

# Tree

## Introduction to TREE data structure

What is a Data Structure -

DS is way of storing/organising data in the memory, in such a way that access, management & modification becomes efficient.

Which DS to use -

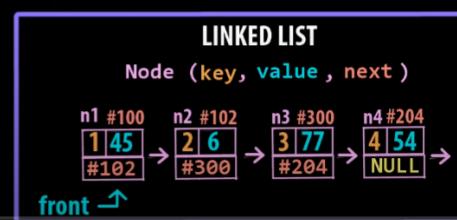
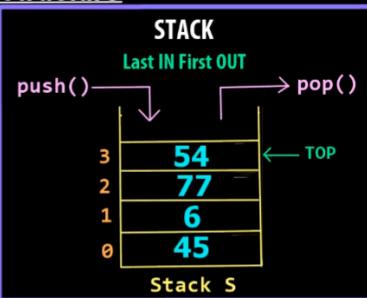
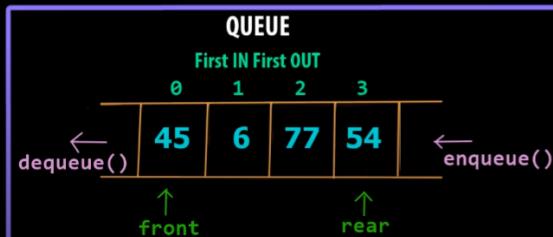
- 1) What type of data needs to be stored
- 2) Cost of operations (read, write, update etc)
- 3) Memory usage & efficiency
- 4) Easy of implementation
- 5) Maintainability

\* Every Data Structure has its own **PROS & CONS** !

## Introduction to TREE data structure

Linear Data Structures -

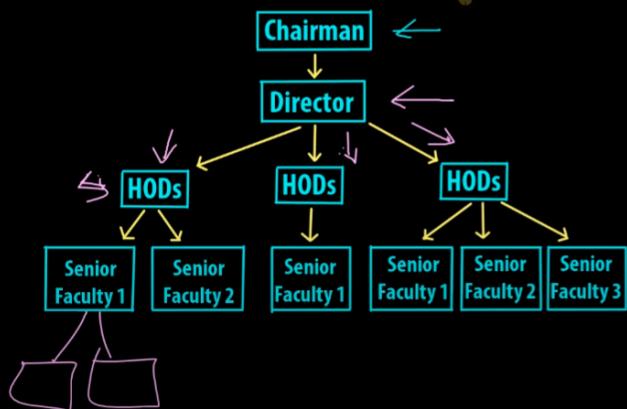
Data is arranged in a sequential manner. We have a logical start and a logical end.  
Every element has a next element and previous element.



## Introduction to TREE data structure

**TREE** - A tree is a non linear data structure that simulates a hierachial tree structure with a root value & subtrees of children with parent node, represented as set of linked nodes.

### Real world example of hierachial tree



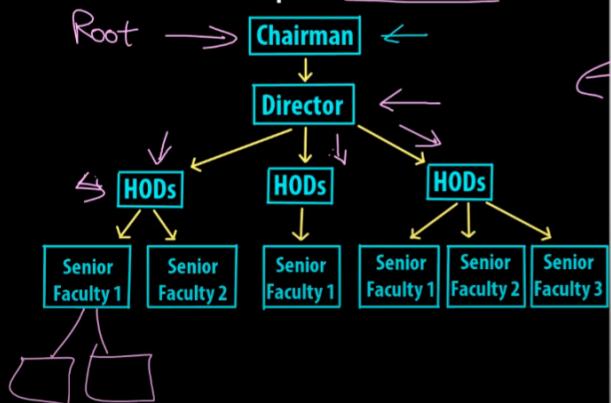
### Company Hierarchy -



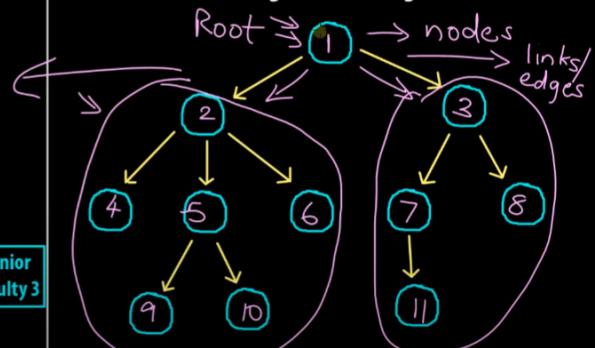
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### Real world example of hierachial tree



### Logical Tree Diagram

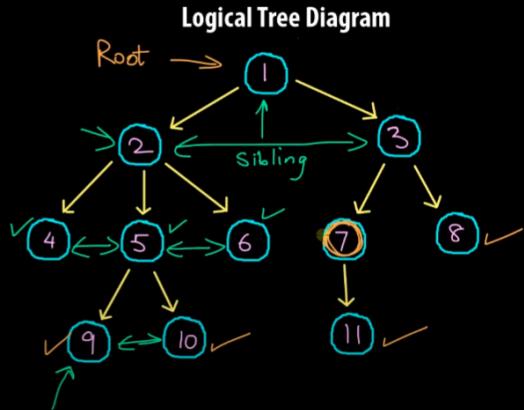


## Introduction to **TREE** data structure

**TREE** - *A tree is a non linear data structure that simulates a hierachial tree structure with a root value & subtrees of children with parent node, represented as set of linked nodes.*

Important TREE terms -

1. Root - Root is a special node in a tree. The entire tree is referenced through it. It does not have a parent.
2. Parent Node - Parent node is an immediate predecessor of a node.
3. Child Node - All immediate successors of a node are its children.
4. Siblings - Nodes with the same parent are called Siblings.
5. Leaf - Last node in the tree. There is no node after this node.
6. Edge - Edge is a connection between one node to another. It is a line between two nodes or a node and a leaf.
7. Path - Path is a number of successive edges from source node to destination node.
8. Degree of Node - Degree of a node is equal to number of children, a node have.



## Introduction to **TREE** data structure

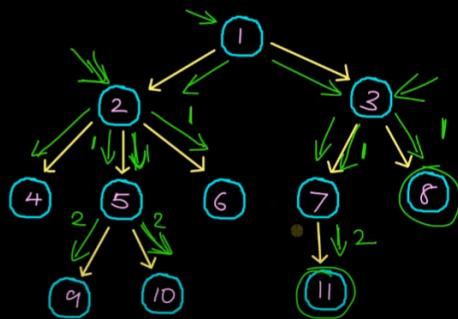
**TREE** - *A tree is a non linear data structure that simulates a hierachial tree structure with a root value & subtrees of children with parent node, represented as set of linked nodes.*

Important TREE terms / properties -

- 1) Tree can be termed as a **RECURSIVE** data structure.
- 2) In a valid tree for **N Nodes** we have **N-1 Edges/Links**.
- 3) Depth of Node - Depth of a node represents the number of edges from the tree's root node to the node.
- 4) Height of Node - Height of a node is the number of edges on the longest path between that node & a leaf.
- 5) Height of Tree - Height of tree is the height of its root node.

$$\begin{aligned} \text{Depth}(6) &= 2 & \text{Height}(3) &= 2 \\ \text{Depth}(9) &= 3 & \text{Height}(2) &= 2 \end{aligned}$$

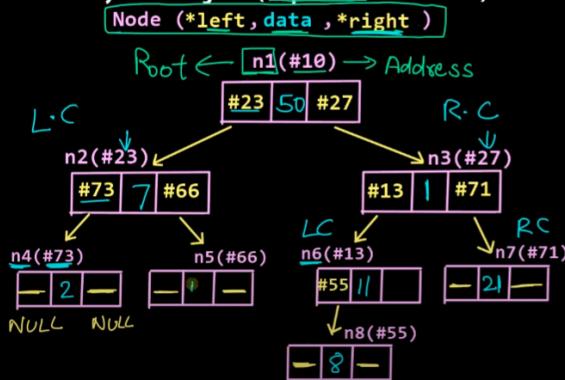
## Logical Tree Diagram



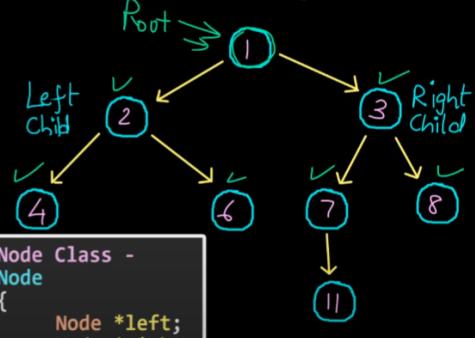
## Introduction to **TREE** data structure

**TREE** - A tree is a non linear data structure that simulates a hierachial tree structure with a root value & subtrees of children with parent node, represented as set of linked nodes.

Binary Tree Diagram (Implementation View)



Binary Tree Diagram



## Introduction to **TREE** data structure

**TREE** - A tree is a non linear data structure that simulates a hierachial tree structure with a root value & subtrees of children with parent node, represented as set of linked nodes.

Types of Trees -

1. General Tree ✓
2. Binary Tree ✓
3. Binary Search Tree ✓
4. AVL Tree ✓
5. Spanning Tree ✓
6. B-Tree ✓
7. B+ Tree ✓
8. Heap ✓

Applications of Tree Data Structure -

1. Store hierarchical data, like folder structure, organization structure data.
2. Binary Search Tree is a tree that allows fast search, insert, delete on a sorted data.
3. Heap is a tree data structure which is implemented using arrays and used to implement priority queues.
4. B-Tree and B+ Tree are used to implement indexing in databases.
5. Used to store router-tables in routers.
6. Used by compilers to build syntax trees.
7. Used to implement expression parsers and expression solvers.

# Binary Tree

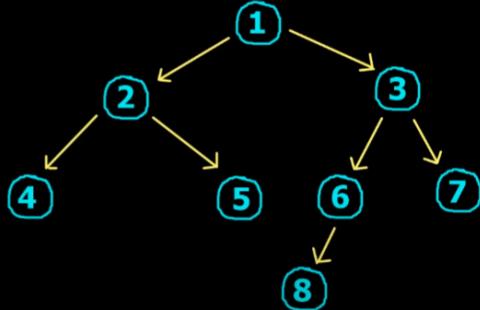
## Binary Tree Data Structure

**Binary Tree** - A binary tree is a tree data structure in which each node has at most two children, which are referred to as the left child (LC) and the right child (RC).

## **Important Binary Tree Terms & Properties -**

- 1) A binary tree is called **STRICT/PROPER** binary tree, when each node has 2 or 0 children.
  - 2) A binary tree is called **COMPLETE** binary tree if all levels except the last are completely filled and all nodes are as left as possible.
  - 3) A binary tree is called **PERFECT** binary tree if all levels are completely filled with 2 children each.
  - 4) Max number of nodes at level 'x' =  $2^X$
  - 5) For a binary tree, maximum no of nodes with height 'h' =  $2^0 + 2^1 + \dots + 2^h$   
 $= 2^{(h+1)} - 1$

## Binary Tree Diagram



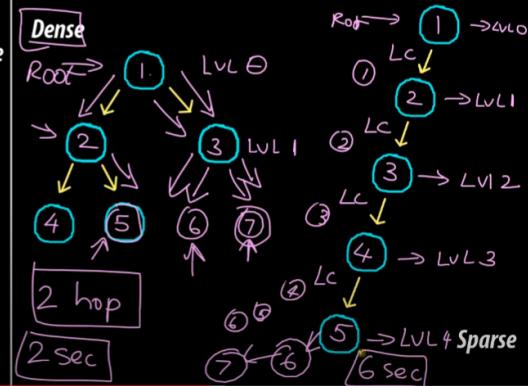
## Binary Tree Data Structure

**Binary Tree** - *A binary tree is a tree data structure in which each node has at most two children, which are referred to as the left child (LC) and the right child (RC).*

## Important Binary Tree Terms & Properties -

- 6) A binary tree is called **BALANCED** binary tree, if the difference between the height of left and right subtree for every node is not more than k (usually 1).

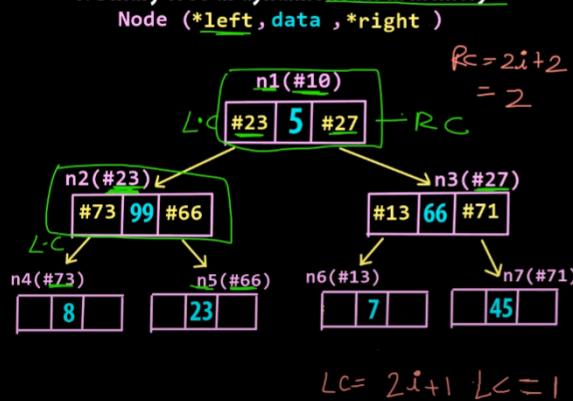
## Binary Tree Variations



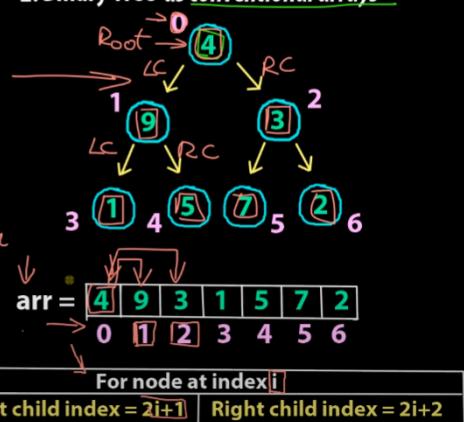
## Binary Tree Data Structure

**Binary Tree** - A binary tree is a tree data structure in which each node has at most two children, which are referred to as the left child (LC) and the right child (RC).

### 1. Binary Tree as dynamic nodes in memory



### 2. Binary Tree as conventional arrays



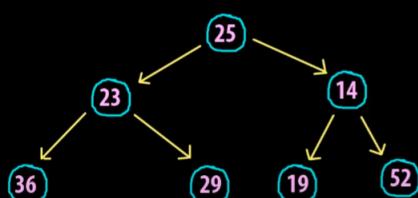
## Binary Search Tree

### Binary SEARCH Tree Data Structure

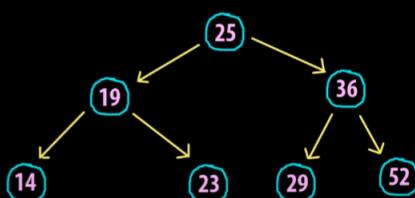
**Binary search tree** - BST is a binary tree data structure, in which the values in the left subtrees of every node are smaller and the values in the right subtrees of every node are larger.

Values/Keys = {25, 23, 14, 36, 29, 19, 52}

#### Binary Tree



#### Binary SEARCH tree



### Binary SEARCH Tree Data Structure

**Binary search tree** - BST is a binary tree data structure, in which the values in the left subtrees of every node are smaller and the values in the right subtrees of every node are larger.

Values/Keys = {25, 23, 14, 36, 29, 19, 52}

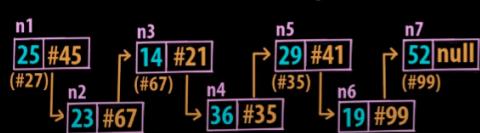
#### Array

$arr = [25, 23, 14, 36, 29, 19, 52]$

#### Time Complexity (Big O)

O(N) linear search	Insert			Delete		
	Start	Random	End	Start	Random	End
$O(1)$ random access	$O(N)$	$O(N)$	$O(1)$	$O(N)$	$O(N)$	$O(1)$

#### LinkedList



$O(N)$

$O(1)$     $O(N)$     $O(N)$     $O(1)$     $O(N)$     $O(N)$

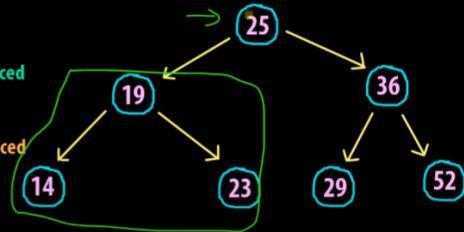
### Binary SEARCH Tree Data Structure

Binary search tree - *BST is a binary tree data structure, in which the values in the left subtrees of every node are smaller and the values in the right subtrees of every node are larger.*

→ Values/Keys = {25, 23, 14, 36, 29, 19, 52}

	Time Complexity (Big O)		
	Search/Access/Update	Insert/Add	Delete/Remove
Avg case	<b>O(log N)</b>	<b>O(log N)</b>	<b>O(log N)</b>
Worst case	<b>O(N)</b>	<b>O(N)</b>	<b>O(N)</b>
	0 1 2 3 4 5 6		
sorted_arr =	[14   19   23   25   29   36   52]		

### Binary SEARCH tree balanced



### Binary SEARCH Tree Data Structure

Binary search tree - *BST is a binary tree data structure, in which the values in the left subtrees of every node are smaller and the values in the right subtrees of every node are larger.*

Values/Keys = {25, 23, 14, 36, 29, 19, 52}

	Time Complexity (Big O)		
	Search/Access/Update	Insert/Add	Delete/Remove
Avg case	<b>O(log N)</b>	<b>O(log N)</b>	<b>O(log N)</b>
Worst case	<b>O(N)</b>	<b>O(N)</b>	<b>O(N)</b>
	0 1 2 3 4 5 6		
sorted_arr =	[14   19   23   25   29   36   52]		

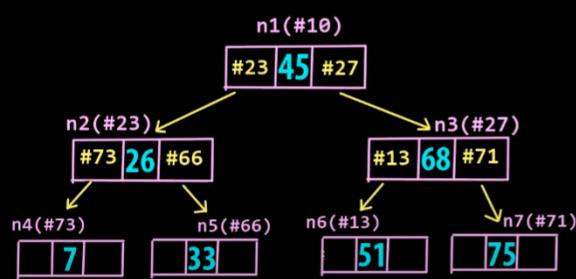
### Binary SEARCH tree unbalanced



## Implementation

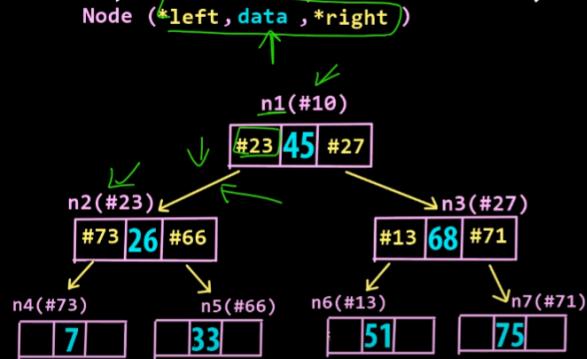
### Binary SEARCH Tree Data Structure (Implementation)

1. Binary **SEARCH** Tree as dynamic nodes in memory  
Node (\*left, data , \*right )



## Binary **SEARCH** Tree Data Structure (Implementation)

### 1. Binary **SEARCH** Tree as dynamic nodes in memory



Tree Node Class -

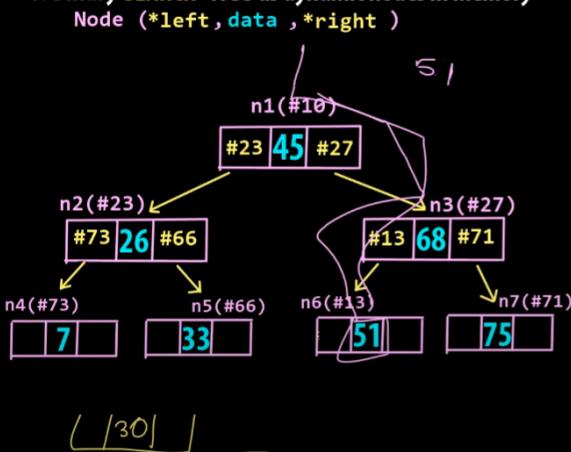
```

class TreeNode
{
    → Data Members -
    1. int value
    2. TreeNode * left // pointers in C++
    → 3. TreeNode * right // pointers in C++

    → Member Functions -
    → 1. TreeNode() // default constructor
    → 2. TreeNode(int v) // parameterized constructor
}
    
```

## Binary **SEARCH** Tree Data Structure (Implementation)

### 1. Binary **SEARCH** Tree as dynamic nodes in memory



BST Class -

```

class BST
{
    Data Members -
    1. TreeNode *root
    Member Functions -
    1. bool isEmpty() // check if empty
    2. void insertNode(TreeNode *new_node)//insertion
    // depth first traversal approach
    3a. void printPreorder(TreeNode* r)//print & traverse
    3b. void printInorder(TreeNode* r)//print & traverse
    3c. void printPostorder(TreeNode* r)//print & traverse
    3d. void print2D(TreeNode* r,int space)//print & traverse
    // breadth-first traversal approach
    3e. void printLevelOrder(TreeNode* r)//print & traverse
    4. TreeNode* search(TreeNode* r, int v) // search
}
    
```

# BST INSERTION

**BST INSERT Pseudocode -**

```

void insertNode(new_node)
{
    1. IF root == NULL THEN -> root = new_node
    2. ELSE
        2.1 SET temp=root
        2.2 WHILE temp != NULL
            2.2.1 IF new_node.value == temp.value
                2.2.1.1 THEN -> return // no duplicates allowed
            2.2.2 ELSE IF (new_node.value < temp.value) && (temp.left == NULL)
                2.2.2.1 THEN -> temp.left = new_node // value inserted on left
                2.2.2.2      break // get out of the function
            2.2.3 ELSE IF (new_node.value < temp.value)
                2.2.3.1 THEN -> temp = temp.left
            2.2.4 ELSE IF (new_node.value > temp.value) && (temp.right == NULL)
                2.2.4.1 THEN -> temp.right = new_node // value inserted on right
                2.2.4.2      break // get out of the function
            2.2.5 ELSE
                2.2.5.1 temp = temp.right
            END WHILE
        END IF
}

```

**Binary SEARCH Tree Data Structure (Insertion)**

## Tree Traversal

**Tree Traversal**

**1. Depth-first search/traversal (DFS)**

(These searches are referred to as depth-first search (DFS), since the search tree is deepened as much as possible on each child before going to the next sibling.)

- 1. Pre-order (NLR)
- 2. In-order (LNR)
- 3. Post-order (LRN)

N → Node  
L → Left Child  
R → Right Child

**2. Breadth-first search/traversal (BFS)**

Trees can also be traversed in level-order, where we visit every node on a level before going to a lower level. This search is referred to as breadth-first search (BFS), as the search tree is broadened as much as possible on each depth before going to the next depth.

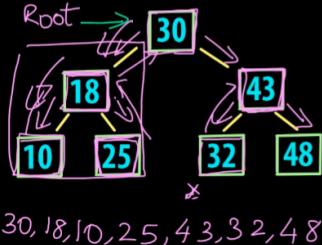
### Binary Tree Data Structure (Traversal Techniques)

→ Depth-first search/Traversal of binary tree (DFS)

These searches are referred to as depth-first search (DFS), since the search tree is deepened as much as possible on each child before going to the next sibling.

#### 1. Pre-order (NLR) (node - left - right)

- ↳ Access the data part of the current node.
- ↳ Traverse the left subtree by recursively calling the pre-order function.
- ↳ Traverse the right subtree by recursively calling the pre-order function.



#### 2. In-order (LNR) (left - node - right)

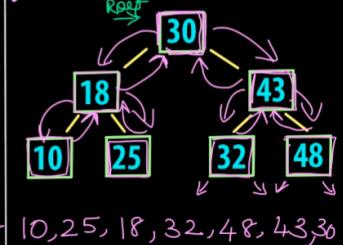
- ↳ Traverse the left subtree by recursively calling the in-order function.
- ↳ Access the data part of the current node.
- ↳ Traverse the right subtree by recursively calling the in-order function.

In BST in-order traversal retrieves the keys in ascending sorted order.



#### 3. Post-order (LRN) (left - right - node)

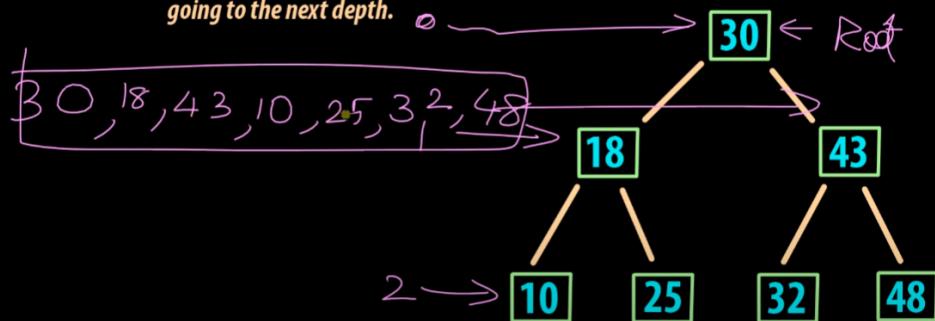
- ↳ Traverse the left subtree by recursively calling the post-order function.
- ↳ Traverse the right subtree by recursively calling the post-order function.
- ↳ Access the data part of the current node.



### Binary Tree Data Structure (Traversal Techniques)

Breadth-first search /level order (BFS)

Trees can also be traversed in level-order, where we visit every node on a level before going to a lower level. This search is referred to as breadth-first search (BFS), as the search tree is broadened as much as possible on each depth before going to the next depth.



### Binary Tree Data Structure (Traversal Techniques)

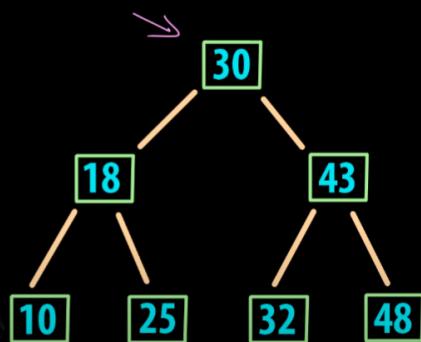
**Tree Traversal :** Tree traversal (also known as tree search and walking the tree) refers to the process of visiting (checking and/or updating) each node in a tree data structure, exactly once. Such traversals are classified by the order in which the nodes are visited.

#### DFS -

1. Pre-order (NLR) - 30, 18, 10, 25, 43, 32, 48
2. In-order (LNR) - 10, 18, 25, 30, 32, 43, 48
3. Post-order (LRN) - 10, 25, 18, 32, 48, 43, 30

#### BFS -

- 30, 18, 43, 10, 25, 32, 48



## PreOrder

**Tree Traversal Technique - Pre-Order Technique(Pseudocode & C++ Program)**

**1. Pre-order (NLR) (node - left - right)**

- Access the data part of the current node.
- Traverse the left subtree by recursively calling the pre-order function.
- Traverse the right subtree by recursively calling the pre-order function.

**1. Pre-order (Pseudocode) -**

```
void printPreorder(TreeNode* r)
{
    1. IF r==NULL THEN -> return
    2. PRINT r->value
    3. printPreorder(r->left)
    4. printPreorder(r->right)
}
```

**OUTPUT -**

The tree structure is as follows:

```

graph TD
    Root[n1(30)] --> n2[16]
    Root[n1(30)] --> n3[99]
    n2[16] --> n4[6]
    n2[16] --> n5[26]
    n5[26] --> n6[83]
  
```

**Tree Traversal Technique - Pre-Order Technique(Pseudocode & C++ Program)**

**1. Pre-order (NLR) (node - left - right)**

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```
void printPreorder(TreeNode* r)
{
    1. IF r==NULL THEN -> return
    2. PRINT r->value
    3. printPreorder(r->left)
    4. printPreorder(r->right)
}
```

**OUTPUT -**

The execution steps and output are:

1. printPreorder(n1)
2. printPreorder(n2)
3. printPreorder(n4)
4. printPreorder(NULL)
5. printPreorder(NULL)
6. printPreorder(n5)
7. printPreorder(NULL)
8. printPreorder(NULL)
9. printPreorder(n3)
10. printPreorder(NULL)
11. printPreorder(n6)
12. printPreorder(NULL)
13. printPreorder(NULL)

The final output sequence is: 30|18|10|25|45|65|

## InOrder

**Tree Traversal Technique - IN-Order Technique(Pseudocode & C++ Program)**

**2. In-order (LNR) (left - node - right)**

- 1 - Traverse the left subtree by recursively calling the in-order function.
- 2 - Access the data part of the current node.
- 3 - Traverse the right subtree by recursively calling the in-order function.

*In BST in-order traversal retrieves the keys in ascending sorted order.*

**2. In-Order (Pseudocode) -**

```
void printInorder(TreeNode* r)
{
    1. IF r==NULL THEN -> return
    2. printInorder(r->left)
    3. PRINT r->value
    4. printInorder(r->right)
}
```

OUTPUT -

**Tree Traversal Technique - IN-Order Technique(Pseudocode & C++ Program)**

**2. In-order (LNR) (left - node - right)**

- Traverse the left subtree by recursively calling the in-order function.
- Access the data part of the current node.
- Traverse the right subtree by recursively calling the in-order function.

*In BST in-order traversal retrieves the keys in ascending sorted order.*

**2. In-Order (Pseudocode) -**

```
void printInorder(TreeNode* r) -> n6
{
    1. IF r==NULL THEN -> return X
    2. printInorder(r->left) -> n6
    3. PRINT r->value
    4. printInorder(r->right)
}
```

Root

OUTPUT -

## PostOrder

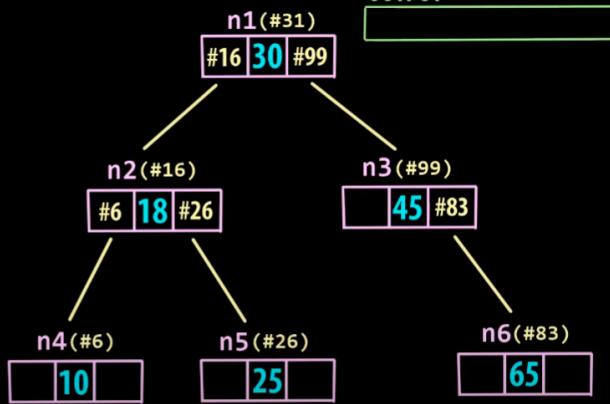
### Tree Traversal Technique : POST-Order Technique(Pseudocode & C++ Program)

#### 3. Post-order (LRN) (left - right - node)

- Traverse the left subtree by recursively calling the post-order function.
- Traverse the right subtree by recursively calling the post-order function.
- Access the data part of the current node.

#### 3. Post-Order (Pseudocode) -

```
void printPostorder(TreeNode* r)
{
    1. IF r==NULL THEN -> return
    2. printPostorder(r->left)
    3. printPostorder(r->right)
    4. PRINT r->value
}
```

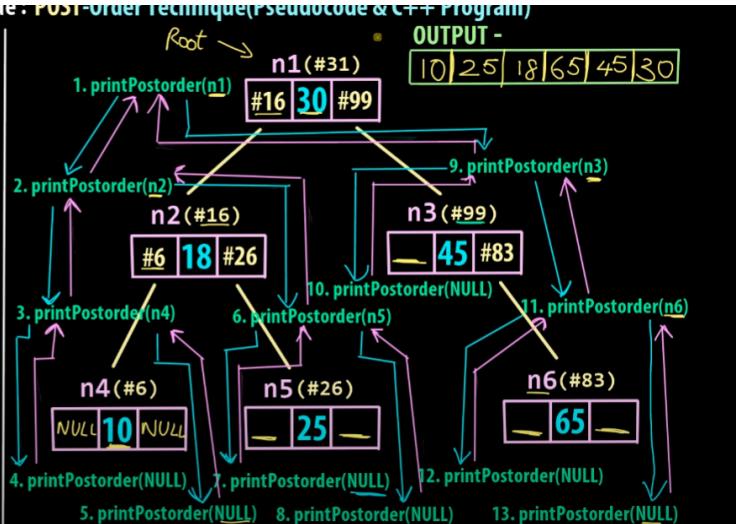


#### 3. Post-order (LRN) (left - right - node)

- Traverse the left subtree by recursively calling the post-order function.
- Traverse the right subtree by recursively calling the post-order function.
- Access the data part of the current node.

#### 3. Post-Order (Pseudocode) -

```
void printPostorder(TreeNode* r)
{
    1. IF r==NULL THEN -> return
    2. printPostorder(r->left)
    3. printPostorder(r->right)
    4. PRINT r->value
}
```



## Search

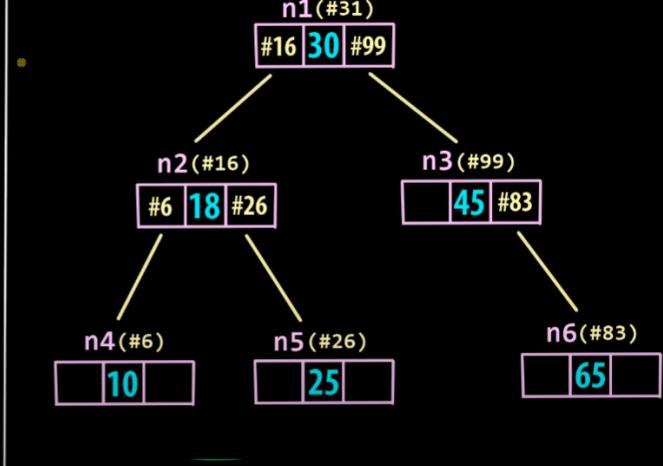
### ⇒ ITERATIVE Search in BST (Pseudocode & C++ Program)

**Iterative Search Operation on BST -**

```

Pseudocode - Val
TreeNode* iterativeSearch(TreeNode* root)
{
    1. IF root==NULL then -> return root
    2. ELSE
        2.1. SET temp=root
        2.2. WHILE temp!=NULL
            2.2.1. IF val == temp.value
                2.2.1.1. THEN -> return temp
            2.2.2. ELSE IF val < temp.value
                2.2.2.1. THEN -> temp = temp.left
            2.2.3. ELSE
                2.2.3.1. temp = temp.right
        END WHILE
    2.3. return NULL
}

```



## Height Of BST

### Finding Height of Binary Tree (with C++ Program)

**Height of Tree -**

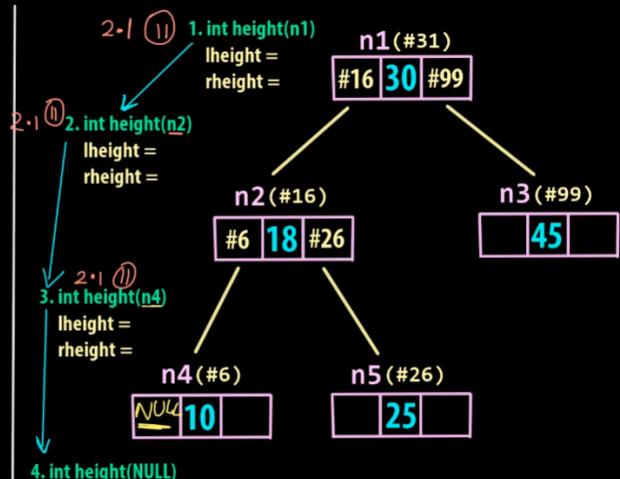
*The height of a binary tree is the number of edges between the tree's root and its furthest leaf.*

**Height of Tree (Pseudocode) -**

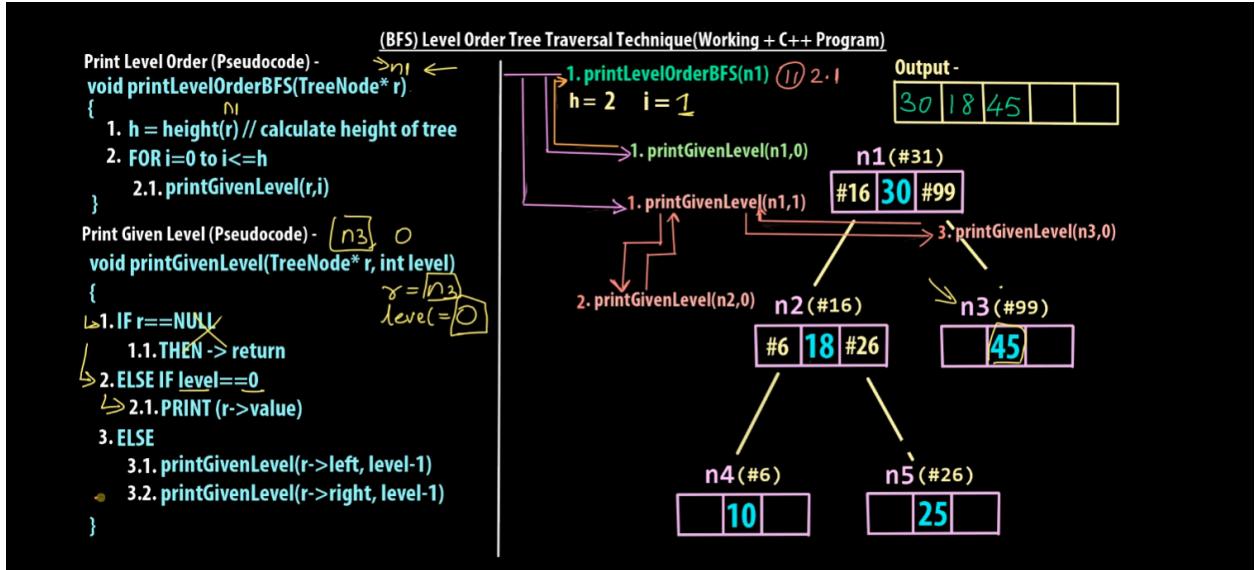
```

int height(TreeNode* r) → ↗
{
    1. IF r==NULL
        1.1. THEN -> return -1
    2. ELSE
        2.1. lheight = height(r->left)
        2.2. rheight = height(r->right)
        2.3. IF lheight>rheight
            2.3.1. THEN -> return (lheight + 1)
        2.4. ELSE
            2.4.1. return (rheight + 1)
}

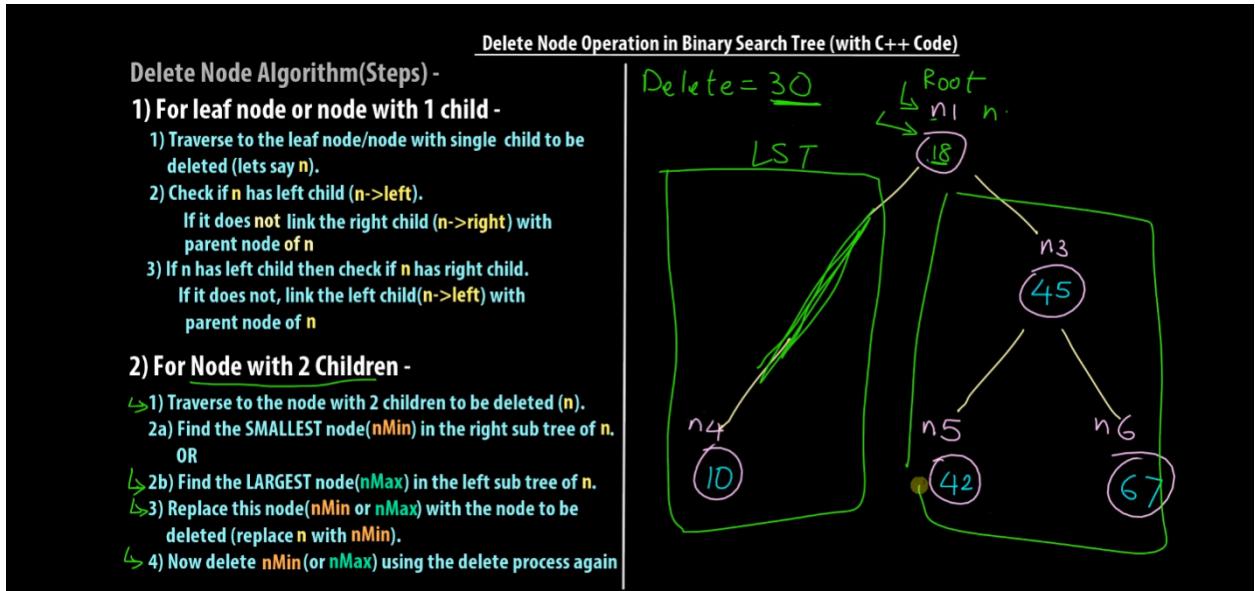
```



## BFS OR LEVEL ORDER TRAVERSAL



## DELETE



> Delete Node(Pseudocode) -

```

TreeNode* deleteNode(TreeNode* r, int v) {
    1. IF r==NULL THEN -> // Base condition
        1.1 return r
    2. ELSE IF v < r->value THEN -> // if value smaller go left sub tree
        2.1 r->left = deleteNode(r->left, v)
    3. ELSE IF v > r->value THEN -> // if value larger go right sub tree
        3.1 r->right = deleteNode(r->right, v)
    4. ELSE // if value matches
        4.1 IF r->left==NULL THEN -> // node with only right child
            4.1.1 temp = r->right // OR no child
            4.1.2 delete r
        4.1.3 return temp
        4.2 ELSE IF r->right==NULL THEN -> // node with only left child
            4.2.1 temp = r->left
            4.2.2 delete r
            4.2.3 return temp
        4.3 ELSE // node with TWO children
            4.3.1 temp = minValueNode(r->right)
            4.3.2 r->value = temp->value
            4.3.3 r->right = deleteNode(r->right, temp->value)
    5. return r
}

```

**Delete Node Operation in Binary Search Tree (with C++ Code)**

## Balancing a binary tree and why do we need balancing

Balancing a Binary Search Tree - WHAT is balancing & WHY do we need it ?

A **Binary Tree** is called a **BALANCED** binary tree, if the difference between the HEIGHT of left & right subtree for every node is not more than k (usually  $k=1$ ). Height of a tree is the number of edges on the longest path between root node & leaf node.

**Binary Tree Diagram 1**

**Binary Tree Diagram 2**  $k=1$

UNBALANCED

$LST = 2$

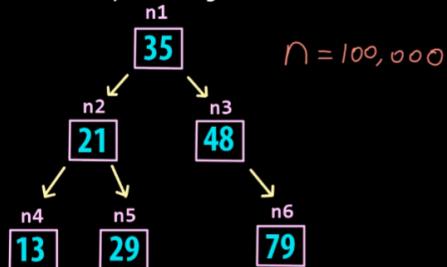
$RST = 0$

$LST - RST = 2 - 0 = 2$

Balancing a Binary Search Tree - WHAT is balancing & WHY do we need it ?

A Binary Tree is called a **BALANCED** binary tree, if the difference between the HEIGHT of left & right subtree for every node is not more than  $k$  (usually  $k = 1$ ) Height of a tree is the number of edges on the longest path between root node & leaf node.

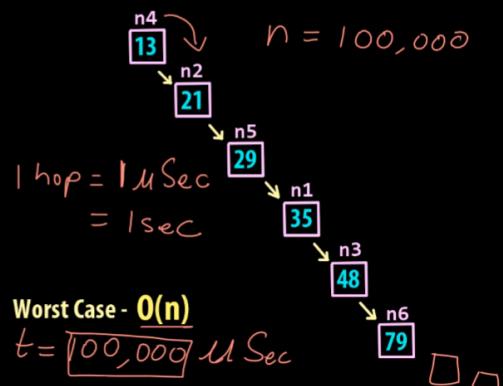
Binary Tree Diagram 1



Avg Case -  $O(\log n)$

$$\log_2(100,000) = \boxed{16.60} \text{ μSec}$$

Binary Tree Diagram 2



□ □

AVL Tree is a self balancing tree.