Physics 2213 Problem Set 1

Due 11:59 PM EST Friday, 2/4/2022

To be submitted via Canvas

1. **Linear, Surface, and Volume densities**

   This problem will give you practice in dealing with linear, surface, and volume densities which come up frequently in electromagnetism. This problem will deal with mass densities, but we'll see that charge densities can be handled in a very similar way.

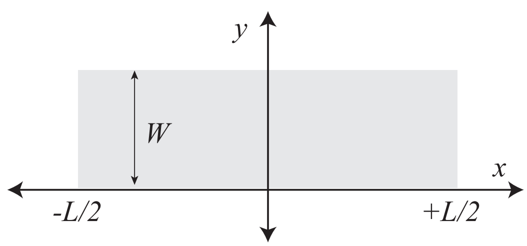
   Parts (a) and (b) deal with a thin wire of length L.

   (a) First, assume the wire has a constant, linear mass density (mass per unit length) λ_0. Give a formula for the total mass m of the wire.

   (b) Now assume the wire has a variable density that is position-dependent, $\lambda(x) = a\cos(\frac{\pi}{L}x)$. The wire is centered at $x = 0$. Calculate the mass of the wire.

   For the next part, consider a thin sheet of width W and length L, as shown below.

   (c) Assume the sheet has a uniform mass per unit area of σ_0. Calculate the mass of the sheet.



   (d) Now let's consider a thin circular sheet of radius R. Its density depends only on the radial distance r from the center of the sheet: $\sigma(r) = ae^{-\frac{r^2}{b^2}}$. Find the total mass of the sheet. *Hint: you should use polar coordinates to solve this problem. The area differential element in polar coordinates is $dA = r\,dr\,d\theta$.*

   The last two parts deal with volume density for a sphere. Use spherical coordinates, in which the differential volume element is $dV = (r\,d\theta)(r\sin\theta\,d\phi)(dr)$. We must add up the masses from each chunk outwards from the center ($r = 0$ to $r = R$) while sweeping around ($\phi = 0$ to $\phi = 2\pi$) at every different latitude on the sphere ($\theta = 0$ to $\theta = \pi$). Note that this is the "physics convention", not the "math convention" which has ϕ and θ reversed. In physics, θ is the polar angle and ϕ is the azimuthal angle. You will have to get used to the physics convention, which we use exclusively in this course.

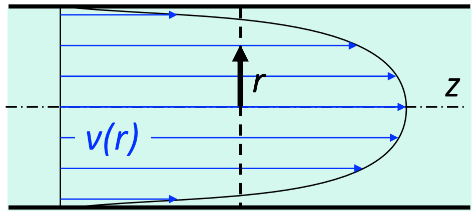
   (e) Consider a solid spherical object of radius R. Assume the sphere has a uniform volume density of ρ_0. Calculate the mass of the sphere.

   (f) Now assume that the sphere has a non-uniform density which varies with position, such that $\rho(r, \theta, \phi) = a\cos^2\theta$. Calculate the total mass of the sphere.

2. **Understanding Flux**

   A liquid flows through a cylindrical pipe of radius a centered on the z-axis. Due to a special case of the viscous flow, the velocity of the fluid varies with distance r from the axis as follows:

$$\vec{\mathbf{v}} = C\cos(\frac{\pi}{2a}r)\hat{\mathbf{k}}$$



This equation defines a vector field. Mathematically, the flux Φ of a vector field $\vec{\mathbf{v}}(\vec{\mathbf{r}})$ through a surface S is defined by this surface integral:

$$\Phi = \iint_S \vec{\mathbf{v}} \cdot d\vec{\mathbf{A}}$$

(a) Find the flux of the velocity through a circular cross section of the pipe's interior.

(b) The physical significance of the flux of $\vec{\mathbf{v}}$ through a cross section of the pipe is the volume rate of flow of the fluid through that cross section. Check that the dimensions of your expression are those of a volume rate of flow.

3. **Coulomb's Law and Approximations**

Consider this collection of point charges in the xy plane:

- charge q is at $(x, y) = (0, d)$
- charge $-2q$ is at $(x, y) = (0, 0)$
- charge q is at $(x, y) = (0, -d)$
- charge Q is at $(x, y) = (x, 0)$

where q, Q, d, and x are all positive.

(a) Find an expression for the force on charge Q due to the other charges. Also find the direction of this force.

(b) Find an approximate expression for this force valid when $x \gg d$. Your expression should be in the form Cx^n, where C is an expression that can involve any of the given constants (but not x) and n is not necessarily an integer.

For help with this part, watch Video 1A: Limiting Cases and read the handout "An Introduction to the Art of Approximation" posted on Canvas.