

C2_W4_Assignment

November 22, 2023

1 Assignment 4: Word Embeddings

Welcome to the fourth (and last) programming assignment of Course 2!

In this assignment, you will practice how to compute word embeddings and use them for sentiment analysis. - To implement sentiment analysis, you can go beyond counting the number of positive words and negative words. - You can find a way to represent each word numerically, by a vector. - The vector could then represent syntactic (i.e. parts of speech) and semantic (i.e. meaning) structures.

In this assignment, you will explore a classic way of generating word embeddings or representations. - You will implement a famous model called the continuous bag of words (CBOW) model.

By completing this assignment you will:

- Train word vectors from scratch.
- Learn how to create batches of data.
- Understand how backpropagation works.
- Plot and visualize your learned word vectors.

Knowing how to train these models will give you a better understanding of word vectors, which are building blocks to many applications in natural language processing.

1.1 Important Note on Submission to the AutoGrader

Before submitting your assignment to the AutoGrader, please make sure you are not doing the following:

1. You have not added any *extra print* statement(s) in the assignment.
2. You have not added any *extra* code cell(s) in the assignment.
3. You have not changed any of the function parameters.
4. You are not using any global variables inside your graded exercises. Unless specifically instructed to do so, please refrain from it and use the local variables instead.
5. You are not changing the assignment code where it is not required, like creating *extra* variables.

If you do any of the following, you will get something like, **Grader not found** (or similarly unexpected) error upon submitting your assignment. Before asking for help/debugging the errors in your assignment, check for these first. If this is the case, and you don't remember the changes you have made, you can get a fresh copy of the assignment by following these [instructions](#).

1.2 Outline

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1. The Continuous bag of words model

Let's take a look at the following sentence: >'I am happy because I am learning'.

- In continuous bag of words (CBOW) modeling, we try to predict the center word given a few context words (the words around the center word).
- For example, if you were to choose a context half-size of say $C = 2$, then you would try to predict the word **happy** given the context that includes 2 words before and 2 words after the center word:

C words before: [I, am]

C words after: [because, I]

- In other words:

$$context = [I, am, because, I]$$

$$target = happy$$

The structure of your model will look like this:

Figure 1

Where \bar{x} is the average of all the one hot vectors of the context words.

Figure 2

Once you have encoded all the context words, you can use \bar{x} as the input to your model.

The architecture you will be implementing is as follows:

$$h = W_1 X + b_1 \quad (1)$$

$$a = \text{ReLU}(h) \quad (2)$$

$$z = W_2 a + b_2 \quad (3)$$

$$\hat{y} = \text{softmax}(z) \quad (4)$$

(1)

```
[ ]: # Import Python libraries and helper functions (in utils2)
import nltk
from nltk.tokenize import word_tokenize
import numpy as np
from collections import Counter
from utils2 import sigmoid, get_batches, compute_pca, get_dict
import w4_unittest

nltk.download('punkt')
```

```
[ ]: # Download sentence tokenizer
nltk.data.path.append('.')
```

```
[ ]: # Load, tokenize and process the data
import re # Load the
    ↳Regex-modul
with open('./data/shakespeare.txt') as f:
    data = f.read() # Read in
    ↳the data
data = re.sub(r'[!?!;-]', '.', data) #
    ↳Punktuations are replaced by .
data = nltk.word_tokenize(data) # Tokenize
    ↳string to words
data = [ ch.lower() for ch in data if ch.isalpha() or ch == '.'] # Lower
    ↳case and drop non-alphabetical tokens
print("Number of tokens:", len(data), '\n', data[:15]) # print
    ↳data sample
```

```
[ ]: # Compute the frequency distribution of the words in the dataset (vocabulary)
fdist = nltk.FreqDist(word for word in data)
print("Size of vocabulary: ", len(fdist) )
print("Most frequent tokens: ", fdist.most_common(20) ) # print the 20 most
    ↳frequent words and their freq.
```

Mapping words to indices and indices to words We provide a helper function to create a dictionary that maps words to indices and indices to words.

```
[ ]: # get_dict creates two dictionaries, converting words to indices and viceversa.
word2Ind, Ind2word = get_dict(data)
V = len(word2Ind)
print("Size of vocabulary: ", V)
```

```
[ ]: # example of word to index mapping
print("Index of the word 'king' : ",word2Ind['king'] )
print("Word which has index 2743: ",Ind2word[2743] )
```

2 Training the Model

1.2.1 Initializing the model

You will now initialize two matrices and two vectors. - The first matrix (W_1) is of dimension $N \times V$, where V is the number of words in your vocabulary and N is the dimension of your word vector. - The second matrix (W_2) is of dimension $V \times N$. - Vector b_1 has dimensions $N \times 1$ - Vector b_2 has dimensions $V \times 1$. - b_1 and b_2 are the bias vectors of the linear layers from matrices W_1 and W_2 .

The overall structure of the model will look as in Figure 1, but at this stage we are just initializing the parameters.

Exercise 01 Please use `numpy.random.rand` to generate matrices that are initialized with random values from a uniform distribution, ranging between 0 and 1.

Note: In the next cell you will encounter a random seed. Please **DO NOT** modify this seed so your solution can be tested correctly.

```
[ ]: # UNIT TEST COMMENT: Candidate for Table Driven Tests
# UNQ_C1 GRADED FUNCTION: initialize_model
def initialize_model(N,V, random_seed=1):
    '''
    Inputs:
        N: dimension of hidden vector
        V: dimension of vocabulary
        random_seed: random seed for consistent results in the unit tests
    Outputs:
        W1, W2, b1, b2: initialized weights and biases
    '''

    ### START CODE HERE (Replace instances of 'None' with your code) ###
    np.random.seed(random_seed)
    # W1 has shape (N,V)
    W1 = np.random.rand(N,V)
    # W2 has shape (V,N)
    W2 = np.random.rand(V,N)
    # b1 has shape (N,1)
    b1 = np.random.rand(N,1)
    # b2 has shape (V,1)
```

```

b2 = np.random.rand(V,1)

### END CODE HERE ###
return W1, W2, b1, b2

```

```

[ ]: # Test your function example.
tmp_N = 4
tmp_V = 10
tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(tmp_N,tmp_V)
assert tmp_W1.shape == ((tmp_N,tmp_V))
assert tmp_W2.shape == ((tmp_V,tmp_N))
print(f"tmp_W1.shape: {tmp_W1.shape}")
print(f"tmp_W2.shape: {tmp_W2.shape}")
print(f"tmp_b1.shape: {tmp_b1.shape}")
print(f"tmp_b2.shape: {tmp_b2.shape}")

```

Expected Output

```

tmp_W1.shape: (4, 10)
tmp_W2.shape: (10, 4)
tmp_b1.shape: (4, 1)
tmp_b2.shape: (10, 1)

```

```

[ ]: # Test your function
w4_unittest.test_initialize_model(initialize_model)

```

2.1 Softmax Before we can start training the model, we need to implement the softmax function as defined in equation 5:

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{i=0}^{V-1} e^{z_i}} \quad (5)$$

- Array indexing in code starts at 0.
- V is the number of words in the vocabulary (which is also the number of rows of z).
- i goes from 0 to $|V| - 1$.

Exercise 02 **Instructions:** Implement the softmax function below.

- Assume that the input z to **softmax** is a 2D array
- Each training example is represented by a vector of shape $(V, 1)$ in this 2D array.
- There may be more than one column, in the 2D array, because you can put in a batch of examples to increase efficiency. Let's call the batch size lowercase m , so the z array has shape (V, m)
- When taking the sum from $i = 1 \cdots V - 1$, take the sum for each column (each example) separately.

Please use - `numpy.exp` - `numpy.sum` (set the axis so that you take the sum of each column in z)

```
[ ]: # UNIT TEST COMMENT: Candidate for Table Driven Tests
# UNQ_C2 GRADED FUNCTION: softmax
def softmax(z):
    """
    Inputs:
        z: output scores from the hidden layer
    Outputs:
        yhat: prediction (estimate of y)
    """
    ### START CODE HERE (Replace instances of 'None' with your own code) ###
    # Calculate yhat (softmax)
    e_z = np.exp(z)
    yhat = e_z/np.sum(e_z,axis=0)
    ### END CODE HERE ###
    return yhat

[ ]: # Test the function
tmp = np.array([[1,2,3],
                [1,1,1]
                ])
tmp_sm = softmax(tmp)
display(tmp_sm)
```

Expected Output

```
array([[0.5      , 0.73105858, 0.88079708],
       [0.5      , 0.26894142, 0.11920292]])
```

```
[ ]: # Test your function
w4_unittest.test_softmax(softmax)
```

2.2 Forward propagation

Exercise 03 Implement the forward propagation z according to equations (1) to (3).

$$h = W_1 X + b_1 \tag{1}$$

$$h = \text{ReLU}(h) \tag{2}$$

$$z = W_2 h + b_2 \tag{3}$$

$$\tag{2}$$

For that, you will use as activation the Rectified Linear Unit (ReLU) given by:

$$f(h) = \max(0, h) \tag{6}$$

Hints

You can use `numpy.maximum(x1,x2)` to get the maximum of two values

Use `numpy.dot(A,B)` to matrix multiply A and B

```
[ ]: # UNIT TEST COMMENT: Candidate for Table Driven Tests
# UNQ_C3 GRADED FUNCTION: forward_prop
def forward_prop(x, W1, W2, b1, b2):
    """
    Inputs:
        x: average one hot vector for the context
        W1, W2, b1, b2: matrices and biases to be learned
    Outputs:
        z: output score vector
    """

    ### START CODE HERE (Replace instances of 'None' with your own code) ###
    # Calculate h
    h = np.dot(W1,x)+b1

    # Apply the relu on h (store result in h)
    h = np.maximum(0,h)

    # Calculate z
    z = np.dot(W2,h)+b2
    ### END CODE HERE ###

    return z, h
```

```
[ ]: # Test the function

# Create some inputs
tmp_N = 2
tmp_V = 3
tmp_x = np.array([[0,1,0]]).T
#print(tmp_x)
tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(N=tmp_N,V=tmp_V,
↳random_seed=1)

print(f"x has shape {tmp_x.shape}")
print(f"N is {tmp_N} and vocabulary size V is {tmp_V}")

# call function
tmp_z, tmp_h = forward_prop(tmp_x, tmp_W1, tmp_W2, tmp_b1, tmp_b2)
print("call forward_prop")
print()
# Look at output
print(f"z has shape {tmp_z.shape}")
print("z has values:")
```

```

print(tmp_z)

print()

print(f"h has shape {tmp_h.shape}")
print("h has values:")
print(tmp_h)

```

Expected output

```

x has shape (3, 1)
N is 2 and vocabulary size V is 3
call forward_prop

```

```

z has shape (3, 1)
z has values:
[[0.55379268]
 [1.58960774]
 [1.50722933]]

```

```

h has shape (2, 1)
h has values:
[[0.92477674]
 [1.02487333]]

```

```

[ ]: # Test your function
w4_unittest.test_forward_prop(forward_prop)

```

2.3 Cost function

- We have implemented the *cross-entropy* cost function for you.

```

[ ]: # compute_cost: cross-entropy cost function
def compute_cost(y, yhat, batch_size):

    # cost function
    logprobs = np.multiply(np.log(yhat), y)
    cost = - 1/batch_size * np.sum(logprobs)
    cost = np.squeeze(cost)
    return cost

```

```

[ ]: # Test the function
tmp_C = 2
tmp_N = 50
tmp_batch_size = 4
tmp_word2Ind, tmp_Ind2word = get_dict(data)
tmp_V = len(word2Ind)

```



```

tmp_x, tmp_y = next(get_batches(data, tmp_word2Ind, tmp_V, tmp_C,
    ↳ tmp_batch_size))

print(f"tmp_x.shape {tmp_x.shape}")
print(f"tmp_y.shape {tmp_y.shape}")

tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(tmp_N, tmp_V)

print(f"tmp_W1.shape {tmp_W1.shape}")
print(f"tmp_W2.shape {tmp_W2.shape}")
print(f"tmp_b1.shape {tmp_b1.shape}")
print(f"tmp_b2.shape {tmp_b2.shape}")

tmp_z, tmp_h = forward_prop(tmp_x, tmp_W1, tmp_W2, tmp_b1, tmp_b2)
print(f"tmp_z.shape: {tmp_z.shape}")
print(f"tmp_h.shape: {tmp_h.shape}")

tmp_yhat = softmax(tmp_z)
print(f"tmp_yhat.shape: {tmp_yhat.shape}")

tmp_cost = compute_cost(tmp_y, tmp_yhat, tmp_batch_size)
print("call compute_cost")
print(f"tmp_cost {tmp_cost:.4f}")

```

Expected output

```

tmp_x.shape (5778, 4)
tmp_y.shape (5778, 4)
tmp_W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp_b1.shape (50, 1)
tmp_b2.shape (5778, 1)
tmp_z.shape: (5778, 4)
tmp_h.shape: (50, 4)
tmp_yhat.shape: (5778, 4)
call compute_cost
tmp_cost 8.9542

```

2.4 Training the Model - Backpropagation

Exercise 04 Now that you have understood how the CBOW model works, you will train it. You created a function for the forward propagation. Now you will implement a function that computes the gradients to backpropagate the errors.

```

[ ]: def back_prop(x, yhat, y, h, W1, W2, b1, b2, batch_size):
    '''
    Inputs:
        x: average one hot vector for the context

```

```

    yhat: prediction (estimate of y)
    y: target vector
    h: hidden vector (see eq. 1)
    W1, W2, b1, b2: matrices and biases
    batch_size: batch size
Outputs:
    grad_W1, grad_W2, grad_b1, grad_b2: gradients of matrices and biases
'''

# Compute l1 as W2^T (Yhat - Y)
l1 = np.dot(W2.T, yhat - y)
l1 = np.maximum(0, l1) # relu

grad_W1 = np.dot(l1, x.T)
grad_W1 = 1 / batch_size * grad_W1

grad_W2 = 1 / batch_size * np.dot(yhat - y, h.T)

grad_b1 = 1 / batch_size * np.sum(l1, axis=1, keepdims=True) #

grad_b2 = 1 / batch_size * np.sum(yhat - y, axis=1, keepdims=True)

return grad_W1, grad_W2, grad_b1, grad_b2

```

```

[ ]: # Test the function
tmp_C = 2
tmp_N = 50
tmp_batch_size = 4
tmp_word2Ind, tmp_Ind2word = get_dict(data)
tmp_V = len(word2Ind)

# get a batch of data
tmp_x, tmp_y = next(get_batches(data, tmp_word2Ind, tmp_V, tmp_C,
    tmp_batch_size))

print("get a batch of data")
print(f"tmp_x.shape {tmp_x.shape}")
print(f"tmp_y.shape {tmp_y.shape}")

print()
print("Initialize weights and biases")
tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(tmp_N, tmp_V)

print(f"tmp_W1.shape {tmp_W1.shape}")
print(f"tmp_W2.shape {tmp_W2.shape}")
print(f"tmp_b1.shape {tmp_b1.shape}")

```

```

print(f"tmp_b2.shape {tmp_b2.shape}")

print()
print("Forwad prop to get z and h")
tmp_z, tmp_h = forward_prop(tmp_x, tmp_W1, tmp_W2, tmp_b1, tmp_b2)
print(f"tmp_z.shape: {tmp_z.shape}")
print(f"tmp_h.shape: {tmp_h.shape}")

print()
print("Get yhat by calling softmax")
tmp_yhat = softmax(tmp_z)
print(f"tmp_yhat.shape: {tmp_yhat.shape}")

tmp_m = (2*tmp_C)
tmp_grad_W1, tmp_grad_W2, tmp_grad_b1, tmp_grad_b2 = back_prop(tmp_x, tmp_yhat,
    ↳tmp_y, tmp_h, tmp_W1, tmp_W2, tmp_b1, tmp_b2, tmp_batch_size)

print()
print("call back_prop")
print(f"tmp_grad_W1.shape {tmp_grad_W1.shape}")
print(f"tmp_grad_W2.shape {tmp_grad_W2.shape}")
print(f"tmp_grad_b1.shape {tmp_grad_b1.shape}")
print(f"tmp_grad_b2.shape {tmp_grad_b2.shape}")

```

Expected output

```

get a batch of data
tmp_x.shape (5778, 4)
tmp_y.shape (5778, 4)

Initialize weights and biases
tmp_W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp_b1.shape (50, 1)
tmp_b2.shape (5778, 1)

Forwad prop to get z and h
tmp_z.shape: (5778, 4)
tmp_h.shape: (50, 4)

Get yhat by calling softmax
tmp_yhat.shape: (5778, 4)

call back_prop
tmp_grad_W1.shape (50, 5778)
tmp_grad_W2.shape (5778, 50)
tmp_grad_b1.shape (50, 1)

```

```
tmp_grad_b2.shape (5778, 1)
```

```
[ ]: # Test your function
w4_unittest.test_back_prop(back_prop)
```

Gradient Descent

Exercise 05 Now that you have implemented a function to compute the gradients, you will implement batch gradient descent over your training set.

Hint: For that, you will use `initialize_model` and the `back_prop` functions which you just created (and the `compute_cost` function). You can also use the provided `get_batches` helper function:

```
for x, y in get_batches(data, word2Ind, V, C, batch_size):
```

```
...
```

Also: print the cost after each batch is processed (use batch size = 128)

```
[ ]: def gradient_descent(data, word2Ind, N, V, num_iters, alpha=0.03):

    '''
    This is the gradient_descent function

    Inputs:
        data:      text
        word2Ind:  words to Indices
        N:         dimension of hidden vector
        V:         dimension of vocabulary
        num_iters: number of iterations
    Outputs:
        W1, W2, b1, b2: updated matrices and biases

    '''
    W1, W2, b1, b2 = initialize_model(N,V, random_seed=282)
    batch_size = 128
    iters = 0
    C = 2
    for x, y in get_batches(data, word2Ind, V, C, batch_size):
        z, h = forward_prop(x, W1, W2, b1, b2)

        yhat = softmax(z)

        cost = compute_cost(y, yhat, batch_size)

        if ( (iters+1) % 10 == 0):
            print(f"iters: {iters + 1} cost: {cost:.6f}")

    # Get gradients
```

```

    grad_W1, grad_W2, grad_b1, grad_b2 = back_prop(x, yhat, y, h, W1, W2,
    ↪ b1, b2, batch_size)

    # Update weights and biases
    W1 = W1 - grad_W1 * alpha
    W2 = W2 - grad_W2 * alpha
    b1 = b1 - grad_b1 * alpha
    b2 = b2 - grad_b2 * alpha

    iters += 1
    if iters == num_iters:
        break
    if iters % 100 == 0:
        alpha *= 0.66

    return W1, W2, b1, b2

```

```

[ ]: # test your function
# UNIT TEST COMMENT: Each time this cell is run the cost for each iteration
    ↪ changes slightly (the change is less dramatic after some iterations)
# to have this into account let's accept an answer as correct if the cost of
    ↪ iter 15 = 41.6 (without caring about decimal points beyond the first decimal)
# 41.66, 41.69778, 41.63, etc should all be valid answers.
C = 2
N = 50
word2Ind, Ind2word = get_dict(data)
V = len(word2Ind)
num_iters = 150
print("Call gradient_descent")
W1, W2, b1, b2 = gradient_descent(data, word2Ind, N, V, num_iters)

```

Expected Output

```

iters: 10 cost: 11.714748
iters: 20 cost: 3.788280
iters: 30 cost: 9.179923
iters: 40 cost: 1.747809
iters: 50 cost: 8.706968
iters: 60 cost: 10.182652
iters: 70 cost: 7.258762
iters: 80 cost: 10.214489
iters: 90 cost: 9.311061
iters: 100 cost: 10.103939
iters: 110 cost: 5.582018
iters: 120 cost: 4.330974
iters: 130 cost: 9.436612

```

```
iters: 140 cost: 6.875775
iters: 150 cost: 2.874090
```

Your numbers may differ a bit depending on which version of Python you're using.

```
[ ]: # Test your function
w4_unittest.test_gradient_descent(gradient_descent, data, word2Ind, N=10,
    ↪V=len(word2Ind), num_iters=15)
```

3.0 Visualizing the word vectors

In this part you will visualize the word vectors trained using the function you just coded above.

```
[ ]: from matplotlib import pyplot
%config InlineBackend.figure_format = 'svg'
words = ['king', 'queen', 'lord', 'man', 'woman', 'dog', 'wolf',
        'rich', 'happy', 'sad']

embs = (W1.T + W2)/2.0

# given a list of words and the embeddings, it returns a matrix with all the
    ↪embeddings
idx = [word2Ind[word] for word in words]
X = embs[idx, :]
print(X.shape, idx) # X.shape: Number of words of dimension N each
```

```
[1]: result= compute_pca(X, 2)
pyplot.scatter(result[:, 0], result[:, 1])
for i, word in enumerate(words):
    pyplot.annotate(word, xy=(result[i, 0], result[i, 1]))
pyplot.show()
```

```

    ↪-----

NameError                                Traceback (most recent call
    ↪last)

<ipython-input-1-bc7754381651> in <module>
----> 1 result= compute_pca(X, 2)
      2 pyplot.scatter(result[:, 0], result[:, 1])
      3 for i, word in enumerate(words):
      4     pyplot.annotate(word, xy=(result[i, 0], result[i, 1]))
      5 pyplot.show()

NameError: name 'compute_pca' is not defined
```

You can see that man and king are next to each other. However, we have to be careful with the interpretation of this projected word vectors, since the PCA depends on the projection – as shown in the following illustration.

```
[ ]: result= compute_pca(X, 4)
      pyplot.scatter(result[:, 3], result[:, 1])
      for i, word in enumerate(words):
          pyplot.annotate(word, xy=(result[i, 3], result[i, 1]))
      pyplot.show()
```

```
[ ]:
```

```
[ ]:
```