Bryant Har

Q1 a) $\vec{E}_0 = \frac{Q_{enc}}{\varepsilon_0 A}\hat{r} = -\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$. $\vec{E}_{net} = \frac{E_0}{\kappa} = -\frac{Q}{4\kappa\pi\varepsilon_0 r^2}\hat{r}$

b) $\Delta V = -\int_a^b \vec{E}_{net}\cdot d\vec{r} = \int_a^b \frac{Q\hat{r}}{4\kappa\pi\varepsilon_0 r^2}\, d\vec{r} = \frac{Q}{4\kappa\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$

$C = \frac{Q}{V} = 4\kappa\pi\varepsilon_0\left(\frac{1}{a}-\frac{1}{b}\right)^{-1}$. When $\kappa = 1$, $C = \frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}}$ ✓ checks out

c) $Q' = Q - Q_{eff}$ $\quad Q_{eff} = \frac{Q}{\kappa} \Rightarrow Q' = \frac{Q(\kappa-1)}{\kappa}$

d) $U = \iiint u\, dV = \iiint \frac{1}{2}\kappa\varepsilon_0\left(-\frac{Q}{4\kappa\pi\varepsilon_0 r^2}\right)^2 dV = \int_a^b\int_0^{2\pi}\int_0^{\pi} \frac{Q^2 r^2 \sin(\theta)}{32\kappa\pi^2\varepsilon_0 r^4}\, d\theta\, d\phi\, dr$

$= \int_a^b \frac{Q^2}{8\kappa\pi\varepsilon_0 r^2}\, dr = \frac{Q^2}{8\kappa\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$.

$U = \frac{1}{2}QV = \frac{1}{2}Q\left(\frac{Q}{4\kappa\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)\right) = \frac{Q^2}{8\kappa\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$ ✓ checks out

Q2 $\frac{Q}{\varepsilon_0} = EA \Rightarrow E = \frac{Q}{2\pi r L\varepsilon_0}$, $\quad \Delta V = -\int_b^a E_{eff}\, dr = \frac{Q}{2\kappa\varepsilon_0\pi L}\ln\left|\frac{b}{a}\right|$

$E_{eff} = \frac{Q}{2\kappa\pi r L\varepsilon_0}$

$C = \frac{Q}{V} = \frac{2\kappa\pi L\varepsilon_0}{\ln\left|\frac{b}{a}\right|}$

Q3 a) add in parallel, $\frac{1}{C_{eff}} = \frac{1}{C_1}+\frac{1}{C_2}$ in series.

$\Leftrightarrow$ A C_1 [C_2 | C_5] B $\Rightarrow C_1^{-1} = \frac{1}{5}+\frac{1}{3.5} \Rightarrow C_1 = \frac{35}{17}\mu F$ $\quad C_2 = 8\mu F$ $\quad C_3^{-1} = \frac{1}{1.5}+\frac{1}{1.5}+\left(0.75+\left(\frac{1}{15}+\frac{1}{3.5}\right)^{-1}\right)^{-1}$

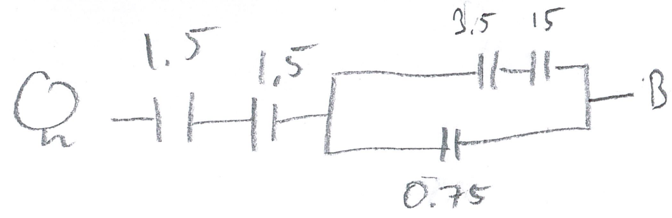
$C_3 = \left[\frac{2}{1.5}+\left(0.75+\frac{105}{37}\right)^{-1}\right]^{-1} = \frac{531}{856}\mu F$

$= 0.62$

$C_1 + C_2 + C_3 = \boxed{10.68\,\mu F}$

b) ~~A - 1.5 - 1.5 - 0.75 - B~~

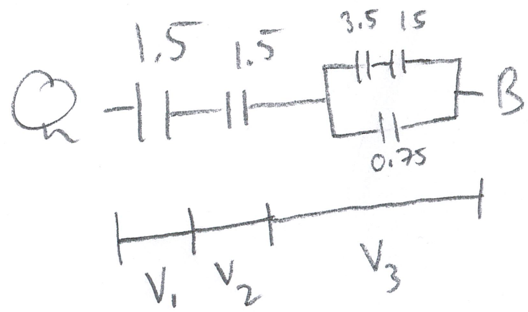
Q3) b)



$Q_T = CV = 12(10.68) = 128.16\,\mu C$

C_3 found earlier

$Q_1 = C_3 V = 12(0.62) = 7.444\,\mu C$



Voltage drops sum to 12

$V_1 + V_2 + V_3 = 12$

$V_3 = 12 - \frac{7.444}{1.5} - \frac{7.444}{1.5} = 2.075\,V$ (μ's cancel out)

Voltage drop across parallel capacitors is equal to voltage drop across either capacitor.

So voltage across $0.75\mu F$ capacitor $= V_3$

$\boxed{\text{Voltage across } 0.75\mu F \text{ capacitor} = 2.075\,V}$