Problem set 12

Applied & Engineering Physics 3330

Problem 1 only is due 5pm Tues Nov. 21;

Remember to explain your answers!

Review some things you should already know from class while skimming the first two sections of Taylor's chapter 10, and then read sections 10.3-10.9, with the exception that the second part of 10.8 entitled "Motion of a Body with Two Equal Moments: free precession" is optional. Also look at Taylor's question 10.24 and understand in your own mind why the first equation in 10.117 is equivalent to the parallel axis theorem you already know, but there is another part of the parallel axis theorem presented there for off-diagonal elements. We may get to section 10.10 by the end of the term.

Problem 1: a) In ps11 you found that inertia tensor elements found with a COM origin could be written as $I_{ij} = \sum_\alpha m_\alpha \left[\delta_{ij}|\vec{r}'_\alpha|^2 - x_{\alpha i}x_{\alpha j}\right]$ if the position of each bit of that rigid object with respect to a COM origin is $\vec{r}'_\alpha = (x_{\alpha 1}, x_{\alpha 2}, x_{\alpha 3})$. Write out what should be the middle row in the matrix below to familiarize yourself with its elements. [Recall that you found the {ith component of $\underset{orig}{\vec{L}_{com}}$} $= \sum_j I_{ij}\omega_j$]

$$\overleftrightarrow{I} = \begin{pmatrix} \sum_\alpha m_\alpha[x_{\alpha 2}^2 + x_{\alpha 3}^2] & -\sum_\alpha m_\alpha[x_{\alpha 1}x_{\alpha 2}] & -\sum_\alpha m_\alpha[x_{\alpha 1}x_{\alpha 3}] \\ & & \\ -\sum_\alpha m_\alpha[x_{\alpha 1}x_{\alpha 3}] & -\sum_\alpha m_\alpha[x_{\alpha 2}x_{\alpha 3}] & \sum_\alpha m_\alpha[x_{\alpha 1}^2 + x_{\alpha 2}^2] \end{pmatrix}$$

If it is possible to have a set of axes firmly attached to a rigid object while it rotates and still have **the origin of that attached axis system be stationary in the inertial frame**, then this is a "stationary pivot" case and exactly the same math steps show that {ith component of $\underset{orig}{\vec{L}_{pivot}}$} $= \sum_j I_{ij}\omega_j$

For either origin Taylor glues on x,y,z axes and uses $\vec{r}'_\alpha = (x_\alpha, y_\alpha, z_\alpha)$ for the position of a piece of the rigid object wrt that origin. Thus, the top left element of his matrix is $I_{xx} = \sum_\alpha m_\alpha \left[\delta_{ij}[x_\alpha^2 + y_\alpha^2 + z_\alpha^2] - x_\alpha x_\alpha\right]$ and the middle component of his matrix is $I_{yy} = \sum_\alpha m_\alpha \left[\delta_{ij}[x_\alpha^2 + y_\alpha^2 + z_\alpha^2] - y_\alpha y_\alpha\right]$

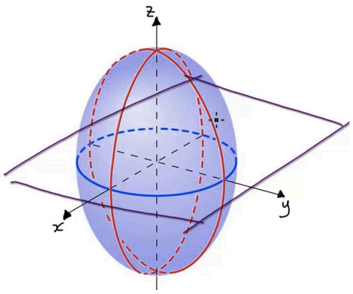
If an object has an axis of rotational symmetry, it looks exactly the same if we rotate it by an arbitrary amount about that axis.

b) If our glued-on 3 axis (same as Taylor's z axis) is an axis of rotational symmetry for a given object and if we calculate the inertia tensor using an origin on that 3 axis, give the relationship between the inertia tensor components I_{11} (Taylor's I_{xx}) and I_{22} (Taylor's I_{yy}), AND then give a corresponding relationship between I_{11} and $.5(I_{11} + I_{22})$.

c) Recall that we can switch from a sum over tiny m_α pieces to a volume integral that includes $dm = (density)dV$ if we simultaneously replace $\vec{r}_\alpha$ components with $\vec{r}$ components. Do this using Taylor's xyz notation, switch to cylindrical coordinates of position $\vec{r}$, remember that in cylindrical $dV = \rho d\phi d\rho dz$, and arrive at the following result for an object of uniform density γ that extends from zmin to zmax and from ρmin to ρmax while having rotational symmetry around the z axis we stuck to it:

$$I_{11} = I_{xx} = \frac{\gamma}{2}\int_0^{2\pi} d\phi \int_{zmin}^{zmax} dz \int_{\rho min}^{\rho max} (\rho^2 + 2z^2)\,\rho d\rho$$

d) **Give a symmetry argument** to help you find the value of $I_{12} = I_{xy}$ for such a case. **What is the value**?



e) Recall that the equation $x = x_{scale}\sqrt{1-\frac{z^2}{z_{sc}^2}}$ describes half of a particular ellipse in the x-z plane. (The half with positive x – the parameters x_{scale}&z_{sc} determines how fat&tall the ellipse is). Notice that the highest z value on the ellipse is z_{sc} and the lowest z value on the ellipse is $-z_{sc}$. Imagine rotating that curve around the z axis and filling the inside of that rotated shape. In cylindrical coordinates you would get an object that extends outward from the z axis to $\rho_{max} = \rho_{sc}\sqrt{1-\frac{z^2}{z_{sc}^2}}$ at any height z, and the whole object would go from its low point $-z_{sc}$ to its top point z_{sc}. Assume that this object has uniform density γ, find the components of the inertia tensor for this object, if we assume its COM is at the origin of the rotating axes stuck to the object.

[Your answer will have $\gamma = M/volume$ in it. In your mind make sure you could switch to $\overleftrightarrow{I}$ components that show M*(distance squared) units if I told you the volume of this object is $\frac{4\pi\gamma}{3}\rho_{sc}^2 z_{sc}$.]

f) If we rotate this object around that z axis glued onto its axis of rotational symmetry so $\vec{\omega} = \omega\hat{z}$, what is $\vec{L}_{\substack{com\\orig}}$ for the object you considered in e (as a function of ω)? [Notice freshman physics would give same answer.]

Check back later for parts of this assignment that will be due during the lecture period after Thanksgiving.]