Network Theory

Basic Circuit Laws, Frequency Analysis, and Network Parameters

Course Code: 3EC4-06

Course Overview

Network Theory (3EC4-06)

1. **Network Simplification**
   Apply basic circuit laws and theorems for network simplification
2. **Frequency-Domain Analysis**
   Utilize frequency-domain techniques in network analysis
3. **Laplace Transforms**
   Perform steady-state and transient analysis using Laplace transforms
4. **Two-Port Networks**
   Analyze transient responses and compute two-port network parameters
5. **Resonance & Filters**
   Examine resonance conditions and design passive filters

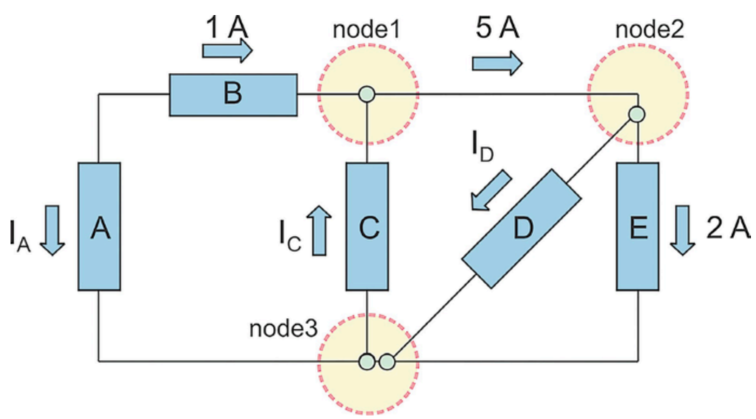
CO-1: Basic Circuit Laws

Apply basic circuit laws and theorems for network simplification

Kirchhoff's Laws

KCL: $\Sigma I_{in} = \Sigma I_{out}$

Current Law: The algebraic sum of currents entering a node equals the sum of currents leaving it.



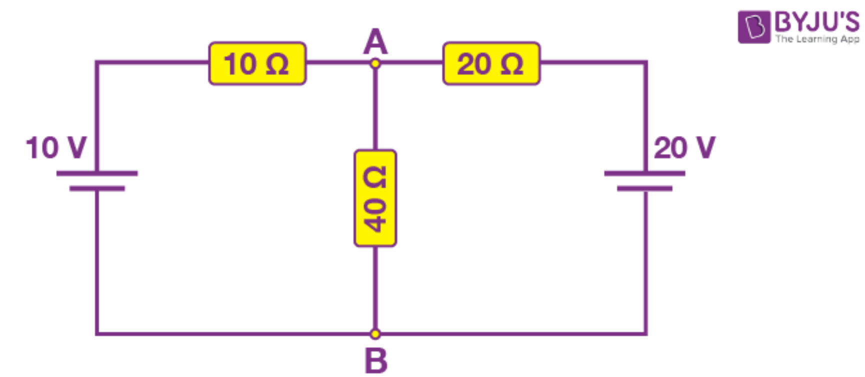
KVL: $\Sigma V = 0$ (closed loop)

Voltage Law: The algebraic sum of voltages around any closed loop is zero.

Ohm's Law

$V = I \times R$

The voltage across a resistor is directly proportional to the current flowing through it.



Applications:

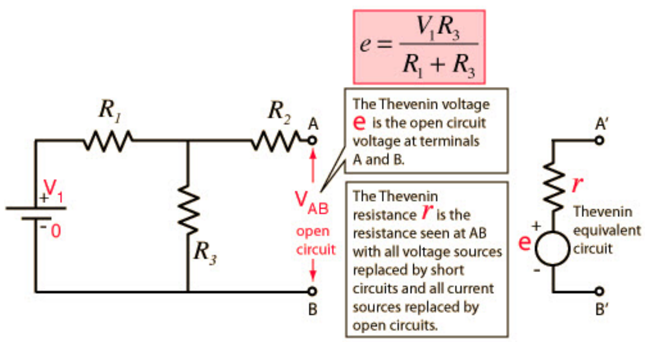
- Calculate voltage, current, or resistance
- Determine power dissipation: $P = V^2/R = I^2R$
- Design voltage dividers

CO-1: Network Theorems

Thevenin's and Norton's Theorems, Superposition Theorem

Thevenin's Theorem

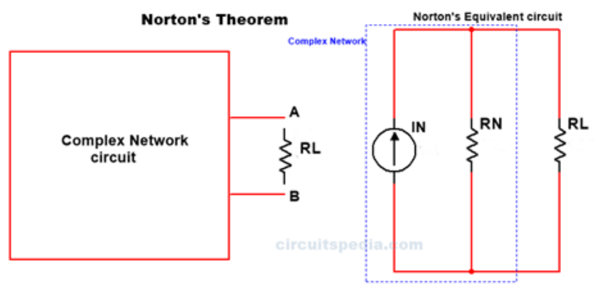
Any **linear network** can be replaced by an equivalent circuit with a single voltage source (V_{th}) in series with a resistance (R_{th}).



1. Remove load resistor
2. Calculate V_{th} (open-circuit voltage)
3. Calculate R_{th} (with sources replaced by their internal resistance)

Norton's Theorem

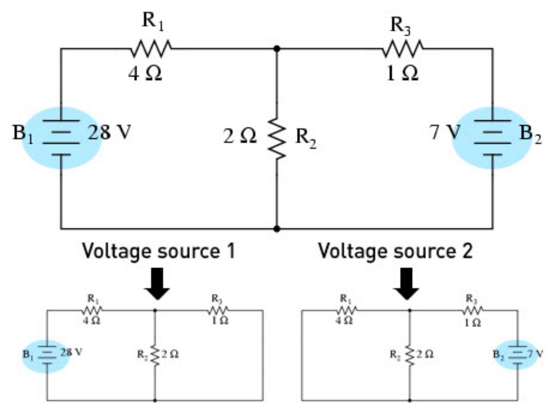
Any **linear network** can be replaced by an equivalent circuit with a current source (I_N) in parallel with a resistance (R_N).



1. Remove load resistor
2. Calculate I_N (short-circuit current)
3. Calculate R_N (with sources replaced by their internal resistance)

Superposition Theorem

In a **linear circuit** with multiple sources, the total response is the sum of individual responses from each source acting alone.



1. Turn off all sources except one
2. Calculate response (voltage/current)
3. Repeat for each source
4. Sum all individual responses

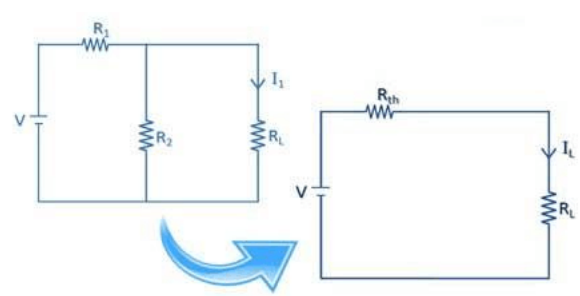
CO-1: Additional Network Theorems

Maximum Power Transfer, Millman's Theorem, and Source Transformation

Maximum Power Transfer

Maximum power is transferred from source to load when **load resistance equals source resistance**

$$P_{max} = V^2_{th}/4R_{th}$$

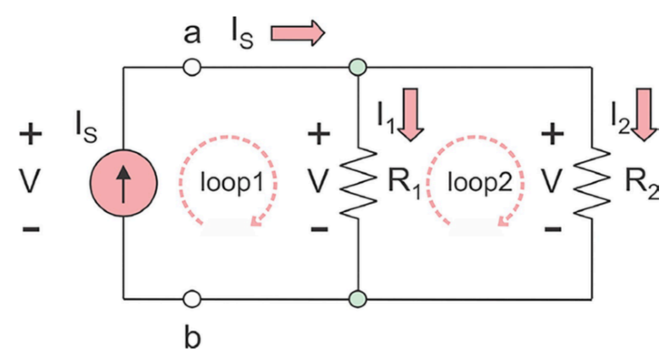


1. Find Thevenin equivalent circuit
2. Set $R_L = R_{th}$
3. Calculate P_{max}

Millman's Theorem

Simplifies circuits with **multiple parallel voltage sources** into a single equivalent source

$$V_{eq} = \Sigma(V_i/R_i) \;/\; \Sigma(1/R_i)$$

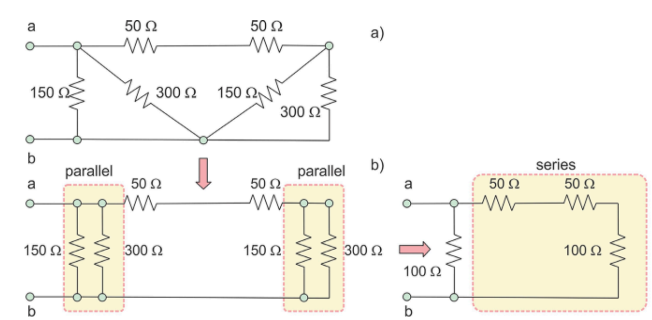


1. Convert all sources to current sources
2. Sum all currents and conductances
3. Calculate equivalent voltage source

Source Transformation

Technique to convert between **voltage and current sources** while maintaining equivalent circuit behavior

$$V_s = I_s \times R_s$$



1. Identify source type (voltage/current)
2. Apply transformation formula
3. Simplify resulting circuit

CO-2: Frequency Domain Fundamentals

Time domain vs. frequency domain representation, phasor representation, and complex numbers in AC circuits

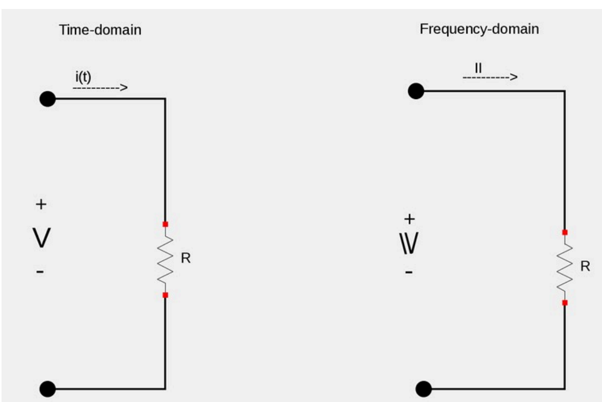
Time vs. Frequency Domain

Time Domain

- Signals expressed as function of time
- Direct representation of waveform
- Amplitude vs. time plot
- Transient behavior visible

Frequency Domain

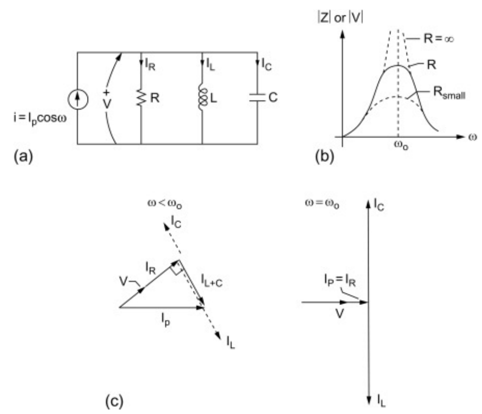
- Signals expressed as sum of sinusoids
- Amplitude and phase vs. frequency
- Simplifies AC circuit analysis
- Steady-state behavior focus



Phasor Representation

Phasor: Complex representation of sinusoidal signals in frequency domain

$$v(t) = V_m \cos(\omega t + \phi) \leftrightarrow V = V_m \angle \phi$$



Key Benefits:

- Simplifies differential equations to algebraic equations
- Enables easy calculation of magnitude and phase relationships
- Reduces AC circuit analysis to DC-like methods

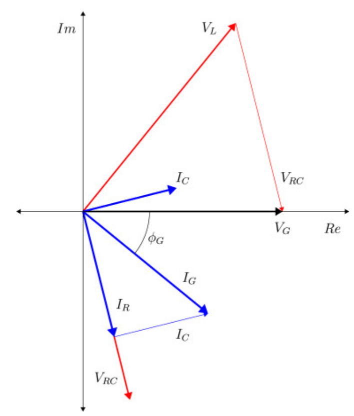
Σ Complex Numbers in AC

Impedance (Z): Complex opposition to AC current

$$Z = R + jX = |Z| \angle \theta$$

Component Impedances:

- Resistor: Z^R= R
- Inductor: Z^L= jωL
- Capacitor: Z^C= 1/(jωC)



CO-2: Impedance and Admittance

Complex impedance, admittance in AC circuits, complex power calculations, and frequency response analysis

Complex Impedance

Impedance (Z): Total opposition to AC current flow

$$Z = R + jX = |Z|\angle\theta$$

Component Impedances:

Component	Impedance	Phase
Resistor (R)	$Z_R = R$	0°
Inductor (L)	$Z_L = j\omega L$	+90°
Capacitor (C)	$Z_C = 1/(j\omega C)$	-90°

Key Applications:

- Simplifies AC circuit analysis
- Enables phasor-based calculations

Admittance & Complex Power

Admittance (Y): Reciprocal of impedance, measures how easily current flows

$$Y = 1/Z = G + jB = |Y|\angle\phi$$

Complex Power (S):

$$S = P + jQ = V\cdot I^* = |V||I|\angle(\theta_V - \theta_I)$$

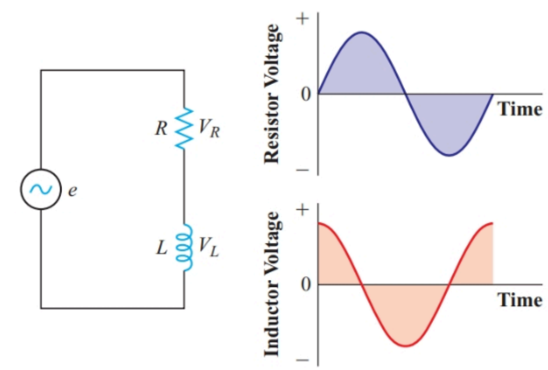
Power Components:

- **P (Real Power):** Energy consumed (W)
- **Q (Reactive Power):** Energy stored/released (VAR)
- **|S| (Apparent Power):** Vector sum (VA)

Frequency Response

Transfer Function (H(jω)): Ratio of output to input in frequency domain

$$H(j\omega) = |H(j\omega)|\angle\phi(\omega)$$



Key Metrics:

- **Magnitude Response:** $|H(j\omega)|$ vs. frequency
- **Phase Response:** $\phi(\omega)$ vs. frequency
- **Bandwidth:** Frequency range where $|H(j\omega)| \geq 0.707|H|_{max}$

CO-3: Laplace Transform Fundamentals

Definition, properties, and transformation of circuit elements to s-domain

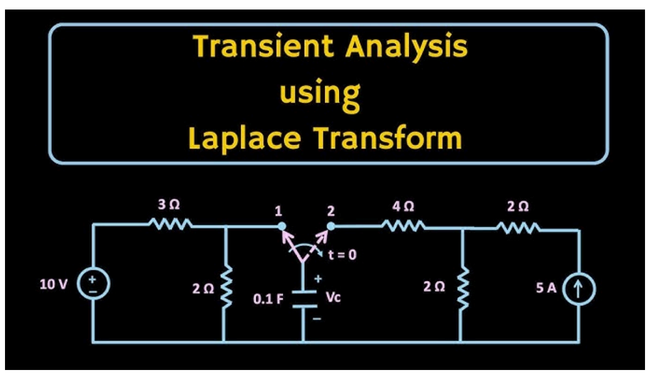
Σ Definition & Properties

Laplace Transform: Converts time-domain functions to complex frequency domain

$$F(s) = L\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$$

Key Properties:

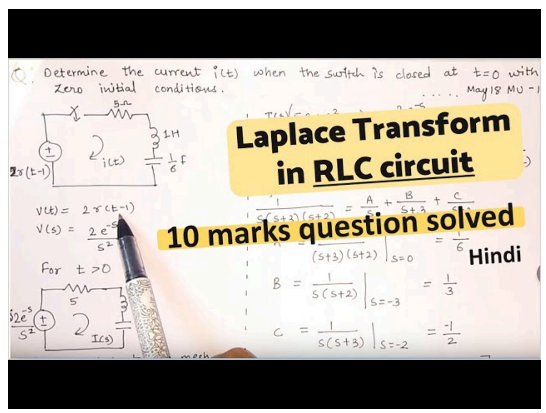
- **Linearity:** $L\{af(t)+bg(t)\} = aF(s)+bG(s)$
- **Differentiation:** $L\{df/dt\} = sF(s)-f(0)$
- **Integration:** $L\{\int f(t)dt\} = F(s)/s$
- **Time Shift:** $L\{f(t-a)u(t-a)\} = e^{-as}F(s)$



Time to s-Domain

Advantages of s-Domain:

- Transforms differential equations to algebraic equations
- Handles initial conditions automatically
- Enables analysis of transient and steady-state responses
- Facilitates system stability analysis



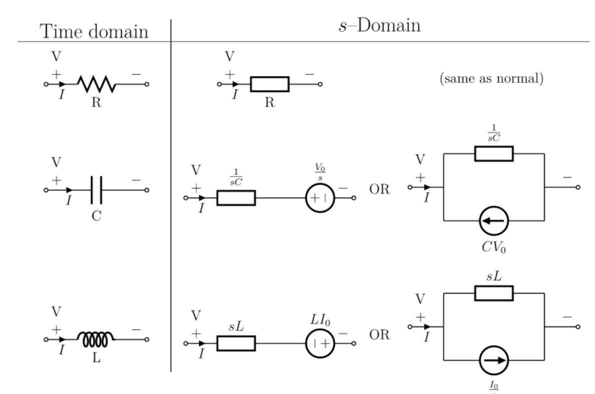
Common Transform Pairs:

- $e^{-at} \leftrightarrow 1/(s+a)$
- $\sin(\omega t) \leftrightarrow \omega/(s^2+\omega^2)$
- $\cos(\omega t) \leftrightarrow s/(s^2+\omega^2)$

Circuit Element Transformations

Transforming Circuit Elements:

Element	Time Domain	s-Domain
Resistor	v = Ri	V(s) = RI(s)
Inductor	v = L(di/dt)	V(s) = sLI(s)-Li(0)
Capacitor	i = C(dv/dt)	I(s) = sCV(s)-Cv(0)



CO-3: Transient and Steady-State Analysis

Initial and final value theorems, solving differential equations using Laplace transforms, and circuit examples

Value Theorems

Initial Value Theorem: Determines f(0+) from F(s)

$$f(0+) = \lim_{s \to \infty} sF(s)$$

Final Value Theorem: Determines f(∞) from F(s)

$$f(\infty) = \lim_{s \to 0} sF(s)$$

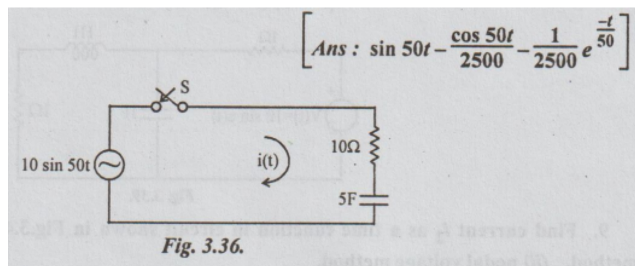
Applications:

1. Find initial conditions without inverse transform
2. Determine steady-state behavior directly
3. Verify circuit response at extreme conditions

Solving Differential Equations

Solution Process:

1. Transform circuit to s-domain
2. Apply circuit laws in s-domain
3. Solve for unknown quantities
4. Apply inverse Laplace transform

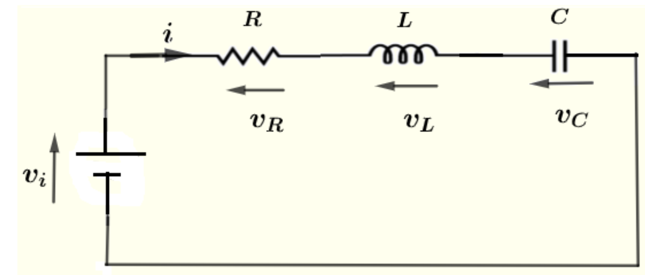


Key Advantage: Transforms differential equations to algebraic equations

Transient & Steady-State

Response Components:

Transient Response	Steady-State Response
Temporary behavior during transition between states Contains exponential terms with negative real parts	Long-term behavior after transients decay Contains sinusoidal terms at input frequency



Poles & Zeros:

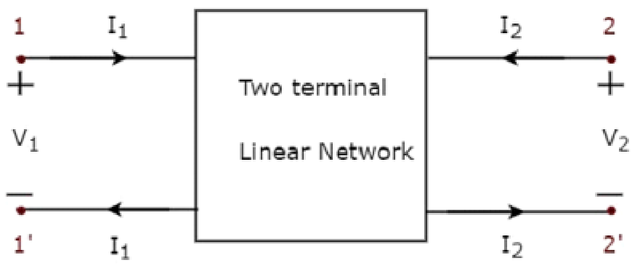
1. Poles determine natural response and stability
2. Zeros affect amplitude and phase characteristics

CO-4: Two-Port Network Fundamentals

Definitions, Z-parameters (impedance parameters), and Y-parameters (admittance parameters) with circuit diagrams

Two-Port Network Basics

Definition: Electrical network with two pairs of terminals (ports) where input is applied to one port and output is taken from the other



Key Variables:

- V^1, V^2: Port voltages
- I^1, I^2: Port currents
- Positive convention: Current entering the port

Applications:

- Amplifier and filter design
- Transmission line analysis
- Network interconnection

Z-Parameters

Impedance Parameters: Relate port voltages to port currents

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

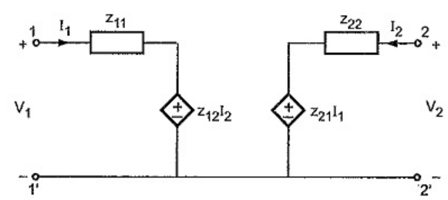


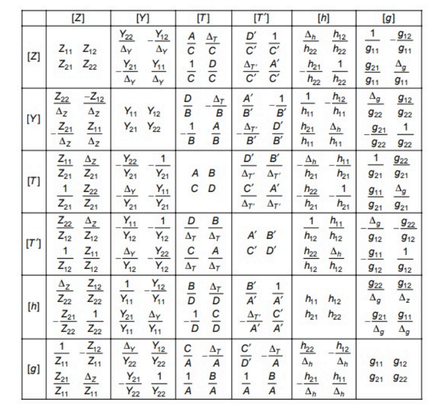
Fig. 6.3 Equivalent network of a two port network interms of z-parameters

Parameter	Definition	Test Condition
Z_{11}	V_1/I_1	$I_2 = 0$ (open circuit)
Z_{12}	V_1/I_2	$I_1 = 0$ (open circuit)

Y-Parameters

Admittance Parameters: Relate port currents to port voltages

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$



	[Z]	[Y]	[T]	[T′]	[h]	[g]
[Z]	$\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$	$\begin{matrix} \frac{Y_{22}}{\Delta_Y} & -\frac{Y_{12}}{\Delta_Y} \\ -\frac{Y_{21}}{\Delta_Y} & \frac{Y_{11}}{\Delta_Y} \end{matrix}$	$\begin{matrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{D'}{C'} & \frac{1}{C'} \\ \frac{\Delta_{T'}}{C'} & \frac{A'}{C'} \end{matrix}$	$\begin{matrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{11}} & -\frac{g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta_g}{g_{11}} \end{matrix}$
[Y]	$\begin{matrix} \frac{Z_{22}}{\Delta_Z} & \frac{-Z_{12}}{\Delta_Z} \\ -\frac{Z_{21}}{\Delta_Z} & \frac{Z_{11}}{\Delta_Z} \end{matrix}$	$\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}$	$\begin{matrix} \frac{D}{B} & -\frac{\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{A'}{B'} & -\frac{1}{B'} \\ -\frac{\Delta_{T'}}{B'} & \frac{D'}{B'} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta_g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ -\frac{g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{matrix}$
[T]	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta_Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$	$\begin{matrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta_Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$	$\begin{matrix} \frac{D'}{\Delta_{T'}} & \frac{B'}{\Delta_{T'}} \\ \frac{C'}{\Delta_{T'}} & \frac{A'}{\Delta_{T'}} \end{matrix}$	$\begin{matrix} -\frac{\Delta_h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$	$\begin{matrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta_g}{g_{21}} \end{matrix}$
[T′]	$\begin{matrix} \frac{Z_{22}}{Z_{12}} & \frac{\Delta_Z}{Z_{12}} \\ \frac{1}{Z_{12}} & \frac{Z_{11}}{Z_{12}} \end{matrix}$	$\begin{matrix} -\frac{Y_{11}}{Y_{12}} & -\frac{1}{Y_{12}} \\ -\frac{\Delta_Y}{Y_{12}} & -\frac{Y_{22}}{Y_{12}} \end{matrix}$	$\begin{matrix} \frac{D}{\Delta_T} & \frac{B}{\Delta_T} \\ \frac{C}{\Delta_T} & \frac{A}{\Delta_T} \end{matrix}$	$\begin{matrix} A' & B' \\ C' & D' \end{matrix}$	$\begin{matrix} \frac{1}{h_{12}} & \frac{h_{11}}{h_{12}} \\ \frac{h_{22}}{h_{12}} & \frac{\Delta_h}{h_{12}} \end{matrix}$	$\begin{matrix} -\frac{\Delta_g}{g_{12}} & -\frac{g_{22}}{g_{12}} \\ -\frac{g_{11}}{g_{12}} & \frac{1}{g_{12}} \end{matrix}$
[h]	$\begin{matrix} \frac{\Delta_Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta_Y}{Y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} \frac{B'}{A'} & \frac{1}{A'} \\ -\frac{\Delta_{T'}}{A'} & \frac{C'}{A'} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} \frac{g_{22}}{\Delta_g} & \frac{g_{12}}{\Delta_z} \\ -\frac{g_{21}}{\Delta_g} & \frac{g_{11}}{\Delta_g} \end{matrix}$
[g]	$\begin{matrix} \frac{1}{Z_{11}} & -\frac{Z_{12}}{Z_{11}} \\ \frac{Z_{21}}{Z_{11}} & \frac{\Delta_Z}{Z_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta_Y}{Y_{22}} & \frac{Y_{12}}{Y_{22}} \\ -\frac{Y_{21}}{Y_{22}} & \frac{1}{Y_{22}} \end{matrix}$	$\begin{matrix} \frac{C}{A} & -\frac{\Delta_T}{A} \\ \frac{1}{A} & \frac{B}{A} \end{matrix}$	$\begin{matrix} \frac{C'}{D'} & -\frac{\Delta_T}{A} \\ \frac{1}{A} & \frac{B}{A} \end{matrix}$	$\begin{matrix} \frac{h_{22}}{\Delta_h} & -\frac{h_{12}}{\Delta_h} \\ -\frac{h_{21}}{\Delta_h} & \frac{h_{11}}{\Delta_h} \end{matrix}$	$\begin{matrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{matrix}$

Parameter	Definition	Test Condition
Y_{11}	I_1/V_1	$V_2 = 0$ (short circuit)
Y_{12}	I_1/V_2	$V_1 = 0$ (short circuit)

CO-4: Additional Two-Port Parameters

H-parameters (hybrid parameters), Transmission parameters (ABCD parameters), and interconnection of two-port networks

H-Parameters

Hybrid Parameters: Mix of impedance and admittance parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Name	Express	In Terms of	Defining Equations
Impedance	V_1, V_2	I_1, I_2	$V_1 = z_{11}I_1 + z_{12}I_2$ and $V_2 = z_{21}I_1 + z_{22}I_2$
Admittance	I_1, I_2	V_1, V_2	$I_1 = y_{11}V_1 + y_{12}V_2$ and $I_2 = y_{21}V_1 + y_{22}V_2$
Hybrid	V_1, I_2	I_1, V_2	$V_1 = h_{11}I_1 + h_{12}V_2$ and $I_2 = h_{21}I_1 + h_{22}V_2$
Transmission	V_1, I_1	$V_2, -I_2$	$V_1 = A\,V_2 - B\,I_2$ and $I_1 = C\,V_2 - D\,I_2$

Parameter	Definition	Test Condition
h_{11}	V_1/I_1	$V_2 = 0$ (short circuit)
h_{21}	I_2/I_1	$V_2 = 0$ (short circuit)

Applications: Transistor modeling

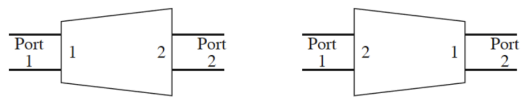
Transmission Parameters

ABCD Parameters: Relate input voltage/current to output voltage/current

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

ABCD matrix representation is $\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$

a. For the ABCD network shown below prove that reversing the network causes the ABCD parameters to change as indicated.



$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ $\begin{bmatrix} D & B \\ C & A \end{bmatrix}$

Parameter	Definition	Test Condition
A	V_1/V_2	$I_2 = 0$ (open circuit)
D	I_1/I_2	$V_2 = 0$ (short circuit)

Applications: Transmission lines, cascaded networks

Network Interconnection

Series Connection: Port 2 of first network connects to port 1 of second network

- Z-parameters: $Z = Z^A + Z^B$
- Used for cascaded amplifiers

Parallel Connection: Both ports of networks connected in parallel

- Y-parameters: $Y = Y^A + Y^B$
- Used for parallel filters

Cascade Connection:

- ABCD-parameters: $T = T^A \times T^B$
- Used for transmission lines

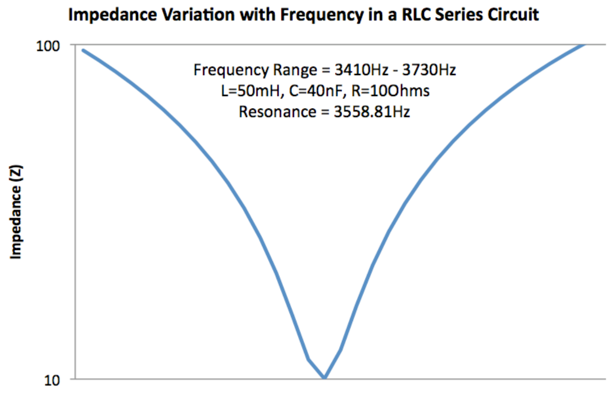
CO-5: Resonance in RLC Circuits

Series and parallel resonance, resonant frequency, quality factor, and frequency response curves

Series Resonance

Condition: $X_L = X_C$

$$\omega_0 = 1/\sqrt{LC}$$

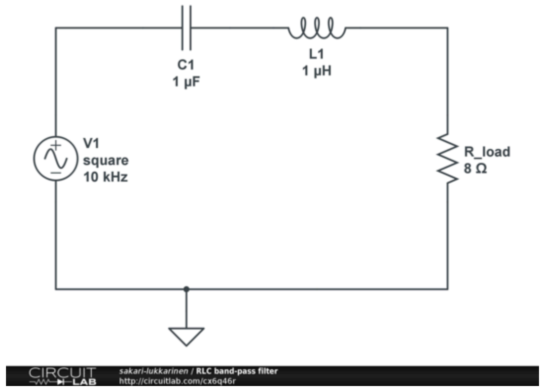


Parameter	Value at Resonance
Impedance (Z)	Minimum = R
Current (I)	Maximum = V/R
Power Factor	Unity (1)

Parallel Resonance

Condition: $B_L = B_C$

$$\omega_0 = 1/\sqrt{LC}$$



Parameter	Value at Resonance
Impedance (Z)	Maximum
Current (I)	Minimum
Power Factor	Unity (1)

Quality Factor & Bandwidth

Quality Factor (Q): Measure of selectivity

$$Q = \omega_0 L/R = 1/(\omega_0 RC)$$

Bandwidth (BW): Frequency range where response ≥ 0.707 of maximum

$$BW = \omega_0/Q$$

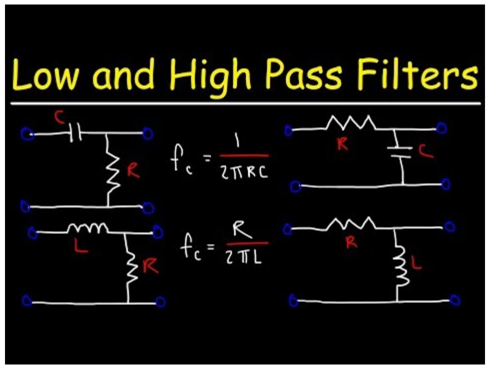
Applications:

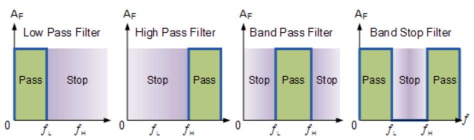
- Radio tuning circuits
- Signal filtering
- Oscillator design
- Power factor correction

CO-5: Passive Filter Design

Low-pass, high-pass, band-pass, and band-stop filters with circuit diagrams and frequency response characteristics

Low-Pass Filter

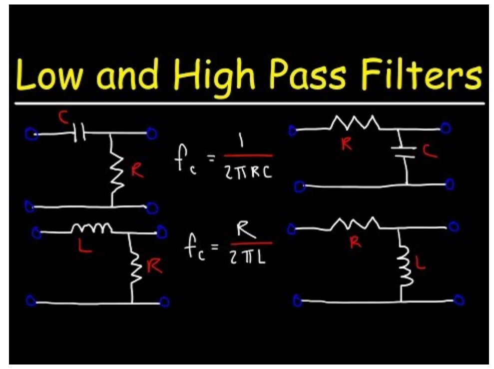


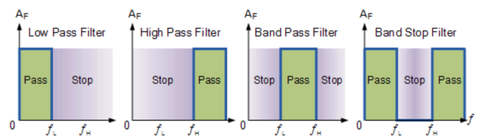


$f_c = 1/(2\pi RC)$

- **Function:** Passes low frequencies, attenuates high frequencies
- **Applications:** Audio systems, anti-aliasing, power supplies

High-Pass Filter

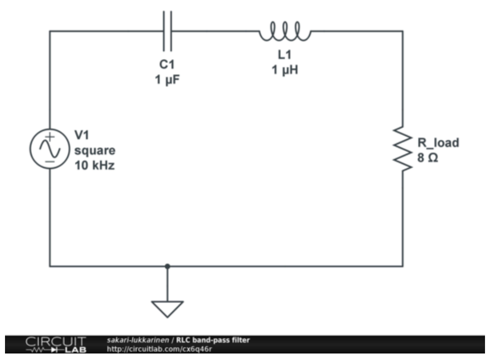


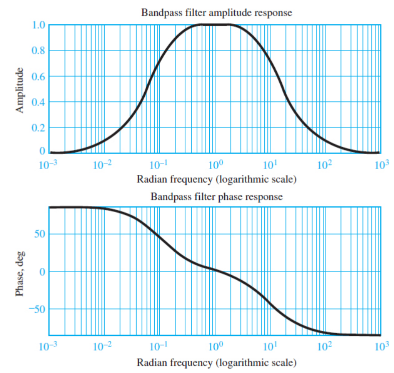


$f_c = 1/(2\pi RC)$

- **Function:** Passes high frequencies, attenuates low frequencies
- **Applications:** Audio crossovers, AC coupling, DC blocking

Band-Pass Filter

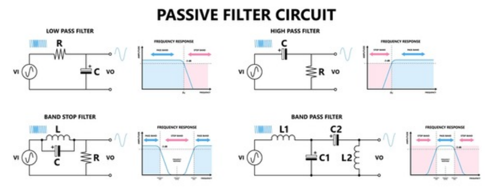


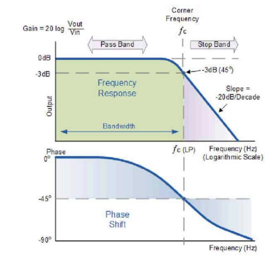


- **Function:** Passes frequencies within a specific band
- **Applications:** Radio tuning, audio equalizers, communication systems

$f_0 = 1/(2\pi\sqrt{LC})$

Band-Stop (Notch) Filter





- **Function:** Rejects frequencies within a specific band
- **Applications:** Noise cancellation, harmonic filtering, audio systems

$f_0 = 1/(2\pi\sqrt{LC})$

Summary

Key concepts and interconnections in Network Theory

Network Simplification

- **Kirchhoff's Laws:** Current and voltage conservation
- **Thevenin/Norton:** Equivalent circuit representations
- **Superposition:** Linear response to multiple sources
- **Maximum Power Transfer:** $R_L = R_{th}$

Frequency Domain Analysis

- **Phasors:** Complex representation of sinusoids
- **Impedance/Admittance:** $Z = R + jX$, $Y = G + jB$
- **Complex Power:** $S = P + jQ = V \cdot I^*$
- **Frequency Response:** $H(j\omega) = |H(j\omega)| \angle\phi(\omega)$

Laplace Transform Analysis

- **Transform:** $F(s) = \int f(t)e^{-st}dt$
- **Circuit Elements:** $R \rightarrow R$, $L \rightarrow sL$, $C \rightarrow 1/sC$
- **Value Theorems:** Initial/final values from $F(s)$
- **Transient/Steady-State:** Natural and forced responses

Two-Port Networks

- **Z-Parameters:** $V = Z \cdot I$ (impedance)
- **Y-Parameters:** $I = Y \cdot V$ (admittance)
- **H-Parameters:** Mixed impedance/admittance
- **ABCD-Parameters:** Transmission parameters

Resonance & Filters

- **Resonance:** $\omega_0 = 1/\sqrt{LC}$
- **Quality Factor:** $Q = \omega_0 L/R$
- **Filter Types:** LP, HP, BP, BS
- **Frequency Response:** Passband/stopband characteristics

Key Interconnections

Simplification → Analysis

Network theorems enable complex circuit simplification before applying frequency or Laplace domain analysis

Frequency → Laplace

Laplace transforms extend frequency domain analysis to include transient responses and initial conditions

Two-Port → Filters

Two-port network parameters provide systematic methods for filter design and cascade connection

Thank You

For your attention to this Network Theory presentation

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Further Study

- Network Analysis - M.E. Van Valkenburg
- Engineering Circuit Analysis - Hayt, Kemmerly, Durbin
- Network Theory - N. C. Jagan & C. Lakshmi
- Network Theory - A. Sudhakar & Shyammohan S. Palli
- IEEE Transactions on Circuits and Systems