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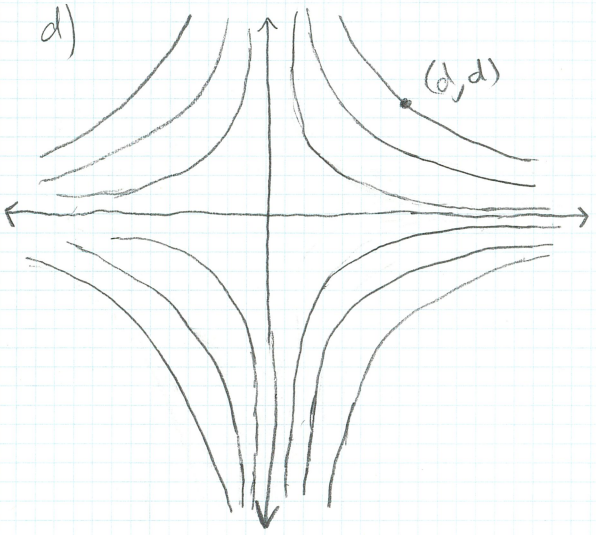
Q1 a) $\nabla \times \vec{E} = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 2Axy - 2Axy = 0$

b) $\nabla \cdot \vec{E} = \frac{\partial E_y}{\partial y} + \frac{\partial E_x}{\partial x} = Ax^2 + Ay^2 = A(x^2+y^2) = A|\vec{r}|^2 = \frac{\rho}{\varepsilon_0} \Rightarrow \boxed{\rho = A\varepsilon_0 |\vec{r}|^2}$

c) $V = -\int_{(0,0)}^{(x,y)} \vec{E} \cdot d\vec{r} = -\left[\int_{(0,0)}^{(x,0)} Axy^2\,dx + \int_{(x,0)}^{(x,y)} Ax^2y\,dy\right] = \boxed{-\frac{Ax^2y^2}{2}}$ because curl = 0

$-\nabla V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j}\right) = Axy^2\hat{i} + Ax^2y\hat{j}$ ✓

d)



e) It will accelerate toward (0,0) and oscillate about (0,0). It will come to rest at (d, d), (-d, -d), and all subsequent times it reaches those points.

$PE = qV \Rightarrow -\frac{Ad^2d^2}{2}(-e) = KE = \frac{1}{2}mv^2$ (at (d,d) vs at (0,0))

$Ad^4e = mv^2 \Rightarrow \boxed{v = \sqrt{\frac{Ad^4e}{m}}}$ at (0,0)

no units given

Q2 a) $E_z = \int \frac{kz}{(r^2+z^2)^{3/2}}dq = \int_a^b \frac{kz}{(r^2+z^2)^{3/2}}(\sigma 2\pi r\,dr) = kz\sigma 2\pi\left((z^2+a^2)^{-\frac{1}{2}} - (z^2+b^2)^{-\frac{1}{2}}\right)$ by calculator

$V = -\int_\infty^z E_z\,dz = k\sigma 2\pi\left(\sqrt{z^2+b^2} - \sqrt{z^2+a^2}\right)$ by calculator

$\sigma = \frac{Q}{\pi b^2 - \pi a^2} \Rightarrow \boxed{V = \frac{2kQ}{b^2-a^2}\left(\sqrt{z^2+b^2} - \sqrt{z^2+a^2}\right)} = 2kQ\left(\frac{z^2+b^2-z^2-a^2}{(b^2-a^2)\left(\sqrt{z^2+a^2}+\sqrt{z^2+b^2}\right)}\right)$

When $|z| >>> b > a$, $V \approx 2kQ\left(\frac{1}{\sqrt{z^2}+\sqrt{z^2}}\right) = \boxed{\frac{kQ}{|z|}}$ same as point charge

b) $PE = qV = K\varepsilon$ far away $\Rightarrow \frac{1}{2}mv^2 = \frac{2kQe}{b^2-a^2}\left(\sqrt{d^2+b^2} - \sqrt{d^2+a^2}\right)$

$$\boxed{v = \sqrt{\frac{4kQe}{m(b^2-a^2)}\left(\sqrt{d^2+b^2} - \sqrt{d^2+a^2}\right)} \approx \sqrt{\frac{4kQe}{m(b+a)}}} \quad \text{for } d << a < b$$

Q3 a) H-OH ignored since within molecule not broken $\Rightarrow$ Energy to break $= \left|\frac{k(e)(-e)(0.35)^2}{10^{-9}(0.1+0.1+0.17)} + 2\left(\frac{ke^2(0.35)^2}{10^{-9}(0.1+0.17)}\right) + \frac{k(-e)(e)(0.35)^2}{10^{-9}(0.17)}\right| = \boxed{3.32\cdot10^{-20}\ J/bond}$

Q3) b) $1\,kg \cdot \frac{1000\,g}{kg} \cdot \frac{1\,mol}{18\,g} \cdot \frac{6.022\cdot10^{23}\,molecules}{mol} \cdot \frac{3\,bonds}{2\,molecules} \cdot \frac{3.32\cdot10^{-20}\,J}{bond} = \boxed{1.67\cdot10^{6}\,J}$

The heat of vaporization of water is $2.257\cdot10^{6}$ J. It's no coincidence that they're similar. On top of reaching the boiling point, water molecules must overcome interactions like hydrogen bonds to become gas. This extra required energy is accounted for by the heat of vaporization. Our value is less, because there are other interactions outside of hydrogen bonds that need to be overcome as well.

Q4] a) outside $\Leftrightarrow$ point charge $\Rightarrow V = \frac{kQ}{r} = \frac{4k\sigma\pi a^2}{r}$

inside $E=0 \Rightarrow -\nabla V = 0 \Rightarrow \nabla V = 0$.

So, $\boxed{V(r) = \begin{cases} 4k\sigma\pi a & 0 \le r < a \\ \frac{4k\sigma\pi a^2}{r} & a \le r \end{cases}}$

b) $U_{tot} = \frac{1}{2}\int_0^q V\,dq = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi} 4k\sigma\pi a\,(\overbrace{\sigma a^2 \sin\theta\,d\theta\,d\phi}^{dq}) = 8k\sigma^2 a^3 \pi^2 = \boxed{\frac{kq^2}{2a}}$ given $q = 4\pi a^2\sigma$

c) $m_e c^2 = 9.11\cdot10^{-31}\cdot(3\cdot10^{8})^2 = \frac{k(-e)^2}{2r_e} = 1.15072\cdot10^{-28}$

$= U_{tot}$

$\Rightarrow r_e = \frac{ke^2}{2mc^2} = \boxed{1.4035\times10^{-15}\,m > 10^{-18}\,m}$

Our model yields a radius greater than the upper limit. As such, it is not reasonable and cannot be an accurate estimate of the true radius. Experimental results have refuted this model, and more accurate models may be more complicated.