Homework 1
Due: Friday, February 4th, 2022, 11:59pm

Notes: (1) Please submit your solution in a single PDF file named HW1.pdf; (2) Please put your name and NetID on the top of the first page; (3) It is important that you show how you arrived at your answers.

Problem 1.
Simplify the following expressions using Boolean algebra theorems to minimize the number of literals. Please show all the steps and name the Boolean algebra theorem used in each step.

(a) $(A' + BC')(A + B'C)$

(b) $A' + B' + C' + ABC$

(c) $(A + B + C)(A + B' + C)(A' + B' + C)$

Problem 2.
Prove the consensus theorem algebraically. Please show all the steps and name the Boolean algebra theorem used in each step.

$$X \bullet Y + X' \bullet Z + Y \bullet Z \;=\; X \bullet Y + X' \bullet Z$$

Problem 3.

Who is the liar? You are at a party with Alice, Bob, Carol and Dan. Somehow you know that three of them always tell the truth, and one of them always lies. Just as you are trying to find out the liars, you overheard the following conversation:

Alice: "I trust Dan. Dan is telling the truth."
Bob: "I think Carol is lying."
Carol: "No, I am telling the truth!"
Dan: "Bob must be lying."

Now, as an enthusiastic ECE2300 student, you immediately realize that you can use Boolean logic to find out the liar!

So you decide to use Boolean variables A, B, C, and D to denote whether Alice, Bob, Carol or Dan is lying. For instance, $A=1$ if Alice is lying, and $A=0$ if Alice is honest.

You think to yourself: "For Alice's statement, there are only two possibilities. Either both Alice and Dan are honest, or Alice is lying as well as Dan." So you write down the Boolean expression for Alice's statement: $AD + A'D'$.

Similarly, you write down the Boolean expression for Bob's statement: _______________. For Carol' statement, you have: _____________. Finally for Dan, you write down: _____________.

You realize that these four Boolean expressions all have to be true at the same time, so you combine them together using logical AND and get: _______________________________________.

You then convert the expression into canonical sum using Boolean algebra theorems:
____
____
____

Looking at the simplified expression, you immediately figure out that ________ is the liar!

Problem 4.

Design a fingerprint safe consisting of 3 fingerprint readers. Each reader (A, B, and C) when activated, is pre-programmed to identify a particular fingerprint. If a reader detects a correct finger print, it generates a ‘1’, otherwise a ‘0’. The safe can be unlocked (i.e., U = 1) if and only if all the three readers detect correct fingers. The safe shouldn’t do anything when only two correct fingers are detected. However, when one or less correct finger is detected, the safe should raise an alarm (i.e., W = 1).

a) Construct a truth table that includes both U and W

b) Write the outputs of the circuits as canonical sums

c) Write the outputs of the circuits as canonical products

Problem 5.

Use Karnaugh maps to simplify (1) sum-of-products for F_1 and F_2, and (2) product-of-sums for F_3 and F_4.

(a) $F_1 = \sum_{x,y,z} \quad (1,2,3)$

(b) $F_2 = \sum_{w,x,y,z} \quad (0,2,3,4,6,7,8,9,10,11,12,14,15)$

(c) $F_3 = \prod_{x,y,z} \quad (1,4,5,6,7)$

(d) $F_4 = \prod_{w,x,y,z} \quad (0,1,2,4,5,6,8,10,12,13,14)$

Problem 6.
Table 1 shows the partial truth table for a mysterious two-input operator, denoted as the ▷ operator.

Table 1. Incomplete truth table for the ▷ operator

inputs		*output*
A	***B***	***F***
0	0	
0	1	1
1	0	
1	1	

For some reason, we happen to known:

a. ▷ is associative, i.e., (X ▷ Y) ▷ Z = X ▷ (Y ▷ Z).
b. ▷ is **not** `commutative`, i.e., `X ▷ Y ≠ Y ▷ X`, `for X ≠ Y`.

Based on the information above, can you complete the truth table for the ▷ operator? Please concisely describe how you arrived at your conclusion.