HW 9
COEN 240

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(a)

$$\mathcal{E}(n) = \frac{1}{2}\sum_{K} e_K^2(n)$$

$$e_K(n) = d_K(n) - y_K(n)$$

$$V_K(n) = \sum_{j=0}^{m} \omega_{Kj}(n)\, y_j(n)$$

$$y_K(n) = \phi(V_K(n))$$

$$\frac{\partial \mathcal{E}(n)}{\partial \omega_{Kj}(n)} = \frac{\partial \mathcal{E}(n)}{\partial e_K(n)} \cdot \frac{\partial e_K(n)}{\partial y_K(n)} \cdot \frac{\partial y_K(n)}{\partial V_K(n)} \cdot \frac{\partial V_K(n)}{\partial \omega_{Kj}(n)}$$

$$\frac{\partial \mathcal{E}(n)}{\partial e_K(n)} = e_K(n)$$

$$\frac{\partial e_K(n)}{\partial y_K(n)} = -1$$

$$\frac{\partial y_K(n)}{\partial V_K(n)} = \phi'(V_K(n))$$

$$\frac{\partial V_K(n)}{\partial \omega_{Kj}(n)} = y_j(n)$$

$$\frac{\partial \mathcal{E}(n)}{\partial \omega_{Kj}(n)} = e_K(n).(-1).\phi'(V_K(n)).y_j(n)$$

$$\Delta \omega_{Kj}(n) = -\eta \frac{\partial \mathcal{E}(n)}{\partial \omega_{Kj}(n)}$$

$$\Rightarrow \eta\, \underbrace{e_K(n).\phi'(V_K(n))}_{\delta_K(n)}.y_j(n)$$

$$\Rightarrow \boxed{\eta\, \delta_K(n)\, y_j(n)}$$

$$\delta_K(n) = e_K(n) \cdot \phi'(V_K(n)) \quad — \quad (1)$$

$$\phi'(V_K(n)) = \phi(V_K(n))(1 - \phi(V_K(n)))$$

$$\phi'(V_K(n)) = y_K(n)(1 - y_K(n)) \quad — \quad (2)$$

replace (2) in (1)

$$\delta_K(n) = e_K(n) \cdot [y_K(n)(1 - y_K(n))]$$

$$\boxed{\delta_K(n) = [d_K(n) - y_K(n)] \cdot [y_K(n)(1 - y_K(n))]}$$

(b) for a single w_{ji}, error will back propogate from all e_K's. So, we will sum all these.

$$\mathcal{E}(n) = \frac{1}{2} \sum_K e_K^2(n)$$

$$e_K(n) = d_K(n) - y_K(n)$$

$$V_K(n) = \sum_{j=0}^{m} w_{Kj}(n)\, y_j(n)$$

$$y_K(n) = \phi(V_K(n))$$

$$y_j(n) = \phi(V_j(n))$$

$$V_j(n) = \sum_{i=0}^{m} w_{ji}(n)\, y_i(n)$$

Path K : Error back Propogation

$$\frac{\partial \mathcal{E}(n)}{\partial w_{ji}(n)} = \frac{\partial \mathcal{E}(n)}{\partial e_K(n)} \cdot \frac{\partial e_K(n)}{\partial y_K(n)} \cdot \frac{\partial y_K(n)}{\partial v_K(n)} \cdot \frac{\partial v_K(n)}{\partial y_j(n)} \cdot \frac{\partial y_j(n)}{\partial v_j(n)} \cdot \frac{\partial v_j(n)}{\partial w_{ji}(n)}$$

$$\frac{\partial \mathcal{E}(n)}{\partial e_K(n)} = e_K(n) \qquad \frac{\partial y_j(n)}{\partial v_j(n)} = \phi'(v_j(n))$$

$$\frac{\partial e_K(n)}{\partial y_K(n)} = -1 \qquad \frac{\partial v_j(n)}{\partial w_{ji}(n)} = y_i(n)$$

$$\frac{\partial y_K(n)}{\partial v_K(n)} = \phi'(v_K(n))$$

$$\frac{\partial v_K(n)}{\partial y_j(n)} = w_{Kj}(n)$$

$$\frac{\partial \mathcal{E}(n)}{\partial w_{ji}(n)} = e_K(n) \cdot (-1) \cdot \phi'(v_K(n)) \cdot w_{Kj}(n) \cdot \phi'(v_j(n)) \cdot y_i(n)$$

Similarly for Path K-1 : Error back Propogation

$$\frac{\partial \mathcal{E}(n)}{\partial w_{ji}(n)} = e_{K-1}(n) \cdot (-1) \cdot \phi'(v_{K-1}(n)) \cdot w_{Kj}(n) \cdot \phi'(v_j(n)) \cdot y_i(n)$$

Hence, summing for all e_K's gives:

$$\frac{\partial \mathcal{E}(n)}{\partial w_{ji}(n)} = -\sum_{K} e_K(n) \cdot \phi'(v_K(n)) \cdot w_{Kj}(n) \cdot \phi'(v_j(n)) \cdot y_i(n)$$

$$\Delta w_{ji}(n) = -\eta \frac{\partial \mathcal{E}(n)}{\partial w_{ji}(n)}$$

$$\Rightarrow \eta \underbrace{\sum_{K} e_K(n) \cdot \phi'(v_K(n)) \cdot w_{Kj}(n) \cdot \phi'(v_j(n))}_{\delta_j(n)} \cdot y_i(n)$$

$$\Rightarrow \boxed{\Delta w_{ji}(n) = \eta \delta_j(n) \cdot y_i(n)}$$

$$\delta_j(n) = \sum_{K} \underbrace{e_K(n) \cdot \phi'(v_K(n))}_{\delta_K(n)} \cdot w_{Kj}(n) \cdot \phi'(v_j(n))$$

$\delta_K(n)$ — from Equation ① in (a)

$$\delta_j(n) = \sum_{K} \delta_K(n) \cdot w_{Kj}(n) \cdot \phi'(v_j(n))$$

$$\delta_j(n) = \sum_{K} \delta_K(n) \cdot w_{Kj}(n) \cdot \phi(v_j(n)) \cdot (1 - \phi(v_j(n)))$$

$$\boxed{\delta_j(n) = \sum_{K} \delta_K(n) \cdot w_{Kj}(n) \cdot y_j(n)(1 - y_j(n))}$$