

$$\text{Poisson}(x|\lambda) = e^{-\lambda} \frac{\lambda^x}{x!} \quad \text{for } x \in \{0, 1, 2, \dots\}$$

$$L(\lambda) = \prod_{i=1}^n e^{-\lambda} \frac{\lambda^{x_i}}{x_i!}$$

$$l(\lambda) = \log(L(\lambda))$$

$$= \log\left(\prod_{i=1}^n e^{-\lambda} \frac{\lambda^{x_i}}{x_i!}\right)$$

$$= \sum_{i=1}^n \log\left(e^{-\lambda} \frac{\lambda^{x_i}}{x_i!}\right)$$

$$= \sum_{i=1}^n \left[\log(e^{-\lambda}) + \log(\lambda^{x_i}) - \log(x_i!) \right]$$

$$= \sum_{i=1}^n \left[-\lambda + x_i \log \lambda - \log(x_i!) \right]$$

$$l(\lambda) = -n\lambda + \log \lambda \sum_{i=1}^n x_i - \sum_{i=1}^n \log(x_i!)$$

$$\text{So } \frac{\partial l(\lambda)}{\partial \lambda} = 0 \quad \text{for } \lambda_{MLE}$$

$$\frac{\partial \ell(\lambda)}{\partial \lambda} = -n + \frac{1}{\lambda} \sum_{i=1}^n x_i$$

$$-n + \frac{1}{\lambda} \sum_{i=1}^n x_i = 0$$

$$\sum_{i=1}^n x_i = n\lambda$$

$$\lambda_{MLE} = \frac{1}{n} \sum_{i=1}^n x_i$$

which is mean of the
n observations.