Assignment 05

Newton Raphson Method

Please follow the guidelines mentioned below:

1. Create a separate folder corresponding to each problem. For example, for problem 3, make a folder: "Problem 3". All the codes and output files for problem 3, for example, should be inside this folder.
2. **Write separate C functions for function evaluation and derivative evaluation for all problems.**
3. Do not hard code data inside the code. All the parameters should be defined before the main () function.
4. All the results should be saved as text/pdf/png files in the same folder as that of the problem. This means, for example, the output files of problem no 2 should be in the folder "Problem 2".
5. All output files should be named as instructed in the problems below.
6. While submitting the assignment, please put all the problem folders in one main folder. The name of the folder should be your roll number. For example, if your roll number is 12345678, then the folder should be named: "12345678". Then, zip the folder as "12345678.zip". You should send this zip file. DO NOT SEND MULTIPLE EMAILS FILES.
7. NOT FOLLOWING THE GUIDELINES WILL ATTRACT PENALTIES.

Question – 1: Find out the value of $\sqrt[3]{11}$ using Newton-Raphson Method. Print your results output file "Output_Problem_1.txt". The output file should have three columns. The first column is the iteration number, and the second column is the value of $\sqrt[3]{11}$ calculated by the NR method, the third column should be the error in the corresponding NR step. Your answer should be **accurate to 5 decimal places**. [10]

Question – 2: Find the value of $\boldsymbol{\pi}$ using the Newton-Raphson Method. Print your results output file "Output_Problem_2.txt". The output file should have three columns. The first column is the iteration number; the second column is the value of $\boldsymbol{\pi}$ calculated by the NR method; the third column should be the error in the corresponding NR step. Your answer should be **accurate to 5 decimal places**. [10]

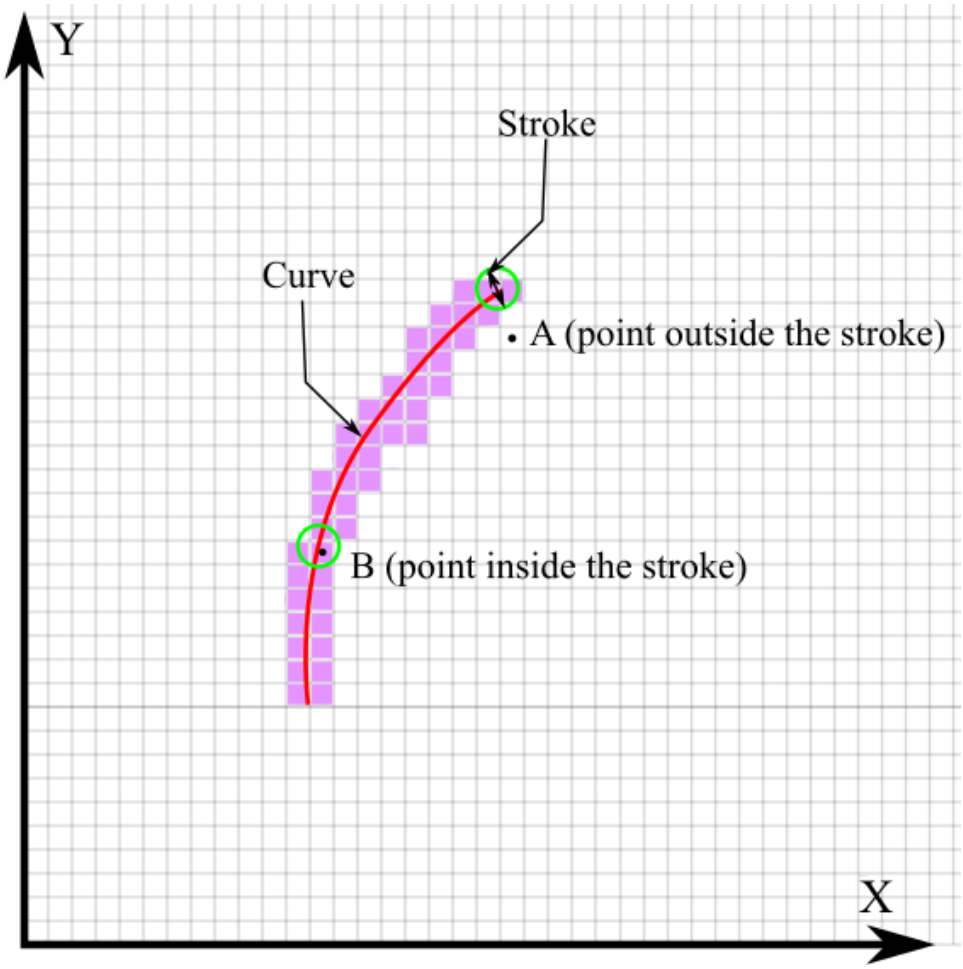

Figure 1

Question – 3: Ritam has developed a 2D CAD representation system that can represent curves with different strokes. It illuminates a pixel when its center falls inside the circle of diameter 'd,' with the center on the curve, as shown in figure 1. [10]

At what stroke value (d) (**accurate to 5 decimal places**) a pixel (2, 0) will illuminate while displaying a curve $y = x^2(1 + \sin^2 x)$?

a) Prepare a one-page document to define the problem statement "formulation_3.pdf".
b) Print your results output file "Output_Problem_3.txt". The output file should have three columns. The first column is the iteration number, the second column is the value of the stroke calculated by the NR method, and the third column should be the error in the corresponding NR step.
c) Plot your curve with the calculated stroke in GeoGebra and verify your answer. Add results as "GeGe_Problem_3.png".

Question – 4: Solve the following system of non-linear equations using the NR method.

$$e^{x_1} - 8x_1 \sin(x_2) = 0,$$
$$x_1 + x_2 - 1 = 0,$$
$$(x_3 - 1)^3 = 0.$$

Start with $x^0 = \begin{bmatrix} 0.1 \\ 0.5 \\ 0.5 \end{bmatrix}$ and tolerance $TOL = 1 \times 10^{-10}$. Observe the error $E = \left|\left|x^k - x^{k-1}\right|\right|_\infty$ for each value of k. Print your results in the output file "output_problem_4.txt". The output file should have five columns. The first column is the iteration number, and the second, third, and the fourth column should be the values of x_1, x_2 and x_3 and the fifth column should be the error E in the corresponding Newton-Raphson step.

Question – 5: Through previous examples, you might have observed that the process of finding roots by Newton Raphson's method can be uncertain for some equations, and the outcome can be extremely sensitive to the location of the initial guess. One such interesting equation is $4x^4 - 4x^2 = 0$.

a) Plot the graph of equation $y = 4x^4 - 4x^2$ in interval $x = [-1, +1]$ with step 0.001.

b) Write a code for the NR method and find out the roots of $4x^4 - 4x^2 = 0$ corresponding to all initial guesses between $[-1, +1]$ with step size 0.001 and the tolerance as $TOL = 1 \times 10^{-10}$. The output file should have four columns. The first column is the initial guess; the second column is the iteration number; the third column is the value of $4x^4 - 4x^2$ and the fourth column should be the error in the corresponding step, Output_problem_5.txt.

c) **With the help of a graph, show that**: If any initial guess (with 3 decimal places) is in the interval $\left(-1, -\frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{21}}{7}, \frac{\sqrt{21}}{7}\right)$, and $\left(\frac{\sqrt{2}}{2}, 1\right)$, it leads to roots -1, 0, and +1, respectively. If the initial guess is $= \pm\frac{\sqrt{21}}{7}$ it leads to each other. If the initial guess is between $\frac{\sqrt{21}}{7}$ and $\frac{\sqrt{2}}{2}$, then there are infinitely many open intervals of points attracted to -1 and open intervals of points attracted to $+1$. Save the file as Zones_5.png.
Hint: You can plot the initial guess as a point $(x_0, 0)$ in; red if it leads to root -1, green if it leads to root 0, blue if it leads to root −1, and black if it does not converge. This graph will have different zones on X-axis in different colors.

d) Prepare a **one-page** document to explain the fundamental phenomena behind this behavior of the NR method for this particular problem with the help of sketches and text, "explaination_5.pdf".