of higher order devices. This is an example of the exaptation of technology discussed by Arthur (2009). Technology originally invented for one task becomes repurposed as it enables the development of higher order technologies. After day 300 in this model, neither tools nor tool-making machines directly contribute to agent utility or increase extraction capacity, yet the agents spend considerable time making both and the entire economy becomes dependent on both. Device-making devices are intermediate processes used to speed the construction of lower-order components, giving agents more time to extract resources with their higher order devices.

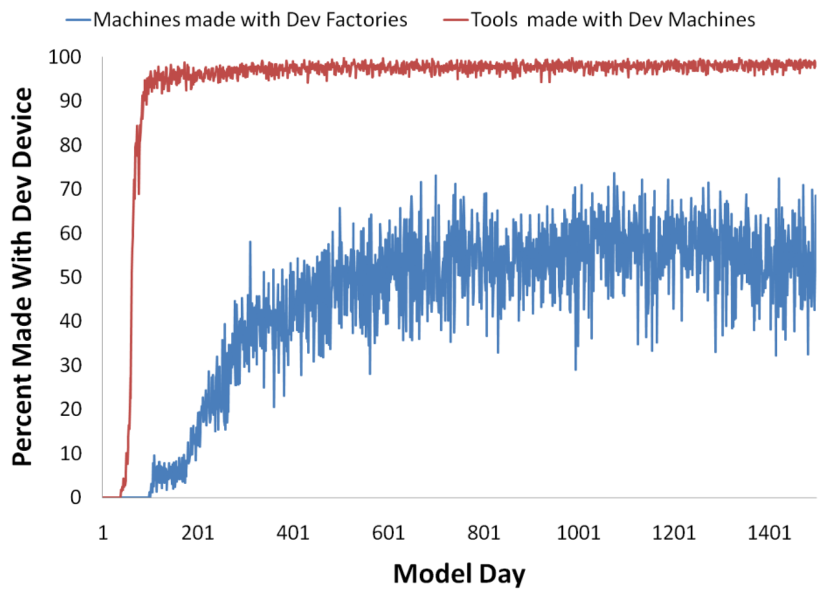

Figure 10. Use of Device-Making Devices by Agents

Interdependence in real world economies is difficult to quantify. Here we display three measures of interdependence in our model.

First, to quantify agent specialization, we calculate the Herfindahl-Hirschman Index (HHI), which is the sum of the squares of the percent of a given resource produced by each agent on a given day. (Rhoades 1995) In runs with 24 agents and 24 resources, the median HHI rises quickly with the invention of tools and machines, and then settles to 0.5 (Fig. 11). In other words, the average resource is being extracted by only two agents. In each run of the model, there are some resources for with the HHI is 1. When that is the case, there is only one agent extracting that resource. Such a phenomenon is only possible with trade and specialization. The lone agent extracting the resource with the HHI of 1 trades away most of the units it extracts each day; if not, other agents would have no access to that resource and would begin extracting it for themselves.

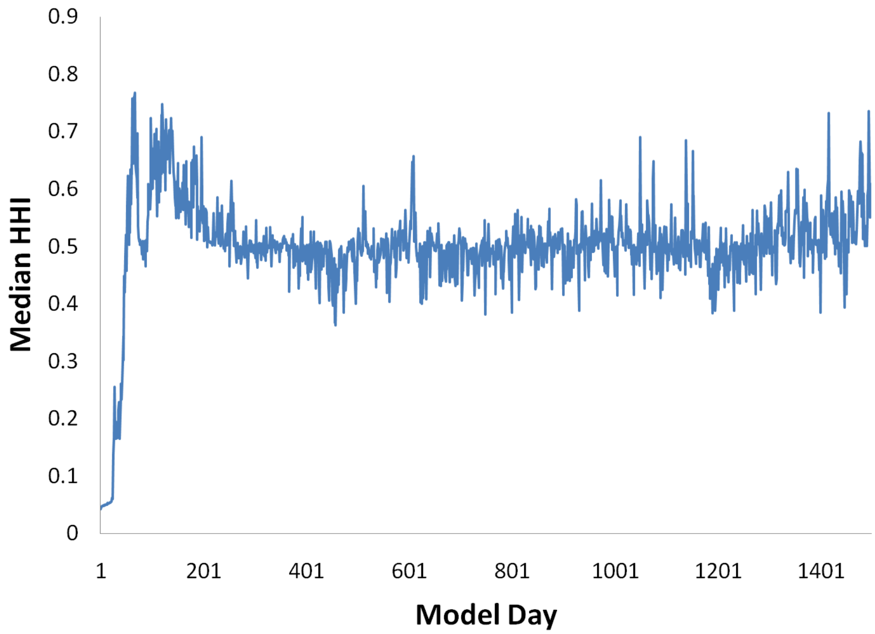

Figure 11. Agent Specialization

The existence of resources with high HHIs reveals that specific agents are important for the utility-generating function of the system. For most resources, at least half of the units are being extracted by a single agent. If this agent was removed, other agents would have to spend less time extracting the resources they were producing before – presumably those in which they have the most experience – to make up the missing production of that resource.

Second, we measure the interdependence of resources used to extract other resources. In much the same way that trade and specialization make individual agents important to the functioning of the system, devices make individual resources important. Because devices are made with resources, the extraction of each resource depends on other resources. With 5 units of resources going into a tool, and 6 tools going into a machine, a machine is made of 30 units of various resources. Factories are made of 180 units of various resources; industries 720 units. For this model, we define "Resource Interdependence" to be the average number of units of resources required in the extraction of resources. It is calculated by the percent of units extracted by each type of device multiplied by the number of units of resources that go into that device. The average Resource Interdependence as a function of "day" is shown in Figure 12. Resource interdependence starts at zero as agents extract "by hand." When tools become common but machines are rare, average resource dependence is 5.

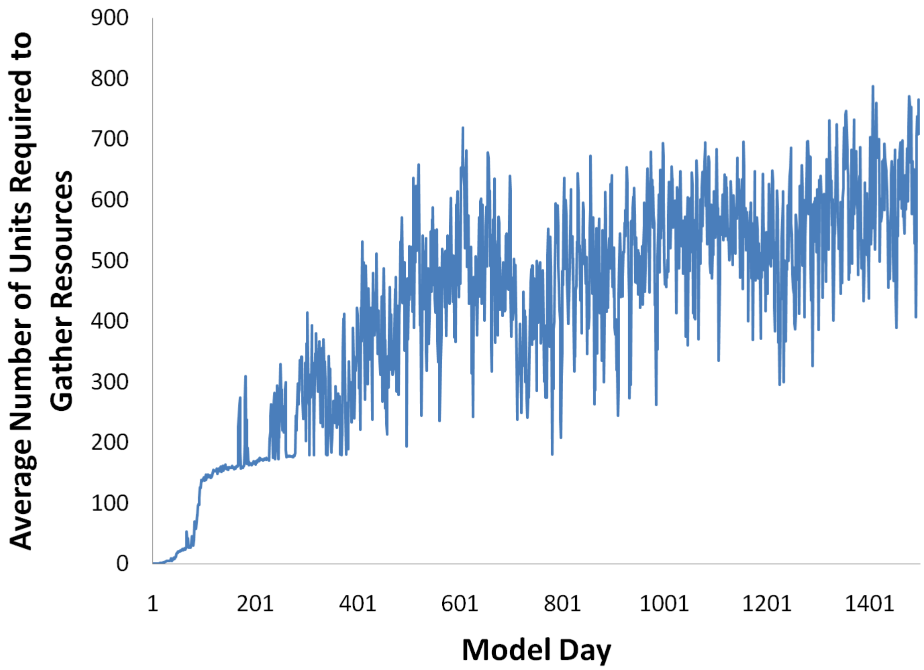

Figure 12. Resource Interdependence

Since there are only 24 different resources in these runs of the model, any resource dependence value over 24 indicates redundancies. A machine made of six tools will have units of some resources as components of more than one tool. By the time factories and industries are developed, there are many redundancies. The complexity of the system reaches the point where extraction of each resource requires every other resource.

A third way to measure interdependence – and a connection to the original motivation of the model – is to examine what happens to an economy when a random shock occurs that restricts access to a particular resource. One might argue that the financial meltdown occurred as a result of a severe restriction on access to borrowed funds. (Yerex 2011) The recent tsunamis and Hurricane Katrina impacted access to critical resources as well. The presence of interdependence in this model provides insight into what can happen to a society in such events and what parameters might magnify or dampen the effect of random shocks. In several runs with 24 agents and 24 resources, one resource was removed at day 3,000. The level of income (measured by agent utility) collapses in half (Fig. 13). The corresponding resource extraction speeds (measured by units of resources) drop by nearly a factor of 10. Agents must re-invent all tools which had that resource as a component, then reinvent machines, then higher order devices. In these runs, the economy recovered to where it was at day 3000 in about 500 days.

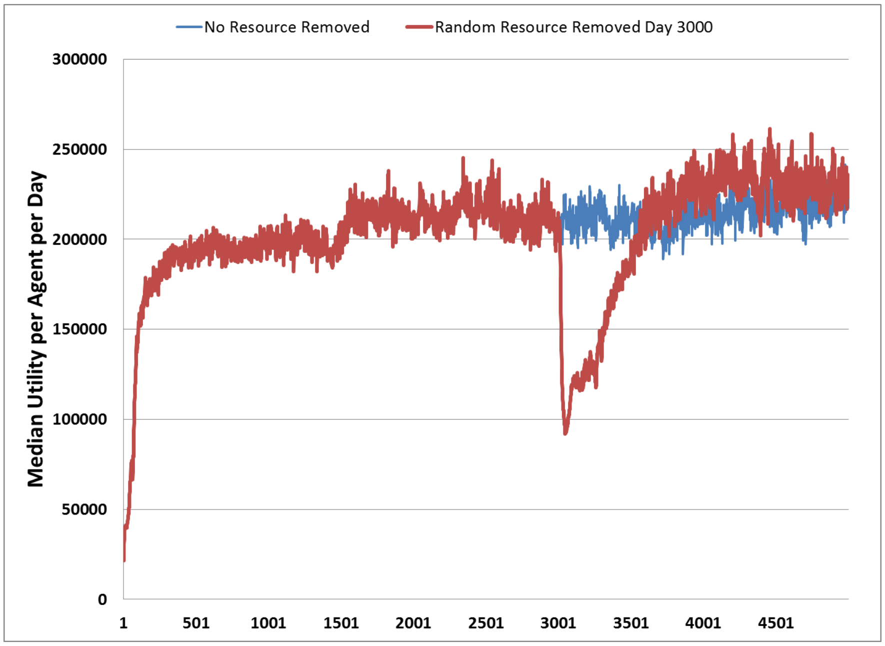

Figure 13. Interdependence causes societal income to collapse and slowly rebuild when access to a resource is removed at model day 3000.

• Conclusion

This model traces out paths of economic growth in modern industrial societies in order to examine the relationship of income and inequality in the presence of technology and high levels of interdependence. It shows that technological impulses – new inventions that are gradually diffused throughout the economy – can produce rises, falls, and steady-state income inequality in the shape of the Kuznets curve. Individual agents are given opportunities to invent labor-saving devices and to gain temporarily from their good fortune. As their technological knowledge is diffused throughout the economy through imitation, their income advantage is diffused as well, although not completely. Our model simulations suggest that in technologically sophisticated and interdependent societies, higher steady-state inequality remains after an initial technological impulse at least in part because specialization and interdependence lock in some of the inequalities. Path-dependence and positive feedback mechanisms exacerbate inequality in contemporary economies.

In addition, our model demonstrates how self-interested, decentralized decisions can lead to the highly interdependent society that exists today. No central planner or perfect foresight is required. Bounded utility-maximizing agents make decisions based on simple rules about what to extract, build, and trade. Interdependence arises from the combination of trade and technology. As society evolves technologically, the amount of interdependence, income, and inequality all increase.

Agent capabilities and heterogeneity are muted ("dialed down") in this paper to show that income, inequality, and interdependence can arise even when there is limited agent capacity and great homogeneity among agents and resources. Agents have extremely limited foresight and memory. During resource extraction they do not anticipate gains from trade or gains from specialization. During resource trading, they do not anticipate gains from the next day's resource extraction. They are only aware of how they value resources themselves and not how other agents value them. When deciding on the value of a device which increases resource extraction, they do not anticipate gains from trade, and they only have foresight regarding gains from extraction for one or two days. During device trading, they only remember prices of trades from the previous five days. In the real world agents are much more sophisticated and heterogeneous, particularly in their skills and access to resources and to other agents (e.g., by geographic location).

Agents in this version of our model are also perfectly rational, self-interested, equally powerful bargainers, honest, and healthy. In the real world, agents make mistakes, act altruistically, have differential power, are dishonest, and get sick. This model provides a unique tool to explore strategic behavior, institutions, and prices on income and inequality, interdependence, and technology. Additionally, extensions to the model will model private costs of investment in R&D, as well as social costs such as pollution and insurance to demonstrate their effect on income and inequality. Extended versions of the model can include increased capabilities; heterogeneity across agents, resources, and location; agent vices and virtues; pollution and investment. These can easily be implemented in to isolate their effects on income, inequality, interdependence and technology.

Our model is available for download under the GPL license at http://code.google.com/p/societies/. (Norman and Ditta 2011) Instructions for replication and extension are available at that location and from the corresponding author.

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