Name:_____

Mathematics 263
Unofficial Practice Final Examination

Instructions: The unrestricted use of calculators and tables on this examination necessitates requiring that **all supporting work be shown in order to receive credit**. Experimental work should be performed on the scratch paper provided and will not be graded. The numbered problems are equally weighted in value. Good luck.

1. (a) Find the angle θ in Figure 12 Below.

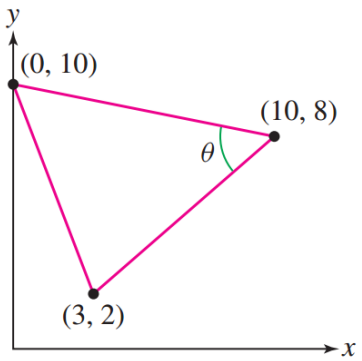

FIGURE 12

(b) Let $\vec{v} =< 4, -1, 5 >$ and $\vec{v} =< 2, 1, 1 >$. Find the decomposition $\vec{v} = \vec{v}_{\perp\vec{w}} + \vec{v}_{\perp\vec{w}}$.

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2. Find the area enclosed by the cardioid given in polar coordinates by $r = r(\theta) = 1 + \cos(2\theta)$, $0 \leq \theta \leq 2\pi$. Provide a rough sketch to support your work.

3. Evaluate these integrals using any method or combination of methods that we have learned throughout this semester.

(a) $\int cos^3\theta \; sin^8\theta \; d\theta$ Hint: p.440 #3

(b) $\int \frac{dx}{x - x^-1}$ Hint: p.440 #10

(c) $\int \frac{dx}{x^2 + 4x - 5}$ Hint: p.440 # 12

4. (a) Classify the following series as **divergent, absolutely convergent,** or **conditionally convergent**. Show all work.

$$\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+4}$$

(b) Classify the following series as **convergent** or **divergent**. State which test(s) you are using.

$$\sum_{n=1}^{\infty}(\frac{n}{2n+3})^n$$

5. (a) Prove whether the series converges or diverges. If the series converges find the interval of convergence.

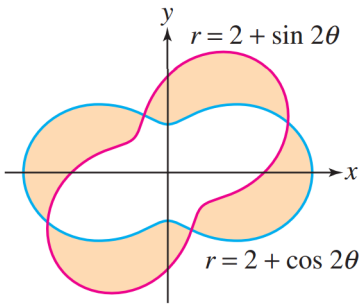
$$\sum_{n=1}^{\infty} n(x-3)^n$$

(b) Find the interval of convergence of the following series.

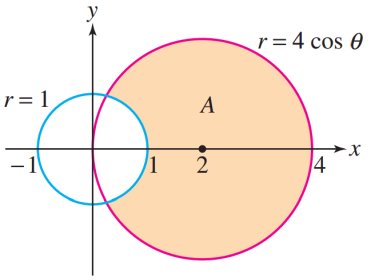
$$\sum_{n=12}^{\infty} e^n(x-2)^n$$

6. (1) Recall the useful observation that $1 + x + x^2 + x^3 + \cdots$ is a geometric series, converging to $\dfrac{1}{1-x}$ on the interval $|x| < 1$. (2) Recall also that $\frac{d}{dx}\tanh^{-1}(x) = \frac{1}{1-x^2}$. Combining these two facts, you can effortlessly find the Maclaurin series for $\frac{d}{dx}\tanh^{-1}(x)$. Do so. Then integrate the resulting series term-by-term to get the Maclaurin series for $f(x) = \tanh^{-1}(x)$. There will be a constant C of integration, it will be useful to know that $\tanh^{-1}(0) = 0$ in order to determine the value of C.

7. (a) Find the area of the region that lies inside one but not both of the curves. **Hint:** pg. 608 #22



(b) Find the area of the shaded region A in the figure below. **Hint:** pg. 608 #13



8. (a) Find the first four non-zero terms of the Taylor series for $f(x) = 4\sqrt{x}$ centered at $a = 4$.

(b) Find the MacLaurin series for $\dfrac{1}{1+4x^2} + \dfrac{3}{1-2x^2}$

9. (a) Find the Taylor series centered at c and the interval on which the expansion is valid for

$$f(x) = x^4 + 3x - 1, c = 2$$

(b) Now find the Taylor series centered around $c = 0$.

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10. Find the Maclaurin series for the function $f(x) = 2 + 4x + 6x^5 + 8x^9 + 10x^{11}$.