UNIT-1

Game — is a ~~invo~~ competitive activity.
Game — involves multiple players/agents.
Game — aim — maximize profit/utility
Game — has → rules

Game is a competitive activity in which multiple agents contest or compete to maximize their profit according to a set of rules.

Real life ex.

market — agents — firms
market — strategy — setting prices, adjusting Qty
market — aim — Maximize profit/share.
market — rules — anti collusion (to fix prices), Quality

politics — agents — political parties or (contestants)
politics — campaining strategy
politics → Spending limit, Campaign laws.
politics — votes/win, fame/power.

Wireless commⁿ — people/devices
Wireless commⁿ — max bandwidth/spectrum
Wireless commⁿ — to send → messages, video, voice, pictures etc.
Wireless commⁿ — rules → registered devices, Interferences.

Auction — Art, antiques, Spectrum, coal blocks
Auction — agents — Bidders
Auction — max utility.
Auction — rules → min bid value, increasing bid values, Bank guarentees, auctioneer.

Q. What does game theory deal with?
Game theory helps us to understand situations where decision makers interact. In real life situations people interact with each other. Your decision will affect what I shall get out of it. In GT we try to figure out

how this interaction takes place & what are the outcome can be generated

Prisoner's Dilemma - accused of a major crime, No eyewi...

P_1 — use one of them as witness / ~~...~~ or ~~use one~~ to get one/both to confess.

P_2.

Interagators devise a plan.

Both prisoners are interogated in separate rooms - No comm^n allowed b/n them.

Actions → Confess → Quiet
→ Deny (Do not Confess) → Fink.

Utility → 1) Both deny - Sentence for minor crime - 1y Both

2) Both confess - each gets a prison sentence of 3 years.

3) If one confesses & other denies, one who denies get sentence of 4 years.

PD — Prisoners
— Confess or deny
— minimize prison sentence
— No comm^n.

Use of PD

Retail price war — 2 shops
— set price H or L

Game Table - to analyze

P_1 ↓ / P_2 →	C	D
C	(3,3)	(0,4)
D	(4,0)	(1,1)

Strategic

Payoff - is determined by his action & other person's action.

Each one is trying to minimize his present term.

Since $u_1(C,D) = 0$, $u_2(C,D) = 0$
P1 con. — Player 1 con — P2
$u_1(C,C) = -3$

R_1 ↓ / R_2 →	H (Deny)	L (Confess)
(Deny) H	500,500	0,750
(Conf) L	750,0	250,250

1) Both H = +500
2) Both L = +250
3) LH / HL - (+750, 0) / (0, +750)

Model
1) Set of players $\{P_1, P_2\}$
2) Action set
— A_i - action set of player i Ex: $A_1 = \{C, D\}$, $A_2 = \{C, D\}$

Pay off - $\{u_i(o)\}$ → outcomes.
$u_i(a_i, a_{-i})$ — a_i: ith player action; a_{-i}: action of rest

3) Set of rules:-
4) Set of outcomes - $O = A_1 \times A_2$

Strategic interaction
→ Pay off is determined by action of individual together with actions of competitors.

Strategi... 2) Politi... parties 3) Bidding

Strategy:-

If P_2 chooses C, then it is better for P_1 to choose C. → Best response (BR)

If P_2 chooses D, then it is better for P_1 to choose C.

Best response Dynamic/strategy

$BR_i(a_{-i})$ — of player i; a_{-i}: given fixed action of all other players

P_1 \ P_2	C	D
C	$\boxed{-3}$, -3	$\boxed{0}$, -4
D	-4, 0	-1, -1

Best responses intersect

$\square$ – Best response of P_1

O – " " of P_2.

$BR_1(C) = C$. $BR_2(C) = C$

$BR_1(D) = C$. $BR_2(D) = C$

B. Nash Equilibrium of a game (NE)

↳ Intersection of best responses.

Each player is playing his best response to the action of all the other players.

NE has interesting implications:-

NE is a 'self sustaining/enforcing' agreement

(D, D) (-1, -1) is not self sustaining because,

for P_1 has incentive to deviate to C.

& for P_2 " " to deviate to C.

	C	D
C	-3, -3	0, -4
D	-4, 0	-1, -1

reduce

reduce.

For (C, D)

P_2 has incentive to deviate to C. (reduce 1 year) -4 to -3.

For (D, C)

P_1 has incentive to deviate to C. (reduce from -4 to -3)

(C, C) is SEA.

NE - No player has an incentive to deviate unilaterally (by himself)

$P_1 \backslash P_2$	C	D
C	-3, -3	0, -4
D	-4, 0	-1, -1

NE - outcome from which no player has an incentive to deviate 'unilaterally'.

NE - is a no regret outcome.

C, C → no regret by both (you can't further improve payoff or utility)

C, D - P_2 regrets

D, C - P_1 regrets.

D, D - P_1 & P_2 regrets.

NE -

$a_1^*, a_2^*, \ldots a_N^*$ — NE if for each player i

N Player game

fixed all other a_i — for all i — for fixed i.

his $u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*)$ has to hold for each player i, each action i.

Market

$F_1 \backslash F_2$	H	L
H	500, 500	0, $\boxed{750}$
L	$\boxed{750}$, 0	$\boxed{250}$, $\boxed{250}$

NE.

1) if both set H - both get higher profit 500.

2) if both set L - both get lower profit 250.

if one firm sets high(H), while other sets Low(L) price, firm which sets high price gets 0, while firm which sets low price gets 750 since it captures the entire market.

$BR_1(H) = 750 = L$ (if F_2) $\qquad BR_2(H) = L$ (if F_1)

$BR_1(L) = 250 = L \qquad BR_2(L) = L$

$\boxed{\text{NE} - (L, L)}$

$BR_1(L) = L = BR_2(L)$. At outcome (L, L) each firm is playing best response.

-ting low prices is indeed a Nash Equilibrium!

NE- Self enforcing agreement

Price war

$F_1 \backslash F_2$	H	L
H	500,500	~~0, 750~~
L	750,0	250,250

→ Not Pareto optimal outcome.

Prior (H, H)

F_1 → incentive to deviate to L

F_2 → " to " to L

(H, L)

F_1 – incentive to deviate to L

(L, H)

F_2 – incentive to deviate to H.

Nash equilibrium outcome is (L, L).

Pareto optimal outcome - Given any outcome, if there is no other outcome such that both players can simultaneously improve their pay offs, it is known as a Pareto optimal outcome.

Ex:- (H, H) is Pareto optimal outcome.

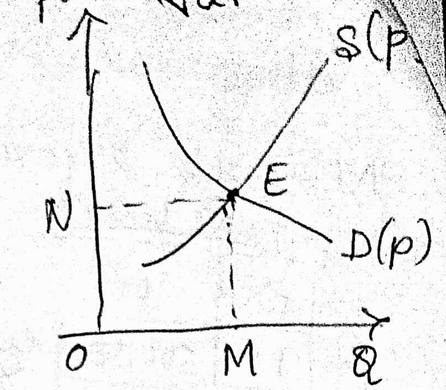
(H, L) is " optimal

(L, H) " "

(L, L) is not Pareto optimal = NE.

In economics

P - independent variable



Model of market.

Behavioral equation.

Demand eqn:

$D = D(P)$

law of demand - $D'(p) < 0$

Demand is inversely $\propto$ to price.

Supply Eqn → If price increases, supplier want to sell more goods.

$S = S(P)$, $S'(P) > 0$

if p is more, Suppliers offer more goods to be sold.

Equilibrium market price is the price in market which will prevail in the market with reasonable stability. If demand side eqn changes drastically then price will change, but as long as demand side remains same & supply side remains same, the price remains same & supply side remains same, the price will be stable at that particular value. That price is to be predicted.

ON - equi price.
OM - " qt - bought & sold.

Model - helps to figure out what is happening in real world through some basic principles. Here, the basic principles were demand equation & supply eqn.
How changes can be done?
It captures essence of real world.

Theory of Rational Choice – is corne stone of economic theory.

In any situation every decision maker has a set of actions available to him & this decision maker or agent, will take that decision that is best for him according to his preferences. ∴ There are 2 components to this RCT. 1. Set of actions

2. Preference

P – preferred
Given set of actions, which action he prefers over other.

Ex:- aPb
bPa
indifferent b/n a & b. aIb.

Preferences are summarized through payoff fun/utility fun. (economics)

$$u(a) > u(b) \text{ iff } aPb$$

payoff fun. → u, (a ↑ action), (b ↑ action)

Advertising: a b → two goods to buy. Actions are a, b.

a } choose a
✓b }

b }
a }

advertisement

$aPb,\ bPc \Rightarrow aPc$. transitivity of preferences.

Transitivity voiolation ex:- 100 diff grey shades 1, 2. . . . 100 with slight changes. If you ask which one you prefer b/n 1, 2. He says it is indifferent to me as both are almost same. Same Qn with 40, 41, he answer same. But if Qn is asked with 1 & 41, he tells 1P41 or 41P1 because now 1 & 41 are not differing by small value. So he is now not saying indifferent.

Demand of Consistency.

Choices must be consistent.

Choices {a, b} – always chooses a.

{a, b, c} – sometimes chooses a.

" " c.

Payoff function (u) are ordinal fun.

relative

	a	b	c
u	100	99	0
v	100	1	0

Ordinal functions emphasize on relative value, not on the absolute value.

Models – are conceptual tools to understand real life situations. These models are based on some basic principles which try to capture the essence of the situation & thirdly we are going to apply the theory of rational choice.

1. What does game theory deal with?

GT helps us to understand situations in which decision makers interact. More specifically GT deals with situations of strategic choice where decision taken by players affect not only their own well being, but that of others. In short it studies strategic interaction.

Debarshi Das

Explain RCT.

1) RCT has two main components.

a) Every decision maker has a set of actions available to her.

b) She has preferences defined over these actions.

Given a & b she takes action best for her. (Preference pattern), 1) choices are consistent. 2) transitivity of pref.

3) Violation of RC

Advertisers are paid to influence choice of customers. So customers do not always know what is best for them.

Lect-2 Interacting Decision makers

u(a

2 people: A & B

u_a = Pay off fun of A

$u_a = u_a(a, b)$ action of B. action of A

$u_b = u_b(a, b)$

Strategic games -

Preferences of Players is ordinal.

In ranking relative

1) 2 political parties – A, B
action – amount they spend if they get elected
x y – affect chances of winning both A & B
2) Firm – Setting prices to High or Low.

Strategic game Components.

- Players – N.
- Actions $i \in N$, A_i – set of actions available to i.
- Preference of each player

$(a_1, a_2 \ldots a_n)$ = Action profile, $a_i \in A_i$.

$u_i(a_1, a_2 \ldots a_n)$

u_i – ordinal pay off fun of i.

Ex:- 2 player: 1 & 2.

$A_1 = \{a_1, a_2\}$

$A_2 = \{b_1, b_2\}$

Action profiles: (a_1, b_1), (a_1, b_2), (a_2, b_1), (a_2, b_2)

Depending on size of action sets, action profile size changes. For each action profile, u_i to be known.

Actions are taken simultaneously.
1. Strategic games are also called simultaneous move games.
2. Beginning of game, they have fixed action & can't change.

It capi...

elicted ?&B

Games - abstract models.

PD :- Prisoner's Dilemma

2 petty thieves — Prison Sentences

(no proof) 1) both Not confessing (Deny) - 1 year each

2) both confess to robbery - 3 year each

3) 1 Confesses - he has robbed bank. - 1 is freed
He is witness

2 does not confesses — 5 years for 2.

4) 2 confesses, 1 does not - 2 is freed, 5y for 1.

- Players : 2 Prisoners P_1, P_2
- Actions - {Confess, Not Confess} = {C, NC}.
- Preference : {(NC, NC), (C, C), (C, NC), (NC, C)}

Action Profile

$$\overset{3}{u_1(C, NC)} > \overset{2}{u_1(NC, NC)} > \overset{1}{u_1(C, C)} > \overset{0}{u_1(NC, C)}.$$

P_1 is freed — imprisoned for 1 year — for 3 years — for 5 years.

$$\overset{3}{u_2(NC, C)} > \overset{2}{u_2(NC, NC)} > \overset{1}{u_2(C, C)} > \overset{0}{u_2(C, NC)}.$$

Payoff Matrix

P_1 \ P_2	C	NC
C	1, 1	3, 0
NC	0, 3	2, 2

Free riding - If there is a good outcome coming out of some coopn. i.e (NC, NC) then from individual Rational perspective each will like to free ride on other.

Pareto Optimality - is a state where it is ~~p~~ not possible to increase the utility/wellbeing of any person without reducing the utility of atleast another person.

1). What is Strategic game? 3 components.

S.G is a model of interacting decision makers.

1. Decision makers - players.
2. Each player has a set of actions to choose from
3. Each player has a specified preference over the action profile. An action profile is a vector of actions - each action taken by an individual players.

St G has no time component. So each player is not informed of choice made by others, when he makes his own choice.
Alternatively, the actions are assumed to have been taken simultaneously.

2) Describe PD.

2 players

Action: C or NC

Pref: $u_1(C,NC) > u_1(NC,NC) > u_1(C,C) > u_1(NC,C)$

$u_2(NC,C) > u_2(NC,NC) > u_2(C,C) > u_2(C,NC)$

P_1 \ P_2	C	NC
C	1,1	3,0
NC	0,3	2,2

Strategic Games & Nash Equilibrium.

High value is best.

PD

		2: C	NC
1	C	1,1	3,0
	NC	0,3	2,2

(C,C) (NC,NC) → Cooperative.

0 - 5 y
1 - 3 year
2 - 1 year.
3 - 0 years.

If player one chooses NC, then it is better for player 2 to choose C.

IIIly If P_1 chooses C, it is better for P_2 to choose C.

If P_2 chooses C, then it is better for P_1 to choose C,

If P_2 chooses NC, then also it is better for P_1 to choose C.

Ex1:- Duopoly -

2 producers/sellers.

Oligopoly - ≥2 sellers.

- Players: 2 producers/sellers
- Action: $\{P_h, P_L\}$
- Preferences:

	P_h NC	P_L C
P_h NC	1000, 1000 (2, 2)	-200, 1200 (0, 3)
P_L C	1200, -200 (3, 0)	500, 500 (1, 1)

$u_1(P_L, P_h) > u_1(P_h, P_h) > u_1(P_L, P_L) > u_1(P_h, P_L)$

$u_2(P_h, P_L) > u_2(P_h, P_h) > u_2(P_L, P_L) > u_2(P_L, P_h)$ PL = C. P_H = NC.

Ex2:- Joint Project.

Players: 2 students.

Action: workhard HW, Shirking S.

{HW, S}.

Preferences:

P1: (S, HW) > (HW, HW) > (S, S) > (HW, S)

P2: (HW, S) > (HW, HW) > (S, S) > (S, HW)

	2: HW	S
1: HW	2, 2 (NC, NC)	0, 3
S	3, 0	1, 1 (C, C)

Common Property

Players: N villages

Action: HG, LG

HG - High grazing

LG - Low "

$u_1(HG, LG, \ldots LG) > u_1(LG, \ldots LG) > u_1(HG, HG \ldots HG) > u_1(LG, HG, \ldots HG)$

(free riding; cooperating)

Battle of Sexes:-

- Players - Husband (H) & Wife (W).
- Action - Boxing Match (B), Opera (O)
- Preferences - Husband - Prefers both going to B.

$U_H(B,B) > U_H(O,O) > U_H(B,O) = U_H(O,B)$

2 1 0 Not enjoy when they are not together.

$U_W(O,O) > U_W(B,B) > U_W(B,O) = U_W(O,B)$

2 1 0

(B, B), (O, O) are better than other outcomes.

H \ W	B	O
B	2,1	0,0
O	0,0	1,2

(B, B) is better for H.

(O, O) is better for W.

Difference b/n PD.

Not Confessing - Cooperation.

In NC - when both both

Confession ~~whether~~ C or NC.

Here it is clear.

Ex:- BoS.

Politician from

2 politicians - Delhi, Bihar of same party.

$(D, D) / (B,B) > (DB) = (B,D)$

P_1 - Party is going to spend money in Delhi

P_2 - " " " " in Bihar.

then people get confused, whether there is a conflict within party.

1) Ex for real life situation PD is applied.

1) Joint project. 2) Price competition.

3) Arms race

2. Describe BoS. How it is diff from PD.

3. Point out elements of co-op^n & conflict in B.S.

4. Give some real life ex similar to BoS.

Adoption of technology when firms are merging.

BoS

There is an incentive if they coperate

Arms Race - Similar to P.D.

Players: Countries A & B.

Actions: {N, R}

N - Build nuclear bombs

R - Refraining.

Preferences:

$$\overset{3}{U_A(N,R)} > \overset{2}{U_A(R,R)} > \overset{1}{U_A(N,N)} > \overset{0}{U_A(R,N)}$$

CC - No cooperation.

$$U_B(R,N) > U_B(R,R) > U_B(N,N) > U_B(N,R)$$

BoS

Ex 2:- 2 Companies - 1 is old, another is new. } they want to me[rge]

Before merging they were using diff technologies A, B

After merging they have to use one single technology.

who is going to adopt technology.

(B, B) { Good for 2. / Not so good for 1.

But better than either (B, A) or (A, B).

Here there is a profit if there is a co-operation.

3rd kind of Game :- Matching Pennies.

Players - 1, 2.

For player 1, he gains 1Rs when both coins turn same, & P2 looses 1R[s]

Actions - {H, T}

Preferences - $U_1(H,H) = U_1(T,T) > U_1(H,T) = U(T,H)$

$U_2(H,T) = U_2(T,H) > U_2(H,H) = U_2(T,T)$

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Player 2 gains 1R, when both coins are different & P1 looses 1Rs.

Match - P_2 pay 1Rs to P_1

Mismatch - P_1 pay 1Rs to P_2

Here there is a pure conflict, i.e., when P_1 is gaining, P_2 is loosing & viceversa. This type of game is also known as Zero sum game. Because in each case the summation of payoffs the players are receiving sums upto 0. Here there is a pure conflict.

Eg:- for Matching Pennies.
1)

Players: 2 companies 1, 2. | ~~2nd~~ 2 company - reputed
Actions:- Appearence of products A & B | ~~1st~~ 1 comp' - new comp.

1st ~~2nd~~ company wants her products to look like ~~2nd~~ 2 comp. for its benefit. & ~~1st~~ 2nd ~~1st~~ company wants products to look different from ~~2nd~~ 1st.

Preferences - $u_1(A,A) = u_1(B,B) > u_1(A,B) = u_1(B,A)$

$u_2(A,B) = u_2(B,A) > u_2(A,A) = u_2(B,B)$

2) Cricket

2 players: Bowler, batsman.

Actions:- Bowler: {fast, slower}

Batsman: {Fast, Slower} play according to fast.

Preferences:-

Batsman prefer - the matching.

Games with no Conflict

Game of Stag hunt
↗ male deer.

No of hunters, goto forest to hunt stag.

Players - n players (hunters)

Actions - {stag (S), hare (H)}

When hunters catch the stag, there is a hare which

passes by. So they can catch hare. One will
after hare and catch and get hare. That for rest
hunters, they will not be benefited as
they could not catch stag.

Preferences:

If all n hunters got get only stag, then each get $\frac{1}{n}$th of stag.

There are many Ha

2 player case

$$\overset{2}{u_1(S,S)} > \overset{1}{u_1(H,S)} = U_1(H,H) > \overset{0}{U_1(S,H)}$$

$$\overset{2}{U_2(S,S)} > \overset{1}{U_2(S,H)} = U_2(H,H) > \overset{0}{U_2(H,S)}$$

(NC,NC)

		2: S	H
1	S	2, 2	0, 1
	H	1, 0	1, 1

Here if (NC,NC) then no change but, in PD 1 has tendency to C.

It is Variant of prisoners Dilemma.

	N	R
N	1, 1	3/2, 0
R	0, 3/2	2, 2

P_1:- $\overset{3}{(N,R)} > \overset{2}{(R,R)} > \overset{1}{(N,N)} > \overset{0}{(R,N)}$

Arms race →

RR = SS

Which case prevail ?

Solution Concept - Nash Equilibrium. - John Nash.

1. Players behave according to theory of rational choice - Each player plays best.

	B	O
B	2,1	0,0
O	0,0	1,2

1 - 13 ~~bey~~ beliefs of player.

NE -
1. Rational choice
2. Beliefs of people regarding other players' actions are correct.

Idea of steady state - is that if the players are taking their action, then for none of them has a tendency to move away from that action. This is the idea of equilibrium. Equilibrium often describe the state of rest. So, here it is a state of rest in this Nash equilibrium also. Given what I started with my expectations are fulfilled & therefore, if since my expectation are fulfilled ∴ I did not change my action next also.

Expectations are formed from experience & expectations are coordinated.

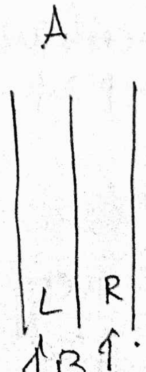
1 beliving 2 b. Also 2 believing 1 a.
a b

Equilibrium is a steady state stable outcome.

Social convention -

Ex:- Traffic

People go from B to A, take left path.



$a \leftarrow b$

- Rational choice
- Beliefs about other players action is correct. $a \rightarrow b$

Beliefs or expectations are co-ordinated.

NE is a situation of Social convention.

a - action profile

$u_i(a)$

a^* is a NE if for each player i playing a_i^* is best compared to all other actions of i (according to her preferences) given any other player j is playing a_j^*

(a_1^*, a_2^*) given a_2^* → i's best possible action is a_1^*.

& viceversa.

In another way, called unilateral deviation is unprofitable.

a^* is NE if for each i

$u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ for every possible $a_i \in A_i$.

other player actions

A_i - set of actions of player i.

$-i$ - players other than i.

Every game does not have a NE.

~~→ atleast one NE exists.~~

Pure strategy - player takes action with probability 1 or 0.

Mixed Strategy - here probability may be 0.75, 0.25.

a^* is NE if for every player i & every action a_i of player i, a^* is atleast as good according to player i's preferences as action profile (a_i, a_{-i}^*).

NE does not guarentee a unique NE.

∴ Mucliple NE are possible.

Does not guarentee that there is atleast one NE.

PD

	C	NC
C	2,2	3,0
NC	3,0	1,1

P_2

	C	NC
P_1 C	1,1	3,0
NC	0,3	2,2

(NC,NC) — P_1 changes to C, P_2 changes to C. → is Pareto superior

(C,NC) — P_2 changes to C.

(NC, C) — P_1 changes to C.

(C,C) → is NE. Here both will not change because it will not improve their pay off's.

But not Pareto superior outcome.

(NC,NC) is called Pareto Superior to (C,C)

Two state X is PS Y. if atleast one is better off compare to Y in X

Joint Projects — (S,S) is NE. S-shirk

Duopoly — (P_L, P_L) is NE.

Arms race — (N,N) is NE.

Common property — graze Large amt of grass.

BoS

(B,B) & (O,O) are NE.

	B	O
B	2,1	0,0
O	0,0	1,2

(Rows: player 1; columns: player 2)

Matching Pennie

No NE.

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

Stag hunt

2

1	S	H
S	2,2	0,1
H	1,0	1,1

(H,H) is NE

(S,S) is also NE.

(H,S) both deviate. (H,H) is Pareto Inferior NE.

(S,H) — P_1 deviate.

ie we can compare NE.

~~(2,2) > (1,1) ie (S,S) is pareto superior to (H,H).~~

Here both (S,S) & (H,H) are not pareto superior.

Ex:

	C	NC
C	1,1	3,0
NC	0,3	2,2

with $\alpha=1$

2

1	C	NC
C	2,2	3,3*
NC	3,3	4,4*

$$U_i(a) = m_i(a) + \alpha\, m_j(a)$$

NE - is an action profile which is played for a no of times tends to get repeated. Steady state of actions.

Boarding later is considered better than both boarding early. If both being on waiting for the other to board, the train leaves. However boarding train before the other Nawab is considered worse than missing the train. Missing it is ~~worse~~ worse from both boarding early. Model this game & Find NE. Does game resemble any game.

Soln. Players – 2 Nawabs (1, 2)

Actions – boarding the train early (E) or late (L)

pay off.

	worst	2 best	3 best	Best
	$u_1(E,L)$	$U_1(E,E)$	$U_1(L,L)$	$U_1(L,E)$
	0	2	1	3

1 \ 2

	E	L
E	2,2	0,3
L	3,0	1,1

Similar to PD.

E = NC

L = C.

(E,E) – Both players have profitable deviation ∴ Not NE

(L E) – P_2 has profitable deviation. ∴ Not NE

(E L) – P_1 has profitable " ∴ Not NE.

(L,L) – is a NE. ∵ No player can deviate & improve her pay off. & is the only NE.

Matching Pennie

No NE.

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Stag hunt

		2	
		S	H
1	S	2,2	0,1
	H	1,0	1,1

(H, H) is NE

(S, S) is also NE.

(H, S) both deviate.

(H, H) is Pareto Inferior NE

(S, H) – P_1 deviate.

ie we can compare NE.

~~(2,2) > (1,1) ie (S,S) is pareto superior to (H,H).~~

Here both (S,S) & (H,H) are not pareto superior.

Ex:-

	C	NC
C	1,1	3, 0
NC	0, 3	2, 2

with $\alpha = 1$

		2	
		C	NC
1	C	2, 2	3,3
	NC	3,3	4,4

$$U_i(a) = m_i(a) + \alpha\, m_j(a)$$

NE – is an action profile which is played for a no of times tends to get repeated. Steady state of actions.

Boarding later is considered better than both boarding early. If both being on waiting for the other to board, the train leaves. However boarding train before the other Nawab is considered worse than missing the train. Missing it is ~~worse~~ worse from both boarding early. Model this game & Find NE. Does game resemble any game.

Soln. Players – 2 Nawabs (1, 2)

Actions – boarding the train early (E) or late (L)

	Worst.	2 best.	3 best.	Best.
payoff.	$u_1(E,L)$	$U_1(E,E)$	$U_1(L,L)$	$U_1(L,E)$
	0.	2	1	3.

1 \ 2

	E	L
E	2,2	0,3
L	3,0	1,1

Similar to PD.

E = NC
L = C.

(E,E) – Both players have profitable deviation ∴ Not NE

(L E) – P2 has profitable deviation. ∴ Not NE

(E L) – P1 has profitable " ∴ Not NE.

(L,L) – is a NE. ∵ No player can deviate & improve her pay off. & is the only NE.

Variation of PD

In PD, suppose each players pay off is solely dependent on what other player is getting.

Variation (Altruism) → selfless concern

$u_1(NC, C) = 3 = u_2(C, NC)$

$u_1(C, C) = 1 = u_2(C, C)$

$u_1(NC, NC) = 2 = u_2(NC, NC)$

$u_1(C, NC) = 0 = u_2(NC, C)$.

for Variation →

	C	NC
C	1, 1	0, 3
NC	3, 0	2, 2

Here ~~No NE exists~~.

(NC, NC) is NE.

(C, NC) – P_1 changes to NC.

(C, C) – P_2 changes to NC, P_1 changes to NC.

(NC, C) – P_2 changes to NC.

(NC, NC) – is a NE ∵ deviation reduces payoff for each player.

Not same as original PD. Also NE has ~~also~~ changed.
In variant, payoffs have changed &

Is altruism always good?
Two Nawab ~~members~~ are waiting to board a train at Lucknow railway St. Out of courtesy each wants the other to board train before him.

it

Find NE

		P_2	
	L	M	R
P_1 U	-1, 3	2, 1	2, 0
P_1 D	0, 2	3, 4	1, 0

(U, L) – ~~NE~~ P_1 deviates to D
(U, M) – P_1 & P_2 deviate.
(U, R) – ~~NE~~ P_2 deviate to L
(D, L) – Both P_2 Deviate to M.
(D, M) – NE
(D, R) – P_1 changes to U, P_2 changes to M.

∴ (D, M) is only NE.

In PD, (1,1) is NE which is suboptimal. This is because people are maximizing his or her individual pay off's. If you relax this assumption, then (2,2) is NE, if there is an altruism.

If there is an altruism, means that an individual cares not only about his own payoff, but also the payoff of the other player, Then we could get an equilibrium which is the best for every one that is the result. Typically, if degree of altruism is high enough, if people care about the other player above a particular critical limit, then we can get ~~that~~ equilibrium which is the best outcome for every one.

Altruism is not always good.

Variations of Stag Hunt:

Original Stag Hunt

1 \ 2	S	H
S	2,2	0,1
H	1,0	1,1

For player1,

$\overset{3}{(S,S)} > \overset{2}{(H,S)} > \overset{1}{(H,H)} > \overset{0}{(S,H)}$.

Variation of Stag Hunt.

	S	H
S	$3^*,3^*$	0, 2
H	2,0	$1^*,1^*$

$\therefore$ (S,S) & (H,H) are NE.

There is a single stag & many Hares in Jungle. If two players go to catch stag, then they will catch it & each have high pay off. If one of them go behind hare, then he is less benifited than previous case, but other person cannot catch stag alone. If both of them go behind hare, both will get hare but payoff's are less than getting stag.

Here (S,S) & (H,H) are both NE.

Variation

If no of hunters is n. — Players.

Actions: {S, H}

Preference: $\frac{1}{n}$ of stag is preferable to a hare.

(S,S . . . S). is a NE.

(H, H, ... H) - If any one deviates from catching H, then tries catch a stag, then he may not be able to catch stag. So it is not profitable. ∴ (H, H, . . . H) is NE.

Any other NE

Any other profile will have atleast one H & one S ∴ there is atleast one H is there, people going after stag will get 0. There is at least one such player playing S & he could better off if he goes behind H.

∴ This action profile is not NE.

2nd variant of stag hunt

n hunters, m need to pursue stag to catch it. & they divide among $2 \le m < n$, hunters who catch stag.

a) Each hunter prefers fraction of $\frac{1}{n}$th of Stag to a hare. Find NE.

b) Assume each hunter prefers fraction $\frac{1}{k}$ of stag to hare, but prefers ~~the~~ hare to any smaller fraction of stag, where k is an integer with $m \le k \le n$. Find NE.

↳ $K = m \ge 2$ is NE.

ie $K = 2 = m$ ie two people going behind ~~stag~~ and getting $\frac{1}{2}$ of Stag is NE.

Hawk Dove Game

2 animals are fighting over some prey. Each can be passive or aggressive. Each prefers to be aggressive if its opponent is passive & passive if its opponent is aggressive; given its own stance, it prefers the outcome in which its opponent is passive to that in which its opponent is aggressive. Formulate this situation as a strategic game & find its NE.

Soln:-

Players - 2 animals - 1, 2

Action - Passive (P), Aggressive (A).

Preferences.

A1: (P, P) (P, A) (A, P) (A, A)

L H H L

2 1 3 0

	P	A
P	2,2	1,3
A	3,1	0,0

(P, A), (A, P) are NE.

$u_1(A, P) > u_1(P, P) > u_1(P, A) > u_1(A, A)$

Co ordination Game -

Variant of BoS. There are 2 players.

Actions: $\{B, O\}$.

Preferences:

Both prefer B over O.

$u_1(B,B) > u_1(O,O) > u_1(B,O) = u_1(O,B)$

$u_2(B,B) > u_2(O,O) > u_2(B,O) = u_2(O,B)$

	B	O
B	2,2	0,0
O	0,0	1,1

(B,B), (O,O) - NE.

This is called co-ordination game.

Here (B,B) is better than (O,O).

Pareto Superior State　　　Pareto Inferior State.

~~This~~ Here we can't say (B,B) is superior to (O,O). Because I can't make others where they have to go. I have command over what I am doing but, unified action is not considered in non-coperative game theory.

(O,O) is Pareto Inferiol NE.

Ex:- QUERTY puzzle?

It is not best but everyone uses it.

In Many situations, people are stuck with actions which n of them wants to take but still they do, because others taking that action — Pareto Inferior NE.

Public goods Game

Road - can be used by all.

If I contribute, I can't restrict others from using it.

Public good game

I i) Each n people choose whether or not to contribute a fixed amt towards provision of a public good. The good is provided iff atleast k people contribute, $2 \le k \le n$;

If it is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows.

i) any outcome in which good is provided & she does not contribute.

ii) any outcome in which good is provided & she contribute.

iii) any outcome in which good is not provided & she does not contribute

iv) any outcome in which good is not provided & she does contribute.

Formulate as a game & find NE.

Soln:-

Players: n

Action: {Contribute (C), NotContribute (NC)}

Preferences

P1: $\overset{3}{(NC,P)} > \overset{2}{(C,P)} > \overset{1}{(NC,NP)} > \overset{0}{(C,NP)}$

P: public good provided ie no of contributors $\geq k$.

NP: " " not provided ie no of con $\leq k$.

C - contributed   NC - Not contributed.

In economics public goods are non excludable, non rivalry.

There is a problem - due to excludable - I cannot make everyone to pay.

Strict NE

NE: for every i $u_i(a^*) \geq u_i(a_i', a_{-i}^*)$ $\forall a_i' \in A_i$.

a^* in NE.

i.e.,

Strict NE:- $\forall i$, $u_i(a^*) > u_i(a_i', a_{-i}^*)$ $\forall a_i' \in A_i$ other than a_i^*.

If he deviates he will have worse pay off in SNE

Ex:- BoS.

Ex/or Non Strict E.

	L	M	R
T	$1^*, 1^*$	$1^*, 0$	$0, 1^*$
B	$1^*, 0$	$0, 1^*$	$1^*, 0$

(T,L) → NE but not strict.

Ex:-

A, B compete an election.

Of n citizens, k support candidate A & m (=n-k) support B. Each citizen decides whether to vote, at a cost, for candidate she supports or to abstain. A citizen who abstains receives payoff of 2 if the candidate she supports wins, 1 if this candidate ties for 1st place & 0 if this candidate loses. Citizen who votes receives payoffs 2-c, 1-c and -c in these three cases, where $0 \leq c < 1$.

a) For k=m=1, is game same as any?

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Choosing a route

Players - 4 people

Action - Choosing route location A-X-B, A-Y-B

Payoff - Negative of total travel time (∵ it is minimization problem).

If all of them choose A-X-B, time spent by each 15+22.7 = 37.7

claim: In NE, two people will travel A-X-B route spending 9+20.9 = 29.9 mins & two people will travel in A-Y-B route spending 9+21 = 30 min.

If someone from A-X-B travels in A-Y-B he spends 12+22 = 34 min > 29.9 min..

|||ly we can show any one in A-Y-B route will not switch to A-X-B.

a) ∴ It is NE.

b) A short route is build b/n X & Y.

Now there are 4 possible paths.

A-X-B, A-Y-B, A-X-Y-B, A-Y-X-B.

claim:- Two persons will take A-X-Y-B route, one each in A-X-B & A-Y-B.

Those travelling in A-X-Y-B, time spent =

12+8+12 = 32 min.

A-X-B route, 12+20 = 32 min.

A-Y-B " , 20+12 = 32 min

∴ Building new road will not improve performance.

remains steady state

Ex for Pareto inferior game - similar to co-ordination game.

2 players, want to meet

	CP	DK
CP	2,2	0,0
DK	0,0	1,1

CP - ~~Condition~~ ought place

DK - Dhdak Kwa.

If they go to different place, they cannot meet.

Here (D,K, DK), (CP, CP) are NE.

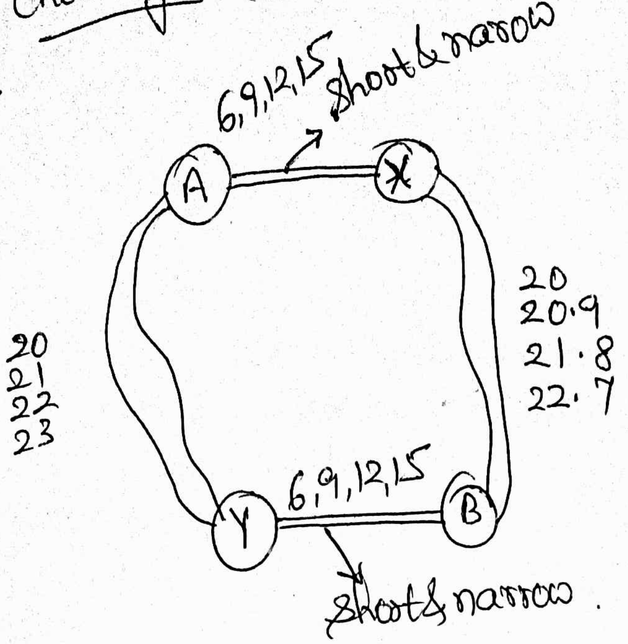
(DK, DK) is pareto inferior NE.

Ex:- Qwerty keyboard - is not efficient but everybody uses.

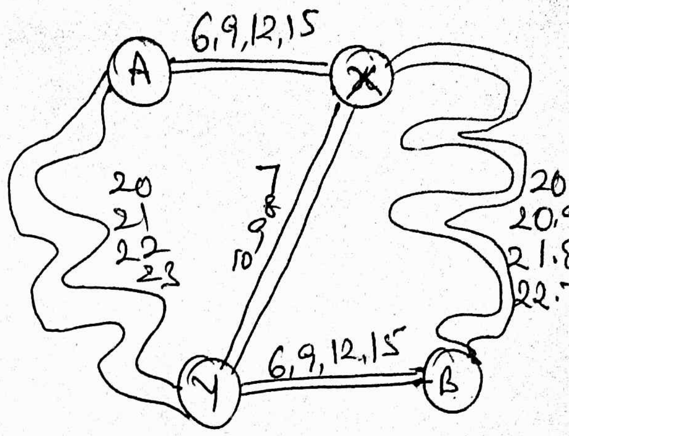
Reason~~able~~ that it is steady state is that no body want to build new one because he has fear that people may not use, but still everybody uses qwerty keyboard only.

∴ Inefficient state is still steady state.

choosing a route



a) Original n/w



b) N/W with new road from X to Y.

a) If there is a single car in route A-X, it takes 6 mins. If 2 cars, then each take 9 mins.

NE: 2 people travel in A-X-B route & → $9+20.9=29.9$
2 people travel in A-Y-B route → $21+9=30$.

b) NE: 2 people will take A-X-Y-B route,
1 person will take A-X-B route &
1 person will take A-Y-B route.

Soln:- In A-X path, there will be 3 people &
Y-B path also, there will be 3 people.

∴ people who choose A-X-Y-B route take
$12+8+12=32$ min.
People who choose A-X-B route take, $12+20=32$ min
Person who choose A-YB route take, $20+12=32$ min.
∴ Building a new road will not improve performance.

Best Response Function

If No of actions may be large, it is difficult to find NE. In those cases use best response fun.

		2	
		B	O
1	B	2,1	0,0
	O	0,0	1,2

When

$B_1(B) = \{B\}$ $\quad B_1(O) = \{O\}$

$B_2(B) = \{B\}$ $\quad B_2(O) = \{O\}$.

Best response of player 2 when player 1 choses B is B.

$B_i(a_{-i}) = \{a_i : u_i(a_i, a_{-i}) \geq u_i(a'_i, a_{-i})$ (atleast) for all $a'_i \in A_i\}$.

Note:- It is a set valued fun.

a^* - action profile $= (a_1^*, a_2^* \ldots\ldots a_n^*)$ will be a NE if for every player i $\quad a_i^* \in B_i(a_{-i}^*)$

Ex:-

1, 2 $\quad (a_1^*, a_2^*)$

$a_1^* \in B_1(a_2^*)$ & $a_2^* \in B_2(a_1^*)$.

$B_1(a_2) = \{b_1(a_2)\}$ $\rightarrow$ if Best response fun is singleton.

$B_2(a_1) = \{b_2(a_1)\}$

$\left.\begin{array}{l} a_1 = b_1(a_2) \\ a_2 = b_2(a_1) \end{array}\right\}$ Solve to find NE.

2 Case I:- Discrete action in each case

	L	C	R
T	1,2	2,1	1,0
M	2,1	0,1	0,0
B	0,1	0,0	1,2

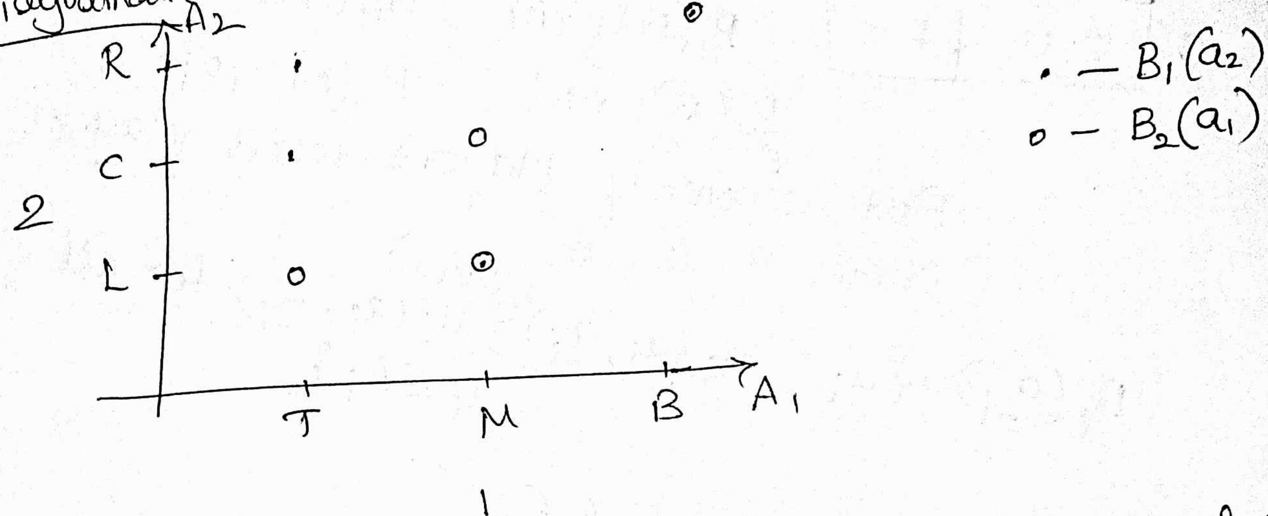
$B_1(L) = \{M\}$ $\quad B_2(T) = \{L\}$

$B_1(C) = \{T\}$ $\quad B_2(M) = \{L, C\}$

$B_1(R) = \{T, B\}$ $\quad B_2(B) = \{R\}$

NE = {(M, L) (B, R)} - Intersection of best responses.

Diagramatic



Case 2:- Action is continuous variables.

Synergic relationship game. | Synergistic - interaction of two or more agents to produce a combined effect greater than sum of separate effects.

2 players

Action :: $0 \le a_1, a_2 < \infty$ {any non-ve no}.

$u_i(a_1, a_2) = a_i(c + a_j - a_i)$ where $c > 0$.

a_i - effort level of i.

Initially if a_i exceeds a_j, he gets more because $a_i < c + a_j$

If I put more effort given the effort level of other player, I should get more benefit out of relnship but if the effort put by other player is constant it is not changing not increasing, then if I go putting more effort, after a point of time, benefit I get goes on declining because, if other person is not contributing, I do not feel very happy.

For 2 players, $u_1(a_1, a_2) = a_1(c + a_2 - a_1)$

$u_2(a_1, a_2) = a_2(c + a_1 - a_2)$

$\underset{a_1}{\text{Max}}\ u_1(a_1, a_2) \rightarrow \underset{a_1}{\text{Maximizing}}\ a_1(c+a_2-a_1)$

(Differential calculus) necessary cond. First order condition: $\frac{\partial}{\partial a_1}\left[a_1(c+a_2-a_1)\right] = 0$.

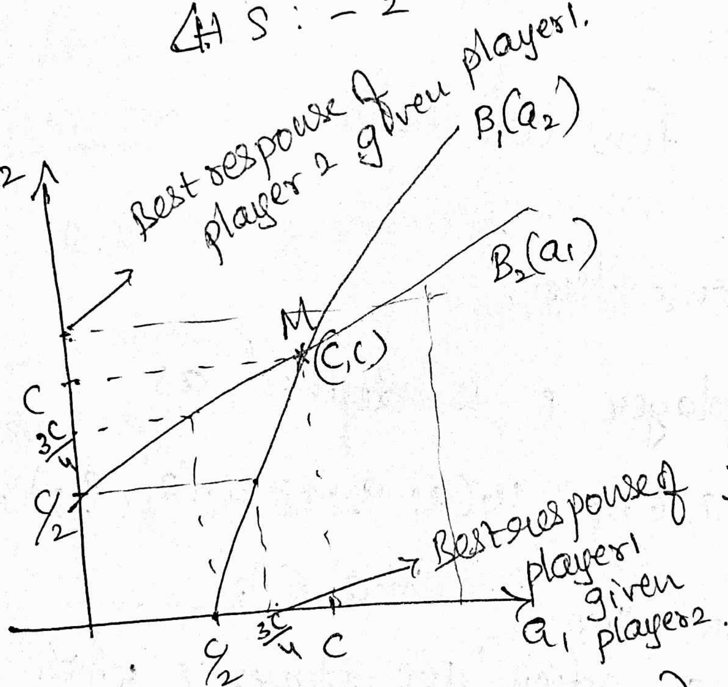
$c+a_2-a_1 + a_1(-1) = 0$ $\left[\text{based on } \frac{d}{dx}(x_1 x_2)\right]$

ie $c+a_2-2a_1 = 0$

$\Rightarrow a_1 = \frac{c+a_2}{2}$ ..... (1) Best Response function for player 1.

Sufficient cond. $\frac{\partial^2}{\partial a_1^2}\left[u_1(a_1, a_2)\right] < 0$ & second order cond is satisfied.

LHS : $-2 < 0$



$a_2 = 2a_1 - c$.
$a_2 = -c + 2a_1$
$y = c + mx$
slope of 2
intercept of $-c$.

if $a_2 = 0$, $a_1 = \frac{c}{2}$.
if $a_2 = c$, $a_1 = c$.

$\underset{a_2}{\text{Max}}\ u_2(a_1, a_2) = a_2(c+a_1-a_2)$

$a_2 = \frac{c+a_1}{2}$ ...... (2)

$a_1 = 2a_2 - c$
or $a_2 = \frac{c}{2} + \frac{a_1}{2}$
$y = c + mx$
slope is $\frac{1}{2}$
intercept $\frac{c}{2}$

if $a_1 = 0$, $a_2 = \frac{c}{2}$
if $a_1 = c$, $a_2 = c$.

$$a_1 = \frac{c + a_2}{2}, \quad a_2 = \frac{c + a_1}{2}$$

Solve,

$$a_1 = \frac{c}{2} + \frac{1}{2}\left(\frac{c}{2} + \frac{a_1}{2}\right)$$

$$a_1 = \frac{c}{2} + \frac{c}{4} + \frac{a_1}{4} = \frac{3}{4}c + \frac{a_1}{4}.$$

$$4a_1 = 3c + a_1$$

$$3a_1 = 3c$$

$$\underline{\underline{a_1 = c.}}$$

$$a_2 = \frac{c + a_1}{2} = \frac{c + c}{2} = \underline{\underline{c}}.$$

$\therefore$ (c, c) is NE.

Here best response fcns are linear.

1) Define Best response fcn.

Ans:- BRF for player i is defined as

$$B_i(a_{-i}) = \{a_i \in A_i : u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}), \ \forall a_i' \in A_i\}$$

It is a set valued fun.

It gives us set of action for player i which are best, given player i's payoff function and given what actions the other players are playing. It could be null, singleton, or many.

Q. How are BRF used to find NE.

a^* is NE iff every players action a_i^* is a best response to the other players actions a_{-i}^*.

$$a_i^* \in B_i(a_{-i}^*) \quad \forall i.$$

In a NE, every players action should be belonging to his or her best response functions and this should be true for every player.

~~Strictly & Weakly Dominated Action.~~

Lecture 10.

Contributing to a public good.

People may or may not contributing. If contributing then whether they contribute more or less.
Here infinite actions. $w_1, w_2 > 0$. (initial money).

- 2 players
- Actions: Contribution to public good : c_1, c_2 $\quad w_1, w_2$ - initial wealth.

$$0 \le c_1, c_2 \le \infty.$$

- Preferences: $u_i(c_1, c_2) = v_i(c_1 + c_2) + w_i - c_i$

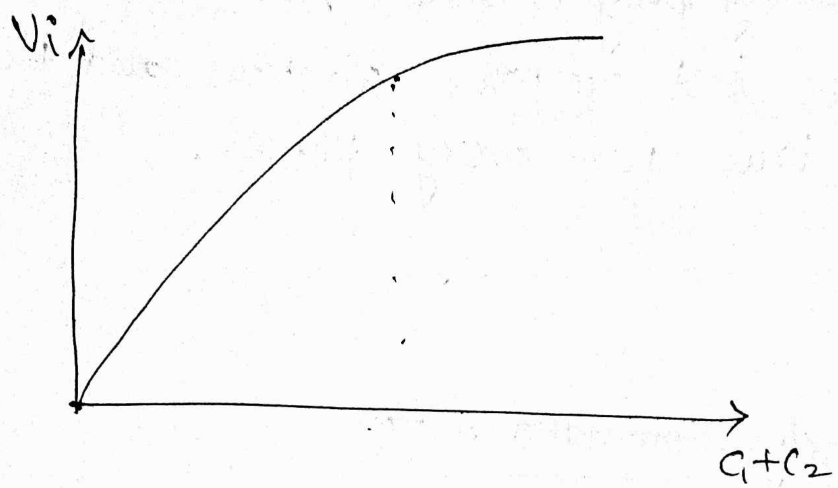
$v_i > 0$

v_i - tells what is the benefit that player i is getting out of that total amount of public good.

In particular, if $c_1 + c_2$ is high, then more

public good will be created & V_p will be high
So I can safely assume that V_i is an increasing fun. So V_i' is +ve that is the reason this assumption

$V_i' > 0, \quad V_i'' < 0$ [Slope decreases].



$C_1(C_2) = B_1(C_2)$ maximizes $U_1(C_1, C_2)$

$\max_{\text{wrt } C_1} U_1(C_1, C_2)$

First order Cond. $\frac{\partial}{\partial c_1} U_1(C_1, C_2) = 0$.

$\frac{\partial}{\partial c_1}\left[V_1(C_1+C_2) + W_1 - C_1\right] = 0$.

V_1' – is derivative of V_1 wrt C_1+C_2

$V_1' \cancel{\frac{\partial}{\partial C_1}} (C_1+C_2) - 1 = 0$.

$\therefore V_1' = 1$ ie., $V_1'(C_1+C_2) = 1$ $\because V_1' > 0, \ C_1+C_2 > 0$. $\therefore V_1' = 1$ & $C_1+C_2 = 1$.

V_1' is fun of C_1, C_2.

$C_1 + C_2 = V_1'^{-1}(1)$

$C_1 = V_1'^{-1}(1) - C_2$ .... ①

$C_1 = B_1(C_2)$

$\boxed{C_1 \propto \frac{1}{C_2}}$

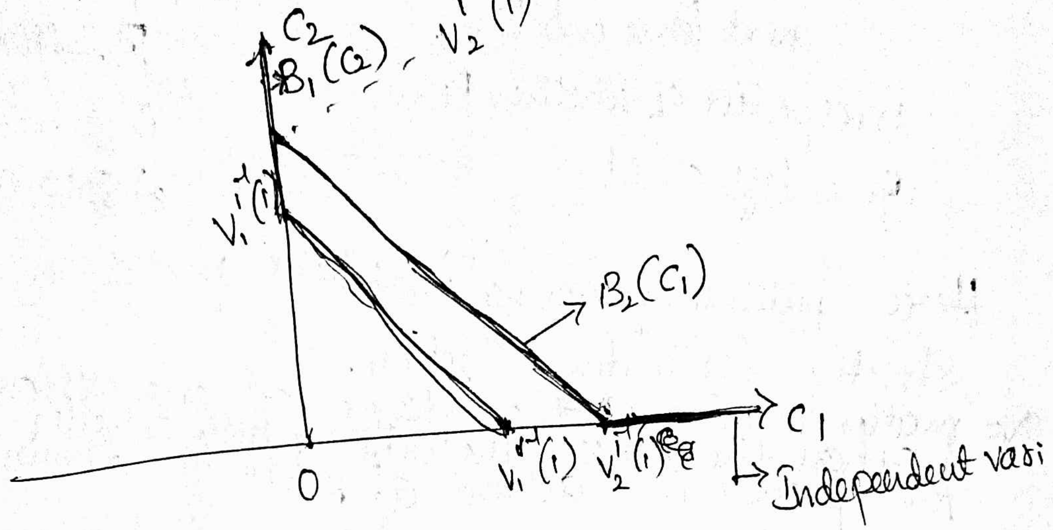
If $C_2 = 0 \Rightarrow C_1 = v_1'^{-1}(1)$

If $C_2 > 0 \Rightarrow$ ①

If $C_2 > v_1'^{-1}(1) \rightarrow C_1 = 0$.

C_2 varies from 0 to $v_1'^{-1}(1)$

$v_1'^{-1}$ is worth of public good to player 1.



$\max_{C_2} u_2(C_1, C_2)$

$$C_2 = v_2'^{-1}(1) - C_1 \quad \text{------} \quad ②$$

$$C_2 = B_2(C_1)$$

Suppose, $v_2'^{-1}(1) > v_1'^{-1}(1)$

NE: $(0, v_2'^{-1}(1))$

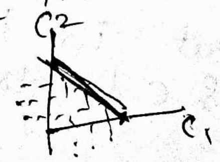
If $C_1 = 0$, $C_2 = v_2'^{-1}(1)$

if $v_1'^{-1}(1) \geq v_2'^{-1}(1)$

NE: $(v_1'^{-1}(1), 0)$.

If $C_2 = 0$, $C_1 = v_1'^{-1}(1)$.

If $v_2'^{-1}(1) = v_1'^{-1}(1)$, then infinite NE exists.



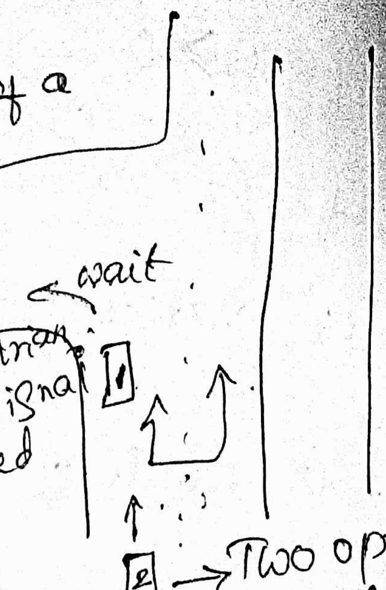
Dominated Actions.

Strict Dominance – An action of a player strictly dominates another action if it is superior, no matter what the other player do.

Player 1 – go left or go st.

wait

pedestrian Signal is red



→ Two opteions
1) go behind
2) goto another lane.

For player 2 – going in next lane is better, irrespective of whether P1 goes left or st.

Here pulling of moving in right lane for 2 is strictly dominating action.

No matter what the other player does, one action is better than the other. The action which is better – strictly dominates/ dominating act

for player i, action a_i' strictly dominates action a_i'' if $u_i(a_i', a_{-i}) > u_i(a_i'', a_{-i})$ for every $a_i', a_i'' \in A_i$, for every possible list a_{-i} of other player's action

Ex:-

1 \ 2	L	R
T	1,1	0,2
M	2,3	1,1
B	7,0	3,1

L-Left R-Right
T-Top, M-Middle, B-Bottom

M strictly dominates T for player 1 but M does not strictly dominates B.

In BoS,

there is no strictly dominated action.

But in PD, NC is strictly dominated action, & C is strictly dominating action

1	C	NC
C	1,1	3,0
NC	0,3	2,2

23

If we have NE, then, in that NE strictly dominated actions are never going to be played. Because in the PD, we found that (C, C) are strictly dominating actions & (NC, NC) are strictly dominated actions.

i.e. In PD for player 1, C dominates NC.
for player 2, also C dominates NC.

Weak Domination – An action weakly dominates other action if it is atleast as good as other action, no matter what the other player do & is better than second action for some actions of the other players.

Here it is not clear whether going behind 1 or 3 is better.

Here waiting behind 1 is weakly dominating waiting behind 3.

For player i, a_i' weakly dominates a_i'' iff $u_i(a_i', a_{-i}) \geq u_i(a_i'', a_{-i})$ for all possible lists of actions a_{-i} of other players.

And there is atleast one action profile a_{-i} such that $u_i(a_i', a_{-i}) > u_i(a_i'', a_{-i})$

		2	
		L	R
1	T	1	0
	M	2	0
	B	2	1

For player 1.

M weakly dominates T.
∵ M is better when L is played but it is same when R is played.

||ly B weakly dominates M. ∵ B is better when R is played but it is same when L is played.

In strict NE, weakly dominated actions are not played.

Weakly dominated actions can be played in non strict NE.

Ex:-

1 \ 2	B	C
B	1,1	2,0
C	0,2	2,2

For 1 : B is weakly dominating C.

For 2 : " " " C.

NE : (B,B), (C,C) $\hookrightarrow$ weakly dominated actions are played.

Ex:-

1. Two people are to divide Rs 10 b/n them. Each calls a whole no b/n 0 & 10. If sum of nos is atmost 10, they get money equal to nos they called. If sum is >10, i) player calling lower no gets same amt of money, other gets balance. ii) each gets 5 if nos are same. Find NE.

Soln:

$i = 1, 2.$

$B_i(0) = \{10\} = 10.$

$B_i(1) = \{9, 10\} = 9$

$B_i(2) = \{8, 9, 10\} = 8$

$B_i(3) = \{7, 8, 9, 10\} = 7$

$B_i(4) = \{6, 7, 8, 9, 10\} = 6$

$B_i(5) = \{5, 6, 7, 8, 9, 10\} = 5$

$B_i(6) = \{5, 6\} = 5$

$B_i(7) = \{6\} = 6$

$B_i(8) = \{7\} = 7$

$B_i(9) = \{8\} = 8$

$B_i(10) = \{9\} = 9.$

P.T in a NE, strictly dominated actions are never going to be played.

Proof: NE: a^* is if for each i,

$$u_i(a^*) \geq u_i(a_i, a_{-i}^*) \text{ for all } a_i \in A_i \quad \text{--- (1)}$$

Suppose there is an action a_i'' which is strictly dominated action & is played.

$$u_i(a_1^*, \dots a_i'', \dots a_n^*) < \qquad a_i'' \neq a_i^*.$$

a_i'' ← dominated

$$u_i(a_1^*, \dots a_i', \dots a_n^*)$$

↑ dominating

The above cond (1) is violated.

$\therefore a_i''$ is not going to be played.

For every ~~player~~ list of actions it is better to play the strictly dominating action.

2. P.T in strict NE weakly dominated actions are not played.

Proof: If a^* is strict NE if for all i

$$u_i(a^*) > u_i(a_i, a_{-i}^*) \text{ for all } a_i \in A_i \text{ other than } a_i^*. \quad \text{---- (2)}$$

Suppose a_i'' is weakly dominated action by a_i'. (a_i' is weakly dominating action)

If player i changes from a_i'' to a_i' then his pay off will remain same or it will go up. It is not going to satisfy ②

$\therefore$ a_i'' is not played.

3. Weakly dominated actions can be played in non strict NE.

Proof:- In non strict NE, if ~~we shift~~ people deviate from a_i' to a_i'', the payoff remains ~~same~~ constant/ or may go down. but should not go up.

In weakly dominated actions also if we shift from weakly dominating (a_i') to weakly dominat~~ing~~ed (a_i'') the payoff remains constant or may go down.

$\therefore$ proof.

Ex:-

		2: B	2: C
1	B	1,1	2,0
	C	0,2	2,2

For player1, B is weakly dominating C.

For player2, B is weakly dominating C.

$\because$ if player1 plays B, B is better & if ~~Pl~~ player1 plays C, ~~he~~ is indifferent b/n B & C.

NE: (B,B) & (C,C)

So (C,C) is NE, but C which is weakly dominated action is played.

Public

~~ublic good game~~

4 NE:
(5,5), (6,6), (5,6), (6,5)

Whether it is possible to have a single NE in which weakly dominated actions are played?

		2		
		L	C	R
1	U	$\frac{1}{2}, \frac{1}{2}$	1,0	0,1
	M	0,1	1,1	0,0
	B	0,½	0,0	½,0

NE: (M,C).

1: M is weakly domiated ~~by~~ action U because if player 2 plays L, U is better than M, if player 2 plays C, U&M give him same pay off.

2: ~~M~~ C is weakly dominated by ~~B~~L.

M & C are weakly dominated actions & they are played.

<u>Voting</u> 2 candidates A, B
n voters, n is odd.
A has more supporters.
Players: n voters.
Action: Voting to A, Voting for B
Preferences:

In all profiles where A is winning, voters are indifferent b/n them.

(A, A, B, .. A ...) A's in Action profile

if B is winning, when no of B's less than no of A's players are indifferent.

indifference b/n profile where A or B is winning.

Result: for a supporter of A (or B) voting for A weakly dominates voting for B(A).

i is a supporter of A:

1) There is a tie (leaving aside i's vote) then voting for A is strictly better/preferable than voting for B.

2) A or B is winning by 2 or more votes. Then vote of i is immaterial, since if i votes A or B, still same person wins. i.e., i is indifferent.

So voting for A is weakly dominates voting for B. [for supporters of A].

III ly for supporters of B, voting for B is weakly dominates voting for A.

Collective Decision-Making.

Players - n people

Actions: Each person's set of action is set of policies i.e. number.

Preference: i prefers the action profile a to a' if the median policy in a is closer to x_i^* than in a'.

See each one of them want common policy which is closest to his favorite policy. (25)

Symmetric Games & Symmetric Equilibrium.

n-odd

Collective decision making - trying to decide common policy.

n-players take common policy - defense expenditure of community be a number x.

$i - x_i^*$ (favorite policy)

Each one randomly picks up a number & median ~~average~~ of all numbers is taken as policy. middle of sorted no.

if everybody asked to give no what he likes to see $(x_1, x_2 \ldots x_n)$ → Median is common policy. Announcements.

Claim: - Giving x_i^* weakly dominates x_i all other actions. Median $= \frac{n+1}{2}$. 1, 5, 9, 20, 21

$x_1^* \; x_2^* \ldots x_m^* \ldots x_n^*$ (Ascending)

1) Can there be an equilibrium where the policy is the median favorite policy? 1, 5, 9, 20 $\frac{n}{2}, \frac{n}{2}+1$ median.

a) NE: Everyone announces $x_m^* : (x_m^*, x_m^* \ldots x_m^*)$

Median $- x_m^*$

If P_1 deviates - he cannot change policy.

$x_1, x_m^* \ldots x_m^*$

median

So he can not improve his x_m^* payoff. b) NE: $(x_1^*, x_2^* \ldots x_m^* \ldots x_n^*)$.

2) Is there a NE in which policy chosen is not median favorite policy?

NE: $(\bar{x}, \bar{x}, \ldots \bar{x})$ $\bar{x} \neq x_m^*$.

Every one announces $\bar{x}$, which is not x_m^*

$n > 2$.

$$\text{Mean} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

<u>Mean</u>:- $x_1, x_2, \ldots x_n \rightarrow$ Mean is the policy.

---

take out ith favorite policy x_i^*. x_i.

$\overbrace{x_1, x_2 \ldots \ldots x_n}^{\text{without } x_i^* =}$ even. Then there will be two medians, one at $\frac{n-1}{2}$ & other at $\frac{n+1}{2}$. Now this x_i can be placed anywhere.

Announce mean $\underline{x} \leftarrow$ Player $\frac{n-1}{2}$, Player $\frac{n+1}{2} \rightarrow \bar{x}$

<u>Case 1</u>: ith player announces x_i, which is $x_i^* < x_i \rightarrow$ strictly greater than x_i^*.

Then announcing x_i is weakly dominated by x_i^*.

a) $x_1 \quad x_2 \ldots x_i^* \quad x_i \ldots \underline{x} \quad \bar{x} \ldots x_n$

a) He announces x_i^* or x_i.

if $x_i^* < x_i \leq \underline{x}$ then $\underline{x}$ becomes median.

i is indifferent b/n x_i^* & x_i.

b) $x_i^* < \underline{x} < x_i < \bar{x}$, then x_i then x_i becomes median.

x_i^* is preferable to x_i.

b) $x_i^* \quad \underline{x} \quad x_i \quad \bar{x}$

$\underline{x}$ median.

c) $x_i^* < \underline{x} < \bar{x} < x_i$

$x_i^* \quad \underline{x} \quad \bar{x} \quad x_i$

Announcing x_i^*, $\underline{x}$ becomes median

Announcing x_i, $\bar{x}$ "

x_i^* is preferable to x_i.

d) $\underline{x} \leq x_i^* < x_i < \bar{x}$

x_i^* is preferable to x_i.

e) $\underline{x} < x_i^* < \bar{x} < x_i$.

(f) $\underline{x} < \bar{x} < x_i^* < x_i$.

$\bar{x}$ median.

I is indifferent.

$\therefore x_i^*$ is weakly dominating x_i.

players - n people

Actions: Each person's set of actions is the set of policies ie numbers.

Preference: i prefers action profile a to a' if median policy in a is closer to x_i^* than median policy in a'.

Claim: For each player announcing x_i^* weakly dominates all other actions.

Soln:-

i is taken out. $\rightarrow x_i^*$.

For all others announcements, arrange in ascending order: [n-1 elements - even].

$x_1, x_2 \ldots\ldots x_n$

Median: Sl no: $\frac{n-1}{2}, \frac{n-1}{2}+1 \Rightarrow \frac{n-1}{2}, \frac{n+1}{2}$

Suppose middle people index values are $\underline{x}$ & $\overline{x}$.

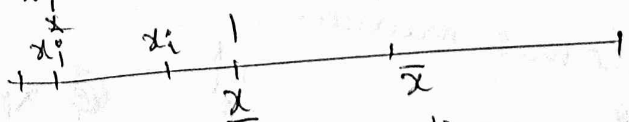
$x_1, x_2 \ldots\ldots \underline{x}, \overline{x}, \ldots\ldots x_n$.

Now i can be any where.

Case 1:- $x_i^* < x_i$. [Announcement value $> x_i^*$]

then announcing x_i is weakly dominated by x_i^*.

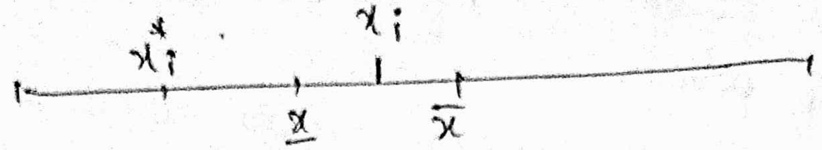
a) $x_i^* < x_i \leq \underline{x}$



Here $\underline{x}$ becomes median.

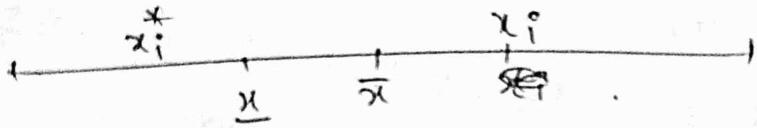
So whether he announces x_i^* or x_i which are $< \underline{x}$, $\underline{x}$ becomes median $\therefore$ i is indifferent b/n announcing x_i^* & x_i.

b) $x_i^* < \underline{x} < x_i < \bar{x}$, then



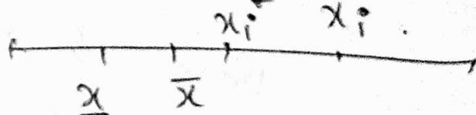
If he announces, x_i^*, then $\underline{x}$ becomes policy.
If he announces, x_i, then x_i becomes policy.
which is preferred?
Median is closer to his favorite profile.
∴ if $\underline{x}$ becomes policy, he prefers.
x_i^* is preferable to x_i

c) $x_i^* < \underline{x} < \bar{x} \le x_i$



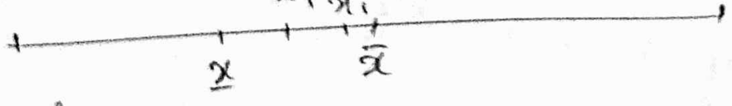
Here also if he announces x_i^*, $\underline{x}$ becomes policy.
If he announces, x_i, $\bar{x}$ becomes policy.
∴ x_i^* is preferable to x_i.

f) $\underline{x} < \bar{x} < x_i^* < x_i$



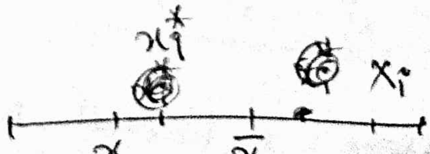
In both cases $\bar{x}$ becomes median. ∴ i is indifferent.

d) $\underline{x} \le x_i^* < x_i < \bar{x}$.



Announces
$x_i^* \longrightarrow x_i^*$ becomes median.
$x_i \longrightarrow x_i$ becomes median.

e) $\underline{x} < x_i^* < \bar{x} < x_i$.



$x_i^* - x_i^*$ becomes median
$x_i \longrightarrow \bar{x}$ becomes median.
x_i^* is preferable.

Cournot Model of Oligopoly

(few)

Depending on sellers: $1 <$ more sellers $< \infty$

MONOPOLY, DUOPOLY, OLIGOPOLY

How do sellers compete – Oligopoly Markets are divided into two kinds of markets

1. Cournot
2. Bertrand

In cournot model, the producers are deciding on their production level. They are not deciding on the price. The price is determined in the market. So, price is outside the control of the sellers.

In Bertrand model, the producers are deciding at what price they are going to sell their goods, but how much they are going to produce, that is going to be decided by the market condition.

So in Cournot model, it is the quantity of goods that the producers are producing is in their control & in Bertrand, it is the price which is in their control of producers.

But both are the cases where the # of producers is few.

Market – 2 sides – Sellers / Buyers.

n producers.

o/p levels of producers are $q_1, q_2 \ldots q_n$

ie $0 \le q_i < \infty$, $i = 1, 2 \ldots n$

goods they are producing are homogeneous. ie there is no brand name as such; there is no label over the products that the producers are producing.

So, it is not, like you have the cold drinks market ie in cold drinks market, you see diff brands of cold drinks produced by diff companies. So difference can be made. But here we are going to assume that, does not matter, where the cold drinks is coming from. The producers are making cold drinks, in such a way that, the buyers cannot distinguish b/n the product of one producer from the product of the other producers. that's why goods are homogeneous.

$Q^s = q_1 + q_2 + \dots q_n$. (supply side)

P = price per unit of good. (demand side)

what is the price that good is sold

<u>Demand:</u> Law of demand: Quantity demanded is inversely related to the price. [Not always true]

Q^d = quantity demanded in entire market.

Here Q^d is not constant. It is dependent on the market price.

$$Q^d = F(P) \qquad \frac{dF(P)}{dP} < 0$$

$Q^d = F(P)$, $F'(P) < 0$, Taking inverse, $P = F^{-1}(Q^d)$

$$Q^s = q_1 + q_2 + \dots q_n$$

ie $P = P(Q^d)$ $\quad F^{-1}(\cdot) = P(\cdot)$ $\quad P'(Q^d) < 0$.

Inverse demand function.

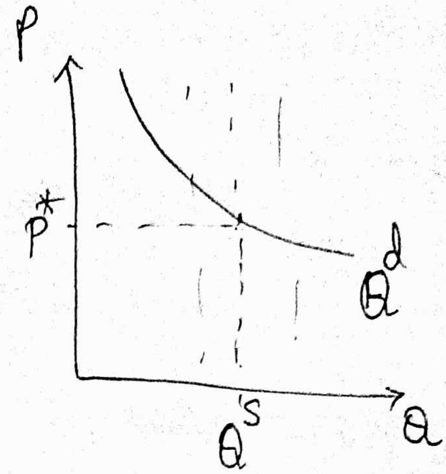
Equilibrium: Quantity demanded = Quantity supplied

If $Q^d < Q^s$, price in market increases.
$Q^d > Q^s$, " in " decreases.

There is a moment.

If $Q^d = Q^s$, then it is a state of rest & is in equilibrium.

$Q^s = \sum_{i=1}^{n} q_i$, $Q^d = F(P)$



Suppose $Q^s = Q^d = Q$

$$\sum_{i=1}^{n} q_i = Q^s = F(P)$$

ie $P = F^{-1}(Q^s) = P(Q)$

So if Q^s increases, price decreases & vice versa.

<u>Cournot Model</u>

n number of producers/firms

q_i - o/p of producer i.

Cost $= C_i(q_i)$ [Cost incurred], $c_i'(q_i) > 0$.

Profit of producer i; $\pi_i(q_i)$ = Total revenue − Total cost

$$\pi_i(q_i) = q_i P - C_i q_i$$

$$= q_i \underbrace{P(q_1 + q_2 + \cdots q_i + \cdots + q_n)}_{\text{inverse demand function}} - C_i(q_i)$$

$$\pi_i(q_1, q_2 \ldots q_i, \ldots q_n)$$

- players: n producers
- Actions: o/p levels $0 \le q_i < \infty$
- preferences: Each producer is trying to maximize his profit function $\pi_i(q_1, q_2 \ldots q_i, \ldots q_n)$.

. Let $n=2$

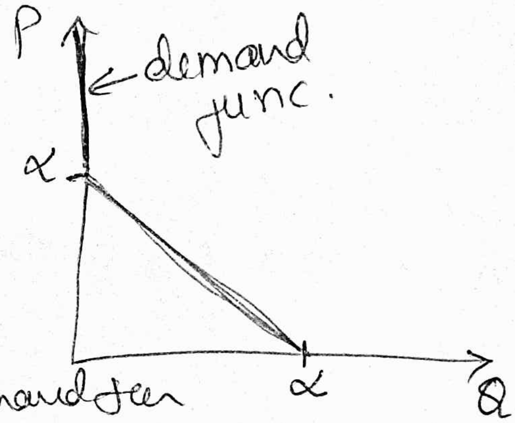
$c_i(q_i) = c \cdot q_i$, $\forall i = 1,2$. [cost funs are same]

$c > 0$. c - unit cost of production.

$P(Q)$ = inverse demand function

$$P(Q) = \begin{cases} \alpha - Q & \text{if } \alpha \ge Q \\ 0 & \text{if } \alpha < Q. \end{cases}$$

Simple linear demand fun



$\underset{q_i}{\text{Max}}\ \pi_i(\ldots) \rightarrow b_i(\ )$

$Q = q_1 + q_2$

$$\pi_1(q_1, q_2) = q_1 \cdot \underbrace{P(q_1+q_2)}_{\text{inverse demand fun}} - c \cdot q_1$$

$$= q_1 [P(q_1+q_2)] - cq_1$$

$$= q_1 \cdot [\alpha - (q_1+q_2)] - cq_1$$

$$\pi_1(q_1,q_2) \begin{cases} q_1(\alpha - c - q_1 - q_2) & \text{if } q_1+q_2 \le \alpha \\ -cq_1 & \text{if } q_1+q_2 > \alpha \end{cases}$$

$$\pi_2(q_1,q_2) = \begin{cases} q_2(\alpha - c - q_1 - q_2) & \text{if } q_1+q_2 \le \alpha \\ -cq_2 & \text{if } q_1+q_2 > \alpha. \end{cases}$$

$\underset{q_1}{\text{Max}}.\ \pi_1(q_1, q_2)$

First order condition : $\frac{\partial}{\partial q_1} \pi_1(q_1, q_2) = 0$

$$-q_1 + \alpha - c - q_1 - q_2 = 0$$

$$\therefore q_1 = \frac{\alpha - c - q_2}{2}$$

Second order condition : $\frac{\partial^2}{\partial q_1^2} \pi_1(q_1, q_2) < 0$.

$$\frac{\partial^2}{\partial q_1^2}\left(\alpha - c - q_2 - 2q_1\right) = -2 < 0.$$

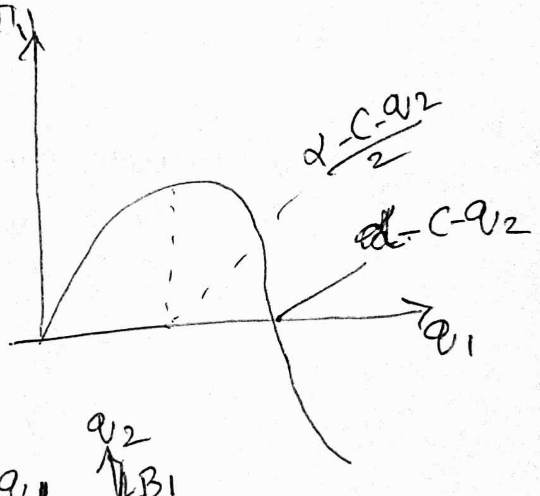
$\therefore$ Best response fun of producer 1 is

$$q_1 = \frac{\alpha - c - q_2}{2}$$

$$\pi_1 = q_1(\alpha - c - q_1 - q_2)$$

if $q_1 = 0$ or $q_1 = \alpha - c - q_2$, then $\pi_1 = 0$.

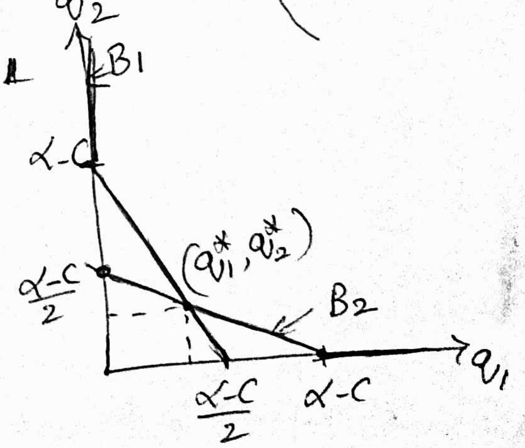
$\frac{\partial^2}{\partial q_1^2} \pi_1 < 0$ [concave curve]



$$q_1 = b(q_2) = \begin{cases} \frac{\alpha - c - q_2}{2} & \text{if } \alpha - c \geq q_2 \\ 0 & \text{if } \alpha - c < q_2 \end{cases}$$

IIIly

$$q_2 = b(q_1) = \begin{cases} \frac{\alpha - c - q_1}{2} & \text{if } \alpha - c \geq q_1 \\ 0 & \text{if } \alpha - c < q_1 \end{cases}$$



NE $= (q_1^*, q_2^*)$

$$q_1^* = \frac{\alpha - c - q_2^*}{2} \to ①, \quad q_2^* = \frac{\alpha - c - q_1^*}{2} \to ②$$

Solving ① & ②,

$$q_1^* = \frac{\alpha - c}{2} - \frac{1}{2}\cdot\left(\frac{\alpha - c - q_1^*}{2}\right)$$

$\therefore\ q_1^* = \frac{\alpha - c}{2} - \frac{\alpha - c}{4} + \frac{1}{4} q_1^*$

$\therefore\ \frac{3}{4} q_1^* = \frac{\alpha - c}{4}$

$\therefore\ q_1^* = \frac{\alpha - c}{3}$

$\therefore\ q_2^* = \frac{\alpha - c}{2} - \frac{1}{2} \cdot \frac{\alpha - c}{3} = \frac{1}{3}(\alpha - c) = \frac{\alpha - c}{3}.$

$\therefore\ NE = \left(\frac{\alpha - c}{3}, \frac{\alpha - c}{3}\right)$.

Q^* - total market o/p in equilibrium

$= q_1^* + q_2^* = \frac{2}{3}(\alpha - c)$

p^* = equilibrium price $= \alpha - Q$

$= \alpha - \frac{2}{3}(\alpha - c)$

$= \frac{1}{3}(3\alpha - 2\alpha + 2c)$

$= \frac{1}{3}(\alpha + 2c)$

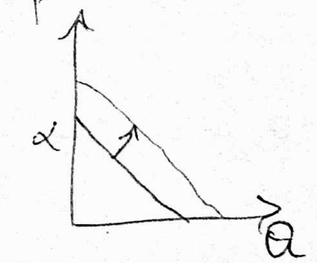
Equilibrium profit of each firm

$\pi_1^* = \pi_2^* = q_1^*(\alpha - c - q_2^*) - q_1^{*2}$

$= q_1^*(\alpha - c - q_2^* - q_1)$

$= \frac{\alpha - c}{3}\left(\alpha - c - \frac{2}{3}(\alpha - c)\right)$

$= \frac{\alpha - c}{3} \cdot \frac{(\alpha - c)}{3} = \frac{(\alpha - c)^2}{9}.$



If α is increasing, demand increases,
q_i^* increases, π_i^* increases.

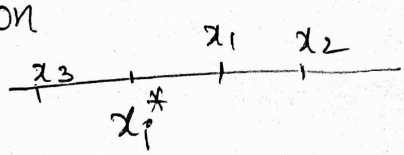
c depends on technology used by a firm.
c decreases as technology improves.
$c\downarrow$, $q_i^*\uparrow$, $p^*\downarrow$, $\pi_i^*\uparrow$, $Q^*\uparrow$.

Electoral Competition.

How people vote, how # of candidates when people are voting, ~~#~~ of are decided, more importantly what are the ajenda said by the candidates if they want to win the election or if they want to win!
In equilibrium, what are ajenda said? who will win?
Candidate has to bare some cost, does it hamper election process? thus # of people who are running for election does that go down. So, these are important issues.

There is a continuum of numbers. Each number represents the favorite position of atleast one voter..

There can be more than one position which is a favorite of more than one voter.

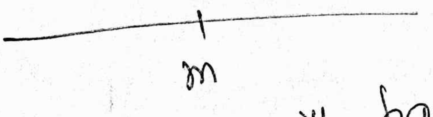


x_1 likes than x_2.

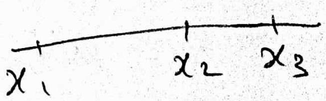
$$u_i(x_1) = -(x_i^* - x_1)^2$$ ← left/right same.

Everyone likes the policy of country to be close to his favorite & towards left/right of favorite position.

m = median



Half of voters will have their favorite position either equal to m or greater than m. that is how median is defined.



Candidates announce positions.

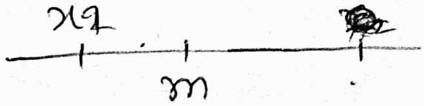
How candidates will choose positions?
How voters vote?
Who will win?

Players: Candidates (n in number)
Actions: Set of positions represented by numbers.
Preferences: . Winning is best represented by n

. k if i tie with (n-k) other candidates in the first place, $1 \leq k \leq n-i$

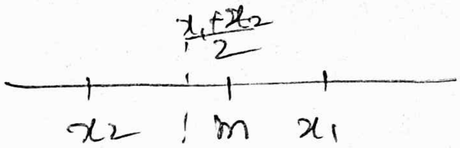
. 0 if i loses.

<u>NE</u>:



Let n=2.

Suppose, announcement by 2, $x_2 < m$



If $x_2 < m$, $x_1 > x_2$.

$\frac{x_1+x_2}{2} < m$.

$\Rightarrow x_1 + x_2 < 2m$

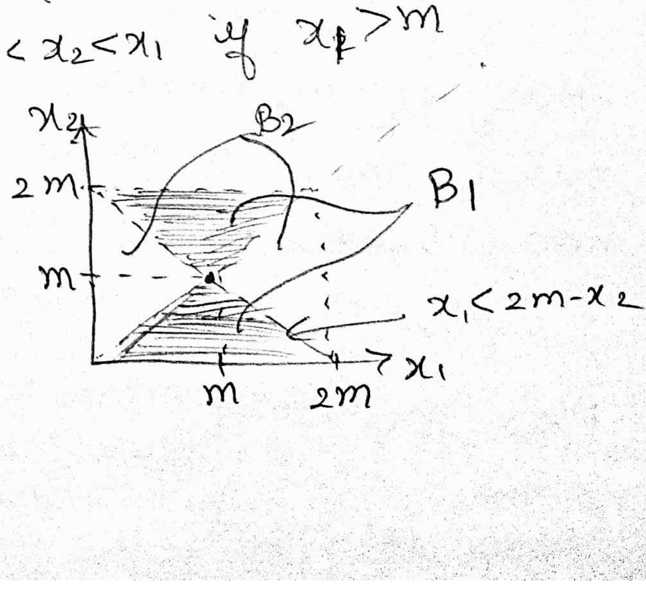
$x_1 < 2m - x_2$.

$\therefore$ If $x_2 < m$, $x_1 = B_1(x_2) = \begin{cases} x_1 : & x_2 < x_1 < 2m - x_2 \} \text{ if } x_2 < m \\ m & ; \text{ if } x_2 = m \\ x_1 : & 2m - x_2 < x_1 < x_2 \} \text{ if } x_2 > m \end{cases}$

Hotelling model of Electoral Competition.

$x_2 = B_2(x_1) = \begin{cases} x_2 : & x_1 < x_2 < 2m - x_1, \text{ if } x_1 < m. \\ m : & \text{if } x_1 = m \\ x_2 : & 2m - x_1 < x_2 < x_1 \text{ if } x_1 > m \end{cases}$

NE: (m, m)
ie both candidates are announcing same position.



Cournot's model of Oligopoly:

A single good is produced by n firms. The cost to firm i of producing q_i units of the goods is $C_i(q_i)$, where C_i is an increasing function (more o/p is more costly to produce). All the output is sold at a single price, determined by the demand for the good and firm's total output. Specifically, if the firms total output is Q, then the market price is $P(Q)$; P is called "inverse demand function". Assume that P is decreasing function when it is positive: if the firms' total output increases, then price decreases. If o/p of each firm i is q_i, then the price is $P(q_1+q_2+\ldots+q_n)$, so that firm i's revenue is $q_i \cdot P(q_1+q_2+\ldots+q_n)$. Thus firm i's profit, is equal to its revenue minus its cost.

ie $\pi_i(q_1, q_2 \ldots q_n) = q_i\, P(q_1+q_2+\ldots q_n) - C_i(q_i)$ → (1)

The model of Oligopoly as strategic game is

Players: The firms

Actions: Each firm's set of actions is the set of its possible outputs (non negative quantities).

Preferences: Each firm's preferences are represented by its profits as in (1)

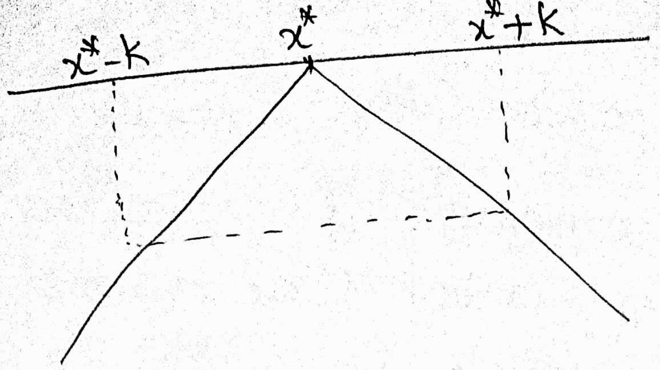
Electoral Competition

Players are candidates and policy is a number, referred to as a "position". After the candidates have chosen positions, each of a set of citizens votes for the candidate whose position she likes the best. The candidate who obtains the most votes wins. Each candidate cares only about winning; no candidate has an ideological attachment to any position. Each candidate prefers to win than to tie for first place (if tie randomly select winner). and prefers to tie for first place than to lose; if she ties for first place, she prefers to do so with as few other candidates as possible.

There is a continuum of voters, each with a favorite position. The distribution of these favorite positions over the set of all possible positions is arbitrary. The distribution may not be uniform - a large fraction of the voters may have favorite positions close to one point, while few voters have favorite positions close to some other point. A position that turns out to have special significance is the median favorite position: the position m with property that exactly half of voters' favorite positions are atmost m and half of the voters' favorite positions are atleast m.

Each voter's ~~distance~~ for any position is given by the distance between that position and her favorite position. ie for any value k, a voter whose favorite position is x^* is indifferent b/n the positions

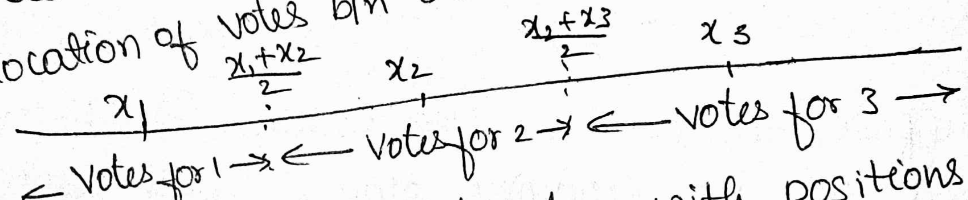
x^*-k and x^*+k as shown below.



← Payoff of a voter whose favorite position is x^*, as a function of winning position, x.

Under this assumption, each candidate attracts the votes of all citizens whose favorite positions are closer to her position than to the position of any other candidate.

Ex:- Allocation of votes b/n 3 candidates with positions x_1, x_2, x_3.



Here there are 3 candidates, with positions x_1, x_2 & x_3. Candidate 1 attracts the voters of every citizen whose favorite position is in interval labelled "Votes for 1" upto mid point $\frac{x_1+x_2}{2}$ of line segment from x_1 to x_2. |||ly for candidate 2, interval is from $\frac{1}{2}(x_1+x_2)$ to $\frac{1}{2}(x_2+x_3)$ & candidate 3 attracts remaining votes. Here we assume that citizens whose favorite position is $\frac{1}{2}(x_1+x_2)$ divide their votes equally b/n candidates 1 and 2 & those whose favorite position is $\frac{1}{2}(x_2+x_3)$ divide their votes b/n 2 & 3. If two or more candidates take the same position, then they share equally the same votes that the position attracts.

This model called as Hotelling's model of electoral competition modeled as strategic game as below:

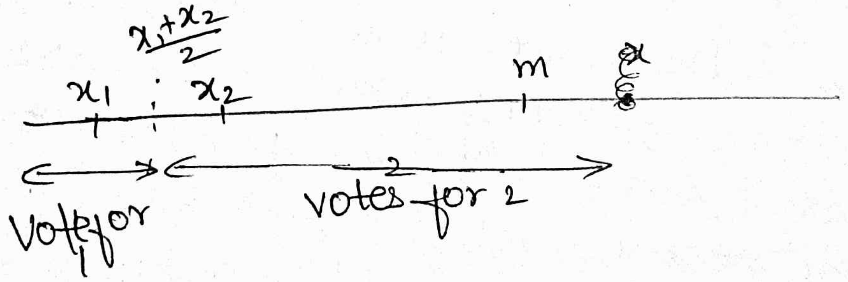
Players: The candidates

Actions: Each candidate's set of actions is the set of positions (numbers).

Preferences: Each candidates preferences are represented by a payoff function that assigns n to every action profile in which she wins outright, k to every action profile in which she ties for first place with n-k other candidates (for $1 \le k \le n-1$) and 0 to every action profile in which she loses,

=.

Suppose there are two candidates. Then we can find NE by studying player's best response functions. Fix position x_2 of candidate 2 & consider the best position for candidate 1. First suppose that $x_2 < m$. If candidate 1 takes a position to the left of x_2, then candidate 2 attracts the votes of all citizens whose favorite positions are to right of $\frac{x_1+x_2}{2}$,



Action profile (x_1, x_2) for which candidate 2 wins.

a set that includes 50% of citizens whose favorite positions are to right of m, & more.

Thus candidate 2 wins & 1 loses.

If candidate 1 takes position to right of x_2.

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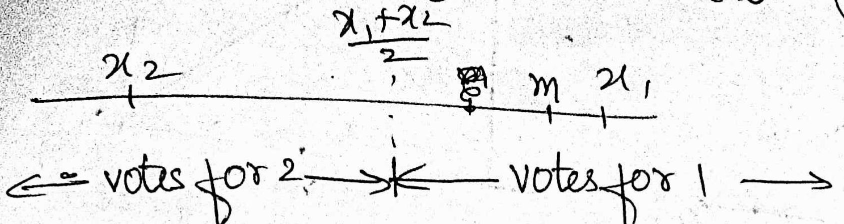
...en she wins as long as the dividing line b/n her supporters and those of candidate 2 is $< m$.



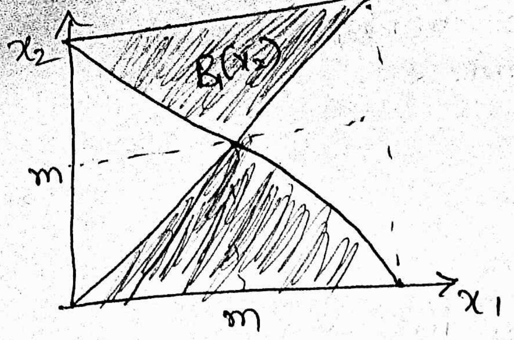
If she is so far to right that this dividing line lies to the right of m, then she loses. She prefers to win than to lose, & is indifferent b/n all outcomes in which she wins, so her set of best responses to x_2 is the set of positions that causes mid point $\frac{x_1+x_2}{2}$ of line segment from x_2 to x_1 to be less than m. The condition $\frac{(x_1+x_2)}{2} < m$ is equivalent to $x_1 < 2m - x_2$. $\therefore$ candidate 1's best responses to x_2 is set of all positions b/n x_2 and $2m - x_2$. (excluding x_2 & $2m - x_2$).

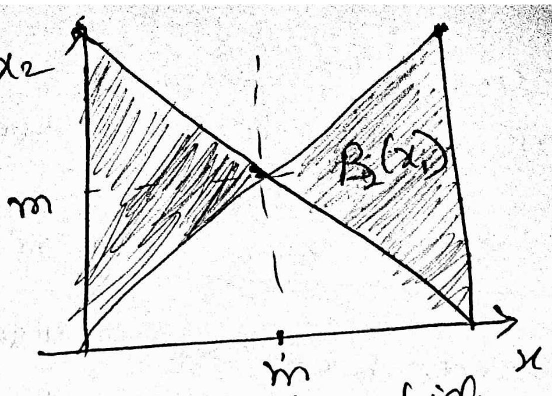
$|||^{ly}$ for $x_2 > m$. In this case candidate 1's best responses to x_2 is set of all positions b/n $2m - x_2$ and x_2.

Lastly if $x_2 = m$. In this case candidate 1's unique best response is to choose same position, m. If she chooses any other position, then she loses, whereas of she chooses m, then she ties for first place.

$$B_1(x_2) = \begin{cases} \{x_1 : x_2 < x_1 < 2m - x_2\} & \text{if } x_2 < m \\ \{m\} & \text{if } x_2 = m \\ \{x_1 : 2m - x_2 < x_1 < x_2\} & \text{if } x_2 > m. \end{cases}$$

Candidate 2 faces exactly same incentives as candidate 1. Best response fns are shown in next pg.





∴ The game has a unique NE, in which both candidates choose the position m, the voters median position. The outcome is that the election is a tie.

Symmetric games & Symmetric Equilibrium.

2 population

1 person from each group is picked & each of these will play & they have knowledge of previous game.

1) In symmetric game, two people come from same population. So action set of players must be the same.

$$A_1 = A_2$$

2) For any pair of actions (a_1, a_2), $u_1(a_1, a_2) = u_2(a_2, a_1)$.

identity of players do not matter.

Suppose $a_1 = a_2 \Rightarrow u_1(a, a) = u_2(a, a)$

If actions are same, their payoffs must also be same.

	A	B
A	x, x	z, k
B	k, z	y, y

Ex:- PD is Symmetric Game

Where as BoS is not Sym. Game.

Symmetric game is notion of game where people/players are coming from same population.

NE - vector of action which is a steady.

Non Symmetric

$A_1 - \{a_1, a_2, a_3\}$ $A_2 = \{b_1, b_2\}$

Suppose NE: (a_1, b_2)

Symmetric $A_1 = A_2 = \{a_1, a_2\}$

1) Can there be a steady state at (a_1, a_2)?

Soln:- No.

NE: (a^*, a^*).

	a_1	a_2
a_1		$\boxed{}$ → NE.
a_2		

Symmetric Nash Equilibrium :- In a n-player game with ordinal preferences, if action sets of players are the same then the nash equilibrium in which all take the same action is called a symmetric NE.

$(a^*, a^*, \ldots, a^*)$

It is a symmetric game. but NE: (a,b) & (b,a) but it is not symmetric nash equilibrium.

1 \ 2	a	b
a	0,0	1,1
b	1,1	0,0

---

1) NE? – (A,A) (A,C), (C,A).

2) It is a symmetric game. ∵ matrix is symmetric over main diagonal.

3) Symmetric NE – (A,A)

1 \ 2	A	B	C
A	$1^*, 1^*$	$2, 1^*$	$4^*, 1^*$
B	$1^*, 2$	$5, 5$	$3, 6^*$
C	$1^*, 4^*$	$6^*, 3$	$0, 0$

Summary of UNIT-I

26-1

Prisoner's Dilemma

NE: (C,C)

Action C strictly dominates NC.

1 \ 2	C	NC
C	1,1	3,0
NC	0,3	2,2

or

	C	NC
C	-3,-3	0,-5
NC	-5,0	-1,-1

Ex. of PD

1) Duopoly/ ~~Arms Race~~

1 \ 2	H	L
H	1000,1000	-200,1200
L	1200,-200	500,500

L = C
H = NC

NE: (L,L)

2) Joint Project.

1 \ 2	HW	S
HW	2,2	0,3
S	3,0	1,1

NE: (S,S)

HW - Hard work - NC

S - Shirking - C

$U_1(S,HW) > U_1(HW,HW) > U_1(S,S) > U_1(HW,S)$

3) Arms Race

1 \ 2	N	R
N	1,1	3,0
R	0,3	2,2

NE: (N,N)

N - Build Nuclear bomb - C

R - Refrain - NC

$U_1(N,R) > U_1(R,R) > U_1(N,N) > U_1(R,N)$

4) Common Property.

1 \ 2	M	L
M	~~1,1~~	3,0
L	0,3	2,2

$U_1(M,L) > U_1(L,L) > U_1(M,M) > U_1(L,M)$ (3, 2, 1, 0)

$U_2(L,M) > U_2(L,L) > U_2(M,M) > U_2(M,L)$ (3, 2, 1, 0)

Type 2

NE: (B,B),(O,O)

Bo.S.

		W	
		B	O
H	B	2, 1	0,0
	O	0,0	1, 2

B - Boxing Match
O - Opera House
H - Husband
W - Wife.

$\overset{2}{U_H(B,B)} > \overset{1}{U_H(O,O)} > \overset{0}{U_H(O,B)} = U_H(B,O)$

Ex:-

1) Political Parties

		2	
		D	B
1	D	2,1	0,0
	B	0,0	1,2

NE: (D,D),(B,B)

D - Delhi Politician - 1
B - Bihar " - 2.

$U_1(D,D) > U_1(B,B) > U_1(B,D) = U_1(D,B)$

2) Merging of Companies/firms race.

		2	
		A	B
1	A	2,1	0,0
	B	0,0	1,2

Company 1 uses A tech.
" 2 uses B tech.

For 1, $\overset{2}{U_1(A,A)} > \overset{1}{U_1(B,B)} > \overset{0}{U_1(A,B)} = U_1(B,A)$

NE: (A,A), (B,B)

26.2

Type 3 / Games with pure conflict.

Matching Pennies.

NE: NIL

		2: H	2: T
1	H	1, -1	-1, 1
	T	-1, 1	1, -1

H - Head

T - Tail

Player 1 prefers both coins turn same.

Ex:-

NE: NIL

1) Company product appearance

		2: A	2: B
1	A	1, -1	-1, 1
	B	-1, 1	1, -1

Product

Company 2 - reputed - ~~A~~ B

Company 1 - new - ~~B~~ A

$U_1(A,A) = U_1(B,B) > U_1(A,B) = U_1(B,A)$.

$U_2(A,B) = U_2(B,A) > U_2(A,A) = U_2(B,B)$

Here company 1, which is new wants her products to look like reputed company product.

2) Cricket.

NE: NIL.

		Bowler: F	Bowler: S
Batsman	F	1, -1	-1, 1
	S	-1, 1	1, -1

F - Fast

S - Slow.

Here batsman prefer matching.

NE:

a^* is NE if

$u_i(a^*) \geq u_i(a_i, -a_i^*)$ for every action a_i of player i.

~~Vari~~

Type 4 (Games without conflict) / Variant PD.

1) Stag Hunt.

NE: (S,S), (H,H)

S - Stag

H - Hare.

1 \ 2	S	H
S	2,2	0,1
H	1,0	1,1

$\overset{2}{U_1(S,S)} > \overset{1}{U_1(H,H) = U_1(H,S)} > \overset{0}{U_1(S,H)}$

Security Dilemma.

2) / Arm's Race. – ~~Both~~ A countries prefer outcome variant in which both countries ~~refrain~~ refrain from arming themselves to the one in which it alone arms itself;

1 \ 2	N	R
N	1,1	2,0
R	0,2	3,3

$\overset{3}{U_1(R,R)} > \overset{2}{U_1(N,R)} > \overset{1}{U_1(N,N)} > \overset{0}{U_1(R,N)}$

Hawk Dove Game.

NE: (A, P), (P, A)

P - Passive

A - Aggressive.

1 \ 2	P	A
P	2, 2	1, 3
A	3, 1	0, 0

$$U_1(\overset{3}{A}, P) > U_1(\overset{2}{P}, P) > U_1(\overset{1}{P}, A) > U_1\overset{0}{(A, A)}$$

$$U_2(P, A) > U_2(P, P) > U_2(A, P) > U_2(A, A)$$

Here each prefers to be aggressive when other is passive and passive when other is aggressive.

Given, each prefers outcome in which opponent is passive rather than aggressive.

Public Good Game.

$(NC, P) > (C, P) > (NC, NP) > (C, NP)$

best to worst preferences.

Person \ Good	P	NP
C	2	0
NC	3	1

1. Each person wants/prefers that good is provided & she does not contribute.
2) good is provided & he contributes.
3) good is not provided & he has not contributed.
4) good is not provided & he has contributed.

ie, when good is provided, each person prefers that he has not contributed & when good is not provided he prefers that he has not contributed. Given, each prefers good provided over not provided

NE: for every i, $u_i(a^*) \geq u_i(a_i, a_{-i}^*)$, $\forall a_i \in A_i$.

Strict NE: $u_i(a^*) > u_i(a_i, a_{-i}^*)$, $\forall a_i \in A_i$ other than a_i^*.

Best Response function:-

$B_i(a_{-i})$ - Best response of player i, when list of other player's action is a_{-i}.

$$B_i(a_{-i}) = \{a_i : u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}), \forall a_i' \in A_i\}.$$

NE in terms of Best Response function.

$a^* = (a_1^*, a_2^*, \ldots a_n^*)$ is NE if for every player i, $a_i^* \in B_i(a_{-i}^*)$.

Dominated Actions

1. Strict Domination - Player i's action a_i'' strictly dominates a_i' if $u_i(a_i'', a_{-i}) > u_i(a_i', a_{-i})$ for every list a_{-i} of other players actions.

Here a_i'' is called strictly dominating action.
a_i' is called strictly dominated action.

NOTE:- A strictly dominated action is not used in any NE.

... is played over and

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Weak Domination - Player i's action a_i'' weakly dominates a_i' if

$$u_i(a_i'', a_{-i}) \geq u_i(a_i', a_{-i}) \text{ for every list } a_{-i} \text{ of other players' actions.}$$

&

$$u_i(a_i'', a_{-i}) > u_i(a_i', a_{-i}) \text{ for some list } a_{-i} \text{ of other players' actions}$$

Here a_i'' - weakly dominating action

a_i' - weakly dominated action.

NOTE: In strict NE, weakly dominated actions are not played.

Ex:- Model paper Q.No. 1.7.

1.7 Find action profile(s) which survive iterative elimination of strictly dominated actions.

$$\begin{array}{cc|c|c|c|}
 & & & 2 & \\
 & & L & \cancel{C} & \cancel{R} \\
\hline
 & T & 4,3 & 5,1 & 6,2 \\
\hline
1 & \cancel{M} & 2,1 & 8,4 & 3,6 \\
\hline
 & \cancel{R} & 3,0 & 9,6 & 2,8 \\
\hline
\end{array}$$

Soln:- 1) For player 2, C is strictly dominated by R.

∴ remove C.

2) For player 1, R is strictly dominated by T.

∴ remove R.

3) For player 1, M is strictly dominated by T.

∴ remove M.

4) For player 2, R is strictly dominated by L. ∴ remove R.

∴ (T, L) is NE.

Symmetric game.

A game is said to be symmetric if players set of actions are the same and the players' preferences represented by payoff functions u_1 and u_2 for which $u_1(a_1, a_2) = u_2(a_2, a_1)$ for every action pair (a_1, a_2)

1 \ 2	A	B
A	w,w	x,y
B	y,x	z,z

Ex:- PD, Stag Hunt.

Symmetric NE :- A NE in which all players take same action is called symmetric NE.

i.e., $(a^*, a^*, \ldots\ldots a^*)$ is symmetric NE

Find NE using Best Response Funs.

1 \ 2	L	R
U	$8^*, 7^*$	4,6
B	6,5	$7^*, 8^*$

1 \ 2	L	R
U	9,9	$1, 10^*$
D	$10^*, 1$	$2^*, 2^*$

1 \ 2	L	R
U	$6^*, 8^*$	$4^*, 7$
D	$7^*, 6$	$3, 7^*$

1 \ 2	L	R
U	$7^*, 6^*$	5,5
D	$4, 5^*$	$6^*, 4$

NE - is an action profile such that no player has incentive to change his action given what the other players are doing.