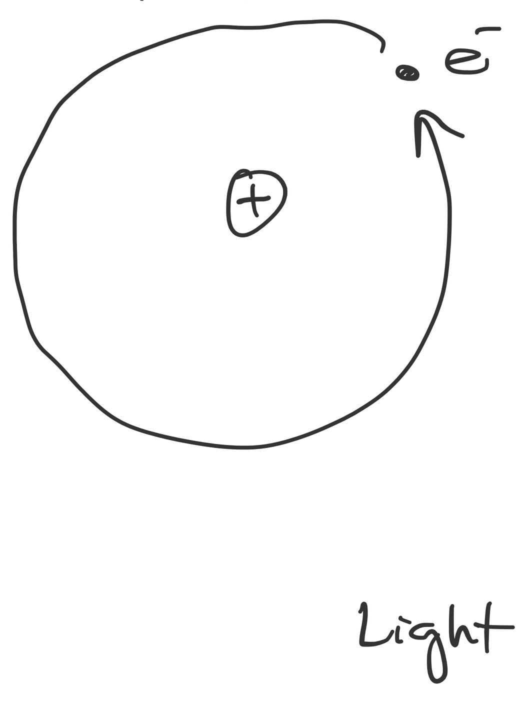
Lecture 4 - de Broglie waves

Monday, August 28, 2023 9:04 AM

- Warm-up Quiz
- HW2 Due Friday
- office hours
  - M - 3-4
  - T - 5-6
  - W - 4-5
  - R - 5-6

Today:
- de Broglie waves
- Schrödinger eqn

Last time: Rutherford → planetary model



Bohr
- orbits → energy level

$$E_n = \frac{-hcR_H}{n^2} \qquad n = 1, 2, 3, \ldots$$

$$\underline{r_n = a_0 n^2} \qquad a_0 \equiv \text{Bohr radius}$$

Light emission from orbital jumps $\Delta E_{n \to m}$

But what makes e^- choose certain orbits, not allowing others?

de Broglie → if Light has wave/particle properties maybe e^-'s are too.

particle ⟷ wave

stationary state ⟷ standing wave

For light:

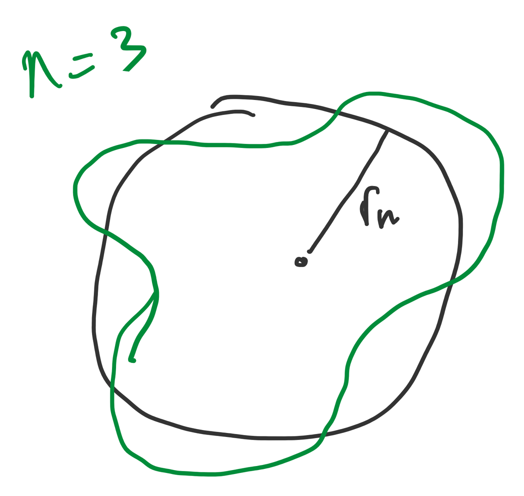
$$E = h\nu = cp$$

$$\frac{h\cancel{c}}{\lambda} = \cancel{c}p \Rightarrow \boxed{\lambda = h/p} \quad ☆$$

$$\vec{p} = \hbar\vec{k}$$

momentum; spacial frequency

Bohr in terms of R_H



$r_n = a_0 n^2$

Circumference $2\pi r_n$

idea - fit inter number of wave lengths in circumference → standing wave

$$\therefore \; 2\pi r_n = n\lambda = \frac{nh}{p} = \frac{nh}{mv}$$

Classical circular orbit

$$\underbrace{\frac{mv_n^2}{r_n}}_{\text{centripetal force}} = \underbrace{\frac{Ze^2}{4\pi\varepsilon_0 r_n^2}}_{\text{Coulomb force}} \Rightarrow v_n = \sqrt{\frac{Ze^3}{4\pi\varepsilon_0 r_n m}}$$

Z: # charges (protons)

$$2\pi r_n = \frac{nh}{mv} = nh\sqrt{\frac{4\pi\varepsilon_0 r_n}{Ze^2 m}}$$

solve for r_n

$$r_n^{\cancel{2}} = \frac{n^2h^2}{\cancel{4\pi^2}} \frac{\cancel{4\pi}\varepsilon_0 \cancel{r_n}}{Ze^2 m}$$

$$r_n = \underbrace{\left(\frac{h^2 \varepsilon_0}{\pi Z e^2 m}\right)}_{a_0} n^2 \qquad \boxed{\text{as before}}$$

in terms of fundamental quanties

Notice angular momentum is $\underline{\text{quantized}}$ ↳ assumption of wave boundary conditions

$$L_n = mv_n r_n = \hbar n$$

Plausibility argument for the Schrödinger equation

$$\vec{p} = \hbar\vec{k} \quad \text{(de Broglie)}$$

$$E = \hbar\omega \quad \text{(Einstein)}$$

$$E = \frac{p^2}{2m} + V \quad \text{(Newton)} \quad [\text{non-relativistic}]$$

We know we need a wave equation

Wave equation solutions $\Psi(x,t)$ have standing wave sol^n and linear superpositions.

$$\Psi(x,t) = C_1\Psi_1(x,t) + C_2\Psi_2(x,t), + \ldots$$

if Ψ_1, Ψ_2 are solutions of the equation.

⇒ only linear terms in Ψ (no Ψ^2, but e.g. $\frac{\partial\Psi}{\partial t}$ ok)

⇒ Look for a wave equation for a massive particle w/ const E, p

→ ω, k have to be constant

Look @ Classical wave equation $\qquad c \equiv$ Velocity $\to = 1$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial y^2}{\partial t^2} \qquad \text{recall } \; \underset{\Psi}{y} = E_0 \cos(kx \mp \omega t)$$

$$\frac{\partial\Psi}{\partial t} = \omega \sin(kx - \omega t) \qquad \frac{\partial^2\Psi}{\partial t^2} = -\omega^2\cos(kx - \omega t)$$

$$\frac{\partial\Psi}{\partial x} = -k\sin(kx - \omega t) \qquad \frac{\partial^2\Psi}{\partial x^2} = -k^2\cos(kx - \omega t)$$

sub in

$$-k^2\cos(kx - \omega t) = \frac{-\omega^2}{c^2}\cos(kx - \omega t)$$

$$\Rightarrow \omega^2 = c^2k^2 \quad \text{or} \quad \underbrace{\omega = c|k|}$$

Equiv. to $E = \hbar c|k|$ ⟷ Linear dispersion of Light

$E \leftrightarrow p$

problem: need $E \propto k^2$ → need to end up w/ $E = \frac{p^2}{2m} + V$

$$E = \frac{\hbar^2 k^3}{2m} + V$$

each $\frac{\partial}{\partial x} \to k$; $\frac{\partial}{\partial t} \to \omega$

$$E\Psi = \frac{p^2}{2m}\Psi + V\Psi$$

$$\omega\Psi = \frac{\hbar^{\cancel{2}}}{2m}k^2\Psi + \frac{V}{\hbar}\Psi$$

single derivative w/ t; 2 der. w/ x

$$\boxed{\underline{\text{postulate}} \qquad \alpha\frac{\partial\Psi}{\partial t} = \beta\frac{\partial^2\Psi}{\partial x^2} + V\Psi}$$