

$$\left(\frac{d^2\vec{r}}{dt^2}\right)_{INERTIAL} = \left(\frac{d^2\vec{r}'}{dt^2}\right)_{ROT} + \left(\frac{d^2\vec{R}}{dt^2}\right)_{INERTIAL} + \dot{\vec{\omega}} \times \vec{r}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}') + 2\vec{\omega} \times \vec{v}_{ROT}$$

$$I_{ij} = \sum_{\alpha} m_{\alpha} \left[\delta_{ij} |\vec{r}_{\alpha}|^2 - x'_{\alpha i} x'_{\alpha j} \right]$$

If $\overleftrightarrow{I}^{CM}$ is the inertia tensor with respect to axes that have a COM origin, and $\overleftrightarrow{I}^{new}$ is the inertia tensor for the same object with respect to parallel axes that have a different pivot origin (that's at (ξ, η, ζ) with respect to the COM origin) are

$$I_{xx}^{new} = I_{xx}^{CM} + M[\eta^2 + \zeta^2] \quad \text{and} \quad I_{yz}^{new} = I_{yz}^{CM} - M\eta\zeta$$

and you should be able to figure out other shifts by analogy.

$$\vec{\omega} = \dot{\phi}\hat{z} + \dot{\theta}\hat{e}_2 + \dot{\psi}\hat{e}_3$$

$$\Gamma_1 = \lambda_1 \dot{\omega}_1 - (\lambda_2 - \lambda_3) \omega_2 \omega_3$$

$$\Gamma_2 = \lambda_2 \dot{\omega}_2 - (\lambda_3 - \lambda_1) \omega_3 \omega_1$$

$$T = \frac{1}{2} M V_{COM}^2 + \frac{1}{2} \sum_i \sum_j I_{ij}^{COM} \omega_i \omega_j \quad \text{or sometimes} \quad T = \frac{1}{2} \sum_i \sum_j I_{ij}^{pivot} \omega_i \omega_j$$

The integral you did in homework was: $\int \frac{\pm(\ell/r^2)dr}{\sqrt{2\mu[E - U(r) - (\ell^2/2\mu r^2)]}}$ for a case

where $U(r) = \frac{-k}{r}$. For this case, you defined $\varepsilon = \sqrt{1 + \frac{2E\ell^2}{\mu k^2}}$ and $c = \frac{\ell^2}{\mu k}$, and

found that you could write your result as $r(\phi) = \frac{c}{1 + \varepsilon \cos \phi}$. Examining the elliptical

r orbits using this result for the case of gravity, Taylor found that the short axis had halflength $\frac{c}{\sqrt{1-\varepsilon^2}}$, and that the constant in Kepler's 3rd law was $\frac{4\pi^2\mu}{k}$. We

plugged c and ε into $\frac{2c}{1-\varepsilon^2}$ and got $\frac{-k}{E}$. I think results above are more handy than the form for d given in Taylor, but I'll give that answer for the distance between the center of the ellipse and its focus: $d = a\varepsilon$.

$$\text{For COM origin can plug } \vec{r}_1 = -\frac{m_2}{m_1} \vec{r}_2 \quad \text{or} \quad \frac{m_1}{m_2} \vec{r}_1 = -\vec{r}_2 \quad \text{into} \quad \vec{r} = \vec{r}_1 - \vec{r}_2$$

We argued

$$\sum_j \left[\frac{\partial H}{\partial q_j} dq_j + \frac{\partial H}{\partial p_j} dp_j \right] + \frac{\partial H}{\partial t} dt = \sum_j [p_j d\dot{q}_j + \dot{q}_j dp_j] - \left\{ \sum_j \left[\frac{\partial \mathcal{L}}{\partial q_j} dq_j + \frac{\partial \mathcal{L}}{\partial \dot{q}_j} d\dot{q}_j \right] + \frac{\partial \mathcal{L}}{\partial t} dt \right\}$$

In phase space $\vec{\nabla} = \sum_j (\hat{q}_j \frac{\partial}{\partial q_j} + \hat{p}_j \frac{\partial}{\partial p_j})$

$drag\ coeff = \frac{f_{drag}}{.5\rho V^2 A_{\perp}}$

$Re = \frac{(lengthcale)(velocity)(fluid\ density)}{viscosity}$

$$\vec{a} = (\ddot{r} - r\dot{\phi}^2)\hat{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{e}_{\phi}$$
$$\vec{a} = (\ddot{r} - r\dot{\phi}^2)\hat{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{e}_{\phi} + \ddot{z}\hat{e}_z$$
$$\vec{a} = (\ddot{r} - r\dot{\phi}^2 \sin^2 \theta - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta} - r\dot{\phi}^2 \sin \theta \cos \theta)\hat{e}_{\theta} + \frac{1}{r \sin \theta} \frac{d}{dt} (r^2 \dot{\phi} \sin^2 \theta) \hat{e}_{\phi}$$

$$\frac{d\vec{L}}{dt} = \vec{R}_{COM} \times M\ddot{\vec{R}}_{COM} + \frac{d}{dt} \left[\sum \vec{r}'_{\alpha} \times m_{\alpha} \vec{r}'_{\alpha} \right]$$

$$\vec{v}_{bit\atop wrt\atop obs} = \vec{v}_{C\atop wrt\atop obs} + \vec{v}_{bit\atop wrt\atop C}$$

$$\vec{a}_{bit\atop wrt\atop obs} = \vec{a}_{O\atop wrt\atop obs} + \dot{\vec{\omega}} \times \vec{r}_{bit\atop wrt\atop O} + \vec{\omega} \times \left[\vec{v}_{bit\atop wrt\atop O} = \vec{\omega} \times \vec{r}_{bit\atop wrt\atop O} \right]$$

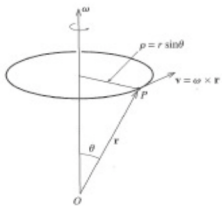
$$dV{=}pd\phi\;d\rho\;dz$$

$$dV{=}r^2sin\theta d\phi\;dr\;d\theta$$

$$\delta_n \rightarrow 4.6692016 \ldots$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x + iy = \sqrt{x^2 + y^2} \exp(i\mathrm{Arctan}(y / x))$$



$$\vec{v}_{bit\atop wrt\atop O} = \vec{\omega} \times \vec{r}_{bit\atop wrt\atop O}$$

$$I_Z^{NEW} = I_Z^{CM} + M(X_{NEW}^2 + Y_{NEW}^2)$$

I made add some moment of inertia values here but also know how to calculate those.