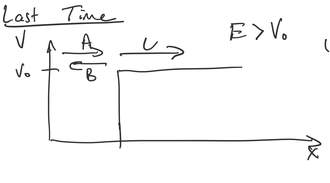
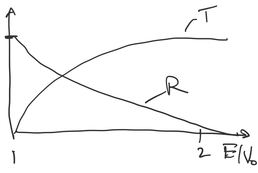
Lecture 14 - Tunneling

Friday, September 22, 2023 8:19 AM

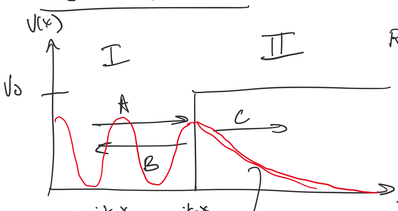
1. Warm-up Quiz
2. HW5 due HW6 out
3. Prelim 1: Oct 3 7:30-9:30 pm PSB 120
4. Today: • T + R coefficients, cont.
   • Tunneling

Last Time





Case 2: $E < V_0$



Region I $k_I = \sqrt{\frac{2m}{\hbar^2}E}$

$k_{II} = \sqrt{\frac{2m}{\hbar^2}(E-V_0)} = i\sqrt{\frac{2m}{\hbar^2}(V_0-E)}$, κ

$\psi_I = Ae^{ik_I x} + Be^{-ik_I x}$

$\psi_{II} = Ce^{ik_{II}x} = Ce^{-\kappa x}$

No $De^{+i\kappa x}$ → not normalizable
No wave from right

Apply B.C.s

$\psi_I(0) = \psi_{II}(0) \Rightarrow A + B = C$ {as before}

$\left.\frac{d\psi_I}{dx}\right|_0 = \left.\frac{d\psi_{II}}{dx}\right|_0 \Rightarrow ik_I(A-B) = \kappa C$ now i's don't cancel

$k_I = k$

$\therefore C = \frac{2k}{k+i\kappa}A$

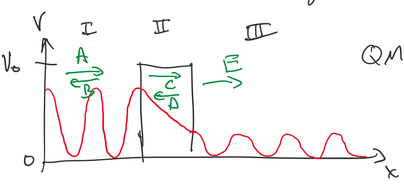
$B = \left[\frac{k-i\kappa}{k+i\kappa}\right]A \Rightarrow |B|^2 = |A|^2$

$R = \frac{k|B|^2}{k|A|^2} = 1$

$E > V_0$ → Propagation w/ some Reflection

$E < V_0$ → only reflection w/ exponential decay ⇒ evanescent wave

What if barrier doesn't go to infinity?



QM Tunneling

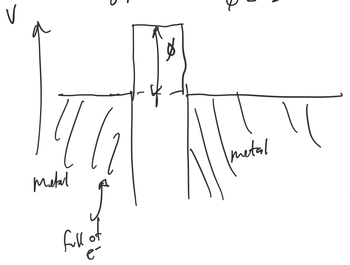
$k_I = k_{III} = \sqrt{\frac{2mE}{\hbar^2}}$ $E > 0$

$k_{II} = i\kappa = i\sqrt{\frac{2m(V_0-E)}{\hbar^2}}$ $E < V_0$

Are there bound stks? no

Solved as before

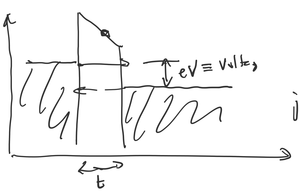
In technology $\phi \equiv$ Barrier height



Total Current density

$J(V) = J_+(V) + J_-(V)$

current $\propto e^{-2\kappa t}$



$\kappa = \sqrt{\frac{2m}{\hbar^2}\left(\phi_0 - \frac{|eV|}{2} - E\right)}$

$j_{QM} = \frac{i\hbar}{2m}\left(\psi\frac{d\psi^*}{dx} - \psi^*\frac{d\psi}{dx}\right)$

- Magnetic Tunnel Junction FM/I/FM sensors, thin drives
- Josephson Junction S/I/S sensors (SQUID)

Many Semiconductor J_s

Esaki (Tunnel) diode