Lecture 9 – Free vs. Bound states

Monday, September 11, 2023 9:00 AM

1. Warm-up Quiz
2. HW 4 due Friday
3. Today
   - Review problem solving approach
   - Localizing a wave problem
   - Momentum state Representation

$$|\Psi|^2 = \Psi^* \Psi = \left(\psi_1^* e^{+iE_1t/\hbar} + \psi_2^* e^{+iE_2t/\hbar}\right)\left(\psi_1 e^{-iE_1t/\hbar} + \psi_2 e^{-iE_2t/\hbar}\right)$$

So Far: SE $\Rightarrow$ TISE + time dep.

Problem $\rightarrow$ define $V(x)$ + initial cond $\Psi(x,0)$

Solve TISE $\Rightarrow$ Find $\psi_n(x)$ (in a bound system these quantized)

$\Rightarrow \Psi_n = \psi_n(x)\, e^{-iE_nt/\hbar}$

Use the fact that General Sol^n $\boxed{\Psi(x,t) = \sum_n c_n \psi_n e^{-iE_nt/\hbar}}$

A particular sol^n requires choosing c_n that match $\Psi(x,0)$

because $\Psi(x,0) = \sum_n c_n \psi_n(x)$

$$\int_{-\infty}^{\infty} \psi_m^* \Psi(x,0)\, dx = \sum_n \int_{-\infty}^{\infty} c_n \psi_m^* \psi_n\, dx = c_m$$

Now you know everything $\rightarrow$ now calc $\langle \hat{Q} \rangle$, etc.

Now move to wavefunctions of free particles ($V(x) = 0$) and boundary conditions?

- still continuous
- $\psi \rightarrow 0$ @ $\pm\infty$ otherwise no.

So TISE

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi \Rightarrow \frac{d^2\psi}{dx^2} = -k^2\psi$$

$k \equiv \frac{\sqrt{2mE}}{\hbar}$

$\omega = E/\hbar$

$$\psi = Ae^{ikx} + Be^{-ikx}$$

$$\Psi(x,t) = Ae^{i(kx-\omega t)} + Be^{-i(kx+\omega t)}$$

No quantization

Problem: Not physical b/c it's not normalizable

$\psi(x) \not\rightarrow \pm\infty$

Not a description of a particle

Sol^{ns} can be linear combinations

$$\Psi = \sum_n c_n \Psi_n(x,t) \Rightarrow \text{move to continuous limit}$$

Construct wave packets.

First a digression:

- recall $v_{ph} = \frac{\omega}{k}$ velocity of const. phase

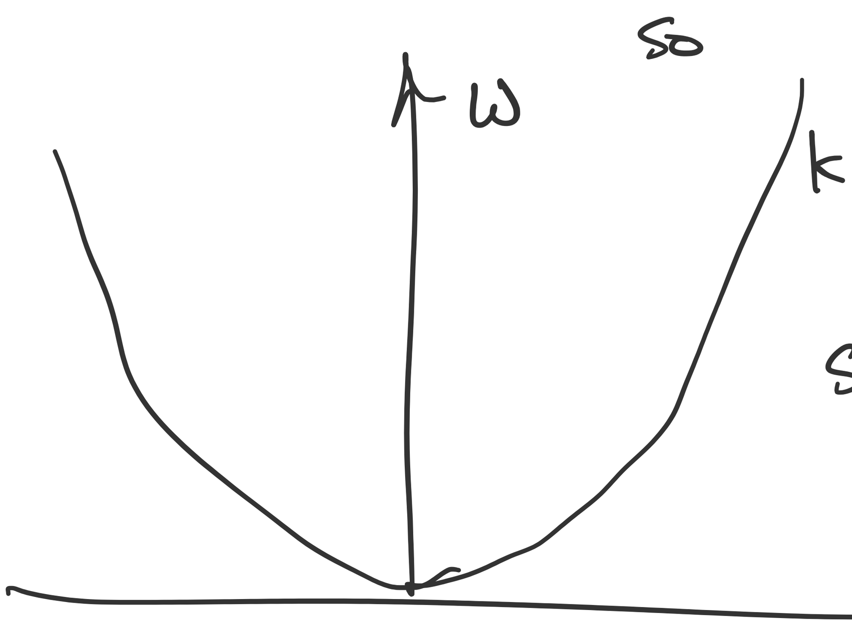
  $e^{i(kx-\omega t)} = e^{ik(x-\frac{\omega}{k}t)}$, $\frac{\omega}{k} = v \rightarrow$ right going

- Plane waves are building blocks (basis functions)

For particle mass m

$$E = \hbar\omega = \frac{p^2}{2m} \quad \text{and} \quad p = \hbar k$$

so $\omega = \frac{\hbar k^2}{2m}$ dispersion



slope $= \frac{d\omega}{dk} = \frac{\hbar k}{m} = \frac{p}{m} = v$ $\boxed{\text{Group Velocity}}$

particle velocity

phase velocity $\frac{\omega}{k} \neq \frac{d\omega}{dk}$

$$v_{ph} = \frac{\frac{\hbar k^2}{2m}}{k} = \frac{\hbar k}{2m} = \sqrt{\frac{E}{2m}}$$

$$v_{gr} = \frac{d\omega}{dk} = \frac{d}{dk}\left(\frac{\hbar k^2}{2m}\right) = \frac{\hbar k}{m} = \sqrt{\frac{2E}{m}}$$

Approach to constructing wave packets $\Rightarrow$ wave interference

Simple example (not physical)

$$\psi(x) = \psi_1 + \psi_2 + \psi_3$$
$$= e^{ik_0x} + \frac{1}{2}e^{i(k_0 - \frac{\Delta k}{2})x} + \frac{1}{2}e^{i(k_0 + \frac{\Delta k}{2})x}$$
$$= \underset{\text{carrier}}{e^{ik_0x}}\underbrace{\left[1 + \cos\left(\frac{\Delta k}{2}x\right)\right]}_{\text{envelope function}}$$