COEN 240

HW8

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(1) $f(x,y) = xy$

$h(x,y) = x+y-10$

$L = f(x,y) + \beta(x+y-10)$

$= xy + \beta(x+y-10)$

$\Rightarrow \frac{\partial L}{\partial x} = y + \beta = 0$ — (1)

$\Rightarrow \frac{\partial L}{\partial y} = x + \beta = 0$ — (2)

$\Rightarrow \frac{\partial L}{\partial \beta} = x + y - 10 = 0$ — (3)

from (1) (2) $y = -\beta,\ x = -\beta$

$-\beta - \beta - 10 = 0$

$-2\beta - 10 = 0$

$\beta = -5$

$\Rightarrow \boxed{x = 5,\ y = 5}$

(2) $f(x) = 7x_1^3 + 8x_2^2 + 9x_1$

s.t.

$h_1(x) \Rightarrow 6x_1 + 11x_2^2 = 13x_2$

$\Rightarrow 6x_1 + 11x_2^2 - 13x_2 = 0$

$\Rightarrow 6x_1 + 11x_2^2 - 13x_2$.

$g_1(x) \Rightarrow 5x_2^2 \leq 4x_1 - 7$

$\Rightarrow 5x_2^2 - 4x_1 + 7 \leq 0$

$g_2(x) \Rightarrow 20x_1 + 5 \geq x_2$

$\Rightarrow x_2 - 20x_1 - 5 \leq 0$

$$L(\alpha, \beta, x) = f(x) + \alpha_1 g_1(x) + \alpha_2 g_2(x) + h_1(x)$$

$$= 7x_1^3 + 8x_2^2 + 9x_1 + \alpha_1(5x_2^2 - 4x_1 + 7)$$
$$+ \alpha_2(x_2 - 20x_1 - 5) + \beta_1(6x_1 + 11x_2^2 - 13x_2)$$

(3)

$$f(x) = \frac{1}{2} \|w\|^2$$

s.t

$y_i(w^T x_i + b) \geq 1 \quad i = 1, 2 \ldots N$

$\Rightarrow g_i(x) = 1 - y_i(w^T x_i + b) \leq 0.$

$$L(w, b, \alpha) = f(x) + \sum_{i=1}^{N} \alpha_i \, g_i(x)$$

$$L = \frac{1}{2} ||w||^2 + \sum_{i=1}^{N} \alpha_i \left(1 - y_i \left(w^T x_i + b\right)\right)$$

$$\frac{\partial L}{\partial w} = w + \sum_{i=1}^{N} \left(-\alpha_i y_i x_i\right) = 0$$

$$w = \sum_{i=1}^{N} \alpha_i y_i x_i \quad \text{—— } (1)$$

$$\frac{\partial L}{\partial b} = \sum_{i=1}^{N} \alpha_i y_i = 0 \quad \text{—— } (2)$$

use w from Equation (1) and plug into L

$$L = \frac{1}{2} w.w^T + \sum_{i=1}^{N} \left(\alpha_i - \alpha_i y_i w^T x_i - \alpha_i y_i b\right)$$

$$= \frac{1}{2} w.w^T + \sum_{i=1}^{N} \alpha_i - \sum_{i=1}^{N} \alpha_i y_i w^T x_i - \underbrace{\cancel{\sum_{i=1}^{N} \alpha_i y_i} \, b}_{0 \text{ (from Eq (2))}}$$

$$= \sum_{i=1}^{N} \alpha_i + \frac{1}{2} \left(\sum_{i=1}^{N} \alpha_i y_i x_i\right) \left(\sum_{i=1}^{N} \alpha_i y_i (x_i)^T\right) - \sum_{i=1}^{N} \alpha_i y_i \left(\sum_{i=1}^{N} \alpha_i y_i (x_i)^T\right) x_i$$

$$= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j (x_i^T x_j) + \sum_{i=1}^{N} \alpha_i - \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j (x_i^T x_j)$$

$$= \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j (x_i^T x_j)$$

$$\theta_p(\omega) = \max_{\alpha, \beta : \alpha_i \geq 0} L(\omega, \alpha, \beta)$$

$$= \max_{\alpha, \beta : \alpha_i \geq 0} L(\omega, b, \alpha)$$

also s.t $\sum_{i=1}^{N} \alpha_i y_i = 0$ (from Eq (2))

$$\Rightarrow \max_{\alpha} J(\alpha) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j (x_i^T x_j)$$

s.t

$\alpha_i \geq 0 \quad , i = 1, 2 \ldots N$

$\sum_{i=1}^{N} \alpha_i y_i = 0 \quad , i = 1, 2 \ldots N.$

4 (a) $\phi(x) = [1, \sqrt{2}x, x^2]^T$

$\phi(x_1) = [1, \sqrt{2} \times 0, (0)^2]^T$

$= [1, 0, 0]^T$

$\phi(x_2) = [1, \sqrt{2} \times \sqrt{2}, (\sqrt{2})^2]^T$

$= [1, 2, 2]^T$

(b) Since there are only 2 points in the dataset, those points are support vectors and to maximize the width of the street. w vector will pass through those 2 points.

hence, width $= 2 \times$ margin

width is distance between those 2 points.

$(1,0,0)$ and $(1,2,2)$

$$\text{distance} = \sqrt{0^2 + 2^2 + 2^2}$$
$$= \sqrt{8}$$

$$\text{and margin} = \frac{\sqrt{8}}{2} = \sqrt{2}$$

$$\underline{\text{margin} = \sqrt{2}}$$

(c). $w = (1-1)t\,\hat{i} + (2-0)t\,\hat{j} + (2-0)t\,\hat{k}$

$w = 2t\,\hat{j} + 2t\,\hat{k}$ $\Rightarrow \boxed{w_1 = 0}$

$$\text{margin} = \frac{1}{\|w\|}$$

$$\sqrt{2} = \frac{1}{\sqrt{(2t)^2 + (2t)^2}} = \frac{1}{\sqrt{8t^2}}$$

$\sqrt{16t^2} = 1 \Rightarrow t = \frac{1}{4}$ $\boxed{\therefore \overline{w} = \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}}$

(d)

$$y_1(w^T \phi(x_1) + w_0) = 1 \quad (1)$$
$$y_2(w^T \phi(x_2) + w_0) = 1 \quad (2)$$

on decision boundary

for (1)

$$\Rightarrow -1\left(\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + w_0\right) = 1$$

$$\Rightarrow -1(0 + w_0) = 1$$

$$\boxed{w_0 = -1}$$

for (2)

$$\Rightarrow 1\left(\begin{bmatrix} 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + w_0\right) = 1$$

$$\Rightarrow 2 \times \frac{1}{2} + 2 \times \frac{1}{2} + w_0 = 1$$

$$\Rightarrow 2 + w_0 = 1$$

$$\boxed{w_0 = -1}$$

(e)

$$f(x) = w_0 + w^T \phi(x)$$

$$f(x) = -1 + \begin{bmatrix} 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ \sqrt{2}x \\ x^2 \end{bmatrix}$$

$$f(x) = -1 + \frac{\sqrt{2}x}{2} + \frac{x^2}{2} = \frac{x^2 + \sqrt{2}x - 2}{2}$$