HW-4
COEN 240

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linear regression cost function

$$J(w_0, w_1) = \frac{1}{2m} \sum_{i=1}^{m} (w_0 + w_1 x^{(i)} - y_i)^2$$

Assumptions: $m=1$, 1-D dataset

$$\Rightarrow J(w_0, w_1) = \frac{1}{2}(w_0 + w_1 x - y)^2$$

$\nabla J(w_0, w_1)$

$$\rightarrow \frac{\partial J}{\partial w_0} = \frac{1}{\cancel{2}} \times \cancel{2}(w_0 + w_1 x - y)$$

$$\rightarrow \frac{\partial J}{\partial w_1} = \frac{1}{\cancel{2}} \times \cancel{2}(w_0 + w_1 x - y)(x)$$

$\nabla^2 J(w_0, w_1)$

$$\rightarrow \frac{\partial^2 J}{\partial w_0^2} = 1$$

$$\rightarrow \frac{\partial^2 J}{\partial w_1^2} = x^2$$

$$\rightarrow \frac{\partial^2 J}{\partial w_0 \partial w_1} = x$$

$$\rightarrow \frac{\partial^2 J}{\partial w_1 \partial w_0} = x$$

$$H = \begin{bmatrix} 1 & x \\ x & x^2 \end{bmatrix}$$

→ Eigen Values of the Hessian Matrix

$$\begin{vmatrix} \lambda-1 & x \\ x & \lambda-x^2 \end{vmatrix} = 0$$

$$(\lambda-1)(\lambda-x^2) - x^2 = 0$$

$$\lambda^2 - \lambda x^2 - \lambda + \cancel{x^2} - \cancel{x^2} = 0$$

$$\lambda(\lambda - x^2 - 1) = 0$$

$$\lambda_1 = 0, \quad \lambda_2 = \underbrace{1 + x^2}_{\downarrow}$$

always Positive

$\lambda_1 = 0$

$\lambda_2 > 0$

⇒ Positive Semi Definite

→ Convex.

Convexity of $J(w_0, w_1)$ is Convex (PSD)