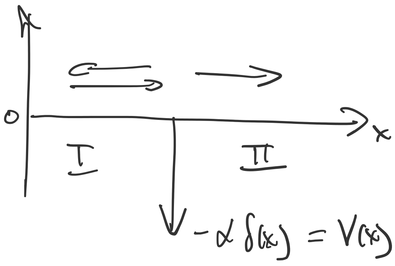
Lecture 15 - Finishing 'Flat' potentials

Monday, September 25, 2023 8:47 AM

1. Warm-up Quiz
2. HW6 due Friday
3. Prelim Oct 3 - Review materials now online
4. Reading: Griffiths 2.3 [Not on prelim]
5. Today:
   - Finish Tunneling / applications
   - Quick δ-function overview?
   - QHO

Gr. sec. 2.5

δ-function potentials



$$\delta(x) = \begin{cases} 0 & x \neq 0 \\ \infty & x = 0 \end{cases}$$

$$\psi_I = Ae^{ikx} + Be^{-ikx} \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\psi_{II} = Ce^{ikx}$$

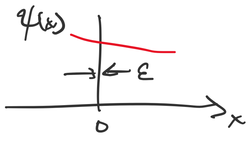
Normal b.c.'s: $\psi(0)$ cont; $\psi'(0)$ cont

$\psi(0)$: $\boxed{A + B = C}$ (1)

$\psi'(0)$:

$$\left.\frac{d\psi}{dx}\right|_{x\to 0^-} = ik(A - B)$$

$$\left.\frac{d\psi}{dx}\right|_{x\to 0^+} = ikC$$



$\Rightarrow$ ψ' cont except when $V(x) = \infty$: here $V(0) = 0$ but has limited area

$$\lim_{\varepsilon\to 0} -\frac{\hbar^2}{2m}\int_{-\varepsilon}^{+\varepsilon}\frac{d^2\psi}{dx^2}\,dx + \int_{-\varepsilon}^{\varepsilon} V(x)\,\psi(x) = E\int_{-\varepsilon}^{\varepsilon}\psi(x)\,dx \to 0$$

$$\Delta\left(\frac{d\psi}{dx}\right)_{x=0} \equiv \lim_{\varepsilon\to 0}\left(\left.\frac{d\psi}{dx}\right|_{-\varepsilon} - \left.\frac{d\psi}{dx}\right|_{+\varepsilon}\right) = \frac{2m}{\hbar^2}\lim_{\varepsilon\to 0}\int_{-\varepsilon}^{\varepsilon} V(x)\,\psi(x)\,dx$$

$V(x) = -\alpha\,\delta(x)$

so $\boxed{\Delta\left(\frac{d\psi}{dx}\right)_{x=0} = -\frac{2m\alpha}{\hbar^2}\psi(0)}$ new boundary condition

$$\boxed{ik(A - B) - ikC = -\frac{2m\alpha}{\hbar^2}(A + B)} \quad (2)$$

let $\gamma = \frac{2m\alpha}{\hbar^2}$, combine w/ (1)

we get $ik(A - B - A - B) = -\gamma(A + B)$

$$2ikB = \gamma(A + B) \Rightarrow B = (2ik - \gamma) = \gamma A$$

$$B = \frac{\gamma A}{2ik - \gamma} \qquad C = \frac{2ikA}{2ikA - \gamma}$$

$$R = \frac{k|B|^2}{k|A|^2} \qquad T = \frac{k|C|^2}{k|A|^2}$$