BHANU PRATAP REDDY

ADVANCED STATISTICS

GRADED PROJECT

SEPTEMBER 17 **2022**

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PROBLEM 1

PROBLEM STATEMENT

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	**35**	**29**	**235**

1.1 What is the probability that a randomly chosen player would suffer an injury?

1.2 What is the probability that a player is a forward or a winger?

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

1.4 What is the probability that a randomly chosen injured player is a striker?

1.5 What is the probability that a randomly chosen injured player is either a forward or an attacking midfielder?

SOLUTION

Before anything we must define what probability is, probability is the estimation of how likely an event is to occur and is mathematically defined as the ratio to the number of favourable events to the total number of possible events. i.e.

$$P = \frac{Number\ of\ Favourable\ events}{Total\ number\ of\ all\ possible\ outcomes}$$

With that in mind let s find the probabilities of the above problem statement.

1.1

In this case the total number of possible outcomes is 235 and the total number of injured players are 145 Therefore,

$$P_{\ Injured\ player} = \frac{145}{235} = 0.62$$

The probability that a randomly chosen player would suffer injury is **0.62**

1.2

In this case the total number of possible outcomes is 235 and the total favourable outcomes is the summation of the number of forwards and wingers therefore the probability is given as,

$$P_{Forward\ or\ Winger} = P_{Forward} + P_{Winger} = \frac{94}{235} + \frac{29}{235} = 0.52$$

The probability that a randomly chosen player is a forward OR a winger is **0.52**

1.3

In this case the total number of possible outcomes becomes the total number of players i.e. 235 and the number of injured players who are also strikers is 45. Therefore, the probability is given as

$$P_{is\ Striker\ and\ Injured} = \frac{45}{235} = 0.19$$

The probability that a randomly chosen player is a striker and is injured is **0.19**

1.4

Here the total number of possible outcomes is the total number of injured players i.e. 145 and among them 45 are injured strikers. Therefore, the probability is given as

$$P_{Injured\ is\ striker} = \frac{45}{145} = 0.31$$

The probability that a randomly chosen injured player is a striker is **0.31**

1.5

In this case the total number of possible outcomes is 145 and the total favourable outcomes is the summation of the number of injured forwards and injured midfielders. Therefore, the probability is given as

$$P_{Injured\ is\ forward\ or\ Mid} = P_{injured\ forward} + P_{injured\ mid} = \frac{56}{145} + \frac{24}{145} = 0.55$$

The probability that the chosen injured player is either a forward or a midfielder is **0.55**

PROBLEM 2

PROBLEM STATEMENT

An independent research organization is trying to estimate the probability that an accident at a nuclear power plant will result in radiation leakage. The types of accidents possible at the plant are, fire hazards, mechanical failure, or human error. The research organization also knows that two or more types of accidents cannot occur simultaneously.

According to the studies carried out by the organization, the probability of a radiation leak in case of a fire is 20%, the probability of a radiation leak in case of a mechanical 50%, and the probability of a radiation leak in case of a human error is 10%. The studies also showed the following:

- The probability of a radiation leak occurring simultaneously with a fire is 0.1%.
- The probability of a radiation leak occurring simultaneously with a mechanical failure is 0.15%.
- The probability of a radiation leak occurring simultaneously with a human error is 0.12%.

On the basis of the information available, answer the questions below:

2.1 What are the probabilities of a fire, a mechanical failure, and a human error respectively?

2.2 What is the probability of a radiation leak?

2.3 Suppose there has been a radiation leak in the reactor for which the definite cause is not known. What is the probability that it has been caused by:

- A Fire.
- A Mechanical Failure.
- A Human Error.

SOLUTION

Based on the question let us first list all the given probabilities) that are known to us from the problem statement (L stands for Leak, F for fire, H for Human Error, and M for Mechanical Error):

- $\boldsymbol{P(L \mid F) = 0.2}$
- $\boldsymbol{P(L \mid M) = 0.5}$
- $\boldsymbol{P(L \mid H) = 0.1}$
- $\boldsymbol{P(L \cap F) = 0.001}$
- $\boldsymbol{P(L \cap M) = 0.0015}$
- $\boldsymbol{P(L \cap H) = 0.0012}$

2.1

In a given set, the probability of simultaneous occurrences of events that are not independent are given as

$$P(A \cap B) = P(A)\,.P(B \mid A) = P(B).P(A \mid B)$$

With that the probabilities of fire, mechanical failure and human error can be given as:

- $P(F) = \frac{P(L \cap F)}{P(L \mid F)} = \frac{0.001}{0.2} = 0.005$

  The Probability of a fire occurring is **0.005** i.e. **0.5%**

- $P(M) = \frac{P(L \cap M)}{P(L \mid M)} = \frac{0.0015}{0.5} = 0.003$

  The Probability of a mechanical failure occurring is **0.003** i.e. **0.3%**

- $P(H) = \frac{P(L \cap H)}{P(L \mid H)} = \frac{0.0012}{0.1} = 0.012$

  The Probability of human error happening is **0.012** i.e. **1.2%**

2.2

The probability of a radiation leak will be the sum of all simultaneous occurrences of the leaks i.e.

$$P(L) = P(L \cap F) + P(L \cap M) + P(L \cap M)$$
$$= 0.0037$$

The Probability of Radiation leak occurring is **0.0037** i.e. **0.37%**

2.3

Given the leak has happened the probabilities of fire, mechanical failure and human error can be derived using bayes theorem, which is given as:

$$P(A \mid B) = \frac{P(A)\,.P(B \mid A)}{P(B)}$$

Hence,

- $\boldsymbol{P(F \mid L) \ = \frac{P(F)\,.P(L \mid F)}{P(L)} \ = \ 0.2703}$

  The Probability of fire occurring given leak has happened is **0.2703** i.e. **27.03%**

- $\boldsymbol{P(M \mid L) \ = \frac{P(M)\,.P(L \mid M)}{P(L)} \ = \ 0.4054}$

  The Probability of mechanical failure occurring given leak has happened is **0.4054** i.e. **40.54%**

- $\boldsymbol{P(H \mid L) \ = \frac{P(H)\,.P(L \mid H)}{P(L)} \ = \ 0.3243}$

  The Probability of human error occurring given leak has happened is **0.3243** i.e. **32.43%**

PROBLEM 3

PROBLEM STATEMENT

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information;

3.1 What proportion of the gunny bags have a breaking strength less than 3.17 kg per sq cm?

3.2 What proportion of the gunny bags have a breaking strength at least 3.6 kg per sq cm.?

3.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

3.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

SOLUTION

In the problem statement it has been stated that the distribution is normal in nature and has also stated the mean and variance of the distribution:

- Mean μ = 5 kg/cm^2
- Standard Deviation σ = 1.5 kg/cm^2

Hence a graphical representation can be given as,

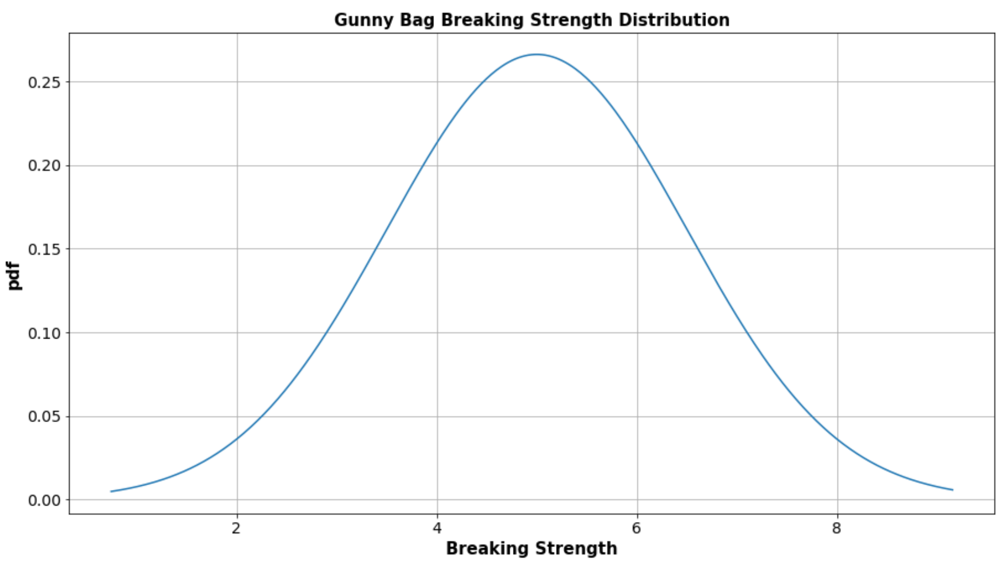

Figure 1

Normal Distribution of Gunny bag breaking strength with mean 5 and std. deviation 1.

<u>3.1</u>

The proportion of gunny bags with strength less than 3.17 kg/cm^2 is given as cumulative probability up to 3.17 kg/cm^2 or the area under the normal distribution curve up to 3.17 kg/cm^2. Graphically represented as:

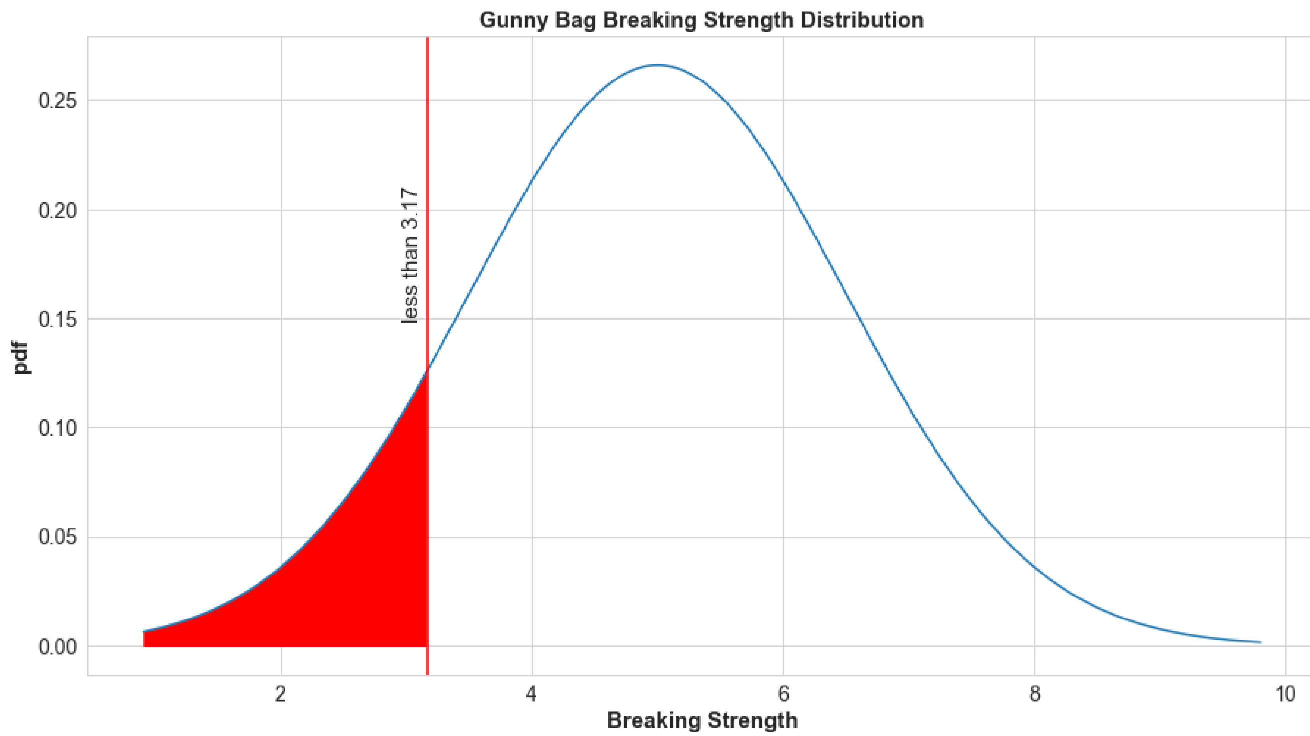

Figure 2

Proportion of gunny bag breaking strength up to 3.17 kg/cm^2

Therefore, the calculated area under the curve i.e. The cumulative probability of breaking strength under 3.17 kg/cm^2 is **0.1112**

3.2

If P is the cumulative probability of gunny bag strength under 3.6 kg/cm^2, then 1 - P is the cumulative probability of gunny bag strength of at least/over 3.6 kg/cm^2. Which is what is required in this case. This is graphically represented as:

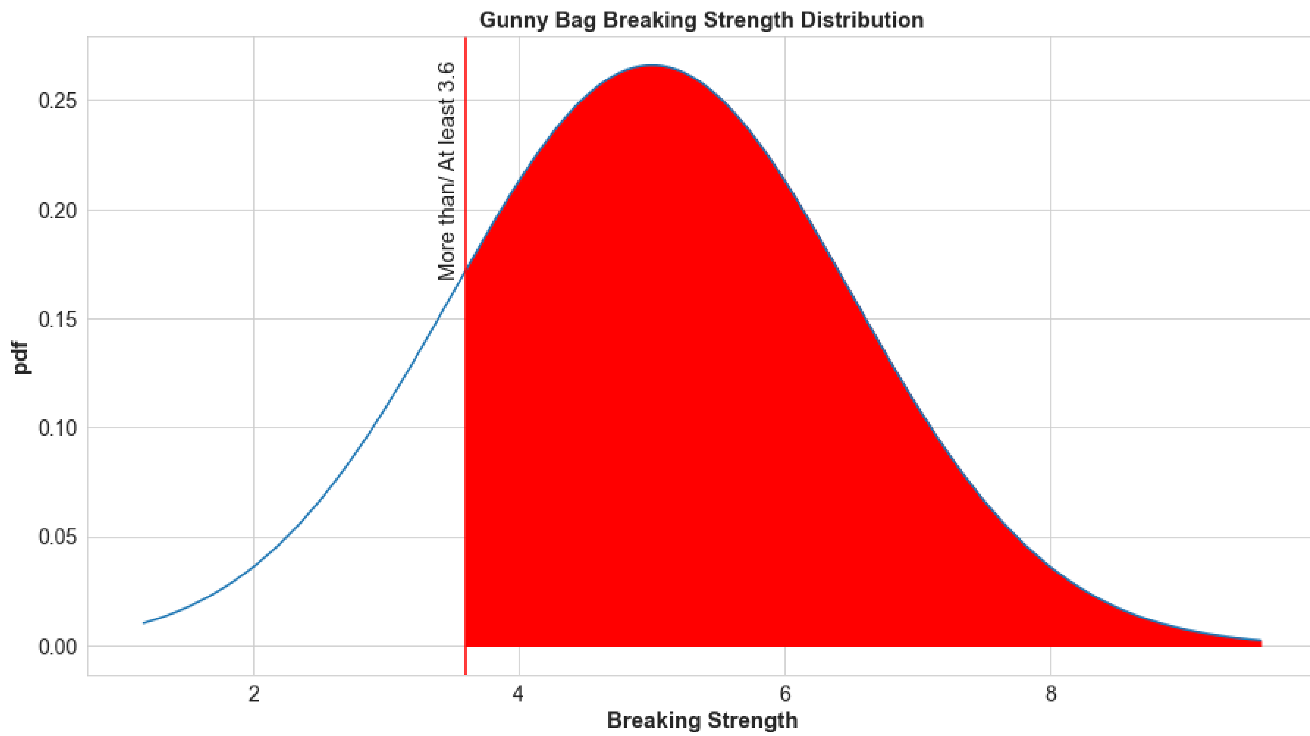

Figure 3

Proportion of gunny bag breaking strength of at least/ over 3.6 kg/cm^2

Therefore, the calculated area under the curve i.e. The cumulative probability of breaking strength over/ at least 3.6 kg/cm^2 is **0.8247.**

3.3

If P_5 is the cumulative probability of gunny bag strength under 5 kg/cm^2, and $P_{5.5}$ is the cumulative probability of gunny bag strength of under 5.5 kg/cm^2. Then $P_{5.5}$ - P_5 is the cumulative probability of gunny bag strength between 5 kg/cm^2 & 5.5 kg/cm^2. Which is what is required in this case. This is graphically represented as:

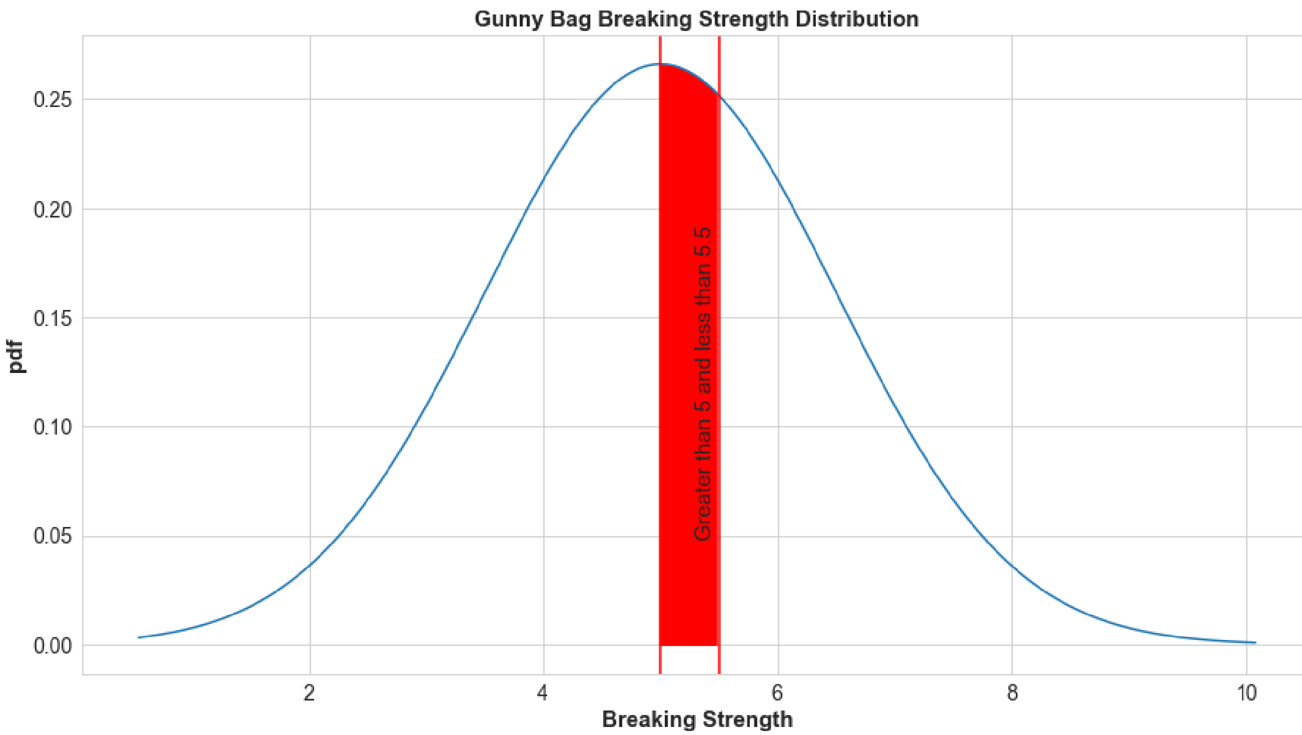

Figure 4

Proportion of gunny bag breaking strength between 5 kg/cm^2 & 5.5 kg/cm^2

Therefore, the calculated area under the curve i.e. The cumulative probability of breaking strength between 5 kg/cm^2 and 5.5 kg/cm^2 is **0.1306**

3.4

If P_3 is the cumulative probability of gunny bag strength under 3 kg/cm^2, and $P_{7.5}$ is the cumulative probability of gunny bag strength of under 7.5 kg/cm^2. Then $P_3 + (1- P_{7.5})$ is the cumulative probability of gunny bag strength **NOT** between 3 kg/cm^2 & 7.5 kg/cm^2. Which is what is required in this case. This is graphically represented as:

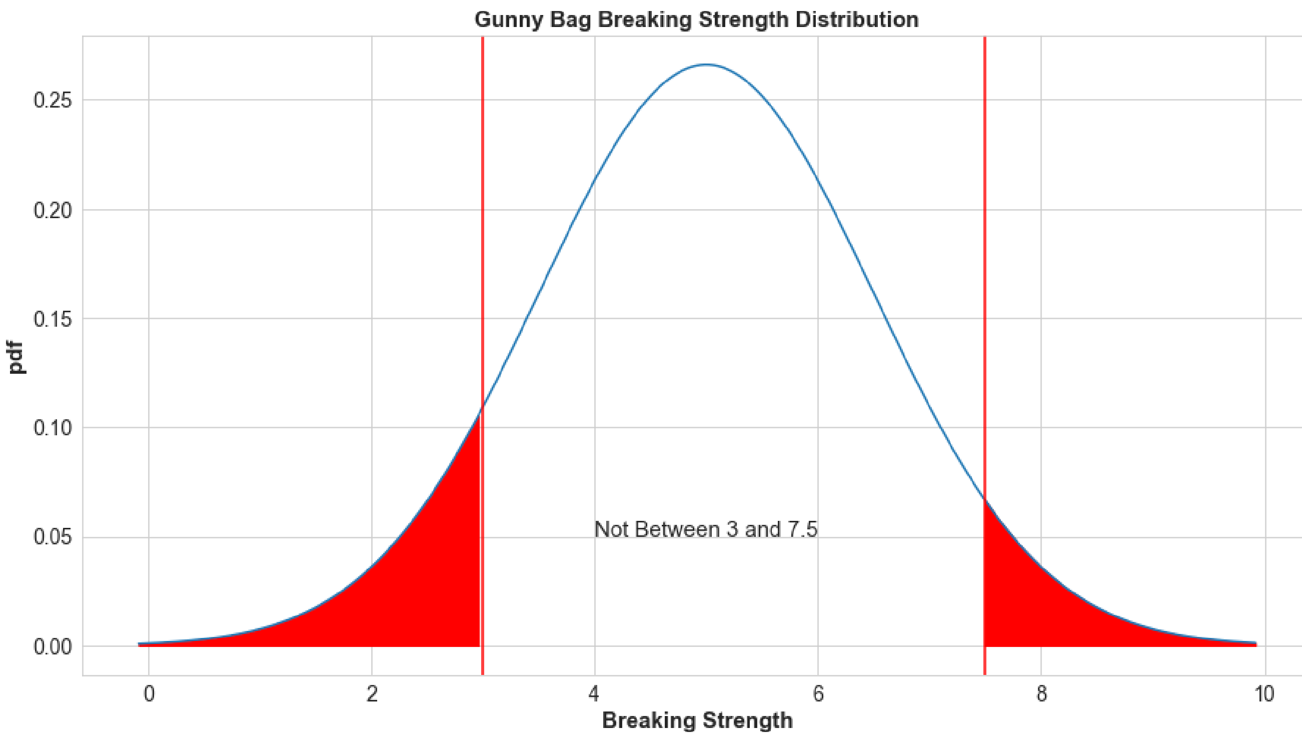

Figure 5

Proportion of gunny bag breaking strength NOT between 3 kg/cm^2 & 7.5 kg/cm^2

Therefore, the calculated area under the curve i.e. The cumulative probability of breaking strength between 3 kg/cm^2 and 7.5 kg/cm^2 is **0.139**

PROBLEM 4

PROBLEM STATEMENT

Grades of the final examination in a training course are found to be normally distributed, with a mean of 77 and a standard deviation of 8.5. Based on the given information answer the questions below.

4.1 What is the probability that a randomly chosen student gets a grade below 85 on this exam?

4.2 What is the probability that a randomly selected student scores between 65 and 87?

4.3 What should be the passing cut-off so that 75% of the students clear the exam?

SOLUTION

In the problem statement it has been stated that the distribution is normal in nature and has also stated the mean and variance of the distribution:

- Mean $\mu = 77$
- Standard Deviation $\sigma = 8.5$

Hence a graphical representation can be given as,

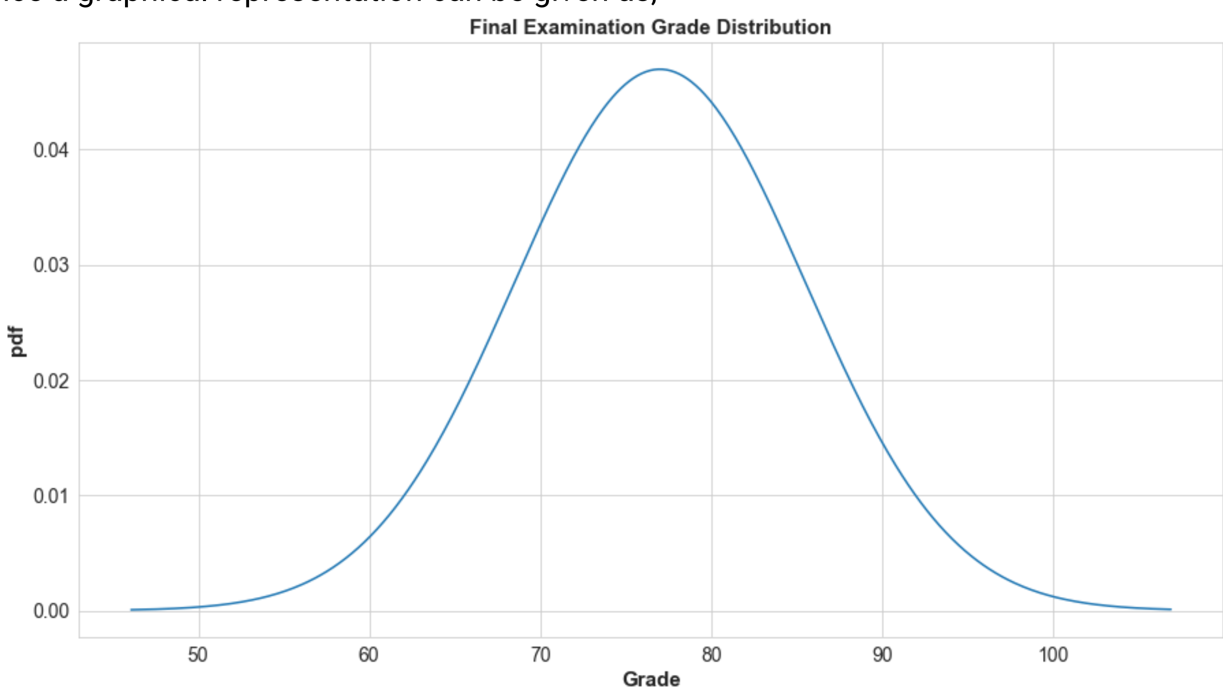

Figure 6

Normal Distribution of grades of a final exam with mean 77 and std. deviation 8.5

4.1

The probability that a randomly chosen student has a grade below 85 is nothing but the cumulative probability up to 85 i.e. area under the curve up to 85 and this is graphically represented as:

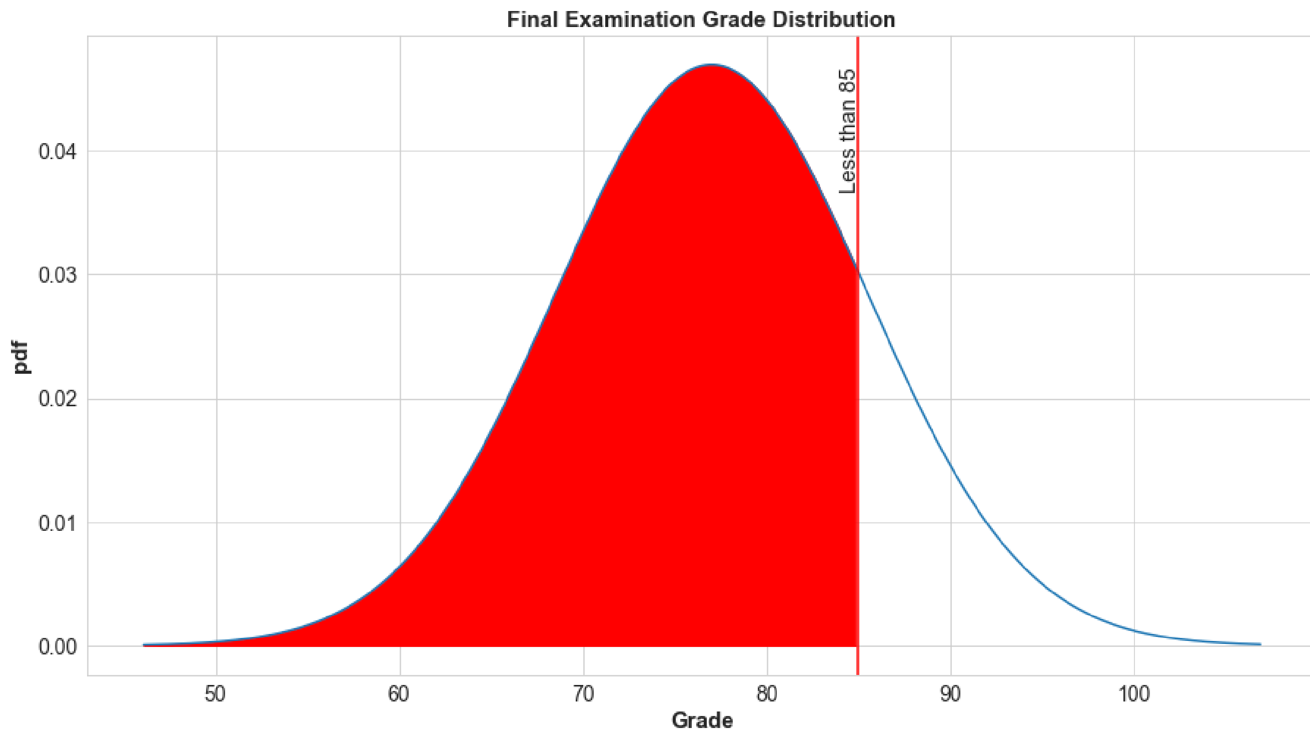

Figure 7

The cumulative probability of grade up to 85

Therefore, the calculated area under the curve i.e. The probability of getting a score under 85 is **0.8267**

4.2

If P_{65} is the cumulative probability of scores below 65 and P_{87} is the cumulative probability of scores below 87 then $P_{87} – P_{65}$ is the cumulative probability of scores between 65 and 87. This is graphically represented as:

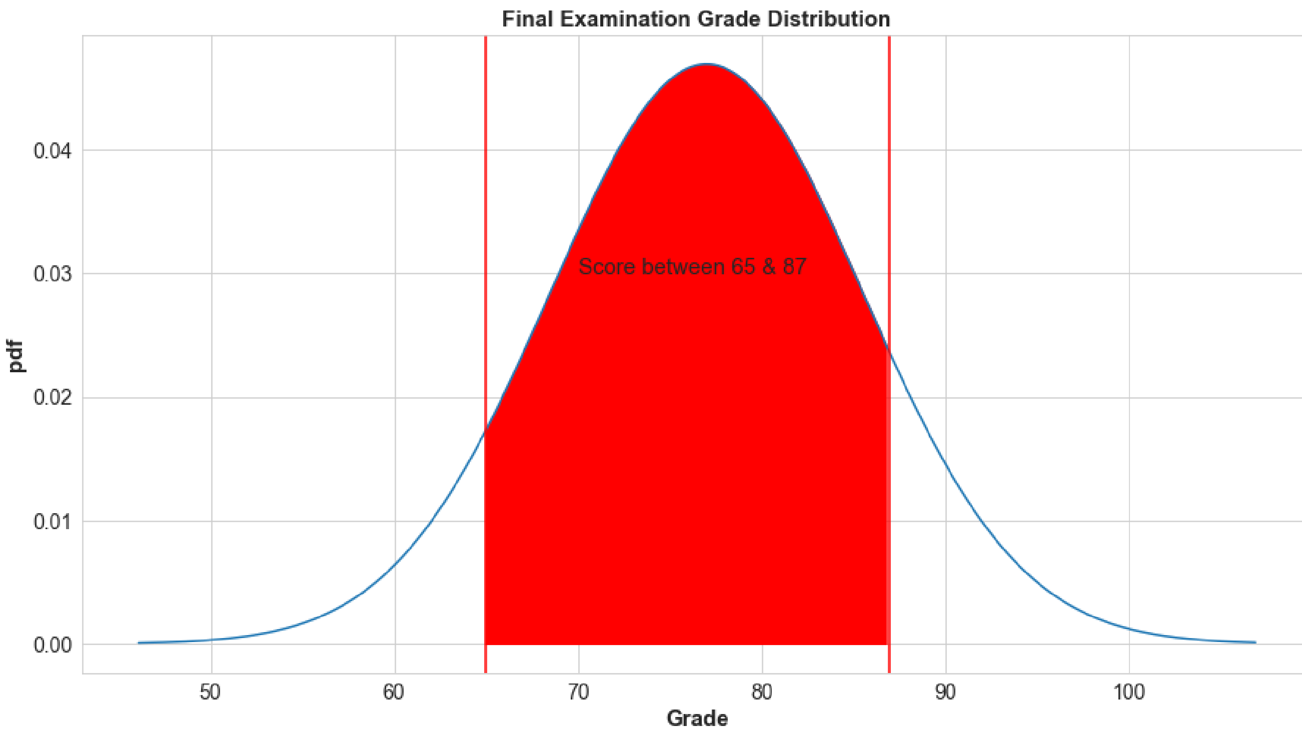

Figure 8

The cumulative probability of grade between 65 and 87

Therefore, the calculated area under the curve i.e. The probability of getting a score between 65 and 87 is **0.8013**

4.3

For 75% percent of the students to pass we must know the probability distribution value at the 25th percentile. This is graphically represented as:

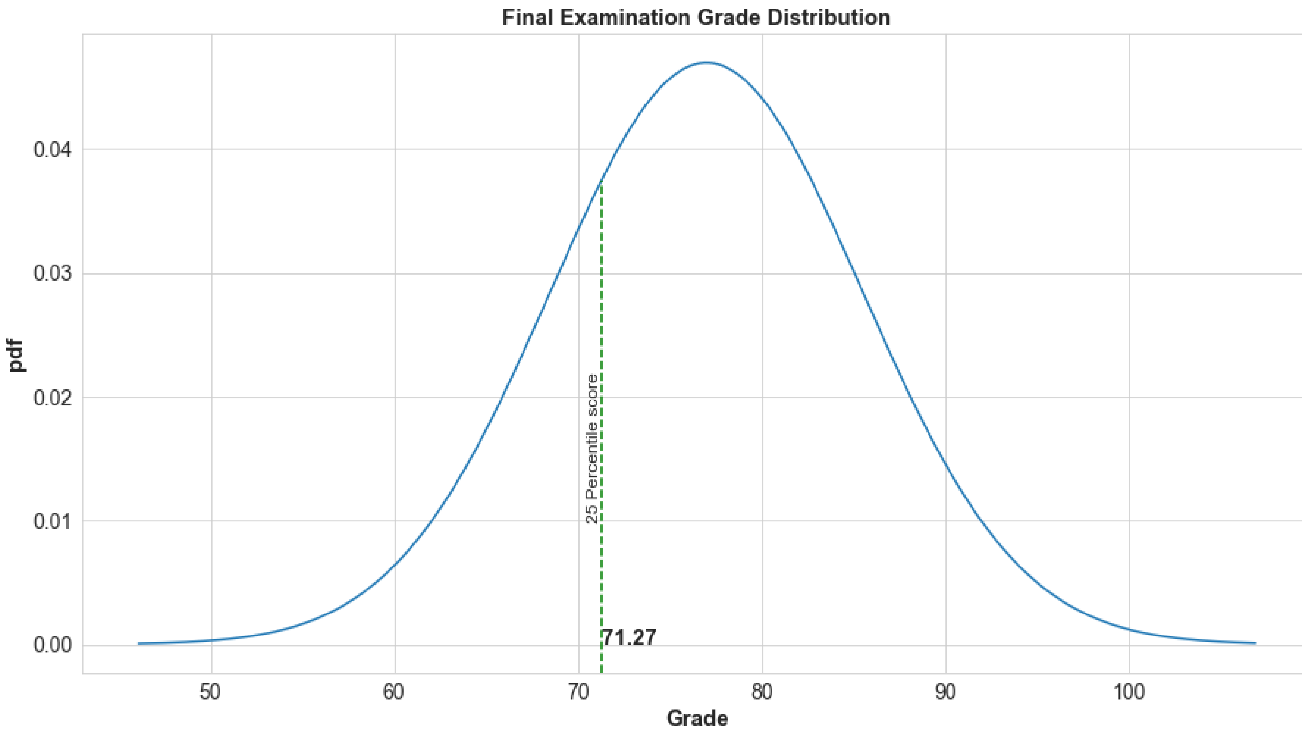

Figure 9
The probability distribution value at the 25th Percentile.

Therefore, the probability distribution value and the score for 75 percent to pass is **71.2668**

PROBLEM 5

PROBLEM STATEMENT

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

5.1 Earlier experience of Zingaro with this particular client is favourable as the stone surface was found to be of adequate hardness. However, Zingaro has reason to believe now that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

5.2 Is the mean hardness of the polished and unpolished stones the same?

SOLUTION

Before we start answering questions, we inspect whether or not the given data is in requirement of any treatment before we process it. And as observed in the notebook all appears to be is order with no nulls or any result changing outliers.

The data was also checked whether or not it was normal and as we can see:

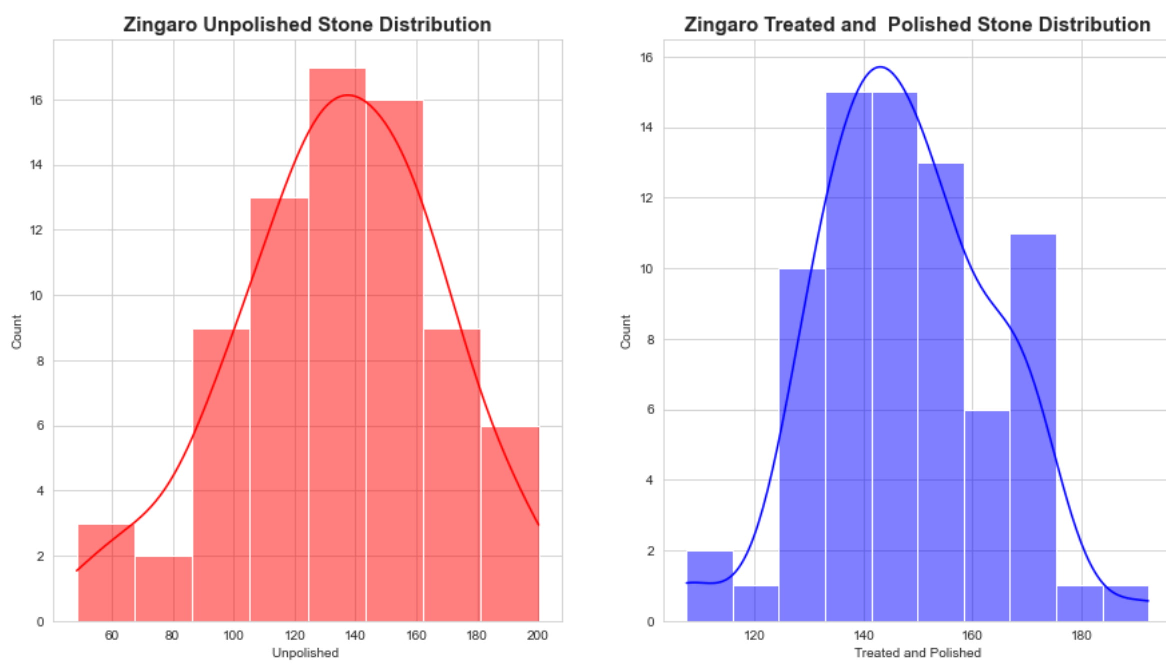

Figure 10

Distribution of the received stones of each category (Both of which are mostly normal)

As seen in the above figure it appears the data is mostly normal and to confirm this mathematically, we perform a Shapario – Wilk Test for which the Null Hypothesis is "Distribution is Normal". Hence, we have:

```
Shapario - Wilk Test Unpolished p - Value =  0.6777130961418152
Shapario - Wilk Test Treated and polished p - Value =  0.331403911113739
```

Figure 11
Shapario - Wilk test results

As seen, since p-value in each case is very much greater than level of significance (α = 0.05) therefore the null Hypothesis holds true i.e. **Both the Distributions are normal**.

5.1

For the given data we know the population mean to be $\mu_0 = 150$ but we do NOT know the standard deviation of the population hence we must perform a one sample t – test.
Let is state our hypothesis

$$H_0: Null\ Hypothesis : Unposlished\ stone\ mean\ \mu\ >=\ \mu_0$$
$$H_A: Alternate\ Hypothesis : Unposlished\ stone\ mean\ \mu\ <\ \mu_0$$

The one sample t -test is mathematically given as:

$$t = \frac{\bar{X} - \bar{\mu}}{\frac{s}{\sqrt{n}}}$$

and then the probability density of the obtained t - statistic from the above formula is calculated or derived from the t - tables and then compared against the level of significance (α = 0.05)

```
t - test result:
t statistic (Unposlished) = -4.164629601426758
p_value (Unposlished) = 8.342573994839285e-05
```

Figure 12

T – Test result for unpolished distribution and its resulting probability obtained from python notebook.

From the t – test as we can see $\boldsymbol{P_t} < \boldsymbol{\alpha}$ and hence we can reject the null hypothesis. Meaning the mean of the samples is lower than the mean of the population. i.e.

Zingaro were right to believe that the unpolished stones were in fact NOT suitable for printing.

<u>5.2</u>

In order to test the hypothesis of whether or not the mean hardness of Unpolished and Treated & Polished stones are same we have to perform a 2 sample t test. The Null and alternate hypothesis if which are stated as:

$$H_0\text{: } Null\ Hypothesis : \ \mu_U - \mu_P = 0 \ \ i.e. \mu_U = \mu_P$$
$$H_A\text{: } Alternate\ Hypothesis : \mu_U - \mu_P \neq 0 \ \ i.e. \mu_U \neq \mu_P$$

The two-sample t test is mathematically given as:

$$t = \frac{\overline{X_u} - \overline{X_p}}{\sqrt{\left(\frac{s_u^2}{n_u} + \frac{s_p^2}{n_p}\right)}}$$

and then the probability density of the obtained t - statistic from the above formula is calculated or derived from the t - tables and then compared against the level of significance (α = 0.05)

```
t - test result:
t statistic (2 - Sample) = 3.242232050141406
p_value (2 - Sample) = 0.001465515019462831
```

Figure 13

two sample t test between unpolished and polished hardness distribution obtained from python notebook

From the t – test as we can see $\boldsymbol{P_t} < \boldsymbol{\alpha}$ and hence we can reject the null hypothesis. Meaning that the **Mean Hardness of unpolished stones is not equal to the mean hardness of polished stones.**

PROBLEM 6

PROBLEM STATEMENT

Aquarius health club, one of the largest and most popular cross-fit gyms in the country has been advertising a rigorous program for body conditioning. The program is considered successful if the candidate is able to do more than 5 push-ups, as compared to when he/she enrolled in the program. Using the sample data provided can you conclude whether the program is successful? (Consider the level of Significance as 5%)

Note that this is a problem of the paired-t-test. Since the claim is that the training will make a difference of more than 5, the null and alternative hypotheses must be formed accordingly.

SOLUTION

Before we start answering questions, we inspect whether or not the given data is in requirement of any treatment before we process it. And as observed in the notebook all appears to be is order with no nulls or any result changing outliers.

The data was also checked whether or not it was normal distribution and as we can see:

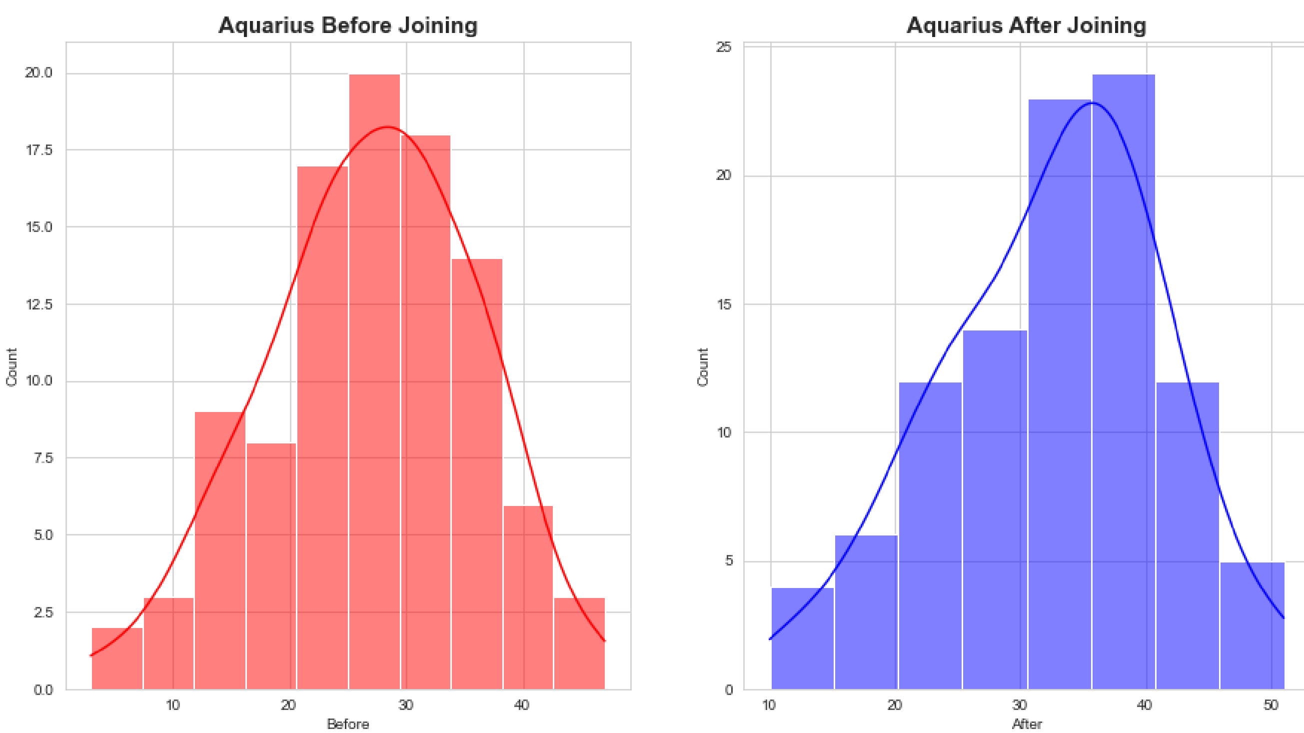

Figure 14
It appears atleast visually the the data is a normal distribution

As seen in the above figure it appears the data is mostly normal and to confirm this mathematically, we perform a Shapario – Wilk Test for which the Null Hypothesis is "Distribution is Normal". Hence, we have:

```
Shapario - Wilk Test Before p - Value =  0.6310831904411316
Shapario - Wilk Test After p - Value =  0.3068997263908386
```

Figure 15
Shapario wilk test

As seen, since p-value in each case is very much greater than level of significance (α = 0.05) therefore the null Hypothesis holds true i.e. **Both the Distributions are normal**.

As the problem states that the program can only be considered successful if the candidate is able at least 5 push-ups. Hence the Null hypothesis can be adjusted accordingly and given as,

$$H_0: Null\ Hypothesis : \mu_{Before} - \mu_{After} - 5 = \mu_D = 0$$
$$H_A: Alternate\ Hypothesis : \mu_{Before} - \mu_{After} - 5 = \mu_D \neq 0$$

Now the t value for a paired t – test is mathematically given as

$$t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}}$$

in this case $\bar{d} = \overline{X_{after}} - \overline{X_{before}} - 5$

From this given data which is calculated out to be as **0.55**

And the standard error $SE = \frac{s_d}{\sqrt{n}}$ which from the data was calculated to be **0.2872**

And hence t value is **1.91489**

Therefore, the probability value p is calculated from the t tables out to be **0.9708.**

Hence, since $\boldsymbol{P_t} > \boldsymbol{\alpha}$ and hence we can accept the null hypothesis. Meaning that **the program is successful, and candidate is able to do more than 5 push-ups, as compared to when he/she enrolled in the program.**

PROBLEM 7

PROBLEM STATEMENT

Dental implant data: The hardness of metal implant in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as on the dentists who may favour one method above another and may work better in his/her favourite method. The response is the variable of interest.

1. Test whether there is any difference among the dentists on the implant hardness. State the null and alternative hypotheses. Note that both types of alloys cannot be considered together. You must state the null and alternative hypotheses separately for the two types of alloys.?

2. Before the hypotheses may be tested, state the required assumptions. Are the assumptions fulfilled? Comment separately on both alloy types.?

3. Irrespective of your conclusion in 2, we will continue with the testing procedure. What do you conclude regarding whether implant hardness depends on dentists? Clearly state your conclusion. If the null hypothesis is rejected, is it possible to identify which pairs of dentists differ?

4. Now test whether there is any difference among the methods on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which pairs of methods differ?

5. Now test whether there is any difference among the temperature levels on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which levels of temperatures differ?

6. Consider the interaction effect of dentist and method and comment on the interaction plot, separately for the two types of alloys?

7. Now consider the effect of both factors, dentist, and method, separately on each alloy. What do you conclude? Is it possible to identify which dentists are different, which methods are different, and which interaction levels are different?

SOLUTION

As usual before we start answering questions, we inspect whether or not the given data is in requirement of any treatment before we process it. By the looks of it seems that even though there are no null values some of the data types are not right.

We convert the data type of ‘Dentist’, ‘Method’, ‘Alloy’ & ‘Temp’ variables to categorical variables.

Since both alloy types cannot be considered together for the analysis, we split the entire data frame into two other data frames in terms of Alloy 1 and Alloy 2 before we perform any kind of analysis.

<u>7.2</u>

Before we test any kind of hypothesis we must first analyze if the given data is suitable or not i.e., we first check for normality in the data for alloy 1:

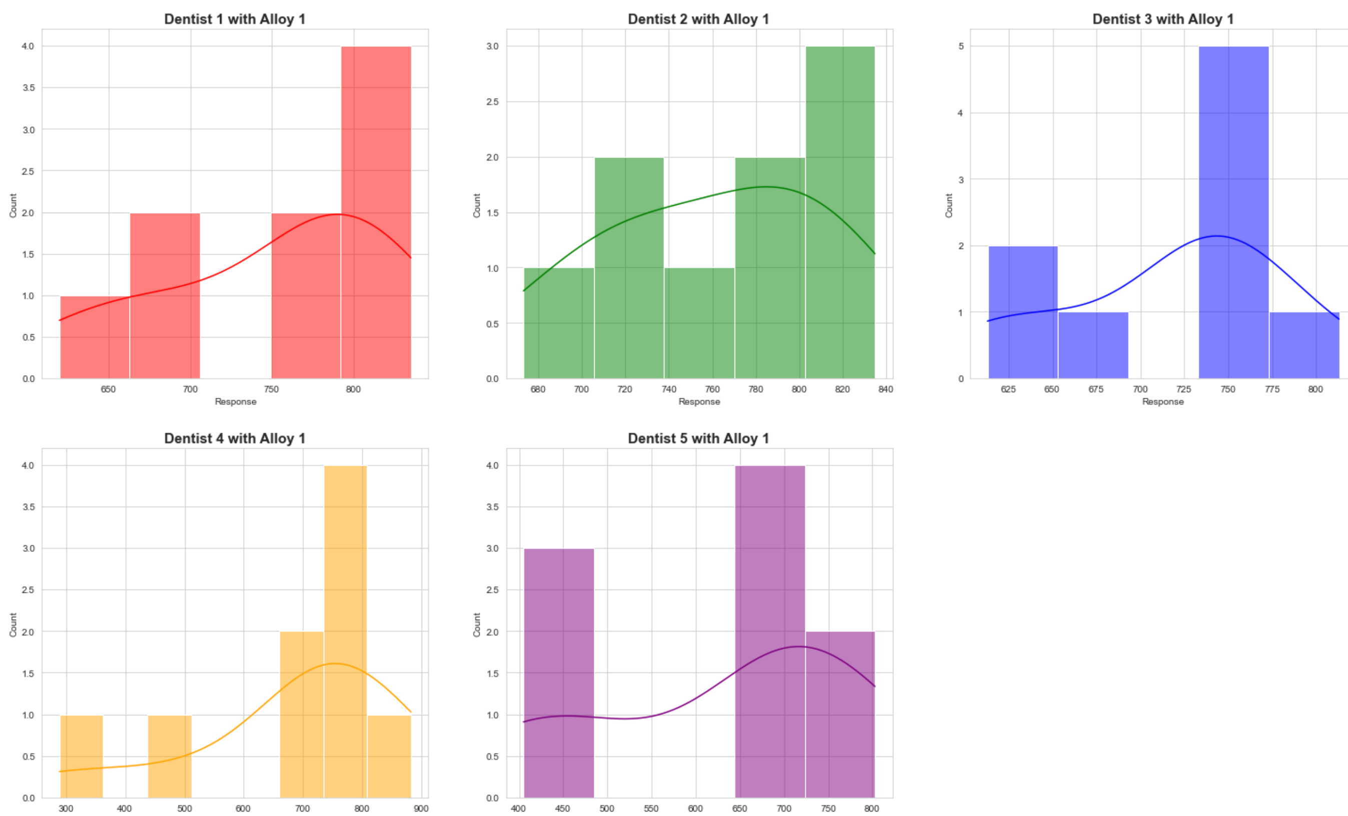

Figure 16

A visual representation of data distribution of the ‘response’ across dentists for Alloy 1

This visual representation doesn’t exactly look convincing enough to comment about the normality, let us try a more accurate one.

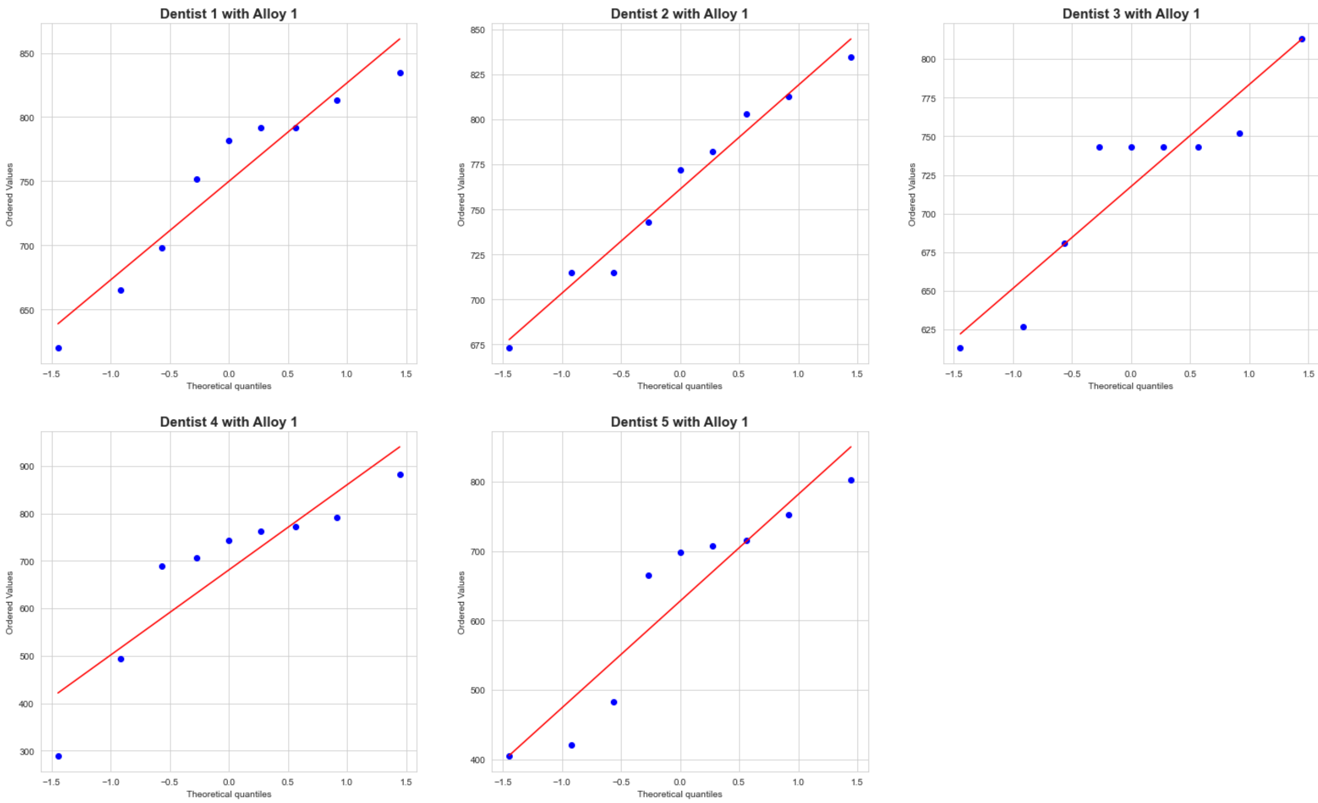

Figure 17

A QQ plot of the distribution gives us a more accurate idea of the normality in the distribution

And the mathematical confirmation is given as:

```
Dentist 1 with Alloy 1 -  ShapiroResult(statistic=0.9113543033599854, pvalue=0.3254694640636444)
Dentist 2 with Alloy 1 -  ShapiroResult(statistic=0.9642463326454163, pvalue=0.8415467143058777)
Dentist 3 with Alloy 1 -  ShapiroResult(statistic=0.8721171617507935, pvalue=0.12953592836856842)
Dentist 4 with Alloy 1 -  ShapiroResult(statistic=0.8368973135948181, pvalue=0.05333660915493965)
Dentist 5 with Alloy 1 -  ShapiroResult(statistic=0.8534294962882996, pvalue=0.08127772063016891)
```

Figure 18

Since all of the p values are greater than the level of significance the null hypothesis is true i.e., They are all Normal distributions

Similarly let's check for normality of data in alloy 2:

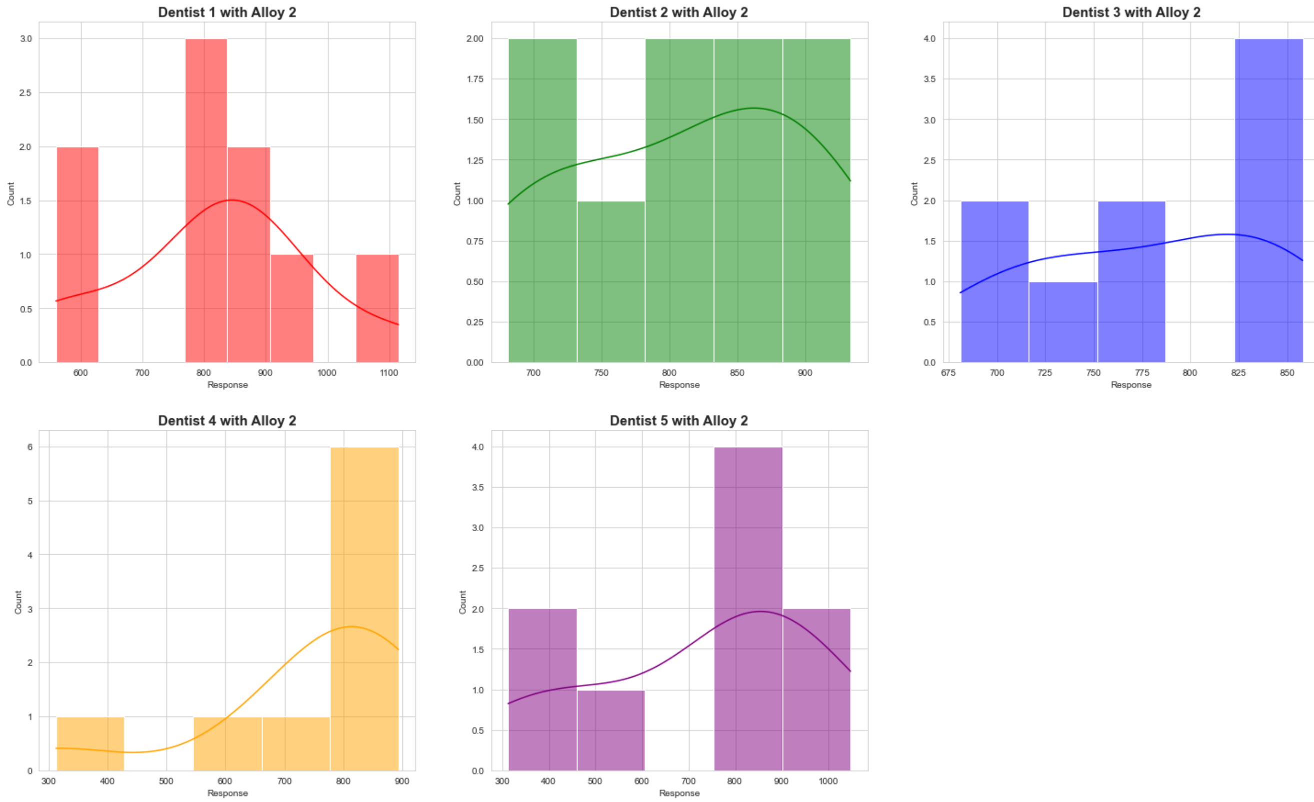

Figure 19

A visual representation of data distribution of the 'response' across dentists for Alloy 2

Trying a Quantile – Quantile plot for the same:

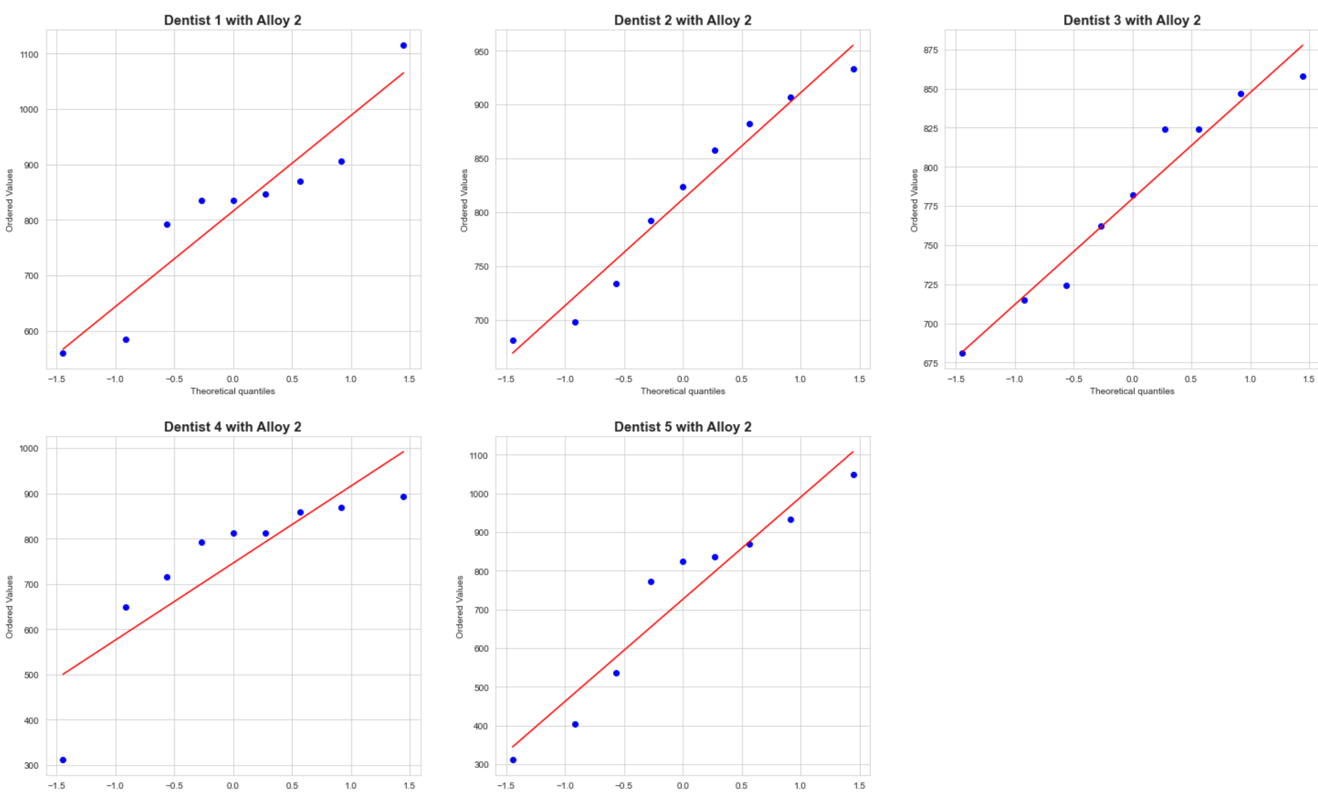

Figure 20

A QQ plot of the distribution gives us a more accurate idea of the normality in the distribution

And the mathematical confirmation is given as:

```
Dentist 1 with Alloy 2 -  ShapiroResult(statistic=0.9039730429649353, pvalue=0.27593860030174255)
Dentist 2 with Alloy 2 -  ShapiroResult(statistic=0.9392002820968628, pvalue=0.5735067129135132)
Dentist 3 with Alloy 2 -  ShapiroResult(statistic=0.9340969920158386, pvalue=0.5213066339492798)
Dentist 4 with Alloy 2 -  ShapiroResult(statistic=0.7613219618797302, pvalue=0.007332703098654747)
Dentist 5 with Alloy 2 -  ShapiroResult(statistic=0.9131584167480469, pvalue=0.33861100673675537)
```

Figure 21

Since all but one of the p values are greater than the level of significance the null hypothesis is true i.e., All but Dentist 4 are normal distributions

Since Shapiro - Wilk test failed for Dentist 4 we must proceed with an Anderson darling test for Dentist 4 on Alloy 2. Which is given as,

```
Dentist 4 with Alloy 2 -  AndersonResult(statistic=0.8877633297431249, critical_values=array([0.507, 0.578, 0.693, 0.808,
0.961]), significance_level=array([15. , 10. ,  5. ,  2.5,  1. ]))
```

Figure 22

Anderson – Darling test for Dentist 4 on Alloy 2 distribution

As seen from the above result it seems that the distribution only passes for normality for a level of significance of 1% (α = 0.01). None the less we shall proceed with the analysis.

7.1 & 7.3

To see if there is an effect on implant hardness ('Response') due to the dentists we perform a one way ANOVA. The hypothesis for such a one way ANOVA is given as:

$\boldsymbol{H_0}: The\ Mean\ of\ Implant\ hardness\ is\ the\ same\ across\ all\ dentists\ for\ alloy\ 1: i.e.,$
$\mu = \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$
$\boldsymbol{H_A}: For\ atleast\ one\ level\ of\ dentists, the\ mean\ hardness\ is\ different\ \ across\ all$
$dentists\ for\ alloy\ 1: i.e.,\ \mu \neq \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4 \neq \mu_5$

The ANOVA table is given as,

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

Figure 23

One way ANOVA table (Alloy 1 & Dentists)

Since the probability value is greater than the level of significance (α = 0.05) we fail to reject the null hypothesis.

Therefore, the mean of hardness is the same across all dentists for alloy 1.

Similarly for alloy 2

$\boldsymbol{H_0}: The\ Mean\ of\ Implant\ hardness\ is\ the\ same\ across\ all\ dentists\ for\ alloy\ 2: i.e.,$
$\mu\ =\ \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$
$\boldsymbol{H_A}: For\ atleast\ one\ level\ of\ dentists, the\ mean\ hardness\ is\ different\ \ across\ all\ dentists\ for\ alloy\ 2: i.e.,\ \ \mu\ \neq\ \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4 \neq \mu_5$

The ANOVA table is given as,

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

Figure 24
One way ANOVA table (Alloy 2 & Dentists)

Since the probability value is greater than the level of significance (α = 0.05) we fail to reject the null hypothesis.

Therefore, the mean of hardness is the same across all dentists for alloy 2.

7.4

To see if there is an effect on implant hardness ('Response') due to the methods we perform a one way ANOVA. The hypothesis for such a one way ANOVA is given as:

$\boldsymbol{H_0}: The\ Mean\ of\ Implant\ hardness\ is\ the\ same\ across\ all\ Methods\ for\ alloy\ 1: i.e.,$
$\mu\ =\ \mu_1 = \mu_2 = \mu_3$
$\boldsymbol{H_A}: For\ atleast\ one\ level\ of\ Methods, the\ mean\ hardness\ is\ different\ \ across\ all$
$Methods\ for\ alloy\ 1: i.e.,\ \ \mu\ \neq\ \mu_1 \neq \mu_2 \neq \mu_3$

The ANOVA table is given as,

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	148472.177778	74236.088889	6.263327	0.004163
Residual	42.0	497805.066667	11852.501587	NaN	NaN

Figure 25
One way ANOVA table (Alloy 1 & Methods)

Since the probability value is lower than the level of significance (α = 0.05) we reject the null hypothesis.

Therefore, the mean of hardness is different across at least one level of methods for alloy 1.

Similarly for alloy 2
$\boldsymbol{H_0}: The\ Mean\ of\ Implant\ hardness\ is\ the\ same\ across\ all\ Methods\ for\ alloy\ 2: i.e.,$
$\mu\ =\ \mu_1 = \mu_2 = \mu_3$
$\boldsymbol{H_A}: For\ atleast\ one\ level\ of\ Methods, the\ mean\ hardness\ is\ different\ \ across\ all$
$Methods\ for\ alloy\ 2: i.e.,\ \ \mu\ \neq\ \mu_1 \neq \mu_2 \neq \mu_3$

The ANOVA table is given as,

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

Figure 26
One way ANOVA table (Alloy 2 & Methods)

Since the probability value is lower than the level of significance (α = 0.05) we reject the null hypothesis.

Therefore, the mean of hardness is different across at least one level of methods for alloy 2.

It is possible to tell from the interaction plot that the mean of all three methods differ.

7.5

To see if there is an effect on implant hardness ('Response') due to the Tempereature we perform a one way ANOVA. The hypothesis for such a one way ANOVA is given as:

$\boldsymbol{H_0}: The\ Mean\ of\ Implant\ hardness\ is\ the\ same\ across\ all\ Temperatures\ for\ alloy\ 1: i.e.,$
$\mu\ =\ \mu_1 = \mu_2 = \mu_3$
$\boldsymbol{H_A}: For\ atleast\ one\ level\ of\ Temperatures\ , the\ mean\ hardness\ is\ different\ \ across\ all$
$Temperatures\ \ for\ alloy\ 1: i.e.,\ \ \mu\ \neq\ \mu_1 \neq \mu_2 \neq \mu_3$

The ANOVA table is given as,

	df	sum_sq	mean_sq	F	PR(>F)
C(Temp)	2.0	10154.444444	5077.222222	0.335224	0.717074
Residual	42.0	636122.800000	15145.780952	NaN	NaN

Figure 27
One way ANOVA table (Alloy 1 & Temperature)

Since the probability value is greater than the level of significance (α = 0.05) we fail to reject the null hypothesis.

Therefore, the mean of hardness is the same across all temperatures for alloy 1.

Similarly for alloy 2,

$\boldsymbol{H_0}: The\ Mean\ of\ Implant\ hardness\ is\ the\ same\ across\ all\ Temperatures\ for\ alloy\ 2: i.e.,$
$\mu\ =\ \mu_1 = \mu_2 = \mu_3$
$\boldsymbol{H_A}: For\ atleast\ one\ level\ of\ Temperatures\ , the\ mean\ hardness\ is\ different\ \ across\ all$
$Temperatures\ \ for\ alloy\ 2: i.e.,\ \ \mu\ \neq\ \mu_1 \neq \mu_2 \neq \mu_3$

The ANOVA table is given as,

	df	sum_sq	mean_sq	F	PR(>F)
C(Temp)	2.0	9.374893e+04	46874.466667	1.883492	0.164678
Residual	42.0	1.045254e+06	24886.996825	NaN	NaN

Figure 28
One way ANOVA table (Alloy 2 & Temperature)

Since the probability value is greater than the level of significance (α = 0.05) we fail to reject the null hypothesis.

Therefore, the mean of hardness is the same across all temperatures for alloy 1.

7.6 & 7.7

The interaction plots for dentists with respect to methods in alloy 1 can be given as:

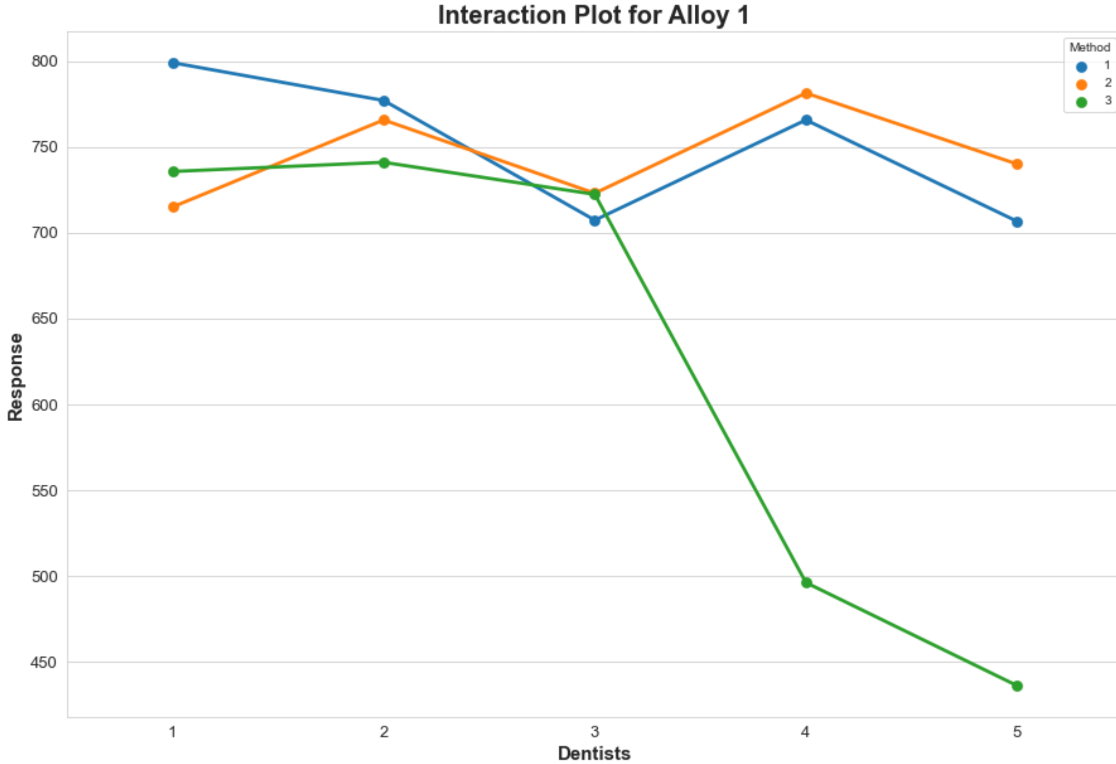

Figure 29
Interaction plot for methods and dentists for alloy 1 (Using seaborn)

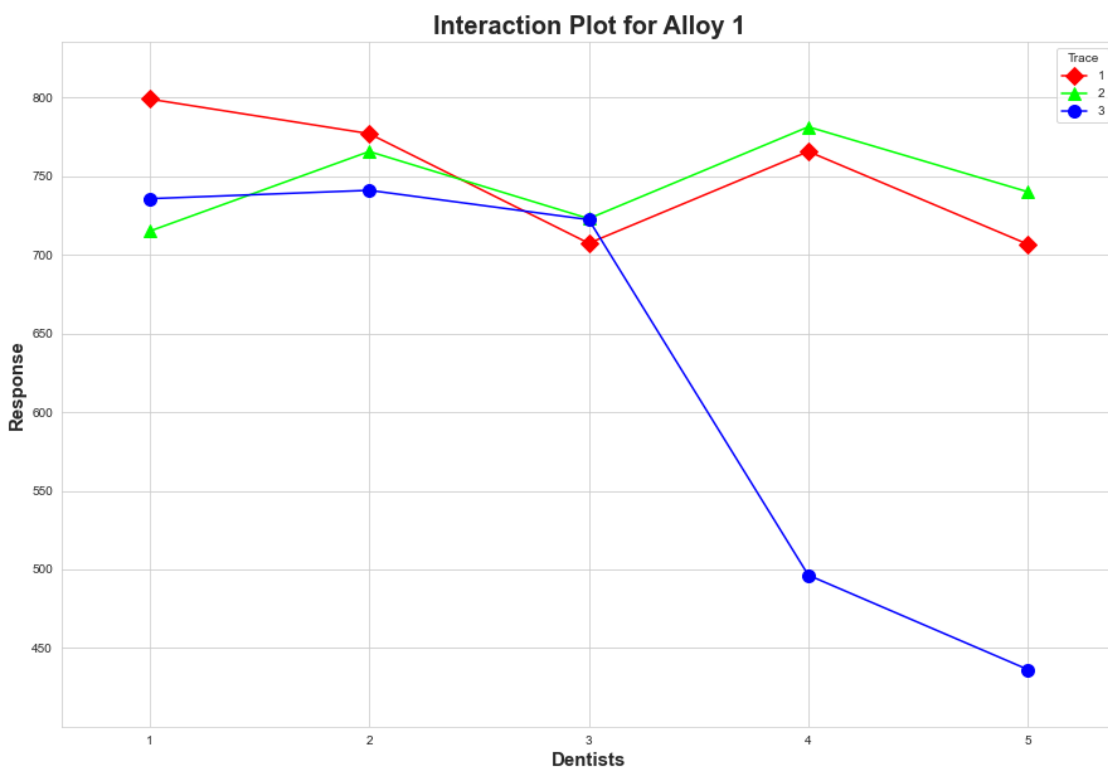

Figure 30
Interaction plot for methods and dentists for alloy 1 (Using interaction_plot)

As seen from the above graphs there is interaction present between all the methods especially among dentists 1,2 and 3 (interaction is defined by if whether or not the plot line cross over each other, if parallel then no interaction).

This can be mathematically observed in a 2 – way ANOVA for Dentists and Methods in alloy 1. Given as,

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	2.591255	0.051875
C(Method)	2.0	148472.177778	74236.088889	7.212522	0.002211
Residual	38.0	391121.377778	10292.667836	NaN	NaN

Figure 31
Two Way ANOVA dentist and method in alloy 1.

Since probability value of Method is lower than the level of significance (α = 0.05) we can state that Method has an effect on the means in alloy 1 just as we characterized it in 7.4.

Similarly for alloy 2,

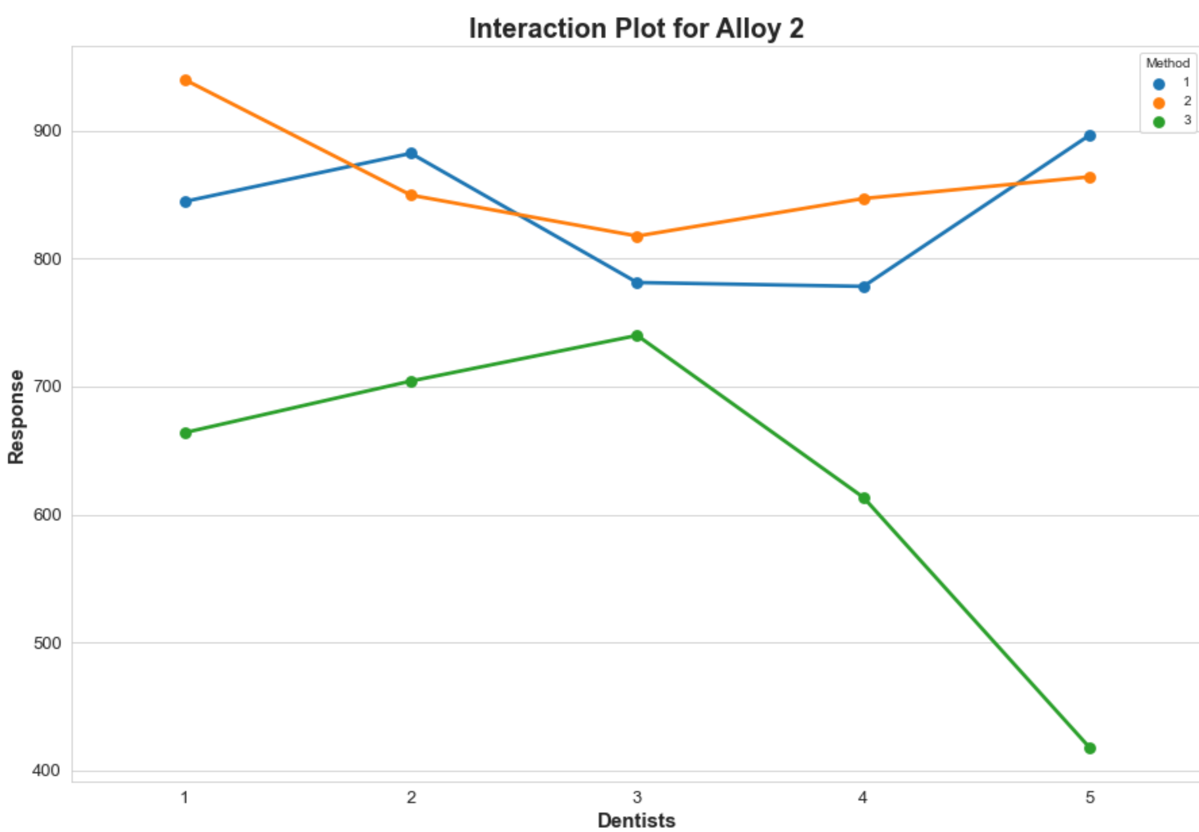

Figure 32
Interaction plot for methods and dentists for alloy 2 (Using seaborn)

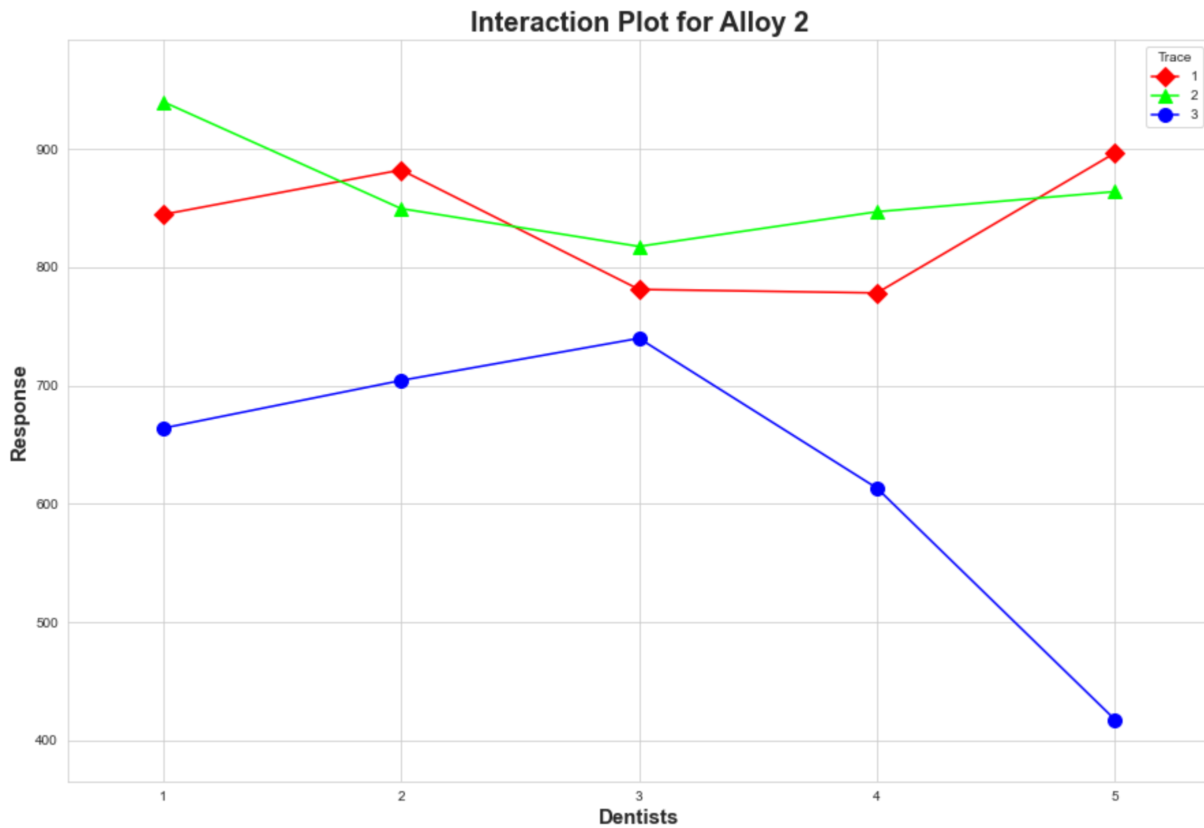

Figure 33
Interaction plot for methods and dentists for alloy 2 (Using interaction_plot)

This can be mathematically observed in a 2 – way ANOVA for Dentists and Methods in alloy 2. Given as,

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	0.926215	0.458933
C(Method)	2.0	499640.400000	249820.200000	16.295479	0.000008
Residual	38.0	582564.488889	15330.644444	NaN	NaN

Figure 34
Two Way ANOVA dentist and method in alloy 2.

Since probability value of Method is lower than the level of significance (α = 0.05) we can state that Method has an effect on the means in alloy 1 just as we characterized it in 7.4.

Let us have a look at the ANOVA for Alloy 1 with Dentist and method and the effect/interaction between each other:

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

Figure 35
Alloy 1 ANOVA interaction between Dentist and Method

Therefore, from all of the above data we can conclude **Dentist 1, 2 and 3 along with Method 1,2 and 3 are different and their interaction levels different for alloy 1.**

Similarly for alloy 2,

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

Figure 36
Alloy 1 ANOVA interaction between Dentist and Method

Therefore, from all of the above data we can conclude **Dentist 2 and 5 along with Method 1 and 2 are different and their interaction levels different for alloy 2.**