IX. You used constant $\ell = \mu r^2\dot{\theta}$ to eliminate $\dot{\theta}$ in constant E=T+U= $\frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + U(r)$

& got E= $\frac{1}{2}\mu\,\dot{r}^2 + \frac{\ell^2}{2\mu r^2} + U(r)$. **a) Must allowed r values have** $E \geq \frac{\ell^2}{2\mu r^2} + U(r)$? Yes or no ?

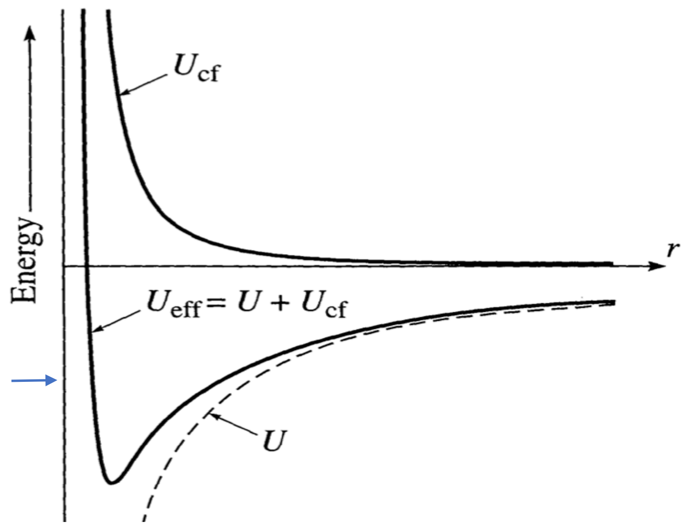
We set $\frac{\ell^2}{2\mu r^2} + U(r) \equiv U_{eff}(r)$ and plot for nonzero ℓ -- qualitative shape is general.

Consider $U(r) = \frac{-k}{r}$ (shown as dashed curve below its U_{eff} in plot). Know $\boldsymbol{r} = |\vec{\boldsymbol{r}}_1 - \vec{\boldsymbol{r}}_2|$

b) Should $\boldsymbol{U_{eff}} = \frac{\boldsymbol{\ell}^2}{2\mu \boldsymbol{r}^2} - \frac{\boldsymbol{k}}{\boldsymbol{r}}$ **approach the dashed U=**$-\frac{\boldsymbol{k}}{\boldsymbol{r}}$ **curve as** $\boldsymbol{r} \to \infty$**?** Y / N

Term from centrifugal effects is called $U_{cf} = \frac{\ell^2}{2\mu r^2}$ in picture; it goes to ∞ as $r \to 0$.

One possible negative E value



c) Does $\boldsymbol{U_{eff}} = \frac{\boldsymbol{\ell}^2}{2\mu \boldsymbol{r}^2} - \frac{\boldsymbol{k}}{\boldsymbol{r}}$ **go to +∞ as** $\boldsymbol{r} \to \boldsymbol{0}$**?**

d) Can separation between m1 and m2 go to zero in $\boldsymbol{\ell} \neq \boldsymbol{0}$ gravity case? Y / N

e) What is the $r \to \infty$ limit of $U_{eff} = \frac{\ell^2}{2\mu r^2} - \frac{k}{r}$?

f) What is minimum E for separation to reach ∞?

g) If E= value of U_{eff} where $\frac{dU_{eff}}{dr} = 0$ what can r do? (r value with $\frac{dU_{eff}}{dr} = 0$ is c below.)

h) For "one possible negative E value" indicated on the energy axis, make marks on the plot indicating the approximate min and max values of r (separation) allowed for that E value.

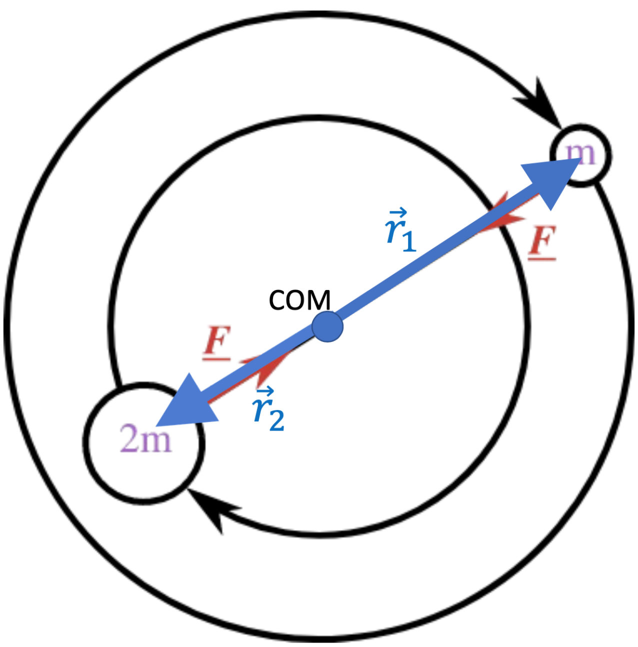
Exact solution has $c = \frac{\ell^2}{\mu k}$ $\quad r = \frac{c}{1+\varepsilon\cos\theta}$ $\quad \varepsilon = \sqrt{1 + \frac{2E\ell^2}{\mu k^2}}$

i) We plugged into $\frac{2c}{1-\varepsilon^2}$ and got $\frac{-k}{E}$. **Relate** $\frac{\boldsymbol{2c}}{\boldsymbol{1-\varepsilon^2}}$ **to** $\boldsymbol{r_{min}}$**+**$\boldsymbol{r_{max}}$**.**

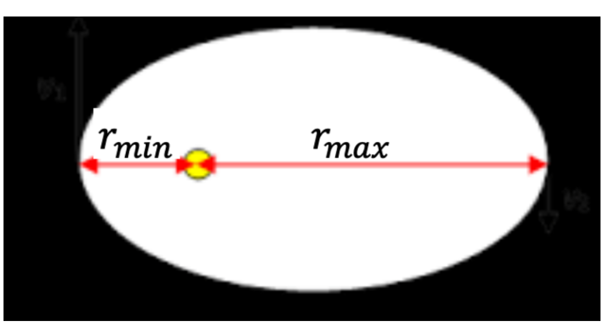
j) You observe round asteroids of mass M and 2M in elliptic orbits around their COM origin and find their min separation=D and max separation=2D. How much energy would have to be added **in the COM frame** (bang!) to allow them to go infinitely far apart eventually?

k) Homework 9 found $m_1\vec{r}_1 = -m_2\vec{r}_2$ meant $\vec{r}\left(\frac{-m_1}{m_1+m_2}\right) = \vec{r}_2$ $\vec{r}\left(\frac{m_2}{m_2+m_1}\right) = \vec{r}_1$

How long is the long axis of the elliptic orbit that JUST m1=M travels around the COM?
[Below is picture for circular orbit case. Bottom ellipse is fake μ orbit.]



l) If m1 << m2 approximate $\vec{r}_1 = \vec{r}\left(\frac{m_2}{m_2+m_1}\right)$



m) If m1 << m2
roughly where is the origin (i.e. the COM)?

n) If m1 << m2 is the m1 orbit like the orbit of the fake particle of mass $\mu = \frac{m_1 m_2}{m_2+m_1}$ at $\vec{r}$? Y or N